Integral Representations for Modified Jacobi Matrix Polynomial

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ABSTRACT
The new matrix polynomial structures of special functions are emerging with wide variety of applications in many of the engineering fields. The focus of this paper is mainly to obtain different types of integral representations for modified Jacobi Matrix Polynomial namely finite and infinite single integral representations, double integral representations of the polynomial.

Keywords: Finite and Infinite Single Integral Representation, Hypergeometric function, Jacobi Polynomial (Modified), Jacobi matrix polynomial.

INTRODUCTION

In the past few years, many authors extended classical polynomials to matrix polynomials. Many authors generalized the hypergeometric series, Appell's hypergeometric functions also to matrix version of the functions. Recently, a good number of authors studied matrix version of Jacobi, Hermite, Legendre and other polynomials [1]-[4]. The theory of matrix polynomials provides a way to solve many problems in mathematical physics which have real time applications. L.Jodar [5]-[8] introduced matrix form of Laguerre and Hermite and Hypergeometric function. Subhi Khan and others extended Laguerre polynomials with two variables [9]. Parihar and Patel [10] introduced the modified Jacobi polynomials and derived generating function and recurrence relations of modified Jacobi polynomial. Later on Sri Lakshmi, V [4] introduced matrix form of modified Jacobi polynomial and derived generating function, recurrence relations of matrix version of modified Jacobi polynomial. Srimanarayana, N et.al [11], [12], derived integral representations for Generalized Hypergeometric function and modified Konhauser's polynomial. In the present study finite and infinite integral representations and double integral representations has been established for $J_n^{(A)}(x,w)$.

Preliminary Definitions

In the present article, we abide by the rules of matrix theory. Assume $P_0, P_1, \ldots, P_n \in C^{m\times m}$, where P_i be the matrix of size 'm' and the matrix polynomial of degree 'n' as $f_n(z) = P_n z^n + P_{n-1} z^{n-1} + \ldots + P_1 z + P_0$, where P_n is not a null matrix. Also, I and O be the identity and null matrices in $C^{m\times m}$. For a matrix $P \in C^{m\times m}$, $\sigma(P)$ is the spectrum of P and the matrix P is a positive stable matrix.

In [13], for all $P \in C^{m\times m}$ and $P + nI$ is invertible, where 'n' is an integer, then the Pochhammer symbol is defined by

$$(P)_n = \begin{cases} P(P+I)\ldots(P+(n-1)I); n \neq 0 \\ \Gamma(P+nI)\Gamma^{-1}(P) \ ; n \neq 0 \\ I \qquad ; n = 0 \end{cases} \quad (1)$$

For preliminary matrix version of Gamma, Beta, and Hypergeometric functions one can refer [2], [5], [6], [7], and [13]. In [10], Parihar and Patel defined modified Jacobi Polynomial using a difference operator [14]. Later, Sri Lakshmi, V, et.al extended the same as modified Jacobi Matrix polynomial, $J_n^{(A)}(x,w)$ as follows [4].

$$J_n^{(A)}(x,w) = \frac{(A+I)_n}{n!} \, {}_2F_1\left[-nI, \frac{x}{w}; I+A; w\right] \quad (2)$$

$$= \frac{(A+I)_n}{n!} \sum_{r=0}^{n} \frac{(-nI)_r \left(\frac{x}{w}\right)_r (w)^r}{r!(I+A)_r} \quad (3)$$

To obtain different integral representations, the following well-known results have been used:
Maclaurin's theorem is

$$f(s) = \sum_{l=0}^{\infty} \frac{f^{(l)}(0)s^l}{l!} \quad (4)$$

so that the coefficients $f^{(l)}(0); l = 0,1,2,---$ are obtained by the integrals

$$f^{(l)}(0) = \frac{l!}{2\pi i} \int^{(0+)} \frac{f(s)ds}{s^{n+1}} \ ; s = 0,1,2\ldots \quad (5)$$

For $\mathrm{Re}(m)$ and $\mathrm{Re}(n-m) > 0$, we have

$$\frac{(m)_j}{(n)_j} = \frac{\Gamma(n)}{\Gamma(m)\Gamma(n-m)} \int_0^1 \xi^{m+j-1}(1-\xi)^{n-m-1} d\xi \quad (6)$$

$$\Gamma(n) = \int_{-\infty}^{\infty} e^{-t^2} t^{2n-1} dt \quad (7)$$

If $\mathrm{Real}(m,n) > 0$, then

$$\int_0^1 \int_0^1 \frac{(1-x)^{k-1} y^k (1-y)^{l-1}}{(1-xy)^{k+l-1}} dxdy = \beta(k,l) \quad (8)$$

If Real (k,l)>0, Real(λ)>0 then

$$\iint_A u^{k-1} v^{l-1} (1-u-v)^{\lambda-1} dudv = \frac{\Gamma(k)\Gamma(l)\Gamma(\lambda)}{\Gamma(k+l+\lambda)} \quad (9)$$

where 'A' is the area lies between u, v ≥ 0 and u+v ≤ 1.

$$\beta(a,b) = 2 \int_0^{\pi/2} (\sin t)^{2a-1} (\cos t)^{2b-1} dt \quad (10)$$

$$(\lambda)_{2r} = 2^{2r} \left(\frac{\lambda}{2}\right)_r \left(\frac{\lambda+1}{2}\right)_r \quad (11)$$

provided Re(s)>0 and Re (α)>0 [15], [16]

Integral Representation For $J_n^{(A)}(x,w)$

A. Integral Representation by a Contour Integral

By the generating function of $J_n^{(A)}(x,w)$, we have

$$\sum_{n=0}^{\infty}\frac{J_n^{(A)}(x,w)}{(I+A)_n}t^n = e^t\ {}_1F_1(\frac{x}{w}, I+A; -wt) \quad (12)$$

Assume $f(u)=e^u\ {}_1F_1(\frac{x}{w}, I+A; -wu)$ (13)

Using Maclaurin's theorem (4) and using (5), we arrive at the following:

Theorem 1: Assume A be any square matrix of order 'n' $A \neq 0$

$$J_n^{(A)}(x,w)=\frac{(A+I)_n}{2\pi i}\int^{0+} u^{-n-1}e^u\ {}_1F_1\left(\frac{x}{w}, I+A; -wu\right)du \quad (14)$$

Where the contour integral is around the u-plane in anti-clockwise sense.

Proof

Using Maclaurin's Theorem, we have

$$f(u)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)u^n}{n!}$$

$$\Rightarrow f^{(n)}(0)=\frac{n!}{2\pi i}\int^{0+}\frac{f(u)}{u^{n+1}}du\,; n=0,1,---$$

If $e^u\ {}_1F_1\left[\frac{x}{w}; I+A; -wu\right]=\sum_{n=0}^{\infty}\frac{J_n^{(A)}(x,w)}{(I+A)_n}u^n$

$\Rightarrow \frac{n!J_n^{(A)}(x,w)}{(I+A)_n} = \frac{n!}{2\pi i}\int^{0+}\frac{f(u)}{u^{n+1}}du$, which leads to

$$J_n^{(A)}(x,w)=\frac{(A+I)_n}{2\pi i}\int^{0+} u^{-n-1}e^u\ {}_1F_1\left(\frac{x}{w}, I+A; -wu\right)du$$

B. Real Integral Representation

If 'A' be any square matrix of order $n\times n$ and $J_n^{(A)}(x,w)$ be the modified Jacobi matrix polynomial, then the real integral representations of this polynomial are as follows:

$$J_n^{(A)}(x,w) = \frac{(A+I)_n}{\pi}\sum_{r,m=0}^{\infty}\frac{\left(\frac{x}{w}\right)_r(-w)^r}{r!(I+A)_r}\int_0^{\pi} cis(m+r-n)\theta\ d\theta \quad (15)$$

Proof

Using (14), it reduces to (by choosing the contour $u=e^{i\theta}$

$$J_n^{(A)}(x,w)=\frac{(A+I)_n}{n!\,(2\pi i)}\int_0^{2\pi} e^{e^{i\theta}}e^{i\theta(-n-1)}\sum_{r=0}^{\infty}\frac{\left(\frac{x}{w}\right)_r w^r\left(e^{i\theta}\right)^r}{r!(I+A)_r}ie^{i\theta}d\theta$$

$$=\frac{(A+I)_n}{n!\,(2\pi)}\int_0^{2\pi} e^{-ni\theta}\sum_{m=0}^{\infty}\frac{(e^{i\theta})^m}{m!}\sum_{r=0}^{\infty}\frac{\left(\frac{x}{w}\right)_r(-w)^r e^{ri\theta}}{r!\,(I+A)_r}d\theta$$

By changing the order of summation and integration, we obtain

$$=\frac{(A+I)_n}{n!\,(2\pi)}\sum_{r,m=0}^{\infty}\frac{\left(\frac{x}{w}\right)_r(-w)^r}{r!\,(I+A)_r}\int_0^{2\pi} e^{(m-n+r)i\theta}\ d\theta$$

$$=\frac{(A+I)_n}{(2\pi)}\sum_{r,m=0}^{\infty}\frac{\left(\frac{x}{w}\right)_r(-w)^r}{r!\,(I+A)_r}\int_0^{2\pi} cis(m+r-n)\,d\theta$$

Consequently, we arrive at

$$J_n^{(A)}(x,w)=\frac{(A+I)_n}{\pi}\sum_{r,m=0}^{\infty}\frac{\left(\frac{x}{w}\right)_r(-w)^r}{r!(I+A)_r}\int_0^{\pi} cis(m+r-n)\theta\,d\theta$$

C. Finite Single Integral Representation

Theorem 2

Assume A be any square matrix of order 'n' $A\neq 0$. If $\mathrm{Re}(a)$ and $\mathrm{Re}(b-a)$ are positive, then

$$J_n^{(A)}(x,w)=\frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)}\int_0^1 t^{a-1}(1-t)^{b-a-1}\,{}_3F_2\left[\begin{matrix}-nI,b,\frac{x}{w};\\ I+A,\ a\ ;\end{matrix}\ wt\right]dt \qquad (16)$$

Proof: By using (3)

$$J_n^{(A)}(x,w)=\frac{(A+I)_n}{n!}\sum_{r=0}^{n}\frac{(-nI)_r\left(\frac{x}{w}\right)_r w^r}{r!\,(I+A)_r}$$

$$=\frac{(A+I)_n}{n!}\sum_{r=0}^{n}\frac{(-nI)_r\left(\frac{x}{w}\right)_r w^r(b)_r(a)_r}{r!(I+A)_r(a)_r(b)_r}$$

On using (1.5), we have

$$=\frac{(A+I)_n}{n!}\sum_{r=0}^{n}\frac{(-nI)_r\left(\frac{x}{w}\right)_r w^r(b)_r\Gamma(b)}{r!(I+A)_r(a)_r\Gamma(a)\Gamma(b-a)}\int_0^1 t^{a+r-1}(1-t)^{b-a-1}dt$$

By interchanging the order of integration and summation, we have

$$=\frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)}\int_0^1 t^{a-1}(1-t)^{b-a-1}\frac{(A+I)_n}{n!}\sum_{r=0}^{n}\frac{(-nI)_r\left(\frac{x}{w}\right)_r(b)_r(wt)^r}{r!\,(I+A)_r\,(a)_r}dt$$

$$=\frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)}\int_0^1 t^{a-1}(1-t)^{b-a-1}\,{}_3F_2\left[\begin{matrix}-nI,b,\frac{x}{w};\\ a,(I+A);\end{matrix}\ wt\right]dt$$

Theorem 3

Assume A be any square matrix of order 'n' $A\neq 0$

If real part of $a,b>-\frac{1}{2}$, then

$$J_n^{(A)}(x,w)=\frac{2\Gamma(a+b)(I+A)}{n!\,\Gamma(a)\Gamma(b)}\int_0^{\pi/2}(\sin t)^{2a-1}(\cos t)^{2b-1}\,{}_4F_3\left[\begin{matrix}-nI,\frac{x}{w},\frac{a+b}{2},\frac{a+b+1}{2};\\ (I+A),\quad a\ ,\quad b;\end{matrix}\ w\sin^2 t\cos^2 t\right]dt$$

(17)

By the equation (3) of $J_n^{(A)}(x,w)$, we have

$$J_n^A(x,w)=\frac{(I+A)_n}{n!}\,{}_2F_1\left[-nI,\frac{x}{w};I+A;w\right]$$

$$=\frac{(I+A)_n}{n!}\sum_{r=0}^{n}\frac{(-nI)_r\left(\frac{x}{w}\right)_r w^r}{(I+A)_r\, r!}\frac{\Gamma(a+b+2r)(a)_r(b)_r}{(a)_r(b)_r\Gamma(a+b+2r)}$$

$$=\frac{(I+A)_n}{n!}\sum_{r=0}^{n}\frac{(-nI)_r\left(\frac{x}{w}\right)_r w^r}{(I+A)_r\, r!}\frac{2\Gamma(a+b+2r)}{\Gamma(a)\Gamma(b)}\int_0^{\pi/2}(\sin t)^{2a+2r-1}(\cos t)^{2b+2r-1}dt$$

On interchanging the order of integration and summation, we have

$$=2\frac{(I+A)_n\Gamma(a+b)}{n!\Gamma(a)\Gamma(b)}\int_0^{\pi/2}(\sin t)^{2a-1}(\cos t)^{2b-1}\sum_{r=0}^{n}\frac{(-nI)_r\left(\frac{x}{w}\right)_r\left(\frac{a+b}{2}\right)_r\left(\frac{a+b+1}{2}\right)_r}{(I+A)_r\, r!\,(a)_r(b)_r}(\sin^2 t)^r(\cos^2 t)^r w^r\,dt$$

$$=\frac{2\Gamma(a+b)(I+A)}{n!\,\Gamma(a)\Gamma(b)}\int_0^{\pi/2}(\sin t)^{2a-1}(\cos t)^{2b-1}\,{}_4F_3\left[\begin{matrix}-nI,\frac{x}{w},\frac{a+b}{2},\frac{a+b+1}{2};\\ (I+A),\quad a\;,\quad b;\end{matrix}\,w\sin^2 t\cos^2 t\right]dt$$

Hence the proof.

D. Infinite Single Integral Representation

If 'A' be any square matrix of order $n\times n$ and $J_n^{(A)}(x,w)$ be the modified Jacobi matrix polynomial, then infinite single integral representation of this polynomial is as follows:

Theorem 4

Assume A be any square matrix of order 'n' $A\neq 0$.

$$J_n^{(A)}(x,w)=\frac{(I+A)_n}{\Gamma(a)n!}\int_{-\infty}^{\infty}e^{-t^2}t^{2a-1}\,{}_2F_2\left[\begin{matrix}-nI,\frac{x}{w};\\ a,I+A;\end{matrix}\,wt^2\right]dt \tag{18}$$

Proof

From the equation (3) of $J_n^{(A)}(x,w)$, we have

$$J_n^A(x,w)=\frac{(I+A)_n}{n!}\,{}_2F_1\left[-nI,\frac{x}{w};I+A;w\right]$$

$$=\frac{(I+A)_n}{n!\,\Gamma(\mathrm{a})}\sum_{r=0}^{n}\frac{(-nI)_r\left(\frac{x}{w}\right)_r w^r\,\Gamma(a+r)}{(I+A)_r\, r!\,(a)_r}$$

$$=\frac{(I+A)_n}{n!\,\Gamma(a)}\sum_{r=0}^{n}\frac{(-nI)_r\left(\frac{x}{w}\right)_r w^r}{(I+A)_r\, r!\,(a)_r}\int_{-\infty}^{\infty}e^{-t^2}t^{2a+2r-1}dt$$

On interchanging the order of integration and summation, we have

$$=\frac{(I+A)_n}{\Gamma(a)n!}\int_{-\infty}^{\infty}e^{-t^2}t^{2a-1}\sum_{r=0}^{n}\frac{(-nI)_r\left(\frac{x}{w}\right)_r (wt^2)^r}{(a)_r(I+A)_r\, r!}\,dt$$

$$= \frac{(I+A)_n}{\Gamma(a)\; n!} \int_{-\infty}^{\infty} e^{-t^2} t^{2a-1} {}_2F_2\left[\begin{matrix} -nI, \frac{x}{w}; \; wt^2 \\ a, I+A; \end{matrix}\right] dt$$

Hence the theorem.

E. Double integral representation

Theorem 5

Assume A be any square matrix of order 'n' $A \neq 0$. If real part of a, b, λ >0 then

$$J_n^{(A)}(x,w) = \frac{\Gamma(\lambda)(I+A)_n}{n!\Gamma(a)\Gamma(b)\Gamma(\lambda-a-b)} \iint_A u^{a-1} v^{b-1} (1-u-v)^{\lambda-a-b-1} {}_4F_3\left[\begin{matrix} -nI, \frac{x}{w}, \frac{\lambda}{2}, \frac{\lambda}{2}+\frac{1}{2}; uvw \\ a, \; b, \; I+A \; ; \end{matrix}\right] dudv \tag{19}$$

Proof: From the equation (3) of $J_n^{(A)}(x,w)$ we have

$$J_n^{(A)}(x,w) = \frac{(I+A)_n}{n!} {}_2F_1\left[-nI, \frac{x}{w}; I+A; w\right]$$

$$= \frac{\Gamma(\lambda)(I+A)_n}{n!\Gamma(a)\Gamma(b)\Gamma(\lambda-a-b)} \sum_{r=0}^{n} \frac{(-nI)_r \left(\frac{\lambda}{2}\right)_r \left(\frac{\lambda}{2}+\frac{1}{2}\right)_r \left(\frac{x}{w}\right)_r w^r}{(I+A)_r \, r!\, (a)_r (b)_r} \iint_A u^{a+r-1} v^{b+r-1} (1-u-v)^{\lambda-a-b-1} du\, dv$$

On interchanging the order of integration and summation, we have

$$= \frac{\Gamma(\lambda)(I+A)_n}{n!\Gamma(a)\Gamma(b)\Gamma(\lambda-a-b)} \iint_A u^{a-1} v^{b-1} (1-u-v)^{\lambda-a-b-1} {}_4F_3\left[\begin{matrix} -nI, \frac{x}{w}, \frac{\lambda}{2}, \frac{\lambda}{2}+\frac{1}{2}; uvw \\ a, \; b, \; I+A \; ; \end{matrix}\right] dudv$$

Hence the theorem.

Applications

If 'A' be any square matrix of order $n \times n$ and $J_n^{(A)}(x,w)$ be the modified Jacobi matrix polynomial, then the applications of the above theorems of this polynomial are as follows:

Theorem 6

$$J_n^{(A)}(x,w) = \int_0^1 \frac{t^{a-1}}{(1-t)} J_n^{(A)}(x,wt)\; dt$$

Proof

By setting a = b, in (15), we come across at

$$J_n^{(A)}(x,w) = \int_0^1 \frac{t^{a-1}(A+I)_n}{(1-t)\; n!} \sum_{r=0}^{n} \frac{(-nI)_r \left(\frac{x}{w}\right)_r (wt)^r}{r!\,(I+A)_r}\; dt$$

$$= \int_0^1 \frac{t^{a-1}}{(1-t)} J_n^{(A)}(x,wt)\; dt$$

Theorem 7

$$J_n^{(A)}(x,w) = \int_0^1 \frac{t^{a-1}}{(1-t)} L_n^{(A)}(xt)\; dt \tag{20}$$

Proof

By considering $w \to 0$ and setting $a = b$ in (15), we arrive at the following:

$$J_n^{(A)}(x,w)=\int_0^1 \frac{t^{a-1}(A+I)_n}{(1-t)\, n!}\sum_{r=0}^{n}\frac{(-nI)_r\,(xt)^r}{n!\,(I+A)_r}\,dt$$

$$=\int_0^1 \frac{t^{a-1}}{(1-t)}L_n^{(A)}(xt)\,dt$$

Where $L_n^{(A)}(x)$ is the Laguerre matrix polynomial of one variable [2].

Theorem 8

$$J_n^{(A)}(x,w)=\frac{(I+A)_n}{n!\,\Gamma(a)}\int_{-\infty}^{\infty} e^{-t^2}t^{2a-1}\,{}_1F_2\left[\begin{matrix}-nI\ ;\\ a,I+A;\end{matrix}\,xt^2\right]dt$$

Proof

By assuming $w\to 0$, we have

$$J_n^{(A)}(x,w)=\frac{(I+A)_n}{n!\,\Gamma(a)}\int_{-\infty}^{\infty} e^{-t^2}t^{2a-1}\sum_{r=0}^{n}\frac{(-nI)_r\,(xt^2)^r}{(a)_r\,(I+A)_r\,r!}dt$$

$$=\frac{(I+A)_n}{n!\,\Gamma(a)}\int_{-\infty}^{\infty} e^{-t^2}t^{2a-1}\,{}_1F_2\left[\begin{matrix}-nI\ ;\\ a,I+A;\end{matrix}\,xt^2\right]dt$$

CONCLUSION

This research paper is intended to give some of integral representations of Modified Jacobi Matrix Polynomials. Also, interesting particular cases as applications to some of our results has been discussed. One can get Laplace Transform of this modified Jacobi Matrix polynomial and its applications. These matrix integral representations emerge to a wide variety of applications in mathematical physics and engineering. These results are significant in nature and are capable of studying further research work.

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