Comparison between revised simplex and usual simplex methods for solving linear programming problems

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ABSTRACT
In this paper, we defined revised simplex method (RSM) and usual simplex method(USM) to solve linear programming problems and suggested their algorithms. The numerical problems were solved by both mention methods , then results are compared. This study confirm that our techniques are valid and can be generalty applied to solve linear programming problems.

Keywords: Simplex method, Revised simplex method (RSM), linear programming problem.

1. INTRODUCTION

In (1843) Irish mathematician Sir William Rowan Hamilton introduced the concept of linear equations, which allowed for the determination of values by linking variables. The Linear Programming Problem (LPP) technique, proposed in (1930)[2], drew inspiration from the work of American economist Wassily Leontief in economics and Russian mathematician Leonid Kantorovich in industrial planning. In (1939), Kantorovich further advanced the development of linear programming for planning and industrial organization. When the author began working on the simplex method for solving linear programs in (1947)[4], aiming to maximize objective functions along convex edges, the concept of the degree derivative emerged.

Linear programming has since become a crucial technique for optimizing various fields such as economics, engineering, logistics, and communications. The Simplex method, developed by George Dantzig in the late (1940)s, revolutionized programming. However, its classical form faced challenges related to computational efficiency and numerical stability, especially with large-scale problems.

To address these issues, the Revised Simplex Method (RSM) was developed by Fiasco and McCormick in (1965)[6].RSM improved upon the original simplex method by introducing enhancements such as updated rotation techniques and the use of sparsely in large linear systems, while retaining the fundamental principles of the simplex approach by seeking optimal solutions at possible polyhedron vertices.Then in 2006 [1] we used a new approach to convert multi objective programming problems to singleobjective programming problems.And work was done again onmean and median valuein 2010[3]. This will continue until 2022, that is new transformation technique to solve multi-objective linear programming problem[5].

This introduction sets the stage for a comprehensive analysis of the updated simplex method, outlining its primary objectives, historical context, and significance. Our aim is to explore modern advancements, practical applications, and the theoretical foundations of RSM to assess its ongoing relevance and impact in the field of optimization theory and practice.

2. Definitions and theorems related with this work

Definition 2.1: Linear programming (LP)

Linear programming (LP) is a method used to optimize a linear objective function while adhering to linear equality and inequality constraints. It involves a polyhedron and a real-valued affine function defined on it. The goal of LP is to find a point on the polyhedron where this function reaches its minimum (or maximum) value, if such a point exists, by examining the vertices of the polyhedron.[7]

A linear program is an optimization problem of the form

Maximize $c^t x$

Subject to

$$Ax = b$$

And

$$x \geq 0$$

Where $c \in R^n$, $b \in R^n$ and $A \in R^m R^n$.[8]

Definition 2.2: Slack variable
We add a slack variable (+S) to guarantee equality when converting an inequality of less than or equal to (<) to an equation.[9]
Definition 2.3: Pivot row
a) Changing the incoming variable to match the outgoing variable in the main column.
b) Create a new summary row by dividing the existing summary row by the summary element's value.
c) For each additional row:
Subtract the pivot column coefficient from the new summary to compute a new row in relation to the current row.**[10]**

Theorem 2.4: Fundamental theorem of linear programming
If the optimal value of the objective function in a linear programming problem exists, then that value (known as the optimal solution) must occur at one or more of the extreme points of the feasible region; see [12]

3. Methods for solving Linear Programming Problem
3.1: Simplex method
When addressing Linear Programming Problems (LPP) with two or more decision variables, the simplex method is applied.[10]

3.2: Revised Simplex Method (RSM)
When using a digital computer to solve a linear programming issue using the traditional simplex method, the computer's memory must have the entire simplex table. Since the table must be recalculated for each iteration, this may not be possible for extremely big issues. merely calculates and keeps the minimal amount of data needed at any given time to evaluate or improve the existing solution. This means keeping only the data required to test and improve the ongoing solution:
Net evaluation line Δj to identify a non-basis variable included in the basis.
• To find a non-base variable contained in the basis, use net evaluation line j.
• The pivot or summary column.
• The values of the current basis variables (XB column) are used to find the lowest positive coefficient and the basis variable that will cause the basis to end.
By using the inverse of the current basis matrix throughout any iteration, this information is obtained directly from the original equations .[11]

4. Algorithms
4.1 : Algorithm for Simplex method
The steps of the simplex method:
Step 1:
Determine a basic, workable solution at first.
Step 2:
Select an input variable by considering the optimality condition. If there isn't an appropriate entering variable, stop the process.
Step 3:
Apply the feasibility criteria to select a leaving variable.

4.2 :Algorithm for Revised Simplex Method
Keeping only the data required to test and improve the ongoing solution:
Net evaluation line Δj to identify a non-basis variable included in the basis.
Step 1:
To find a non-base variable contained in the basis, use net evaluation line j.
Step 2:
The pivot or summary column.
Step 3:
The values of the current basis variables (XB column) are used to find the lowest positive coefficient and the basis variable that will cause the basis to end.

By using the inverse of the current basis matrix throughout any iteration, this information is obtained directly from the original equations.[11]

5: Numerical Examples

5.1 Examples (applied Simplex Method)

1)Max $Z = 6x_1 - 2x_2 + 3x_3$

Such that

$2x_1 - x_2 + 2x_3 \leq 2$

$x_1 \quad + 4x_3 \leq 4$

$x_1 , x_2 , x_3 \geq 0$

Solution: standard form

Max $Z = 6x_1 - 2x_2 + 3x_3 + 0s_1 + 0s_2$

Such that :

$2x_1 - x_2 + 2x_3 + s_1 = 2$

$x_1 + 4x_3 + s_2 = 4$

$x_1 , x_2 , x_3, s_1 , s_2 \geq 0$

Table 1.1. Simplex Table

			6	-2	3	0	0	
Basic variable	C.B	X.B	x_1	x_2	x_3	s_1	s_2	Min ratio
←s_1	0	2	2	-1	2	1	0	2/2=1 ←
s_2	0	4	1	0	4	0	1	4/1=4
	Z=0		-6t	2	-3	0	0	
x_1	6	1	1	-1/2	1	1/2	0	-
←s_2	0	3	0	1/2	3	-1/2	1	3/2 ←
	Z=6		0	-7/3 t	0	3	0	
x_1	6	4	1	0	4	0	1	
x_2	-2	6	0	1	6	-1	2	
	Z=12		0	0	9	2	2	

Since all $\Delta j \geq 0$ then it is optimal solution.

$x_1 = 4 , x_2 = 6 , x_3 = 0$ and Max Z = 12

2) Max $Z = x_1 + x_2 + 3x_3$

Such that :

$3x_1 + 2x_2 + x_3 \leq 3$

$2x_1 + x_2 + 2x_3 \leq 2$

$x_1 , x_2 , x_3 \geq 0$

Solution:standard form

Max $Z = x_1 + x_2 + 3x_3 + 0s_1 + 0s_2$

Such that :

$3x_1 + 2x_2 + x_3 + s_1 = 3$

$2x_1 + x_2 + 2x_3 + s_2 = 2$

$x_1 , x_2 , x_3 , s_1 , s_2 \geq 0$

Table 1.2. Simplex Table

			1	1	3	0	0	
Basic variable	C.B	X.B	x_1	x_2	x_3	s_1	s_2	Min ratio
s_1	0	3	3	2	1	1	0	3/1=3
← s_2	0	2	2	1	2	0	1	2/2=1←
	Z=0		-1	-1	-3t	0	0	
s_1	0	2	2	3/2	0	1	-1/2	
x_3	3	1	1	1/2	1	0	1/2	
	Z=3		2	1/2	0	0	3/2	

Since all $\Delta j \geq 0$ then it is optimal solution .

$x_1 = 0 , x_2 = 0 , x_3 = 1$ and Max Z = 3

5.2 Examples(applied Revised Simplex Method)

1) Max $Z = 6x_1 - 2x_2 + 3x_3$

Such that :

$2x_1 - x_2 + 2x_3 \le 2$

$x_1 \quad + 4x_3 \le 4$

$x_1, x_2, x_3 \ge 0$

Solution: The given problem in the revised simplex form may be expressed by introducing the slack variables $s_1$ands_2 as :

Max $Z = 6x_1 - 2x_2 + 3x_3 + 0s_1 + 0s_2$

s.t. $2x_1 - x_2 + 2x_3 + s_1 = 2$

$x_1 \quad + 4x_3 + s_2 = 4$

$x_1, x_2, x_3, s_1, s_2 \ge 0$

And $Z - 6x_1 + 2x_2 - 3x_3 + 0s_1 + 0s_2 = 0$

$2x_1 - x_2 + 2x_3 + s_1 = 2$

$x_1 \quad + 4x_3 + s_2 = 4$

$x_1, x_2, x_3, s_1, s_2 \ge 0$

The system of constraint equations may be represented in the following matrix form:

$Q_0^{(1)}\ \mathbf{a_1^{(1)}}\ \mathbf{a_2^{(1)}}\ \mathbf{a_3^{(1)}} Q_1^{(1)} Q_2^{(1)}$

$$\begin{bmatrix} 1 & -6 & 2 & -3 & 0 & 0 \\ 0 & 2 & -1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 4 & 0 & 1 \end{bmatrix} * \begin{bmatrix} Z \\ x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$$

The starting revised simplex is given in the table (1.3):

Table 1.3

Basic variable	B_0	B_1	B_2	x_B	x_k	Ratio x_B/x_k
Z	1	0	0	0	-6	-
← s_1	0	1	0	2	2	1←
s_2	0	0	1	4	1	4

a_1	a_2	a_3
-6	2	-3
2	-1	2
1`	0	4

$$Z=(1,0,0) * \begin{bmatrix} -6 & 2 & -3 \\ 2 & -1 & 2 \\ 1 & 0 & 4 \end{bmatrix} = \{-6,2,-3\}$$

$\{\Delta_1, \Delta_2, \Delta_3\}$= min{-6,2,-3}= -6 =$\Delta_1 \rightarrow$ then $x_k = a_1$.

$$x_1 \text{ input} = [-6,2,1]*\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \\ 1 \end{bmatrix}$$

B_1	B_2	x_B	x_3
3	0	6	0
1/2	0	1	1
-1/2	1	3	0

Table 1.4

Basic variable	B_0	B_1	B_2	x_B	x_k	Ratiox_B/x_k
Z	1	3	0	6	-6	
x_1	0	1/2	0	1	-1/2	
← s_2	0	-1/2	1	3	1/2	

x_1	a_2	a_3
0	2	-3
1	-1	2
0`	0	4

$$Z=(1,3,0) * \begin{bmatrix} 0 & 2 & -3 \\ 1 & -1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \min\{3,-1,3\}$$

$\{\Delta_1, \Delta_2, \Delta_3\}$= min{3,-1,3}= -1 =$\Delta_2 \rightarrow$ then $x_k = a_2$

$$x_2 \text{ input} = [2,-1,0]*\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1/2 & 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1/2 \\ 1/2 \end{bmatrix}$$

B_1	B_2	x_B	x_2
2	2	12	0
0	1	4	0
-1	2	6	1

Table 1.5

Basic variable	B_0	B_1	B_2	x_B	x_k	Ratio x_B/x_k
Z	1	2	2	12		
x_1	0	0	1	4		
x_2	0	-1	2	6		

x_1	x_2	a_3
0	0	-3
1	0	2
0`	1	4

$$Z=(1,2,2) * \begin{bmatrix} 0 & 0 & -3 \\ 1 & 0 & 2 \\ 0 & 1 & 4 \end{bmatrix} = \min [2,2,9]$$

Since all $\Delta j \geq 0$ then it is optimal solution
$x_1 = 4$, $x_2 = 6$ and Max Z = 12 .

2) Max $Z = x_1 + x_2 + 3x_3$
Such that :
$3x_1 + 2x_2 + x_3 \leq 3$
$2x_1 + x_2 + 2x_3 \leq 2$
$x_1 , x_2 , x_3 \geq 0$

Solution: The given problem in the revised simplex form may be expressed by introducing the slack variables s_1 and s_2 as :
Max $Z = x_1 + x_2 + 3x_3 + 0s_1 + 0s_2$
s.t. $3x_1 + 2x_2 + x_3 + s_1 = 3$
$2x_1 + x_2 + 2x_3 + s_2 = 2$
$x_1 , x_2 , x_3 , s_1 , s_2 \geq 0$
And $Z - x_1 - x_2 - 3x_3 + 0s_1 + 0s_2 = 0$
$3x_1 + 2x_2 + x_3 + s_1 = 3$
$2x_1 + x_2 + 2x_3 + s_2 = 2$
$x_1 , x_2 , x_3 , s_1 , s_2 \geq 0$
The system of constraint equations may be represented in the following matrix form:
$Q_0^{(1)}$ $\mathbf{a_1^{(1)}}$ $\mathbf{a_2^{(1)}}$ $\mathbf{a_3^{(1)}}$ $Q_1^{(1)}$ $Q_2^{(1)}$

$$\begin{bmatrix} 1 & -1 & -1 & -3 & 0 & 0 \\ 0 & 3 & 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 2 & 0 & 1 \end{bmatrix} * \begin{bmatrix} Z \\ x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

The starting revised simplex is given in thetable (1.6):

Table 1.6

Basic variable	B_0	B_1	B_2	x_B	x_k	Ratio x_B/x_k
Z	1	0	0	0	-3	-
s_1	0	1	0	3	1	3
← s_2	0	0	1	2	2	1←

a_1	a_2	a_3
-1	-1	-3
3	2	1
2`	1	2

$$Z=(1,0,0) * \begin{bmatrix} -1 & -1 & -3 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \min \{-1,-1,-3\}$$

$\{\Delta_1, \Delta_2, \Delta_3\} = \min\{-1,-1,-3\} = -3 = \Delta_3 \rightarrow$ then $x_k = a_3$

$$X_3 \text{ input} = [-3,1,2] * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$$

B_1	B_2	x_B	x_3
0	3/2	3	0
1	-1/2	2	0
0	1/2	1	1

Table 1.7

Basic variable	B_0	B_1	B_2	x_B	x_k	Ratiox_B/x_k
Z	1	0	3/2	3		
s_1	0	1	-1/2	2		
x_3	0	0	½	1		

a_1	a_2	x_3
-1	-1	0
3	2	0
2	1	1

$$Z=(1,0,3/2) * \begin{bmatrix} -1 & -1 & 0 \\ 3 & 2 & 0 \\ 2 & 1 & 1 \end{bmatrix} = \min\,[7/2, 2, 3/2]$$

Since all $\Delta j \geq 0$ then it is optimal solution

$x_1 = 0$, $x_2 = 0$, $x_3=1$ and Max Z = 3 .

6. Experimental results

In this paper that we are working on and as much as we have researched,

When we solved two questions for linear programming problems and two analytical questions, we found that both the final results of maxes and variables are exactly the same without any difference.

This is a pleasant result and makes our work go well.

As we explain better in the following table.

7. Resultscomparation between simplex method and revised simplex

Table (1.8)

Examples	**Simplex Method**	**Revised Simplex Method**
Example (1)	$Z_{opt.} = 12$ $x_1 = 4$ $x_2 = 6$ $x_3 = 0$	Z opt. = 12 $x_1 = 4$ $x_2 = 6$ $x_3 = 0$
Example (2)	$Z_{opt.} = 3$ $x_1 = 0$ $x_2 = 0$ $x_3 = 1$	$Z_{opt.} = 3$ $x_1 = 0$ $x_2 = 0$ x3 = 1

8. CONCLUSION

I was found from this study that the result , which are obtained from solution of numerical examples are identical in both usual Simplex Method and Revised Simplex Method as shown in table (1.8) ,while it was appeared that the result obtained by Revised Simplex Method required less time than that obtained by usual Simplex Method , from then we conclude that the Revised Simplex Method more efficient for application.

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