Numerical Solutions of Fuzzy Two Coupled Nonlinear Differential Equations

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December 26, 2022

Abstract

In this manuscript, we will use the new modified version of the Runge-Kutta method suitable for solving fuzzy two coupled systems of Nonlinear Ordinary Differential Equations (ODE). With the aid of a numerical example, we will demonstrate the accuracy of the $RK-4$ coupled method for solving these two coupled differential equations. To find the analytical solutions we use Laplace Adomian Decomposition Method since it is a semianalytical method used well in many existing studies on dynamical systems. In order to tell the accuracy, we use the error analysis technique. With the help of numerical simulations, we are able to show at what point of t, both $x(t)$ and $y(t)$ will interact in order to support the theory.

Keywords: New theory of Numerical methods; Analytical Solution; Laplace Adomian Decomposition method; Runge-Kutta method; Two Coupled Differential Equations.

1 Introduction

The equivalence relations to a set of the non-crisp data set called fuzzy sets or fuzzy data set obtained by partisioning the existing relation that will not fail to satisfy the oprations satisfied by the crisp data set. The subsequent of differentiation as well as integration of fuzzy defined equations, and the ever existing theorems on existence and with it the uniqueness of FDE solutions in those space of quotients of fuzzy numbers are presented by various existing studies. The unique solution to the FDE's IVP will be well established if fuzzy normed f satisfies Lipschitz condition.

Many recent studies also developed the fuzzy methods and they have been implemented in so many grounds, such as optimization of multi-objective problems with various decision criteria. The development of mathematics has reached a very high level and is still available today.

The need of RK-4 method was very first arisen at the time of Euler methods to solve ODE numerically. Since it was clearly found very first time by the mathematicians Runge-Kutta, that the convergence of Euler method is only about $O(h^2)$ and error existence affects the coincidense of approximate solutions obtained by Euler with that of Exact solutions. $O(h^2)$ is not a good approximation order. So RK-4 methood was developed and found with $O(h^4)$ which provides the confidence of least error and better approximation that coincdes to atleast four decimal places i.e., $O(h^4)$ for solving linear ode. It also helped the researchers to get the better approximate solutions to that of few non-linear problems like non-linear hybrid differential equations. But for two coupled system of differential equations there are still a research going on many fields like mathematical modelling in poplation dynamics, epidemiology etc.,

There are few noticable works are done on nonlinear epidemic modells and RK-4 methods have also been used but it is also to be mentioned that those modells are not completely three coupled differential equations. After this modell has been developed and if got published we hope strongely that it could be applied to get the solutions of three ccoupled or four coupled DE on the epidemic modells epidemic models. Also, The entire manuscript is brought up by the motivation of well established researches and some of the notable works are Allen, [1] gave his way of introduction mathematical biology. Abbasbandy extended a numerical method called Newtons method to deal with the nonlinear system of equations using modified Adomian Decomposition Method (ADM) in [2]. In [3] Bukley et al., researched on fuzzy differential equations (FDEs). Kermack et al., [4] mathematically analyzed theory of epidemics. Makinde et al., [5] applied ADM to a SIR epidemic model with uniform vaccination therapy. Farman [10] presented solution of SEIR epidemic model of meseales with non- integer time fractional derivatives by using LADM. Ongun [11], applied the LADM for solving a model for HIV infection of $CD4^{+}T$ cells. Palese [12] analysed Variation of Influenza A, B, and C. Saberiad [15] applied of Homotopy Perturbation Method for solving Hybrid Fuzzy Differential Equations. Pederson et al., [19] numerically solved hybrid fuzzy differential equation IVPs by a characterisation theorem. [20] Kandel et al., studied Fuzzy dynamical systems and nature of their solutions. In [21], [22], Lakshmikantham et al., Impulsive hybrid systems and stability theory, Theory of fuzzy differential equations and inclusions. In [23] Seikkala, On the fuzzy initial value problem. [24] Sepahvandzadeh et al., applied Variational Iteration method (VIM) for solving Hybrid Fuzzy Differential Equations. Also there are many researchers who are working on different types fuzzy differential equations in his research on hybrid systems, delay systems, epidemic models etc., in [13, 14], [16],[17, 18], [6, 7], [8, 9]. The manuscript consists of preliminaries in 2, fuzzy-two-coupled non-linear differential equations in 3, Analytical Solution, Semi Analytical Solution in 4, modified Fuzzy RK-4 Algorithm in 5, and finally conclusion in 6

2 Preliminaries

Let E^1 represents the set of functions $q : \mathscr{R} \to [0, 1]$ such that

$$q(y) = \begin{cases} 4y-3, & \text{if } \ y \in (0.75, 1], \\ -2y+3, & \text{if } \ y \in (1, 1.5), \\ 0, & \text{if } \ y \notin (0.75, 1.5). \end{cases} \tag{2.1}$$

The r-level set of q in (2.1) can be wriiten as

$$[q; r] = [0.75 + 0.25r,\ 1.5 - 0.5r]. \tag{2.2}$$

We define $\hat{0} \in E^1$ as $\hat{0}(y) = 1$ if $y = 0$ and $\hat{0}(y) = 0$ if $y \neq 0$ for future reference.

From [23] of $y : I \to E^1$ where $I \subset \mathscr{R}$ is an interval. If $\tilde{y}(t) = [\underline{y}(t; r), \overline{y}(t; r)]$ for all $t \in I$ and $r \in [0, 1]$, then $\tilde{y}'(t) = [\underline{y}'(t; r), \overline{y}'(t; r)]$, if $y'(t; r) \in E^1$.
Following IVP,

$$y'(t) = g(t, y(t)),\ y(0) = y_0, \tag{2.3}$$

where $g : [0,\infty) \times \mathscr{R} \to \mathscr{R}$ is continuous. We would like to interpret (2.3) using the Seikkala's derivative and $y_0 \in E^1$. Let $\tilde{y}_0 = [\underline{y}(0;r), \overline{y}(0;r)]$ and $\tilde{y}(t) = [\underline{y}(t;r), \overline{y}(t;r)]$.

2.1 Definitions and Basic Results

This secion consits of important results considered from [25, 23, 3, 26] "Let $G_k(\mathscr{R}^n)$ represents the house of complete nonempty, compacted, convex collection of subsets of $\mathscr{R}^n$. Sum and product in $G_k(\mathscr{R}^n)$ are existing as usual. Let y be a point in $\mathscr{R}^n$ and B be a non-empty sub set of $\mathscr{R}^n$. The distance $D(y, B)$ from y to B is defined by

$$D(y, B) = \inf_{b \in B} \{\|y - b\|\}$$

Let M and N be two nonempty bounded subsets of $\mathscr{R}^n$. The Housdorff separation of M from N is defined by

$$D^*_H(M, N) = \sup_{\mu \in M} \{d(\mu, \nu)\},$$

The Housdorff separation of N from M is defined by

$$D^*_H(N, M) = \sup_{\nu \in N} \{d(\nu, \mu)\},$$

The distance of separation between M and N as understood by the Housdorff sense

$$D_H(M, N) = \max\left\{ \sup_{m\in M} \inf_{n\in N} \|m - n\|, \sup_{n\in M} \inf_{m\in M} \|m - n\| \right\},$$

where $\|\cdot\|$ is the traditional Euclidean norm $||.||$ in $\mathscr{R}^n$. Then it is clear that $(F_k(\mathscr{R}^n), D)$ becomes a complete metric space.

A fuzzy subset of $\mathscr{R}^n$ is explained in terms of a membership arguments which coins to each point $x \in \mathscr{R}^n$, a grade of membership in the fuzzy set. Such a membership function $q : \mathscr{R}^n \to I \in [0, 1]$ is used to denote the corresponding fuzzy set.

To every $r \in (0, 1]$, the r- level set $[q]^r$ of a fuzzy set u is the subset of values $y \in \mathscr{R}^n$ with memberships $q(y)$ of r powers, that is $[q]^r = \{y \in \mathscr{R}^n : q(y) \geq r\}$. The support $[q]^0$ of a fuzzy set is then defined as the closure of the union of all its level sets, that is, $[q]^0 = \overline{\bigcup_{r\in(0,1]} [q]^r}$. An inclusion result arrives spontaneously from the above definitions.

Result 1

To every $0 \leq r_1 \leq r_2 \leq 1$, $[q]^{r_2} \subseteq [q]^{r_1} \subseteq [q]^0$.

Universally, some level sets usually be null in a ordinary fuzzy set. Particularly, the triviality arise when $q(y) \equiv 0$ for all $y \in \mathscr{R}^n$, though the support is null: q is null fuzzy set in this sense. Here we shall pay focus only to the normal fuzzy sets which satisfy.

In view of Result 1. we have

Result 2

$[q]^r$ is a compact subset of $\mathscr{R}^n$ for all $r \in I$.

Result 3

"If u is fuzzy convex, then $[q]^r$ is convex for each $r \in I$.

Let $I = [0, 1] \subseteq R$ be as compact interval and let E^n denote the set of all $q : \mathscr{R}^n \to I$ such that q satisfies the following conditions.

(i) q is normal, that is, there exist an $q_0 \in \mathscr{R}^n$ such that $q_0 = 1$,

(ii) q is fuzzy convex,

(iii) q is upper semicontinuous,

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(iv) $[q]^0 \equiv$ closure of $\{q \in \mathscr{R}^n : q(x) > 0\}$ is compact.
Then, from $(1) - (4)$, it follows that the r-level set $[q]^r \in P_k(\mathscr{R}^1)$ for all $0 \leq r \leq 1$. If $g : \mathscr{R}^n \times \mathscr{R}^n \to \mathscr{R}^n$ is a function, then using Zadeh's extension principle we can extend g to $E^n \times E^n \to E^n$ by the equation"

$$\tilde{g}(q_1, q_2)(z) = \sup_{z=g(x,y)} \min\{q_1(x), q_2(y)\}. \tag{2.4}$$

It is well known that$[\tilde{g}(q_1, q_2)]^r = g([q_1]^r, [q_2]^r)$, for all $q_1, q_2 \in E^n$, $0 \leq r \leq 1$, and continuous function g. Further we have

$$\begin{aligned} [q_1^r + q_2^r] &= ([q_1]^r + [q_2]^r), & (2.5)\\ [kq]^r &= k[q]^r, & (2.6) \end{aligned}$$

where $k \in \mathscr{R}$. The real numbers can be embedded in E^n by the rule $c \to \hat{c}(t)$", where,

$$\hat{c}(t) = \begin{cases} 1 & for \quad t = c, \\ 0 & elsewhere. \end{cases}$$

3 Fuzzy-Two-Coupled Non-linear Differential Equations

For preliminary definitins of fuzzy differential equations authors are encouraged to go through [25, 26], [18], etc., Two Coupled differential Equations have wide range of applications in any mathematicall modell of physical phenomena in epidemiology, ecology,etc., By the application of fuzzy it is used to eliminate the randomness and vagueness that arises in any dynamics of the system.

$$\begin{cases} x'(t) = c_1 x(t) y(t),\ t_0 \leq t \leq t_n \\ y'(t) = c_2 x(t) y(t),\ t_0 \leq t \leq t_n \\ x(t_0) = x_0, \\ y(t_0) = y_0 \end{cases} \tag{3.1}$$

where c_1, c_2 are numeric constants such that they are not equal to zero and also $c_1 \neq c_2$. By using the concept fuzzy, the equation (3.1) becomes,

$$\begin{cases} \tilde{x}'(t) = c_1 \tilde{x}(t) \tilde{y}(t),\ t_0 \leq t \leq t_n \\ \tilde{y}'(t) = c_2 \tilde{x}(t) \tilde{y}(t),\ t_0 \leq t \leq t_n \\ \tilde{x}(t_0) = x_0, \\ \tilde{y}(t_0) = y_0 \end{cases} \tag{3.2}$$

Such that $\tilde{x}(t) = [\underline{x}(t; r), \overline{x}(t; r)]$. In the same way for $\tilde{y}(t)\, \tilde{x}'(t)\, \tilde{y}'(t)$ and also for $\tilde{x}_0$, $\tilde{y}(0)$

4 Analytical Solution, Semi Analytical Solution

The analytical Solution of the system (3.2) is given by

$$\begin{aligned} \tilde{x}(t) &= \tilde{x}_0 e^{c_1 \int_0^t y(s) ds} \\ \tilde{y}(t) &= \tilde{y}_0 e^{c_2 \int_0^t x(s) ds} \end{aligned} \tag{4.1}$$

In order to obtain the semi analytical solution we are here by making use of well known Laplace Adomian Decomposition method (LADM). We prefer this method to compare the solutions of nonlinear coupled differential equations. The method is already defined and described in somany papers previously whereas the RK-4 algorith or method for nonlinear coupled differential equations is not defined clearly yet but found traces of the authors try over it in the literature. The method is taken since we are unable to process the analytical solutions even though its structure is expalined above.

4.1 Fuzzy Laplace Adomian Decomposition Method

$$\begin{aligned} X(k+1) &= L^{-1}(c_1/s \times L(A_k)) \\ Y(k+1) &= L^{-1}(c_2/s^{\alpha 2} \times L(A_k)) \end{aligned} \tag{4.2}$$

Where (A_k) is an Adomian polynomial defined by $A_k = \frac{1}{k!}\frac{d^k}{\lambda^k}(\sum_{l=0}^{k}(\lambda^l.x_l\lambda^l.y_l)|_{\lambda=0}$ i.e.,
$A_0 = x_0y_0$
$A_1 = x_0y_1 + x_1y_0$
$A_2 = x_0y_2 + x_1y_1 + x_2y_0$ and so on.
$x(t) = \sum_{k=0}^{\infty}(x(k))$
$y(t) = \sum_{k=0}^{\infty}(y(k))$

5 Modified Fuzzy RK-4 Algorithm:

We are at present sharing the new algorithm for novel RK-4 method for solving nonlinear coupled differential equations. In this section we are using the fourth order Runge-Kutta method (RK-4). We are finding the values of $\tilde{x}(t)$,$\tilde{y}(t)$, at $h = 0.1$ for the best approximation. For $0 \leq r \leq 1$.
To evaluate x(t), and y(t):
Consider,

$$\begin{aligned} \tilde{x}(t+1) &= (\tilde{x}(t) + (1/6(A_1 + 2A_2 + 2A_3 + K_4))) \\ \tilde{y}(t+1) &= (\tilde{y}(t) + (1/6(L_1 + 2B_2 + 2B_3 + B_4))) \end{aligned} \tag{5.1}$$

To estimate (5.1), consider the following.

$$\begin{aligned} \tilde{A}_1 &= h \times c_1((\tilde{x}(t))(\tilde{y}(t)) \\ \tilde{B}_1 &= h \times c_2((\tilde{x}(t))(\tilde{y}(t)) \\ \tilde{A}_2 &= h \times c_1(\tilde{x}(t) + (\tilde{A}_1/2))(\tilde{y}(t) + (\tilde{B}_1/2)) \\ \tilde{B}_2 &= h \times c_2(\tilde{x}(t) + (\tilde{A}_1/2))(\tilde{y}(t) + (\tilde{B}_1/2)) \\ \tilde{A}_3 &= h \times c_1(\tilde{x}(t) + (\tilde{A}_2/2))(\tilde{y}(t) + (\tilde{B}_2/2)) \\ \tilde{B}_3 &= h \times c_2(\tilde{x}(t) + (\tilde{A}_2/2))(\tilde{y}(t) + (\tilde{B}_2/2)) \\ \tilde{A}_4 &= h \times c_1(\tilde{x}(t) + (\tilde{A}_3))(\tilde{y}(t) + (\tilde{B}_3)) \\ \tilde{B}_4 &= h \times c_2(\tilde{x}(t) + (\tilde{A}_3))(\tilde{y}(t) + (\tilde{B}_3)) \end{aligned} \tag{5.2}$$

For $1 \leq p \leq 4$ and $0 \leq r \leq 1$,
$\tilde{A}_p = \tilde{A}_p(t;r) = [\underline{A}_p(t;r), \overline{A}_p(t;r)]$,
$\tilde{B}_p = \tilde{B}_p(t;r) = [\underline{B}_p(t;r), \overline{B}_p(t;r)]$,
For $0 \leq t \leq n$, $n = 1, 2, 3, ...,$,
and for $\underline{q} = t$, $\overline{q} = t + 1$, $t = 0, 1, 2, 3, ...$
$\tilde{x}(q) = \tilde{x}(q)(t;r) = [\underline{x}_q(t;r), \overline{x}_q(t;r)]$,
$\tilde{y}(q) = \tilde{y}(q)(t;r) = [\underline{y}_q(t;r), \overline{y}_q(t;r)]$,
Where, $[\underline{f}(t;r), \overline{f}(t;r)] = [0.75 + 0.25r, 1.125 - 0.125r]f(t)$.

Table 1: Approximate solution by RK-4 for non-fuzzy case

t	LADM-4		RK-4		Error	
	x(t)	y(t)	x(t)	y(t)	x(t)	y(t)
0	5	3	5	3	0	0
0.1	4.991	3.0045	5.006	2.99325	0.015	0.01125
0.2	4.98201	3.009	4.99701	2.99775	0.015	0.01125
0.3	4.97301	3.01349	4.98802	3.00224	0.01501	0.01125
0.4	4.96402	3.01799	4.97904	3.00673	0.01502	0.01126
0.5	4.95503	3.02248	4.97006	3.01122	0.01503	0.01126
0.6	4.94605	3.02697	4.96108	3.01571	0.01503	0.01126
0.7	4.93707	3.03147	4.9521	3.0202	0.01503	0.01127
0.8	4.92809	3.03595	4.94313	3.02469	0.01504	0.01126
0.9	4.91912	3.04044	4.93416	3.02917	0.01504	0.01127
1.0	4.91014	3.04493	4.92519	3.03366	0.01505	0.01127

Table 2: Approximate solution by RK-4 for fuzzy case

r	x(t;r)		y(t;r)	
	min	max	min	max
0	3.6939	5.625	2.27524	3.41286
0.1	3.81703	5.56917	2.35108	3.37494
0.2	3.94016	5.49671	2.42692	3.33702
0.3	4.06329	5.42447	2.50277	3.2991
0.4	4.18641	5.35246	2.57861	3.26118
0.5	4.30954	5.28068	2.65445	3.22326
0.6	4.43267	5.20913	2.73029	3.18534
0.7	4.5558	5.13781	2.80613	3.14742
0.8	4.67893	5.06671	2.88197	3.1095
0.9	4.80206	4.99584	2.95781	3.07158
1	4.92519	4.92519	3.03366	3.03366

5.1 An Example

Let us consider the following problem and compare the results of the method LADM RKM-4 in both non-fuzzy and as well as fuzzy. In Table 1, we are presenting the values of $x(t)$, $y(t)$, in non fuzzy by means of LADM-4 and RK-4, and also the error analysis between them. In table 1, we have presented only the values for $t \in [0, 1]$ but one can estimate the values for $t \in [0, 100]$. In that way we found that at $t = 15.5$, $x(t) = 3.66502$, $y(t) = 3.66749$, i.e $x(t) \approx y(t)$, $t = 20.$, there is a interaction between $x(t)$, and $y(t)$.

$$\begin{cases} \tilde{x}'(t) = -0.006\tilde{x}(t)\tilde{y}(t), \ 0 \le t \le 1 \\ \tilde{y}'(t) = 0.003\tilde{x}(t)\tilde{y}(t), \ 0 \le t \le 1 \\ \tilde{x}(0) = 5, \\ \tilde{y}(0) = 3. \end{cases} \tag{5.3}$$

We calculate error by means of $Error = |(LADM - 4) - (RK - 4)|$ Let us present below the table value of $x(t)$ and $y(t)$ in terms of fuzzy in Table 2. So that we have $x(t; r)$ and $y(t; r)$ for $t \in [0, 1]$ and $r \in [0, 1]$.

5.2 Numerical Simlations

For non-Fuzzy coupled case of above example:
For Fuzzy coupled case of above example:

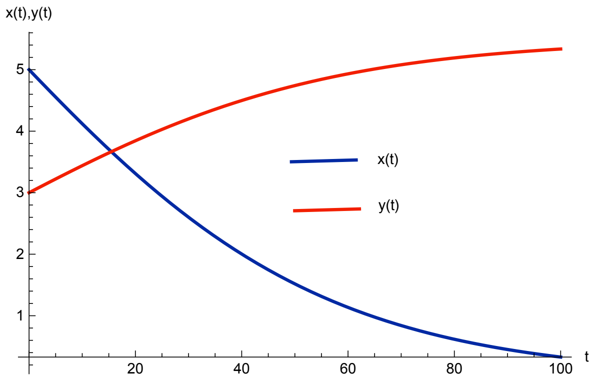

Figure 1: Non-Fuzzy Nonlinear Two Coupled Differential Systems

By the above figures, Figure1 and 2 we are able to understand the travel of

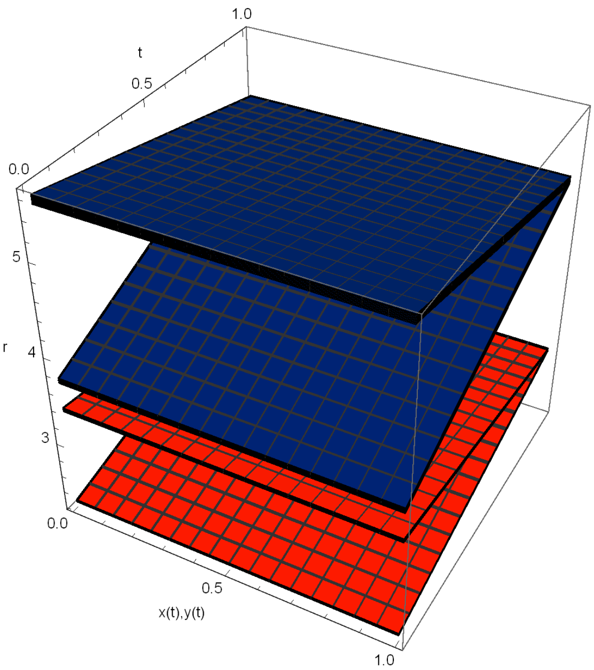

Figure 2: Fuzzy Nonlinear Two Coupled Differential Systems

solutions in $t \in 0, 100$ for non-fuzzy case and for $t \in [0, 1]$ and $r \in [0, 1]$ for fuzzy case respectively.

6 Conclusion

There are numerous numerical methods that one want to use other than Rung-Kutta method when it comes to the need to solve the function with non-linear

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ODE especially for coupled differential equaions we are having so many applications but the methods like Laplace Adomian Decomposition method etc., are used as presented in earlier section. But now we had presented the new coupled form of RK-4 algorithm for solving any kind of nonlinear two coupled nonlinear ODE. We recommend this RK-4 algorithm since its accuracy is of about $O(h^1)$ or one decimal place when it is compared with semianalytical method like LADM. The important aspect is that one can easily see the interaction between $x(t)$ and $y(t)$ in figure 1 which tells us at $t = 15.5$, $x(t) \approx y(t)$. As a future work, we will present this approach on completely coupled fuzzy disease modelling problems.

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Formatting of Funding Sources:
This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.