Optimal Replacement Policy under Partial Product Process in an Alternative Repair Model

T. R. Raajpandiyan[1*], Syed Siddiqua Begum[2], Syed Tahir Hussainy[1], U. Rizwan[1]

[1]Department of Mathematics, Islamiah College (Autonomous) Vaniyambadi Affiliated to Thiruvalluvar University, Vellore.
[2]Department of Mathematics and Actuarial Science, B.S. Abdur Rahman Crescent institute of Science and Technology,Vandalur, Chennai.
Email: raajpandiyan393@gmail.com
*Corresponding Author

Received: 18.04.2024 Revised : 16.05.2024 Accepted: 27.05.2024

ABSTRACT
A common assumption in replacement problems is that the repair of a failed system may yield a functioning system, which may be either as good as new (complete repair) or as old as just prior to failure (partial repair). In this paper, we study the partial product process and replacement model for a deteriorating system under various conditions and a repairable system of an alternative repair model, called the Negligible Or Non-Negligible (NONN) repair times introduced by Thangaraj and Rizwan [2001] to develop a new repair model, a replacement policy T NONN repair time. Furthermore, explicit expressions for the long-run average cost of the T policy is to be derived. An optional replacement policy for a deteriorating system using partial product process is developed.

Keywords: Partial product Process, Replacement policy (T), δ -shock model, Alternative Repair Times

1. INTRODUCTION

The study of maintenance problem is an important topic in reliability. Most maintenance models just pay attention on the internal cause of the system failure, but do not on an external cause of the system failure. A system failure may be caused by some external cause, such as a shock. The shock models have been successfully applied to many different subjects, such as physics, communication, electronic engineering and medicine. Barlow and Proschan [1975] have considered a shock model in which arrival of shock causes a random amount of damage to the system. Many reliability analysts have applied shock models to diverse areas.

In engineering, a precision instrument and meter may fail due to the effect of operation of other equipment. This might be an example of discrete stochastic shock.

In this paper, we study an improved δ - shock model with NONN (Negligible or Non-Negligible) repair times. It is a new model because the threshold of a deadly shock is not a constant but monotone and the repair takes the negligible time with probability p and the repair takes the non-negligible time with probability $(1-p)$. Moreover, the successive repair times after failure form an increasing partial process.

In section 3, we introduce and study the δ -shock model for the maintenance problem of a repairable system with NONN repair times.Explicit expressions for the long-run average cost under T -policy is also derived.

2. Preliminaries

The preliminary definitions and results relevant to this are given below.

Definition 2.1. Given two random variables X and Y, X is said to be stochastically smaller than Y (or Y is stochastically greater than X), if $P(X > \alpha) \leq P(Y > \alpha)$ for all real α. This is written as $X \leq_{st} Y$ or $Y \geq_{st} X$.

Definition 2.2. A stochastic process $\{X_n, n = 1,2,3\ldots\}$ is said to be stochastically decreasing (increasing) if $X_n \geq_{st} (\leq_{st}) X_{n+1}$, for all $n = 1,2,3\ldots$

Definition 2.3. A stochastic process $\{Y_n, n=1,2,3\ldots\}$ is said to be stochastically increasing (decreasing) if $Y_n,, _{st}(\cdot\cdot_{st})Y_{n+1}$, for all $n=1,2,3\ldots.$

Definition 2.4. An integer valued random variable N is said to be a stopping time for the sequence of independent random variables $X_1, X_2, \ldots,$ if the event $\{N=n\}$ is independent of $X_{n+1}, X_{n+2}, \ldots$ for all $n=1,2,\ldots.$

Definition 2.5. Let $\{X_n, n=1,2,3,\ldots\}$ be a sequence of independent and non-negative random variables and let $F(x)$ be the distribution function of X_1. Then $\{X_n, n=1,2,3\ldots\}$ is called partial product process, if the distribution function of X_{k+1} is $F(\alpha_k x)(k=1,2,3\cdots),$ where $\alpha_k > 0$ are real constants and $\alpha_k = \alpha_0\alpha_1\alpha_2\ldots\alpha_{k-1}$. In what follows, $F(x)$ denotes the distribution function of non-negative random variable X_1.

Lemma 2.1. If $\alpha_k = \alpha_0\alpha_1\alpha_2\ldots\alpha_{k-1}$, then $\alpha_k = \alpha_0^{2^{k-1}}$ $(k=1,2,3,\ldots)$.

Lemma 2.2. The partial product process $\{X_n, n=1,2,3\cdots\}$ is

(i) stochastically decreasing, if $\alpha_0 > 1$

(ii) stochastically increacing, if $0 < \alpha_0 < 1$

Definition 2.6. The T Policy

It is a policy under which the system will be replaced whenever the working age of the system reaches T.

Definition 2.7. If the sequence of nonnegative random variables $\{X_1, X_2, \ldots\}$ is independent and identically distributed, then the counting process $\{N(t), t..0\}$ is said to be a renewal process.

Definition 2.8. If a repair to a system after failure is done in negligible or non-negligible time, then it will be called a model with NONN repair times

In this case, whenever the system fails, two possibilities may arise: either, the repair takes Negligible time with probability p ; or Non-Negligible time with probability $1-p$.

Lemma2.3. Let $E(X_1)=\mu,\ \operatorname{var}(X_1)=\sigma^2.$ Then for $k=1,2,3,\ldots.$ $E\left(X_{k+1}\right)=\dfrac{\mu}{\alpha_0^{2^{k-1}}}$ and

$$var(X_{k+1}) = \frac{\sigma^2}{\alpha_0^{2^k}}, \text{ where } \alpha_0 > 0.$$

Theorem 2.1. Wald's equation

If $X_1, X_2, \ldots$ are independent and identically distributed random variables having finite expectations and if N is the stopping time for $X_1, X_2, \ldots$ such that $E[N] < \infty$, then

$$E\left[\sum_{n=1}^{N} X_n\right] = E[N]E\left[X_1\right].$$

Theorem 2.2. Wald's equation for partial product process

Suppose that $\{X_k, k=1,2,3,\ldots\}$ forms a partial product process with ratio α_0 and $E[X_1]=\mu<\infty$ and if $\omega(t)=\sup_{k\in Z^+}\{k : V_k \le t\}$ and $V_k = \sum_{i=1}^{k} X_i$. Then for $t>0,$

$$E\left[V_{\omega(t)+1}\right] = \mu E\left[1+\sum_{k=2}^{\omega(t)+1}\frac{1}{\alpha_0^{2^{k-2}}}\right].$$

3. Model Assumptions

In this section, we introduce and study an improved δ shock model of a repairable system and we use the T-policy in an alternative repair model. Under the replacement policy T, the problem is to determine an optimal replacement policy T^* such that the long run average cost per unit time is minimized. We consider the replacement model for a deteriorating system and make the following assumptions.

Assumption 3.1 At the beginning $t=0$, a new simple repairable system is installed. Whenever the system fails, it will be repaired. The system will be replaced by an identical new one, some times later.

Assumption 3.2 Let X_1 be the first operating time of the system after installation, let $\{X_n, n=2,3,\ldots\}$ be the operating time of the system after the $(n-1)$- st repair in a cycle. The distribution of X_n is denoted by $F_n(\cdot)$.

Assumption 3.3 Let Y_n be the repair time after the n-th failure. Then $\{Y_n, n=1,2,\ldots\}$ is a nondecreasing partial product process with rate $\beta_0, 0<\beta_0<1$ and $E(Y_1)=\mu\geq 0,\ \mu=0$ means that repair time is negligible.

Assumption 3.4 Define

$$\xi_n=\begin{cases} Y_n & if\ \ Y_n>0 \\ 1 & if\ \ Y_n=0 \end{cases}$$

for $n=1,2,\ldots$. We can write

$$\xi_i=Y_i\ I(Y_i>0)+1\cdot I(Y_i=0), i=1,2,\ldots$$

where $I(\cdot)$ denotes the indicator function and assume that $P(Y_i=0)=p$ for $i=1,2,\ldots$.

Assumption 3.5 The shock will arrive according to a Poisson Process with rate θ. If the system has been repaired for n-failures $(n=0,1,2,\ldots)$, the threshold of a deadly shock will be $\alpha_0^n\delta$, where α_0 is the rate and δ is the threshold of a deadly shock for a new system. Whenever the time to the first shock following the n-th repair or an inter arrival time of two successive shocks after the n-th repair is less than $\alpha_0^n\delta$, the system will fail. During the repair time, the system is closed; this means that no shock arrive during repair.

Assumption 3.6 The repair does not cause any damage to the system.

Assumption 3.7 The Poisson Process and the partial product process process (ξ_i) are independent.

Assumption 3.8 Let $\alpha_0>1$ and $0<\beta_0\leq 1$.

Assumption 3.9 Let r be the reward rate per unit time of the system when it is operating and c be the repair cost rate per unit time of the system and the replacement cost is R.

Assumption 3.10 The working-age A of the system at time t is the cumulative life-time given by

$$A(t)=\begin{cases} t-V_n: & U_{n+}V_{n},, t,, U_{n+1}+V_n \\ U_{n+1}: & U_{n+1},+V_{n},, t,, U_{n+1}+V_{n+1} \end{cases}$$

where $U_n=\sum_{k=1}^{n}X_k$ and $V_n=\sum_{k=1}^{n}Y_k$. and $U_0=V_0=0$.

Assumption 3.11 The replacement policy T is adapted under which the system will be replaced whenever its working age reaches T. The replacement time is a random variable Z with $E(Z)=\tau$. Under T-policy, the problem is to determine an optimal T^* such that the long-run average cost per unit time is minimized. Under the aforesaid assumptions, the improved δ-shock model is a maintenance model for a deteriorating system.

3.1 Limitations

Whenever a system fails, it needs to wait for repair. In this model, we study the maintenance problem for a system with one repair facility. In this case, the repair facility will repair the system when it fails, until it

is recovered from failure. Therefore the repair facility will be idle, if the system is operating.
In assumption 3.9, we assume that the repair cost is proportional to the repair time at rate c. Whenever the system is operating, reward will be received, it is also assumed to be proportional to the operating time at rate r. However, the replacement cost is mainly determined by the cost of an identical new system and the replacement time, both are assumed to be invariant, no matter how long the system has been used.
The shocks arrive according to a counting process. If the shocks arrive in a purely random manner, then a Poisson process will be an adequate approximation of the real arrival process of the shocks. On the other hand assumption 3.7 is natural, as the Poisson process is due to an external cause, while the partial product process is determined by the system itself.
In practice many systems are deteriorating because of the aging effect and accumulated wearing. For a deteriorating system, it will be more fragile and easier to break down after repair. Finally, the threshold of a deadly shock of the system will be increasing in n, the number of repairs taken. As the number of repairs n increases, the threshold of a deadly shock of the system will increase accordingly. As an approximation, we may assume that the threshold value increases at rate $\alpha_0 \geq 1$. For a deteriorating system, the successive operating times of the system will be shorter and shorter. Finally, the consecutiverepair times of the system will be longer and longer. A general repair procedure will actually include two steps: inspection for making diagnosis or replacement of some damaged or defective parts of the system. If an older system fails, on the one hand a longer time for inspection will be expected as the failure situation might be more complicated; on the other hand, more parts in the system might be broken with more serious damage, a longer time for replacing these parts will be needed. As a result the consecutive repair times are getting longer and longer. Therefore for a deteriorating system, it is reasonable to assume that the consecutive repair times constitute an increasing partial product process.
A cycle is completed if a replacement is done. Since a cycle is actually the time interval between the installation of the system and the first replacement or the time interval between two consecutive replacements, the successive cycles will form a renewal process. Finally, the successive cycles together with the cost incurred in each cycle will constitute a renewal reward process.

4. The Replacement Policy T with NONN Repair Times

Let T_1 be the first replacement time; in general for $n = 2, 3, \ldots,$ let T_n be the time between the $(n-1)$-st replacement and the n-th replacement. Thus the sequence $\{T_n, n = 1, 2, \ldots\}$ forms a renewal process. By the renewal reward theorem, the long-run average cost per unit time under the replacement policy-T is given by

$$\mathrm{C}(T) = \frac{\textit{the expected costincurred in a cycle}}{\textit{the expected length of acycle}}$$

$$= \frac{cE\left(\sum_{n=1}^{\eta-1}\xi_n\right) + R - rE\left(\sum_{n=1}^{\eta}X_n\right)}{E\left(\sum_{n=1}^{\eta}X_n\right) + E\left(\sum_{n=1}^{\eta-1}\xi_n\right) + E(Z)}, \qquad (1)$$

where η is a random variable denoting the number of failures in time T. Since η is also a stopping time with respect to the σ-field $\{\sigma\langle X_1, X_2, \ldots, X_n\rangle, n = 1, 2, \ldots\}$, by Wald's equation, we have

$$E\left(\sum_{n=1}^{\eta}X_n\right) = E\left[E(X_n \mid \eta = n\right]$$

$$= E\left(E\left[\sum_{k=1}^{\eta}X_k \mid \eta = n\right]\right)$$

$$= \sum_{n=1}^{\infty}\left[\sum_{k=1}^{\eta}E(X_k)\right]P(\eta = n). \qquad (2)$$

Following Lam and Zhang [2004],

$$E(X_n) = \frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{n-1}}\delta)]},$$

so that (2) becomes

$$E\left(\sum_{n=1}^{\eta} X_n\right) = \sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]. \tag{3}$$

Also, we have

$$E(Y_n) = \frac{\mu}{\beta_0^{2^{n-1}}},$$

where $\mu = E(Y_1)$ and $0 < \beta_0 \le 1$. Therefore

$$E(\xi_n) = E(Y_n)P(Y_n > 0) + 1\,P(Y_n = 0)$$
$$= \frac{\mu}{\beta_0^{2^{n-1}}}(1-p) + p,$$

so that

$$E\left[\sum_{n=1}^{\eta-1}\xi_n\right] = E\left[E\left(\sum_{n=1}^{\eta-1} Y_n \mid \eta = n\right)\right]$$
$$= \sum_{n=1}^{\infty}\left(\sum_{n=1}^{\eta-1} E(Y_n)\right)P(\eta = n)$$
$$= \sum_{n=1}^{\infty}\left[E(Y_1) + \sum_{k=2}^{\eta-1} E(Y_k)\right]P(\eta = n)$$
$$= E(Y_1)\sum_{n=1}^{\infty}P(\eta = n) + \sum_{n=2}^{\infty}\left(\sum_{k=1}^{\eta-1}E(Y_k)\right)P(\eta = n)$$
$$= \mu\sum_{n=1}^{\infty}P(\eta = n) + \sum_{n=2}^{\infty}\left(\sum_{k=1}^{n-1}E(Y_{k+1})\right)P(\eta = n)$$
$$= \mu\sum_{n=1}^{\infty}P(\eta = n) + \sum_{n=2}^{\infty}\left(\sum_{k=1}^{n-1}\frac{\mu(1-p)}{\beta_0^{2^{k-1}}} + p\right)P(\eta = n)$$
$$= \mu P(\eta = 1) + \mu\sum_{n=2}^{\infty}\left[1 + \sum_{k=1}^{n-1}\frac{1-p}{\beta_0^{2^{k-1}}} + p\right]P(\eta = n)$$
$$= \mu(G_1 - G_2) + \mu\sum_{n=2}^{\infty}\left[1 + \sum_{k=1}^{n-1}\frac{1-p}{\beta_0^{2^{k-1}}} + p\right]\left(G_n(T) - G_{n+1}(T)\right)$$
$$= \mu G_2(T) + \mu\sum_{n=2}^{\infty}\frac{1-p}{\beta_0^{2^{n-1}}}G_{n+1}(T) + p$$

$$E\left[\sum_{n=1}^{\eta-1}\xi_n\right] = \mu\left[G_2(T) + \sum_{n=2}^{\infty}\frac{G_{n+1}(T)(1-p)}{\beta_0^{2^{n-1}}} + p\right]. \tag{4}$$

where $G_n(\cdot)$ denotes the n-fold convolution of $G(\cdot)$ with itself.

Using the equations (3) and (4), the equation (1) becomes

$$\mathrm{C}(T)=\frac{N_1}{D_1}, \tag{5}$$

where

$$N_1=\left[c\mu\left[G_2(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]+R-r\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]\right]$$

$$D_1=\left[\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]+\mu\left[G_2(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]+\tau\right].$$

Differentiating equation (5) with respect to T, we obtain

$$\mathrm{C}'(T)=\frac{Nr}{Dr}, \tag{6}$$

where

$$Nr=\left[\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]\right.$$

$$\left.+\mu\left[G_2(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]+\tau\right]\times\left[c\mu\left[G_2'(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}'(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]\right.$$

$$\left.-r\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n'(T)-F_{n+1}'(T)\right]\right]$$

$$-\left[c\mu\left[G_2(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]+R\right.$$

$$\left.-r\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{n-1}}\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]\right]$$

$$\times\left[\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n'(T)-F_{n+1}'(T)\right]\right.$$

$$\left.\left.+\mu\left[G_2'(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}'(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]\right]\right]$$

and

$$Dr=\left[\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]\quad+\mu\left[G_2(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]+\tau\right]^2.$$

On equating (6) to zero, we obtain on simplification

$$(c+r)\left[\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]\right.$$

$$\times\mu\left[G_2'(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}'(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]-\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\alpha_0^{2^{k-1}}\delta)]}\right)$$

$$\times\left[F_n'(T)-F_{n+1}'(T)\right]+\mu\left[G_2(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]\Bigg]$$

$$+\ \tau\left[c\mu\left[G_2'(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}'(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]-r\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\alpha_0^{2^{k-1}}\delta)]}\right)\right.$$

$$\left.\times\left[F_n'(T)-F_{n+1}'(T)\right]\right]+R\left[\mu\left[G_2'(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}'(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]\right.$$

$$\left.+\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\alpha_0^{2^{k-1}}\delta)]}\right)\times\left[F_n'(T)-F_{n+1}'(T)\right]\right]=0$$

Again differentiating (6) and using (7), we obtain on simplification that

$$(c+r)\left[\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\alpha_0^{2^{k-1}}\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]\right.$$

$$\times\mu\left[G_2''(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}''(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]-\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\alpha_0^{2^{k-1}}\delta)]}\right)$$

$$\left.\times\left[F_n''(T)-F_{n+1}''(T)\right]\times\mu\left[G_2(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]\right]+\tau$$

$$\left[c\mu\left[G_2''(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}''(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]-r\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\delta)]}\right)\right.$$

$$\left.\times\left[F_n''(T)-F_{n+1}''(T)\right]\right]-\ R\left[\mu\left[G_2''(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}''(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]\right.$$

$$\left.+\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\alpha_0^{2^{k-1}}\delta)]}\right)\times\left[F_n''(T)-F_{n+1}''(T)\right]\right]>0\,, \tag{8}$$

if $C''(T)>0$.

For $C(T)$ to be a minimum, $C'(T)=0$ and $C''(T)>0$. Summarizing the above facts, we have the following.

The long run average cost per unit per unit time $C(T)$ given by

$$C(T)=\frac{c\mu\left[G_2(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]+R-r\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]}{\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]+\mu\left[G_2(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}(T)(1-p)}{\beta_0^{2^{n-1}}}+p\right]+\tau}.$$

for the improved δ-shock maintenance model with NONN repair times under T-policy is minimum, if

(7) and (8) holds

Remark 1. If $p = 0$, that is, the repair times are non-negligible always, then (5) reduces to

$$C_1(T) = \frac{c\mu\left[G_2(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}(T)}{\beta_0^{2^{n-1}}}\right]+R-r\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]}{\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]+\mu\left[G_2(T)+\sum_{n=2}^{\infty}\frac{G_{n+1}(T)}{\beta_0^{2^{n-1}}}\right]+\tau}.$$

Remark 2. If $p = 1$, that is, the repair times are negligible always, then (5) reduces to

$$C_2(T) = \frac{c\mu\left[G_2(T)\right]+R-r\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]}{\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n(T)-F_{n+1}(T)\right]+\mu\left[G_2(T)\right]+\tau}.$$

4.1 Corollary

If T^*, the optimal replacement time that minimizes the long-run average cost per unit per unit time, uniquely satisfying (6) and minimizes $C(T)$ given in (5) exists, then the resulting minimum attained is

$$C(T^*) = \frac{c\mu\left[G_2'(T^*)+\sum_{n=2}^{\infty}\frac{G_{n+1}'(T^*)(1-p)}{\beta_0^{2^{n-1}}}+p\right]-r\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n'(T^*)-F_{n+1}'(T^*)\right]}{\sum_{n=1}^{\infty}\left(\sum_{k=1}^{\eta}\frac{1}{\theta[1-\exp(-\theta\,\alpha_0^{2^{k-1}}\,\delta)]}\right)\left[F_n'(T^*)-F_{n+1}'(T^*)\right]+\mu\left[G_2'(T^*)+\sum_{n=2}^{\infty}\frac{G_{n+1}'(T^*)(1-p)}{\beta_0^{2^{n-1}}}+p\right]}.$$

5. CONCLUSION

In this paper, we have studied the replacement policy under partial product process and various conditions are investigated for a deteriorating system. Using the partial product process technique, we have determined a cost structure by considering T policy, under a alternative repair model. By considering a repairable system, an explicit expression for the long-run average cost per unit time under the univariate T -policy is derived.The conditions for the existence of the optimal replacement policy T^* are also derived.

REFERENCES

[1] Babu, D., Govindaraju, P. and Rizwan, U. [2018], Partial Product Processes and Replacement Problem, International Journal of Current Advanced Research, 71, 139 – 142.

[2] Babu, D., Govindaraju, P. and Rizwan, U. [2019], A Maintenance Model for a Deteriorating System with Imperfect Delayed Repair under Partial Product Process, Journal of Applied Science and Computations, 6 4, 795 – 800.

[3] Babu, D., Govindaraju, P. and Rizwan, U. [2019], A δ-Shock Maintenance Model for a Deteriorating System with Imperfect Delayed Repair under Partial Product Process, Journal of Mathematics and Computer Science, 9 5, 571 – 581.

[4] Barlow, R. E. and F. Proschan [1965]. Mathematical Theory of Reliability, John Wiley, New York.

[5] Barlow, R. E. and F. Proschan [1975]. Statistical Theory of Reliability and Life Testing, John Wiley, New York.

[6] Chen, J. and Z. Li [2008]. An Extended Extreme Shock Maintenance Model for a Deteriorating System, Reliability Engineering and System Safety, 93, 1123 – 1129.

[7] Fagiuoli, E. and F. Pelleray [1994]. Preservation of Certain Classes of Life Distributions Under

Poisson Shock Models, Journal of Applied Probability, 31, 458 – 465.
[8] Feller, W. [1965]. An Introduction to Probability Theory and its Applications, John Wiley and Sons, New York.
[9] Govindaraju, P. , Rizwan, U. and Thagaraj, V.[2011] An extreme shock maintenance model under a Bivariate Replacement Policy, Research Methods in Mathematical Sciences, Ed. U.Rizwan, 1 – 10.
[10] Govindaraju, P. and Rajendiran, R., [2020] A Study on an Optimal Replacement Policy for a Deteriorating System under Partial Product Process, Journal of Computational Mathematica, 4 1, 48 – 54.
[11] Hunter, J. J. [1974]. Renewal Theory in Two Dimensions: Asymptotic Results. Advances in Applied Probability, 6, 546 – 562.
[12] Jhang, J. P. and S. H. Sheu [1999]. Opportunity Based Age Replacement Policy with Minimal Repair , Reliability Engineering and System Safety, 64, 339 – 344.
[13] Lam, Y. [1988b]. A note on the Optimal Replacement Problem, Advances in Applied Probability.,20, 479 – 482.
[14] Lam, Y. [1990]. A Repair Replacement Model, Advances in Applied Probability, 22, 494 – 497.
[15] Lam, Y. [1991]. An Optimal Repairable Replacement Model for Deteriorating Systems, Journal of Applied Probability, 28, 843 – 851.
[16] Lam, Y. and Y. L. Zhang [2004]. A Shock Model for the Maintenance Problem of a Repairable System, Computers and Operations Research, 31, 1807 – 1820.
[17] Lehmann, A. [2006]. Degradation Threshold Shock Models, Springer, New York.
[18] Leung, K. N. G. [2006]. A note on A Bivariate Optimal Replacement Policy for a Repairable System , Engineering Optimization, 38, 621 – 625.
[19] Muth, E. J. [1977]. An Optimal Decision rule for Repair vs. Replacement, IEEE Transactions on Reliability, 3,179 – 181.
[20] Raajpandiyan, T. R., Syed Tahir Hussainy and Rizwan, U.[2022], Optimal Replacement Model under Partial Product Process, Stochastic Modeling and Applications, 26 3, 170 – 176.
[21] Raajpandiyan, T. R., Syed Tahir Hussainy and Rizwan, U.[2024], A Bivariate Replacement Policy (U, N) under Partial Product Process, Asia Pacific Journal of Mathematics, 11 10.
[22] Raajpandiyan, T. R., Syed Tahir Hussainy and Rizwan, U.[2024]. A Bivariate Replacement Policy (T, N) under Partial Product Process. The Scientific Temper, 15 02, 2065 – 2069.
[23] Ross, S. M. [1996]. Stochastic Processes, (2nd ed), John Wiley and Sons, New York.
[24] Rudin, W. [1976]. Principles of Mathematical Analysis, (3rd ed),McGraw-Hill Company, New York.
[25] Shantikumar, J. G. and U. Sumita [1983].General Shock Models Associated with Correlated Renewal Sequences, Journal of Applied Probability,20, 600 – 614.
[26] Tang, Y. and Y. Lam [2006]. A δ Shock Model for a Deteriorating System, European Journal of Operational Research, 168, 541 – 546.
[27] Thangaraj, V. and Revathy Sundarajan [1997]. Multivariate Optimal Replacement Policies for a System Subject to Shocks, Optimization, 41, 173 – 195.
[28] Thangaraj, V. and U. Rizwan [2001]. Optimal Replacement Policies in burn-in Process for an Alternative Repair Model, International Journal of Information Management Sciences, 12(3), 43 – 56.
[29] Wang, G. J. and Y. L. Zhang [2005].A Shock Model with Two-type Failures and Optimal Replacement Policy, International Journal of Systems Science, 36, 209 – 214.
[30] Wang, G. J. and Y. L. Zhang [2007]. An Optimal Replacement Policy for a Two Component Series System assuming Geometric Process Repair, Computers and Mathematics With Applications, 54, 192 – 202.
[31] Zhang, Y. L. [1994]. A Bivariate Optimal Replacement Policy for a Repairable System, Journal of Applied Probability, 31, 1123 – 1127.