An Algorithm for Solving Fully Fuzzy Linear Fractional Programming Problems in Fuzzy Environment

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ABSTRACT
Fuzzy Fractional Linear Programming Problem (FFLPP) is one in which the objective function is a linear fractional function, while the constraints are in the form of linear inequalities. In this paper, we introduce a new algorithm for solving FFLPP. The algorithm is based on the effective choose of incoming vectors. We start with an initial basic feasible solution and keep improving basic feasible solution until the optimal solution is reached. We use separate selection criteria for incoming vectors for problems with non-negative and negative denominator in the objective function. The rule of selection of outgoing vector is the same as for the simplex method. Numerical illustrations are given to demonstrate our method.

Keywords: Fuzzy Fractional Linear Programming Problem (FFLPP), Triangular Fuzzy Number, Simplex Method.

1. INTRODUCTION

This FLFPP was created by Hungarian mathematician B. Matrosin (1960, 1964). Many methods for solving the FFLPP problems are exist. Among the methods, Charnes and Cooper's (1962, 1973) transformation technique, Swarup's method (1964, 2003) and Bitran & Novae's method (1972) are very familiar. Recently Tantawy (2007, 2008) has suggested the concept of duality to solve a FLFPP.
A new, effective, and simple approach with lower computing costs and adequate accuracy to convert MOLFPP to LPP was presented by Mojtaba Borza and Azmin Sham Rambely (2021). A new technique was presented by D. Sahoo, A.K. Tripathy, and J.K. Pati in 2022. With the help of this method, the input file (IFMOLFPP) was turned into an intuitionistic fuzzy multi-objective linear programming problem (IFMOLPP), which was further transformed into a crisp multi-objective linear programming problem (CMOLPP) under a precisely defined accuracy function. In 2019, T. Loganathan and K. Ganesan presented a strategy for handling fully fuzzy linear fractional programming issues. In this case, all of the variables and parameters are fuzzy triangular integers. After converting each triangle fuzzy number into its parametric form, transform the fractional programming issue into a parametric single-objective linear programming problem.
In this article, the preliminaries are given in the second section. The proposed method is thoroughly explained in the third section. The fourth section describes some Numerical examples. The final section summarizes the conclusions.

2. Preliminaries

2.1 Fuzzy set

A fuzzy set $\widetilde{A}$ is defined by $\widetilde{A} = \{(x, \mu_A(x): x \in A,\ \mu_A(x) \in [0,1]\}$.In the pair $\left(x, \mu_A(x)\right)$, the first element x belong to the classical set A, the second element $\mu_A(x)$, belong to the interval $[0,1]$and is called Membership function.

2.2 Fuzzy Number

A fuzzy set $\widetilde{A}$ on R must posses at least the following three properties to qualify as a fuzzy number.

(i) $\widetilde{A}$ must be a normalfuzzy set,
(ii) $\alpha_{\widetilde{A}}$must be closed interval for every $\alpha \in [0,1]$,
(iii) The support of $\widetilde{A}$, $\alpha_{\widetilde{A}}$must be bounded.

2.3 A Triangular Fuzzy Number

A triangular fuzzy number is a fuzzy number represented with three points as follows: $\widetilde{A} = (a_1, a_2, a_3)$. This representation is interpreted as membership functions and holds the following conditions

(i) a_1 to a_2 is an increasing function

(ii) a_2 to $a_3$is a decreasing function

(iii) $a_1 \le a_2 \le a_3$

$$\mu_{\widetilde{A}(x)} = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{for } x > a_3 \end{cases}$$

2.4 Operation of Triangular Fuzzy Number using Function Principle

The following are the four operations that can be performed on triangular fuzzy numbers.

Let $\tilde{C} = (c_1, c_2, c_3)$ and $\widetilde{D} = (d_1, d_2, d_3)$ Then

(i) Addition :$\tilde{C} + \widetilde{D} = (c_1 + d_1,\ c_2 + d_2,\ c_3 + d_3)$

(ii) Subtraction: $\tilde{C} - \widetilde{D} = (c_1 - d_3,\ c_2 - d_2,\ c_3 - d_1)$

(iii) Multiplication:

$$\tilde{C} \times \widetilde{D} = \left(\min(c_1d_1, c_1d_3, c_3d_1c_3d_3), c_2d_2, \max(c_1d_1, c_1d_3, c_3d_1, c_3d_3)\right)$$

(iv) Division:

$$\frac{\tilde{C}}{\widetilde{D}} = \left(\min\left(\frac{c_1}{d_1}, \frac{c_1}{d_3}, \frac{c_3}{d_1}, \frac{c_3}{d_3}\right), \frac{c_2}{d_2},\ \max\left(\frac{c_1}{d_1}, \frac{c_1}{d_3}, \frac{c_3}{d_1}, \frac{c_3}{d_3}\right)\right)$$

2.5Fuzzy Linear Fractional Programming Problem

he mathematical form of an FLFPP is as follow:

$$\text{Max } \tilde{Z} = \frac{\tilde{Z}^1}{\tilde{Z}^2} = \frac{\tilde{c}_1\tilde{x}_1 + \tilde{c}_2\tilde{x}_2 + \cdots + \tilde{c}_n\tilde{x}_n + \tilde{c}_0}{\tilde{d}_1\tilde{x}_1 + \tilde{d}_2\tilde{x}_2 + \cdots + \tilde{d}_n\tilde{x}_n + \tilde{d}_0} = \frac{\tilde{c}_i\tilde{x}_i + \widetilde{\alpha}}{\tilde{d}_i\tilde{x}_i + \tilde{\beta}} \quad \forall\ i = 1,2,, \ldots, n. \quad (1)$$

subject to $\widetilde{A}_{ji}\tilde{x}_i \le \tilde{b}_j$

$\tilde{x}_i \ge (0,0,0)$

where $i = 1,2,, \ldots, n$ and $j = 1,2,, \ldots, m$;

$\tilde{x}_i,\ \tilde{c}_i$ and $\tilde{d}_i \in R^n$; $\tilde{b}_j \in R^m$

$\widetilde{A}_{ji}$is a $m \times n$-matrix of co-efficient (TFN);

$\widetilde{\alpha}$and $\tilde{\beta}$ is a Fuzzy constant.

2.6 The simplex algorithm in fuzzy environment

The common form of an FLPP is

Maximize $\tilde{Z} = \tilde{c}_i\tilde{x}_i$

Subject to $\widetilde{A}_{ji}\tilde{x}_i \le \tilde{b}_j,\ \tilde{x}_i \ge 0$

Here $\widetilde{A}_{ji}$ is a $m \times n$ fuzzy matrix, $\tilde{b}_j \in R^m,\ \tilde{x}_i, \tilde{c}_i \in R^n$.

Procedure for solving FLPP:

Step 1: Check primary the fuzzy objective function of the FLPP is to be maximized or minimized. If it is given minimized then modify it into maximized by using$Max\ \tilde{Z} = -Min(-\tilde{Z})$

Step 2: Check another all $\tilde{b}_i\ (i = 1,2,3, \ldots\ldots, m)$ are non-negative. If any one of $\tilde{b}_i$ is non-positive then multiply the corresponding inequation by -1

Step 3: Change over the given inequation into standard form by giving the slack fuzzy variables.

Step 4: Later changing find the initial fuzzy basic feasible solution to the problem in the form $\tilde{x}_B = \tilde{B}^{-1}\tilde{b}$ and substitute it in the first column of the simplex tableu.

Step 5: Find out $\tilde{z}_j - \tilde{c}_j\ (j = 1,2, \ldots, n)$, using $\tilde{z}_j - \tilde{c}_j = \tilde{C}_B\tilde{y}_j - \tilde{C}_j$ where $\tilde{y}_j = \tilde{B}^{-1}\tilde{a}_j$

a) If all $(\tilde{z}_j - \tilde{c}_j) \ge 0$, then the IBFS is an optimal basic feasible solution.

b) If atleast one of $(\tilde{z}_j - \tilde{c}_j) < 0$, then go to step 6.

Step 6: If there is more than one $(\tilde{z}_j - \tilde{c}_j) < 0$, then select the most negative of them. Let it be for $j = r$

a) If all $\tilde{y}_{ir} \le 0\ (i = 1,2,3, \ldots, m)$ then unbounded solution reached for the given problem.

b) If atleast one of $\tilde{y}_{ir} > 0$ $(i = 1,2,3, \dots, m)$ then the vector corresponding is $\tilde{y}_r$ is known as pivotal column and enter to the basis $\tilde{y}_B$.

Step 7: Calculate the ratios $\left\{\frac{\tilde{x}_{B_i}}{\tilde{y}_{ir}}; \forall\, \tilde{y}_{ir} > 0\right\}$ and choose the minimum ratio. Suppose the vector $\tilde{y}_k$ havethe minimumratio then it is known as pivotal row, will remove from the basis $\tilde{y}_B$. The commonelementof pivotal column and pivotal row is pivotal element $\tilde{y}_{kr}$.

Step 8: Change the pivotal element to unity by dividing its row by pivotal element $\tilde{y}_{kr}$ and make other elements in pivotal column to zero by making use of the relations:

$\tilde{y}_{ij}\,(New) = \frac{\tilde{y}_{kj}\,(Old)}{\tilde{y}_{kr}\,(Old)}$; $\forall\, j = 1,2,3, \dots, n,\ i = k$

$$\tilde{y}_{ij}\,(New) = \tilde{y}_{ij}\,(Old) - \frac{\tilde{y}_{kj}\,(Old) \times \tilde{y}_{ir}(Old)}{\tilde{y}_{kr}\,(Old)};\ \forall\, i = 1,2,3, \dots, m, \quad i \neq k, j = 1,2,3, \dots, n$$

Step 9: Go to step 5 and repeat the procedure until either you obtain an optimum solution or there is an unbounded solution.

3. Proposed Method

Let $\widetilde{\boldsymbol{B}}$ be any $m \times m$ fuzzy submatrix of $\widetilde{\boldsymbol{A}}$ formed from m linearly independent columns of $\widetilde{\boldsymbol{A}}$ and let$\tilde{X}_B = \left[\tilde{x}_{B_1}, \tilde{x}_{B_2}, \dots\dots, \tilde{x}_{B_m}\right]^T$ be an initial basic feasible solution of the above FLFP Problemsuch that

$$\widetilde{\boldsymbol{B}}\tilde{X}_B = \tilde{\boldsymbol{b}} \text{ i.e., } \tilde{X}_B = \widetilde{\boldsymbol{B}}^{-1}\tilde{\boldsymbol{b}} \qquad (2)$$

Also, let$\tilde{Z}^1 = \tilde{c}_0 + \tilde{c}_B\tilde{x}_B$ and $\tilde{Z}^2 = \tilde{d}_0 + \tilde{d}_B\tilde{x}_B$, where$\tilde{c}_B$ and$\tilde{d}_B$ are the vectors havingtheir components associated with the basic variables in the numerator and the denominator of the objective function respectively.

If the columns of matrix $\widetilde{\boldsymbol{A}}$ be denoted by $\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \dots, \tilde{\alpha}_{n+m}$ and columns of submatrix $\widetilde{\boldsymbol{B}}$by $\tilde{\beta}_1, \tilde{\beta}_2, \tilde{\beta}_3, \dots, \tilde{\beta}_m$, then$\widetilde{\boldsymbol{A}} = [\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \dots, \tilde{\alpha}_{n+m}]$ $and \widetilde{\boldsymbol{B}} = \left[\tilde{\beta}_1, \tilde{\beta}_2, \tilde{\beta}_3, \dots, \tilde{\beta}_m\right]$

Using simplex method, we obtain a new basic feasible solution by replacing one of the vectors$\tilde{\beta}_j \in \widetilde{\boldsymbol{B}}$by $\tilde{\alpha}_j$, which is a vector in $\widetilde{\boldsymbol{A}}$ but not in $\widetilde{\boldsymbol{B}}$.

Let the new basic feasible solution be given by

$$\tilde{X}_B' = \left\{\tilde{x}_{B_1} - \tilde{x}_{B_r}\frac{\tilde{y}_{ij}}{\tilde{y}_{rj}}, \tilde{x}_{B_2} - \tilde{x}_{B_r}\frac{\tilde{y}_{2j}}{\tilde{y}_{rj}}, \dots, \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}, \dots\dots, \tilde{x}_{B_m} - \tilde{x}_{B_r}\frac{\tilde{y}_{mj}}{\tilde{y}_{rj}}\right\} \qquad (3)$$

$$where\ \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}} = min_i \frac{\tilde{x}_{B_i}}{\tilde{y}_{ij}} \qquad (4)$$

and other non-basic components are zero.

Now, we proceed to find the criterion to select the incoming vector $\tilde{\alpha}_j \in \widetilde{\boldsymbol{A}}$ such that the valueof the objective function corresponding to the new basic feasible solution is improved.

The value of the objective function for the original basic feasible solution is

$$\tilde{Z} = \frac{\tilde{Z}^1}{\tilde{Z}^2} = \frac{\tilde{c}_0 + \tilde{c}_B\tilde{x}_B}{\tilde{d}_0 + \tilde{d}_B\tilde{x}_B} = \frac{\tilde{c}_0 + \sum_{i=1}^m \tilde{c}_{B_i}\tilde{x}_{B_i}}{\tilde{d}_0 + \sum_{i=1}^m \tilde{d}_{B_i}\tilde{x}_{B_i}} \qquad (5)$$

The value of the objective function for the new basic feasible solution is

$$\tilde{Z} = \frac{\tilde{Z}^1}{\tilde{Z}^2} = \frac{\tilde{c}_0 + \tilde{c}_B{}'\tilde{x}_B{}'}{\tilde{d}_0 + \tilde{d}_B{}'\tilde{x}_B{}'} = \frac{\tilde{c}_0 + \sum_{i=1}^m \tilde{c}_{B_i}{}'\tilde{x}_{B_i}{}'}{\tilde{d}_0 + \sum_{i=1}^m \tilde{d}_{B_i}{}'\tilde{x}_{B_i}{}'} \qquad (6)$$

But

$$\tilde{c}_{B_i}{}' = \tilde{c}_{B_i}(i = 1,2, \dots, m, i \neq r),\ \tilde{c}_{B_r}{}' = \tilde{c}_j, \qquad (7)$$

$$\tilde{d}_{B_i}{}' = \tilde{d}_{B_i}(i = 1,2, \dots, m, i \neq r),\ \tilde{d}_{B_r}{}' = \tilde{d}_j. \qquad (8)$$

Substituting the values of $\tilde{c}_{B_i}{}'$ and $\tilde{d}_{B_i}{}'$ from equations (7) and (8) in equation (6) and using equation (3), we get

$$\bar{\tilde{Z}} = \frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}} = \frac{\tilde{c}_0 + \sum_{i=1}^m \tilde{c}_{B_i}\left(\tilde{x}_{B_i} - \tilde{x}_{B_r}\frac{\tilde{y}_{ij}}{\tilde{y}_{rj}}\right) + \tilde{c}_j\frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}}{\tilde{d}_0 + \sum_{i=1}^m \tilde{d}_{B_i}\left(\tilde{x}_{B_i} - \tilde{x}_{B_r}\frac{\tilde{y}_{ij}}{\tilde{y}_{rj}}\right) + \tilde{d}_j\frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}}$$

$$= \frac{\tilde{c}_0 + \sum_{i=1}^m \tilde{c}_{B_i}\tilde{x}_{B_i} + \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\left(\tilde{c}_j - \sum_{i=1}^m \tilde{c}_{B_i}\tilde{y}_{ij}\right)}{\tilde{d}_0 + \sum_{i=1}^m \tilde{d}_{B_i}\tilde{x}_{B_i} + \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\left(\tilde{d}_j - \sum_{i=1}^m \tilde{d}_{B_i}\tilde{y}_{ij}\right)}$$

$$= \frac{\tilde{Z}^1 + \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\left(\tilde{c}_j - \tilde{Z}^1{}_j\right)}{\tilde{Z}^2 + \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\left(\tilde{d}_j - \tilde{Z}^2{}_j\right)}$$

$$where\ \tilde{Z}^1{}_j = \sum_{i=1}^{m} \tilde{c}_{B_i}\tilde{y}_{ij}\ \ and\ \ \tilde{Z}^2{}_j = \sum_{i=1}^{m} \tilde{d}_{B_i}\tilde{y}_{ij}$$

$$= \frac{\tilde{Z}^1 - \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)}{\tilde{Z}^2 - \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\left(\tilde{Z}^2{}_j - \tilde{d}_j\right)}$$

$$= \frac{\frac{\tilde{Z}^1}{\tilde{Z}^2} - \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\frac{\tilde{Z}^1{}_j - \tilde{c}_j}{\tilde{Z}^2}}{1 - \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\frac{\tilde{Z}^2{}_j - \tilde{d}_j}{\tilde{Z}^2}}$$

$$= \left[\frac{\tilde{Z}^1}{\tilde{Z}^2} - \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\frac{\tilde{Z}^1{}_j - \tilde{c}_j}{\tilde{Z}^2}\right]\left[1 - \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\frac{\tilde{Z}^2{}_j - \tilde{d}_j}{\tilde{Z}^2}\right]^{-1}$$

$$= \left[\frac{\tilde{Z}^1}{\tilde{Z}^2} - \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\frac{\tilde{Z}^1{}_j - \tilde{c}_j}{\tilde{Z}^2}\right]\left[1 + \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\frac{\tilde{Z}^2{}_j - \tilde{d}_j}{\tilde{Z}^2}\right]\text{(neglecting terms of higher powers)}$$

$$= \frac{\tilde{Z}^1}{\tilde{Z}^2} + \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\frac{1}{(\tilde{Z}^2)^2}\left[\tilde{Z}^1\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) - \tilde{Z}^2\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)\right]\text{(neglecting terms of higher powers)}$$

Therefore

$$\frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}} = \frac{\tilde{Z}^1}{\tilde{Z}^2} + \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\frac{1}{(\tilde{Z}^2)^2}\left[\tilde{Z}^1\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) - \tilde{Z}^2\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)\right] \quad (9)$$

Equation (9) implies that $\frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}} > \frac{\tilde{Z}^1}{\tilde{Z}^2}$ only if $\tilde{Z}^1\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) - \tilde{Z}^2\left(\tilde{Z}^1{}_j - \tilde{c}_j\right) > 0$. Hence it followsthat as soon as $\tilde{Z}^1\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) - \tilde{Z}^2\left(\tilde{Z}^1{}_j - \tilde{c}_j\right) \le 0$, no further improvement is possible and the optimalsolution is reached.

Also, equation (9) implies that $\frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}}$ is maximum if $\tilde{Z}^1\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) - \tilde{Z}^2\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)$ is maximum.

Therefore, we can conclude the following:

If $\tilde{Z}^1\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) - \tilde{Z}^2\left(\tilde{Z}^1{}_j - \tilde{c}_j\right) > 0$, then the non-basic vector $\tilde{\alpha}_j \in \tilde{\boldsymbol{A}}$corresponding to $Max\left\{\tilde{Z}^1\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) - \tilde{Z}^2\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)\right\}$is selected as the incoming vector. If $\tilde{Z}^1\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) - \tilde{Z}^2\left(\tilde{Z}^1{}_j - \tilde{c}_j\right) \le 0$, for all the non-basic vectors, then no further improvement is possible and the optimal solution is reached.

Now, we consider the following cases:

Case 1

$\tilde{Z}^1 = 0\ or\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) = 0$. In this case, it follows from equation (4.8) that$\frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}} > \frac{\tilde{Z}^1}{\tilde{Z}^2}$only if $-\tilde{Z}^2\left(\tilde{Z}^1{}_j - \tilde{c}_j\right) > 0$i.e., if $\left(\tilde{Z}^1{}_j - \tilde{c}_j\right) < 0$.

Also, equation (8) implies that$\frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}}$ is maximum if $-\tilde{Z}^2\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)$is maximum i.e., if $\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)$is minimum.

Therefore, we can conclude the following:

If $\left(\tilde{Z}^1{}_j - \tilde{c}_j\right) < 0$, then the non-basic vector $\tilde{\alpha}_j \in \tilde{\boldsymbol{A}}$corresponding to $Min\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)$ isselectedas the incoming vector. If $\left(\tilde{Z}^1{}_j - \tilde{c}_j\right) \ge 0$ for all the non-basic vectors, then no further improvementis possible and the optimal solution is reached.

Case 2

$\tilde{Z}^1 \ne 0$ and also $\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) \ne 0$. In this case, equation (8) can be written as follows:

$$\frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}} = \frac{\tilde{Z}^1}{\tilde{Z}^2} + \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\tilde{Z}^2\left(\tilde{Z}^2{}_j - \tilde{d}_j\right)\left[\frac{\tilde{Z}^1}{\tilde{Z}^2} - \frac{\tilde{Z}^1{}_j - \tilde{c}_j}{\tilde{Z}^2{}_j - \tilde{d}_j}\right]$$

i.e.,

$$\frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}} = \frac{\tilde{Z}^1}{\tilde{Z}^2} + \frac{\tilde{x}_{B_r}}{\tilde{y}_{rj}}\tilde{Z}^2\left(\tilde{Z}^2{}_j - \tilde{d}_j\right)\left[\frac{\tilde{Z}^1}{\tilde{Z}^2} - \tilde{R}_j\right] \quad (10)$$

Where $\tilde{R}_j = \frac{\tilde{Z}^1{}_j - \tilde{c}_j}{\tilde{Z}^2{}_j - \tilde{d}_j}$

Now, consider the following sub-cases:

Sub-case 1

$\tilde{Z}^1 \ne 0$ and $\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) > 0$

In this case, equation (10) implies that $\frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}} > \frac{\tilde{Z}^1}{\tilde{Z}^2}$ only if $\tilde{R}_j < \frac{\tilde{Z}^1}{\tilde{Z}^2}$
Also, equation (10) implies that $\frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}}$ is maximum if $\left(\frac{\tilde{Z}^1}{\tilde{Z}^2} - \tilde{R}_j\right)$ is maximum; i.e., if $\tilde{R}_j$ is minimum.
Therefore, we can conclude the following:
If $\tilde{Z}^1 \neq 0$ and also $\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) > 0$ and $\tilde{R}_j < \frac{\tilde{Z}^1}{\tilde{Z}^2}$, then the non-basic vector $\tilde{\alpha}_j \in \tilde{\boldsymbol{A}}$ corresponding to $Min\ \tilde{R}_j$ is selected as the incoming vector. If this condition is not satisfied, then no furtherimprovement is possible and the optimal solution is reached.

Sub-case 2
$\tilde{Z}^1 \neq 0$ and $\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) < 0$.
In this case, equation (10) implies that $\frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}} > \frac{\tilde{Z}^1}{\tilde{Z}^2}$ only if $\tilde{R}_j > \frac{\tilde{Z}^1}{\tilde{Z}^2}$
Also, equation (10) implies that $\frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}}$is maximum if $\left(\frac{\tilde{Z}^1}{\tilde{Z}^2} - \tilde{R}_j\right)$ is minimum; i.e., if $\tilde{R}_j$ is maximum.
Therefore, we can conclude the following:
If $\tilde{Z}^1 \neq 0$ and also $\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) < 0$ and $\tilde{R}_j > \frac{\tilde{Z}^1}{\tilde{Z}^2}$, then the non-basic vector$\tilde{\alpha}_j \in \tilde{\boldsymbol{A}}$ correspondingto $Max\ \tilde{R}_j$is selected as the incoming vector. If this condition is not satisfied, then no furtherimprovement is possible and the optimal solution is reached.

4. Algorithm for the Proposed Method
Step 1: Find an initial basic feasible solution of the given FLFP problem.
Step 2: Compute the values of $\tilde{Z}^1$, $\tilde{Z}^2$and $\frac{\tilde{Z}^1}{\tilde{Z}^2}$
Step 3: Compute the values of $\tilde{Z}^1{}_j - \tilde{c}_j$ and $\tilde{Z}^2{}_j - \tilde{d}_j$ for all the non-basic vectors.
Step 4: Check whether $\tilde{Z}^1 = 0\ or\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) = 0$ for the non-basic vectors holds or not. If yes, go to Step 5; else, Step 6.
Step 5: If either $\tilde{Z}^1 = 0\ or\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) = 0$ for all the non-basic vectors holds, then calculate $\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)$for all the non-basic vectors. If either of the above two holds, then calculate $\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)$ for all the non-basic vectors.
If $\left(\tilde{Z}^1{}_j - \tilde{c}_j\right) < 0$, then the non-basic vector $\tilde{\alpha}_j \in \tilde{\boldsymbol{A}}$ corresponding to $Min\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)$ is selectedas the incoming vector. Go to Step 7.
If $\left(\tilde{Z}^1{}_j - \tilde{c}_j\right) \geq 0$, for all the non-basic vectors, then no further improvement is possible and the optimal solution is reached.
Step 6: If neither $\tilde{Z}^1 = 0\ nor\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) = 0$ for the non-basic vectors, then check whether

$$\tilde{Z}^2{}_j - \tilde{d}_j > 0\ or\ \tilde{Z}^2{}_j - \tilde{d}_j < 0.$$

Now, calculate $\tilde{R}_j = \frac{\tilde{Z}^1{}_j - \tilde{c}_j}{\tilde{Z}^2{}_j - \tilde{d}_j}$,for the non-basic vectors for which$\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) \neq 0$.
Step 6a: Check whether the conditions $\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) < 0$and $\tilde{R}_j > \frac{\tilde{Z}^1}{\tilde{Z}^2}$ for one or more non- basicvectors are satisfied or not. If yes, then the non-basic vector $\tilde{\alpha}_j \in \tilde{\boldsymbol{A}}$ corresponding to $Min\ \tilde{R}_j$isselected as the incoming vector. Go to Step 7; else, Step 6(b).
Step 6b: Check whether the conditions $\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) > 0$and $\tilde{R}_j < \frac{\tilde{Z}^1}{\tilde{Z}^2}$ for one or more non- basic vectors are satisfied or not. If yes, then the non-basic vector corresponding to $Max\ \tilde{R}_j$is selected as theincoming vector. Go to Step 7; else, no further improvement is possible and the optimal solution is reached.
Step 7: Using simplex method, the outgoing vector is selected and a new basic feasible solution is obtained. The process is continued till the criterion of optimality is satisfied.

Solution of FLFP Problems with Negative Denominator in the Objective Function
In the method of solution discussed above, it is supposed that the denominator is positive for all feasible solutions. Now we consider the case when the denominator is negative for some feasible solutions.
Let us suppose that $\tilde{Z}^2 < 0$, for the FLFP Problem (1) for some feasible solution. To select the incoming vector in this case, we proceed as follows:
Equation (9) $\Longrightarrow \frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}}$ is maximum if $\tilde{Z}^1\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) - \tilde{Z}^2\left(\tilde{Z}^1{}_j - \tilde{c}_j\right)$is maximum.
$\Longrightarrow \frac{\overline{\tilde{Z}^1}}{\overline{\tilde{Z}^2}}$ is maximum if $\tilde{Z}^2\left(\tilde{Z}^2{}_j - \tilde{d}_j\right)\left[\frac{\tilde{Z}^1}{\tilde{Z}^2} - \frac{\tilde{Z}^1{}_j - \tilde{c}_j}{\tilde{Z}^2{}_j - \tilde{d}_j}\right]$is maximum.

$\Rightarrow \frac{\tilde{Z}^1}{\tilde{Z}^2}$ is maximum if $\left(\tilde{Z}^2{}_j - \tilde{d}_j\right)\left[\frac{\tilde{Z}^1}{\tilde{Z}^2} - \frac{\tilde{Z}^1{}_j - \tilde{c}_j}{\tilde{Z}^2{}_j - \tilde{d}_j}\right]$ is minimum.

$\Rightarrow \frac{\tilde{Z}^1}{\tilde{Z}^2}$ is maximum if $\left(\tilde{Z}^2{}_j - \tilde{d}_j\right)\left[\frac{\tilde{Z}^1}{\tilde{Z}^2} - \tilde{R}_j\right]$ is minimum.

Where $\tilde{R}_j = \frac{\tilde{Z}^1{}_j - \tilde{c}_j}{\tilde{Z}^2{}_j - \tilde{d}_j}$

$\Rightarrow \frac{\tilde{Z}^1}{\tilde{Z}^2}$ is maximum if we take $\begin{cases} Max\ \tilde{R}_j\ for\ \left(\tilde{Z}^2{}_j - \tilde{d}_j\right) > 0\ and\ \tilde{R}_j > \frac{\tilde{Z}^1}{\tilde{Z}^2} \\ Min\ \tilde{R}_j\ for\ \left(\tilde{Z}^2{}_j - \tilde{d}_j\right) < 0\ and\ \tilde{R}_j < \frac{\tilde{Z}^1}{\tilde{Z}^2} \end{cases}$

Therefore we conclude the following:

1. If $\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) > 0\ and\ \tilde{R}_j > \frac{\tilde{Z}^1}{\tilde{Z}^2}$, then the non-basic vector $\tilde{\alpha}_j \in \tilde{\boldsymbol{A}}$ corresponding to $Max\ \tilde{R}_j$ is selected as the incoming vector.
2. If $\left(\tilde{Z}^2{}_j - \tilde{d}_j\right) < 0\ and\ \tilde{R}_j < \frac{\tilde{Z}^1}{\tilde{Z}^2}$, then the non-basic vector $\tilde{\alpha}_j \in \tilde{\boldsymbol{A}}$corresponding to$M$ in $\tilde{R}_j$is selected as the incoming vector.

If neither of the above two conditions is satisfied, then no further improvement is possible and the optimal solution is reached.
The outgoing vector is selected using simplex method and a new basic feasible solution is obtained.
The process is continued till the criterion of optimality is satisfied.

5. Numerical Illustrations

Numerical Example:1

$$Max\tilde{Z} = \frac{(1,2,3)\tilde{x}_1 + (2,3,4)\tilde{x}_2}{(1,1,1)\tilde{x}_1 + (1,1,1)\tilde{x}_2 + (1,1,1)}$$

subject to

$(1,1,1)\tilde{x}_1 + (1,1,1)\tilde{x}_2 \leq (2,3,4)$
$(1,1,1)\tilde{x}_1 + (1,2,3)\tilde{x}_2 \leq (2,3,4)$
$\tilde{x}_1, \tilde{x}_2 \geq \tilde{0}$

Rewriting the above FFLPP problem in to standard form, we get

$$Max\tilde{Z} = \frac{(1,2,3)\tilde{x}_1 + (2,3,4)\tilde{x}_2 + (0,0,0)\tilde{x}_3 + (0,0,0)\tilde{x}_4}{(1,1,1)\tilde{x}_1 + (1,1,1)\tilde{x}_2 + (0,0,0)\tilde{x}_3 + (0,0,0)\tilde{x}_4 + (1,1,1)}$$

subject to

$(1,1,1)\tilde{x}_1 + (1,1,1)\tilde{x}_2 + (1,1,1)\tilde{x}_3 + (0,0,0)\tilde{x}_4 = (2,3,4)$
$(1,1,1)\tilde{x}_1 + (1,2,3)\tilde{x}_2 + (0,0,0)\tilde{x}_3 + (1,1,1)\tilde{x}_4 = (2,3,4)$
$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0$, where $\tilde{x}_3, \tilde{x}_4 \geq 0$ are slack fuzzy variables.

Now we will solve the above FLFPP by our proposed method

Initial Table

BV	$\tilde{C}_B$	$\tilde{D}_B$	$\tilde{X}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\tilde{x}_3$	(0,0,0)	(0,0,0)	(2,3,4)	(1,1,1)	(1,1,1)	(1,1,1)	(0,0,0)
$\tilde{x}_4$	(0,0,0)	(0,0,0)	(2,3,4)	(1,1,1)	(1,2,3)	(0,0,0)	(1,1,1)
$\tilde{Z}^{(1)} = \tilde{C}_B\tilde{X}_B + \tilde{\alpha} = (0,0,0)$			$\tilde{Z}_j{}^1 - \tilde{c}_j$	$-(1,2,3)$	$-(2,3,4)\uparrow$	(0,0,0)	(0,0,0)
$\tilde{Z}^{(2)} = \tilde{D}_B\tilde{X}_B + \tilde{\beta} = (1,1,1)$			$\tilde{Z}_j{}^2 - \tilde{d}_j$	$-(1,1,1)$	$-(1,1,1)$	(0,0,0)	(0,0,0)

Iteration 1

BV	$\tilde{C}_B$	$\tilde{D}_B$	$\tilde{X}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\tilde{x}_3$	(0,0,0)	(0,0,0)	$(-2,1.5,3.34)$	(0,0.5,0.67)	(0,0,0)	(1,1,1)	$-(0.33,0.5,1)$
$\tilde{x}_2$	(2,3,4)	(1,1,1)	(0.66,1.5,4)	(0.33,0.5,1)	(1,1,1)	(0,0,0)	(0.33,0.5,1)
$\tilde{Z}^{(1)} = \tilde{C}_B\tilde{X}_B + \tilde{\alpha} = (1.32,4.5,16)$			$\tilde{Z}_j{}^1 - \tilde{c}_j$	$(-2.34,-0.5,3)$	(0,0,0)	(0,0,0)	(0.66,1.5,4)
$\tilde{Z}^{(2)} = \tilde{D}_B\tilde{X}_B + \tilde{\beta} = (1.66,2.5,5)$			$\tilde{Z}_j{}^2 - \tilde{d}_j$	$-(0,0.5,0.67)$	(0,0,0)	(0,0,0)	(0.33,0.5,1)

The present solution in optimal

$\tilde{x}_1 = (0,0,0), \ \ \tilde{x}_2 = (0.66,1.5,4)$

$$Max\tilde{Z} = \frac{(1,2,3)(0,0,0) + (2,3,4)(0.66,1.5,4)}{(1,1,1)(0,0,0) + (1,1,1)(0.66,1.5,4) + (1,1,1)}$$

$$= \frac{(2,3,4)(0.66,1.5,4)}{(1,1,1)(0.66,1.5,4) + (1,1,1)}$$

$$= \frac{(1.32,4.5,16)}{(0.66,1.5,4) + (1,1,1)}$$

$$= \frac{(1.32,4.5,16)}{(1.66,2.5,5)}$$

$Max\tilde{Z} = (0.26,1.8,9.66)$

The optimal solution is $\tilde{x}_1 = (0,0,0), \ \ \tilde{x}_2 = (0.66,1.5,4)$ and $Max\tilde{Z} = (0.26,1.8,9.66)$

Numerical Example:2

$$Max\tilde{Z} = \frac{(1,1,1)\tilde{x}_1 + (2,3,4)\tilde{x}_2 + (1,2,3)\tilde{x}_3}{(1,2,3)\tilde{x}_1 + (1,1,1)\tilde{x}_2 + (3,4,5)\tilde{x}_3 + (1,1,1)}$$

subject to

$(1,1,1)\tilde{x}_1 + (2,3,4)\tilde{x}_2 + (5,6,7)\tilde{x}_3 \leq (7,8,9)$

$(1,2,3)\tilde{x}_1 + (1,1,1)\tilde{x}_2 + (3,4,5)\tilde{x}_3 \leq (4,5,6)$

Rewriting the above FFLPP problem in to standard form, we get

$$Max\tilde{Z} = \frac{(1,1,1)\tilde{x}_1 + (2,3,4)\tilde{x}_2 + (1,2,3)\tilde{x}_3 + (0,0,0)\tilde{x}_4 + (0,0,0)\tilde{x}_5}{(1,2,3)\tilde{x}_1 + (1,1,1)\tilde{x}_2 + (3,4,5)\tilde{x}_3 + (0,0,0)\tilde{x}_4 + (0,0,0)\tilde{x}_5 + (1,1,1)}$$

subject to

$(1,1,1)\tilde{x}_1 + (2,3,4)\tilde{x}_2 + (5,6,7)\tilde{x}_3 + (1,1,1)\tilde{x}_4 + (0,0,0)\tilde{x}_5 = (7,8,9)$

$(1,2,3)\tilde{x}_1 + (1,1,1)\tilde{x}_2 + (3,4,5)\tilde{x}_3 + (0,0,0)\tilde{x}_4 + (1,1,1)\tilde{x}_5 = (4,5,6)$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5 \geq 0$, where $\tilde{x}_4, \tilde{x}_5 \geq 0$ are slack fuzzy variables.

Initial Table

BV	$\tilde{C}_B$	$\tilde{D}_B$	$\tilde{X}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$
$\tilde{x}_4$	(0,0,0)	(0,0,0)	(7,8,9)	(1,1,1)	(2,3,4)	(5,6,7)	(1,1,1)	(0,0,0)
$\tilde{x}_5$	(0,0,0)	(0,0,0)	(4,5,6)	(1,2,3)	(1,1,1)	(3,4,5)	(0,0,0)	(1,1,1)
$\tilde{Z}^{(1)} = \tilde{C}_B\tilde{X}_B + \tilde{\alpha} = (0,0,0)$			$\tilde{Z}_j^{\,1} - \tilde{c}_j$	−(1,1,1)	−(2,3,4) ↑	−(1,2,3)	(0,0,0)	(0,0,0)
$\tilde{Z}^{(2)} = \tilde{D}_B\tilde{X}_B + \tilde{\beta} = (1,1,1)$			$\tilde{Z}_j^{\,2} - \tilde{d}_j$	−(1,2,3)	−(1,1,1)	(3,4,5)	(0,0,0)	(0,0,0)

Iteration 1

BV	$\tilde{C}_B$	$\tilde{D}_B$	$\tilde{X}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$
$\tilde{x}_2$	(2,3,4)	(1,1,1)	(1.75,2.67,4.5)	(0.25,0.33,0.5)	$\tilde{1}$	(1.25,2,3.5)	(0.25,0.33,0.5)	$\tilde{0}$
$\tilde{x}_5$	(0,0,0)	(0,0,0)	(−0.5,2.33,4.25)	(0.5,1.67,2.75)	$\tilde{0}$	(−0.5,2,3.75)	−(0.25,0.3,0.5)	$\tilde{1}$
$\tilde{Z}^{(1)} = \tilde{C}_B\tilde{X}_B + \tilde{\alpha} = (3.5,8.01,18)$			$\tilde{Z}_j^{\,1} - \tilde{c}_j$	(−0.5, −0.01,1)	$\tilde{0}$	(−0.5,4,13)	(0.5,0.99,2)	$\tilde{0}$
$\tilde{Z}^{(2)} = \tilde{D}_B\tilde{X}_B + \tilde{\beta} = (2.75,3.7,5.5)$			$\tilde{Z}_j^{\,2} - \tilde{d}_j$	(−2.8, −1.7, −0.5)	$\tilde{0}$	(−3.8, −2,0.5)	(0.25,0.33,0.5)	$\tilde{0}$

The present solution in optimal

$\tilde{x}_1 = (0,0,0), \ \ \tilde{x}_2 = (1.75,2.67,4.5)$ and $\tilde{x}_3 = (0,0,0)$

$$Max\tilde{Z} = \frac{(1,1,1)(0,0,0) + (2,3,4)(1.75,2.67,4.5) + (1,2,3)(0,0,0)}{(1,2,3)(0,0,0) + (1,1,1)(1.75,2.67,4.5) + (3,4,5)(0,0,0) + (1,1,1)}$$

$$= \frac{(2,3,4)(1.75,2.67,4.5)}{(1,1,1)(1.75,2.67,4.5)) + (1,1,1)}$$

$$= \frac{(3.5,8.01,18)}{(1.75,2.67,4.5) + (1,1,1)}$$

$$= \frac{(3.5,8.01,18)}{(2.75,3.67,5.5)}$$

$Max\tilde{Z} = (0.64,2.18,6.55)$
The optimal solution is $\tilde{x}_1 = (0,0,0), \tilde{x}_2 = (1.75,2.67,4.5), \tilde{x}_3 = (0,0,0)$ *and* $Max\tilde{Z} = (0.64,2.18,6.55)$

Numerical Example:3

$$Max\tilde{Z} = \frac{(4,5,6)\tilde{x}_1 + (2,3,4)\tilde{x}_2}{(4,5,6)\tilde{x}_1 + (1,2,3)\tilde{x}_2 + (1,1,1)}$$

subject to

$(2,3,4)\tilde{x}_1 + (1,2,3)\tilde{x}_2 \leq (14,15,16)$
$(4,5,6)\tilde{x}_1 + (1,2,3)\tilde{x}_2 \leq (9,10,11)$

$$\tilde{x}_1, \tilde{x}_2 \geq \tilde{0}$$

Rewriting the above FFLPP problem in to standard form, we get

$$Max\tilde{Z} = \frac{(4,5,6)\tilde{x}_1 + (2,3,4)\tilde{x}_2 + (1,1,1)\tilde{x}_3 + (0,0,0)\tilde{x}_4}{(4,5,6)\tilde{x}_1 + (1,2,3)\tilde{x}_2 + (0,0,0)\tilde{x}_3 + (1,1,1)\tilde{x}_4 + (1,1,1)}$$

$(2,3,4)\tilde{x}_1 + (1,2,3)\tilde{x}_2 + (1,1,1)\tilde{x}_3 + (0,0,0)\tilde{x}_4 = (14,15,16)$
$(4,5,6)\tilde{x}_1 + (1,2,3)\tilde{x}_2 + (0,0,0)\tilde{x}_3 + (1,1,1)\tilde{x}_4 = (9,10,11)$
$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0$, where $\tilde{x}_3, \tilde{x}_4 \geq 0$ are slack fuzzy variables.
Now we will solve the above FLFPP by our proposed method

Initial Table

BV	$\tilde{C}_B$	$\tilde{D}_B$	$\tilde{X}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\tilde{x}_3$	(0,0,0)	(0,0,0)	(14,15,16)	(2,3,4)	(1,2,3)	(1,1,1)	(0,0,0)
$\tilde{x}_4$	(0,0,0)	(0,0,0)	(9,10,11)	(4,5,6)	(1,2,3)	(0,0,0)	(1,1,1)
$\tilde{Z}^{(1)} = \tilde{C}_B\tilde{X}_B + \tilde{\alpha} = (0,0,0)$			$\tilde{Z}_j^{\,1} - \tilde{c}_j$	$-(2,3,4)\uparrow$	$-(1,2,3)$	(0,0,0)	(0,0,0)
$\tilde{Z}^{(2)} = \tilde{D}_B\tilde{X}_B + \tilde{\beta} = (1,1,1)$			$\tilde{Z}_j^{\,2} - \tilde{d}_j$	$-(4,5,6)$	$-(1,2,3)$	(0,0,0)	(0,0,0)

Iteration 1

BV	$\tilde{C}_B$	$\tilde{D}_B$	$\tilde{X}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\tilde{x}_3$	(0,0,0)	(0,0,0)	(3,9,13)	(0,0,0)	$-(0.36,0.8,2)$	(1,1,1)	$-(0.34,0.6,1)$
$\tilde{x}_1$	(4,5,6)	(4,5,6)	(1.5,2,2.75)	(1,1,1)	(0.17,0.4,0.75)	(0,0,0)	(0.17,0.2,0.25)
$\tilde{Z}^{(1)} = \tilde{C}_B\tilde{X}_B + \tilde{\alpha} = (6,10,16.5)$			$\tilde{Z}_j^{\,1} - \tilde{c}_j$	(0,0,0)	(0.52,0.8,1.05)	(0,0,0)	(0.34,0.6,1)
$\tilde{Z}^{(2)} = \tilde{D}_B\tilde{X}_B + \tilde{\beta} = (7,11,17.5)$			$\tilde{Z}_j^{\,2} - \tilde{d}_j$	(0,0,0)	(0.26,0.43,0.93)	(0,0,0)	(0.68,0.6,1.47)
$\tilde{R}_j = \dfrac{\tilde{Z}_j^{\,1} - \tilde{c}_j}{\tilde{Z}_j^{\,2} - \tilde{d}_j}$				– – –	(0.57,1.86,4.04)	– – –	(0.22,0.6,1.47)

Iteration 2

BV	$\tilde{C}_B$	$\tilde{D}_B$	$\tilde{X}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$
$\tilde{x}_3$	(0,0,0)	(0,0,0)	(3.72,13,45.36)	(0.48,2,11.76)	$\tilde{0}$	$\tilde{1}$	(0,0,0)
$\tilde{x}_2$	(2,3,4)	(1,2,3)	(2,5,16.18)	(1.33,2.5,5.88)	$\tilde{1}$	$\tilde{0}$	(1,1,1)
$\tilde{Z}^{(1)} = \tilde{C}_B\tilde{X}_B + \tilde{\alpha} = (4,15,64.72)$			$\tilde{Z}_j^{\,1} - \tilde{c}_j$	$-(0.69,2,11.76)$	$\tilde{0}$	$\tilde{0}$	$(-1.13,0.2,0.88)$
$\tilde{Z}^{(2)} = \tilde{D}_B\tilde{X}_B + \tilde{\beta} = (3,11,49.54)$			$\tilde{Z}_j^{\,2} - \tilde{d}_j$	$-(0.35,1.08,5.41)$	$\tilde{0}$	$\tilde{0}$	$(-0.67,0.78,1.44)$

The present solution in optimal
$\tilde{x}_1 = (0,0,0), \quad \tilde{x}_2 = (2,5,16.18)$

$$Max\tilde{Z} = \frac{(4,5,6)(0,0,0) + (2,3,4)(2,5,16.18)}{(4,5,6)(0,0,0) + (1,2,3)(2,5,16.18) + (1,1,1)}$$

$$= \frac{(2,3,4)(2,5,16.18)}{(1,2,3)(2,5,16.18) + (1,1,1)}$$

$$= \frac{(4,15,64.72)}{(2,10,48.54) + (1,1,1)}$$

$$=\frac{(4,15,64.72)}{(3,11,49.54)}$$

$\text{Max}\tilde{Z} = (0.08,1.36,21.57)$

The optimal solution is $\tilde{x}_1 = (0,0,0)$, $\tilde{x}_2 = (0.66,1.5,4)$ and $\text{Max}\tilde{Z} = (0.26,1.8,9.66)$

6. CONCLUSION

In this paper, assuming triangular fuzzy numbers, we propose a method for solving fuzzy linear fractional programming problem. In order to find an improved basic feasible solution, the method of selection of entering variable is organised in a way that the total computational effort required is minimum. This makes the process easy for manual calculation and saves our time.

Additionally, the proposed method can be usedto find the optimal solution of all linear fractional programming problems, irrespective of the sign of the denominator of the objective function..Numerical examples are included to strengthen the algorithm.

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