Extended Jacobi Elliptic Function Technique: A Tool for Solving Nonlinear Wave Equations with emblematic Software

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Abstract

The Extended Jacobi Elliptic Function Technique (EJET) is a powerful technique for finding the solutions for traveling waves form coming from Non-Linear Waveguides (NLWs). As a result, solitary and shock-wave profiles are obtained simultaneously with corresponding amplitudes and speeds by this method for three types of nonlinear wave equations. A class of nonlinear wave equations of particular interest in mathematical physics have been used to investigate the legality and credibility of this technique. A short script is considered a symbolic software package that calculates traveling wave solutions in exact form.

Key words: Extended Jacobi Elliptic Function Technique; Traveling Waves; solitary and shock-wave profiles; Symbolic Software
Mathematics Subject Classification(2010): 35M10, 65Z05.

[1]

1 Introduction

Nonlinearity is a mesmerizing component of nature, with nonlinear wave phenomena appearing in one way or another in nearly all scientific and engineering fields such as physics (Plasma and Fluid), Ocean Engineering, Chemical Dynamics, Geochemistry and mathematical biology (Population Dynamics) [[1]-[5]] .The nonlinear equations appear in different scenarios in daily real-life situations and very difficult to solve it [[6]-[8]]. Manny method are used to find the solutions (solitary and shock-wave solution) of nonlinear wave phenomena like Tanh-Coth Method [[5],[9]], Expansion method [[10]-[13]] the decomposition method with Integral transformation [[14]-[16]] and so on.
The development of the present paper is as follows. In Section 2, we have outline

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of EJET for solving NLW. In Section-3 (Application 3.1), we apply this Technique to the second order nonlinear partial differential equations (SONLPDE). And also applied in Section 4 (Application 3.2) K.G. equation and In Section 5 (Application 3.3) Population Dynamics equation. In section 6 discussion and numerical Sketch and in section 6 result and conclusions.

2 Outline of Extended Jacobi Elliptic Function Technique

We now present a brief strategy of the technique. Given non-linear wave equation

$$\Re\left(v, v_t, v_x, v_{tt}, v_{xx} \ldots.\right) = 0 \tag{2.1}$$

can be converted to an ordinary differential equation (ODE)

$$P\left(v, v', v'', v''' \ldots.\right) = 0 \tag{2.2}$$

upon using a wave variable $z = \alpha\left(x - ct\right)$where α and c are wave number and wave speed respectively. Introducing a new independent variable

$$v(x,t) = v\left(\alpha(x - ct)\right) = v(z)$$

By the Jacobi elliptic function expansion method, $v\left(z\right)$ can be expressed as a finite series in the form of Jacobi elliptic functions[[10],[17]-[18]] ,

$$u\left(z\right) = \sum_{i=0}^{n} \lambda_i (\psi\left(z\right))^i \tag{2.3}$$

is prepared and its highest degree is $O\left\{\psi\left(z\right)\right\} = n.$ where

$$\psi = \psi\left(z\right)$$

satisfies the eq.(2.1) the following auxiliary equation:

$$\frac{\partial}{\partial z}\left(\psi\left(z\right)\right) = \psi'\left(z\right) = \kappa\sqrt{p\,\psi^4(z) + q\,\psi^2\left(z\right) + r} \tag{2.4}$$

Where $\kappa = \pm 1$ and p, q and r are constants. It holds for $\psi\left(z\right)$ as

$$\left.\begin{array}{l}
\frac{\partial^2}{\partial z^2}\left(\psi\left(z\right)\right) = \psi'' = 2p\,\psi^3(z) + q\,\psi(z) \\
\frac{\partial^3}{\partial z^3}\left(\psi\left(z\right)\right) = \psi''' = \left(6p\,\psi^2(z) + q\right)\psi'(z) \\
\frac{\partial^4}{\partial z^4}\left(\psi\left(z\right)\right) = \psi''''(z) = 24p^2\psi^5(z) + 20p\,q\,\psi^3(z) + \left(12pr + q^2\right)\psi(z) \\
\frac{\partial^5}{\partial z^5}\left(\psi\left(z\right)\right) = \psi'''''(z) = \left(120p^2\psi^4(z) + 60p\,q\,\psi^2(z) + 12pr + q^2\right)\psi'(z) \\
\ldots
\end{array}\right\} \tag{2.5}$$

we present many closed solutions for eq.(2.4) . In fact, these solutions as $\psi(z)$ = Jacobi elliptic functions, can be casted to hypothesis for more solutions eq.(2.1).$sn(\xi) = sn(\xi, m)$, $dn(\xi) = dn(\xi, m)$and $cn(\xi) = cn(\xi, m)$are the Jacobi elliptic function with modulus m, where $0 < m < 1$. These functions are considerable the resulting formulas:

$$sn^2(\xi) + cn^2(\xi) = 1,\ dn^2(\xi) + m^2 sn^2(\xi) = 1$$
$$sn(\xi) = \frac{1}{ns(\xi)},\ cn(\xi) = \frac{1}{nc(\xi)},\ dn(\xi) = \frac{1}{nd(\xi)}$$
$$cs(\xi) = \frac{sn(\xi)}{cn(\xi)},\ ds(\xi) = \frac{dn(\xi)}{sn(\xi)},\ sd(\xi) = \frac{ns(\xi)}{dn(\xi)}$$
$$sn'(\xi) = \frac{d(sn(\xi))}{d\xi} = cn(\xi)\, dn(\xi), dn'(\xi) = \frac{d(dn(\xi))}{d\xi} - m^2 cn(\xi)\, sn(\xi),$$
$$cn'(\xi) = \frac{d(cn(\xi))}{d\xi} - dn(\xi)\, sn(\xi)$$

when $m \to 1$; These functions convert to hyperbolic functions as follows:

$$sn(\xi) \to \tanh(\xi),\ \{cn(\xi), dn(\xi)\} \to \operatorname{sech}(\xi),$$
$$\{sc(\xi), sd(\xi)\} \to \sinh(\xi), \{cd(\xi),\ dc(\xi)\} \to 1$$
$$\{ds(\xi), cs(\xi)\} \to \cos ech(\xi), \{nc(\xi), nd(\xi)\} \to \cosh(\xi), \{ns(\xi)\} \to \coth(\xi)$$

when $m \to 0$ These functions convert to trigonometric functions as follows:

$$\{sn(\xi), sd(\xi)\} \to \sin(\xi),\ \{cn(\xi), cd(\xi)\} \to \cos(\xi),$$
$$\{sc(\xi)\} \to \tan(\xi), \{dn(\xi),\ nd(\xi)\} \to 1$$
$$\{ns(\xi), ds(\xi)\} \to Cosec(\xi), \{cs(\xi)\} \to \cot(\xi) \{nc(\xi), dc(\xi)\} \to \operatorname{Sec}(\xi)$$

Its balancing the highest order derivative term and the nonlinear term and find the value of n in eq. (2.3).

3 Application -3.1

We consider the second order nonlinear partial differential equations with combination Kortewegde Vries (KdV) Equation and BenjaminBonaMahony equation (BBM) Equation of two famous and fundamental nonlinear wave equations. This is as

$$\theta_t + a\,\theta_x + \theta\theta_x + b^2\theta_{x\,x\,x} - c^2\theta_{x\,x\,t} = 0 \tag{3.1}$$

Where $\theta = \theta(x, t)$ unknow wave function with space variable x and time variable t. a, b and c are arbitrary real constant. If $a = 0, c = 0$ then eq. (3.1) is Kortewegde Vries (KdV) Equation, this is one of the most famous non-linear wave equations, it was derived in fluid mechanics to describe shallow water waves in a rectangular channel [[1],[28]]. If $b = 0$ then eq. (3.1) is Benjamin-BonaMahony equation (BBM) Equation, also called regularized long-wave equation (RLWE), this serves as an approximate model in studying the dynamics of small-amplitude surface water waves propagating unidirectionally [[1]]. Suppose that the travelling wave solutions for eq. (3.1) are of the forms as follows

$$\theta(x, t) = \theta(z) = \theta(k(x - \omega t)$$

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where k and ω area constant, put in eq. (3.1) then

$$k\,(a-\omega)\;\theta' + k\,\theta\,\theta' + k^3\left(b^2+c^2\omega\right)\theta''' = 0$$

Integral one time, take constant zero

$$k\,(a-\omega)\;\theta + \frac{k^2}{2}\theta^2 + k^3\left(b^2+c^2\omega\right)\theta'' = 0 \tag{3.2}$$

Balancing θ'' with θ^2 in eq. (3.2) gives $2n = n+2$ i.e.,$n = 2$, then

$$\theta\,(z) = \sum_{i=0}^{2} \lambda_i(\psi\,(z))^i = \lambda_0 + \lambda_1\,\psi\,(z) + \lambda_2\,(\psi\,(z))^2 = \lambda_0 + \lambda_1\,\psi + \lambda_2\,\psi^{\,2}$$

$$\theta''\,(z) = \lambda_1\,\psi'' + 2\lambda_2\,\left\{\psi'^{\,2} + \psi\,\psi''\right\}$$

Put these values in eq. (3.2) with eq. (2.5)

$$\begin{gathered}(a-\omega)\left(\lambda_0 + \lambda_1\,\psi + \lambda_2\,\psi^2\right)\\ + \frac{k^2}{2}\left(\lambda_0^2 + \lambda_1^2\,\psi^2 + \lambda_2^2\,\psi^2 + 2\lambda_0\lambda_1\,\psi + 2\lambda_1\lambda_2\,\psi^3 + 2\lambda_0\lambda_2\,\psi^2\right)\\ + k^3\left(b^2+c^2\omega\right)\left\{2\lambda_2 R + \lambda_1\,q\,\psi + 2\lambda_2 q\,\psi^2 + \psi^3\left(2\lambda_1 p + 2\lambda_2 q\right) + \psi^4 6\lambda_2 p\right\} = 0\end{gathered} \tag{3.3}$$

equating all terms with the powers in ψ , and setting each of the obtained coefficients for ψ to zero, yields the following set of algebraic equations for $\lambda_0, \lambda_1, \lambda_2, k, \omega, a, b\; and\, c$

$$\begin{aligned}
\psi^0&: \quad \lambda_0 k\,(a-\omega) + \tfrac{k^2\lambda_0^2}{2} + 2\lambda_2\,r\,k^3\left(b^2+c^2\omega\right) = 0\\
\psi^1&: \quad \lambda_1 k\,(a-\omega) + \tfrac{k^2\lambda_0\lambda_1}{2} + \lambda_1\,q\,k^3\left(b^2+c^2\omega\right) = 0\\
\psi^2&: \quad \lambda_2 k\,(a-\omega) + \tfrac{k^2\left(\lambda_1^2+2\lambda_0\lambda_2\right)}{2} + 2\lambda_2\,q\,k^3\left(b^2+c^2\omega\right) = 0\\
\psi^3&: \quad 2\lambda_2\lambda_1 + k^3\left(b^2+c^2\omega\right)\left(2\lambda_1 p + 2\lambda_2 q\right) = 0\\
\psi^4&: \quad \lambda_2^2 + 6\lambda_2 p k^3\left(b^2+c^2\omega\right) = 0
\end{aligned}$$

One obtains solution

$$\lambda_0 = -\tfrac{k^2\left(b^2+c^2\omega\right)q+(a-\omega)}{k}, \quad \lambda_1 = k^2\left(b^2+c^2\omega\right)\sqrt{-12p\,q}$$

$$\lambda_2 = -6k^3\left(b^2+c^2\omega\right)p$$

then

$$\begin{aligned}\theta\,(z) = &-\frac{k^2\left(b^2+c^2\omega\right)q+(a-\omega)}{k}\\ &+\left\{k^2\left(b^2+c^2\omega\right)\sqrt{-12p\,q}\right\}\,\psi(z)\\ &-\left\{6k^3\left(b^2+c^2\omega\right)p\right\}\,(\psi(z)\,)^2\end{aligned}$$

We choose p q and r from [[17],[18]], such that
Solution -1 $p\ :\ m^2;\ \ q:\ \ -\left(1+m^2\right)\ \ then\ \psi(z) =\ sn(z)$ thus

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$$\theta(z) = -\frac{k^2\left(b^2+c^2\omega\right)\left(1+m^2\right)-(a-\omega)}{k} + \left\{k^2\left(b^2+c^2\omega\right)\sqrt{12m^2\left(1+m^2\right)}\right\} sn(z) - \left\{6k^3\left(b^2+c^2\omega\right)m^2\right\}\left(sn(z)\right)^2$$

Solution -2 $p\ :\ -m^2\ ,\ \ q:\ \left(2m^2-1\right)\ ,\ \ then\ \psi(z)=\ cn(z)$ thus

$$\theta(z) = -\frac{k^2\left(b^2+c^2\omega\right)\left(2m^2-1\right)+(a-\omega)}{k} + \left\{k^2\left(b^2+c^2\omega\right)\sqrt{12m^2\left(2m^2-1\right)}\right\} cn(z) + \left\{6k^3\left(b^2+c^2\omega\right)m^2\right\}\left(cn(z)\right)^2$$

Solution -3 $p\ :\ -\frac{1}{4}\ ,\ \ q:\ \left(\frac{1+m^2}{2}\right)\ ,\ \ r:\ \left(\frac{1-m^2}{2}\right)^2,\ then\ \psi(z)=mcn(z)\pm dn(z)$ thus

$$\theta(z) = -\frac{k^2\left(b^2+c^2\omega\right)\left(1+m^2\right)+2(a-\omega)}{2k} + \left\{k^2\left(b^2+c^2\omega\right)\sqrt{3\left(\frac{1+m^2}{2}\right)}\right\}\left(mcn(z)\pm dn(z)\right) + \left\{\frac{3k^3\left(b^2+c^2\omega\right)}{2}\right\}\left(mcn(z)\pm dn(z)\right)^2$$

Solution -4 $p\ :\ \frac{m^2}{4}\ ,\ \ q:\ \left(\frac{m^2-2}{2}\right),\ then\ \psi(z)=\ sn(z)+i\,cn(z)$ thus

$$\theta(z) = -\frac{k^2\left(b^2+c^2\omega\right)\left(m^2-2\right)+2(a-\omega)}{2k} + \left\{k^2\left(b^2+c^2\omega\right)\sqrt{3m^2\left(\frac{2-m^2}{2}\right)}\right\}\left\{sn(z)\pm i\,cn(z)\right\} - \left\{\frac{3m^2k^3\left(b^2+c^2\omega\right)}{2}\right\}\left(sn(z)\pm i\,cn(z)\right)^2$$

4 Application -3.2

We consider nonlinear KleinGordon (NKG) [[19]-[20]]. The Klein-Gordon equations play a significant role in solid state physics, plasma physics, nonlinear optics and quantum field theory

$$\theta_{tt} - \theta_{xx} + \theta + \beta|\theta|^2\theta = 0 \tag{4.1}$$

the travelling wave solutions for Eq. (4.1) are of the forms as follows:

$$\theta(x,t) = \theta(z)\,e^{i(\gamma\ (\omega x-t))} = \theta\left(k(x-\omega t)\right)e^{i(\gamma\ (\omega x-t))}$$

where k and ω area constant, put in eq. (2.1) then

$$\left(k^2\omega^2-k^2\right)\theta'' + \left\{\gamma^2\left(\omega^2-1\right)+1\right\}\theta + \beta\theta^3 = 0 \tag{4.2}$$

Balancing θ'' with θ^3 in eq. (4.2) gives $3n=n+2$ i.e.,$n=1$, then

$$\theta(z) = \sum_{i=0}^{1}\lambda_i(\psi(z))^i = \lambda_0 + \lambda_1\,\psi(z) = \lambda_0 + \lambda_1\,\psi$$

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Put these values in eq. (4.2)

$$\theta''(z) = \lambda_1 \psi''$$

Using eq. (2.5) and equating all terms with the powers in ψ , and setting each of the obtained coefficients for ψ to zero, yields set of algebraic equations for $\lambda_0, \lambda_1 k, \omega\, and\, \gamma$,

One obtains solution

$$\lambda_0^2 = -\frac{k^2(\omega^2-1)q+\{\gamma^2(\omega^2-1)+1\}}{3\beta}, \quad \lambda_1^2 = -\frac{2k^2(\omega^2-1)p}{\beta}$$

Then

$$\theta(z) = \sqrt{-\frac{k^2(\omega^2-1)q+\{\gamma^2(\omega^2-1)+1\}}{3\beta}} + \left\{\sqrt{-\frac{2k^2(\omega^2-1)p}{\beta}}\right\}\psi(z)$$

We choose p, q and r from [[17],[18]], such that

Solution -2.1 $p\ :\ m^2\ ,\quad q:\ -\left(1+m^2\right),\ then\ \psi(z) =\ sn(z)$ thus

$$\theta(z) = \left\{\begin{array}{l}\sqrt{-\frac{\{\gamma^2(\omega^2-1)+1\}-k^2(\omega^2-1)(1+m^2)}{3\beta}} \\ +\left\{\sqrt{-\frac{2k^2(\omega^2-1)m^2}{\beta}}\right\} sn\,(k(x-\omega t))\end{array}\right\} e^{i(\gamma\ (\omega x - t))}$$

Solution -2.2 $p\ :\ -m^2\ ,\quad q:\ \left(2m^2-1\right),\ then\ \psi(z) =\ cn(z)$ thus

$$\theta(z) = \left\{\begin{array}{l}\sqrt{-\frac{k^2(\omega^2-1)(2m^2-1)+\{\gamma^2(\omega^2-1)+1\}}{3\beta}} \\ +\left\{\sqrt{-\frac{2k^2(1-\omega^2)m^2}{\beta}}\right\} cn\,(k(x-\omega t))\end{array}\right\} e^{i(\gamma\ (\omega x - t))}$$

Solution -2.3 $p\ :\ \left(\frac{1-m^2}{4}\right)\ ,\quad q:\ \left(\frac{1+m^2}{2}\right),\ ,\ then\ \psi(z) = \frac{cn(z)}{1\pm\, sn(z)}$ thus

$$\theta(z) = \left\{\begin{array}{l}\sqrt{-\frac{k^2(\omega^2-1)(1+m^2)\,+2\{\gamma^2(\omega^2-1)+1\}}{6\beta}} \\ +\left\{\sqrt{-\frac{2k^2(\omega^2-1)(1-m^2)}{4\beta}}\right\}\left(\frac{cn\,(k(x-\omega t))}{1\pm\ sn(k(x-\omega t))}\right)\end{array}\right\} e^{i(\gamma\ (\omega x - t))}$$

Solution -2.4 $p\ :\ \frac{m^2}{4}\ ,\quad q:\ \left(\frac{m^2-2}{2}\right),\ then\ \psi(z) =\ sn(z)+i\,cn(z)$ thus

$$\theta(z) = \left\{\begin{array}{l}\sqrt{-\frac{\{2\gamma^2(\omega^2-1)+1\}+k^2(\omega^2-1)(m^2-2)}{6\beta}} \\ +\left\{\sqrt{-\frac{2k^2(\omega^2-1)m^2}{4\beta}}\right\}(\,sn(z)+i\,cn(z))\end{array}\right\} e^{i(\gamma\ (\omega x - t))}$$

5 Application -3.3

We consider Fisher equation

$$\theta_t = \delta_1\,\theta_{xx} + \delta_2\,\theta\,(1-\theta) \tag{5.1}$$

introduced by Fisher [[21]] to describe the propagation of a virile mutant in an infinitely long habitat. It also represents a model equation for the evolution of a neutron population in a nuclear reactor [[22]-[23]] and a prototype model for a spreading flame [[24]-[25]]. The travelling wave solutions for Eq. (5.1) are of the forms as follows:

$$\theta\,(x,t) = \theta\,(z) = \theta\,(k(x-\omega t)$$

where k and area constant, put in eq. (5.1) then

$$k\omega\theta' + k^2\delta_1\,\theta'' + \delta_2\,\theta - \delta_2\,\theta^2 = 0 \tag{5.2}$$

Balancing θ'' with θ^2 in eq. (5.2) gives $2n = n+2$ i.e.,$n = 2$, then

$$\theta\,(z) = \sum_{i=0}^{2} \lambda_i(\psi\,(z))^i = \lambda_0 + \lambda_1\,\psi\,(z) + \lambda_2\,(\psi\,(z))^2 = \lambda_0 + \lambda_1\,\psi + \lambda_2\,\psi^{\,2}$$

$$\theta''\,(z) = \lambda_1\,\psi'' + 2\lambda_2\,\left\{\psi'^{\,2} + \psi\,\psi''\right\}$$

Put these values in eq. (5.2)

$$(k\omega + \delta_2)\,\left(\lambda_0 + \lambda_1\,\psi + \lambda_2\,\psi^{\,2}\right) + k^2\delta_1\,\left(\lambda_1\,\psi'' + 2\lambda_2\,\left\{\psi'^{\,2} + \psi\,\psi''\right\}\right) - \delta_2\left(\lambda_0 + \lambda_1\,\psi + \lambda_2\,\psi^{\,2}\right)^2 = 0 \tag{5.3}$$

Using eq. (2.5) and collecting the coefficients of the same power $\psi^i\,(z)(\psi'(z))^j$ $(j = 0, 1\ i = 0, 1, 2, 3, 4...)$ and setting each of the attained coefficients to be zero we have a set of over determined algebraic equations for $\lambda_0, \lambda_1, \lambda_2, k, \omega, \delta_1$ *and* δ_2. One obtains solution

$$\lambda_0 = \tfrac{4k^2\delta_1 q + \delta_2}{2\delta_2},\ \ \lambda_2 = \tfrac{4k^2\delta_1 p}{\delta_2},\ \ \lambda_1 = 2\lambda_2$$

Then

$$\theta\,(z) = \tfrac{4k^2\delta_1 q + \delta_2}{2\delta_2} + \tfrac{4k^2\delta_1 p}{\delta_2}\,\psi(z) + \tfrac{4k^2\delta_1 p}{2\delta_2}\,\psi^2(z)$$

We choose p q and r from [[17]-[18]], such that
Solution -3.1 $p\ :\ m^2\ ,\quad q:\ -\left(1+m^2\right)\ ,\ then\ \psi(z) =\ sn(z)$ thus

$$\theta\,(z) = \tfrac{\delta_2 - 4k^2\delta_1\left(1+m^2\right)}{2\delta_2} + \tfrac{4k^2\delta_1 m^2}{\delta_2}\,sn(z) + \tfrac{4k^2\delta_1 m^2}{2\delta_2}\,sn^2(z)$$

Solution -3.2 $p\ :\ -m^2\ ,\quad q:\ \left(2m^2-1\right)\ ,\ then\ \psi(z) =\ cn(z)$ thus

$$\theta\,(z) = \tfrac{4k^2\delta_1\left(2m^2-1\right)+\delta_2}{2\delta_2} - \tfrac{4k^2\delta_1 m^2}{\delta_2}\,cn(z) - \tfrac{4k^2\delta_1 m^2}{2\delta_2}\,cn^2(z)$$

Solution -3.3 $p\ :\ \left(\frac{1}{4}\right)\ ,\ \ q:\ \ \left(\frac{1-2m^2}{2}\right)\ ,\ \ ,\ then\ \psi(z) = msn(z) \pm i\, dn(z)$thus

$$\theta\,(z) = \frac{2k^2\delta_1\left(1-2m^2\right)+\delta_2}{2\delta_2} + \frac{k^2\delta_1}{\delta_2}\ \{m\, sn(z) \pm i\, dn(z)\} + \frac{k^2\delta_1}{2\delta_2}\,\{m\, sn(z) \pm i\, dn(z)\}^2$$

Solution -3.4 $p\ :\ 1\ ,\ \ q:\ \ \left(2-4m^2\right)\ ,\ \ ,\ then\ \psi(z) = \frac{sn(z)dn(z)}{cn(z)}$thus

$$\theta\,(z) = \frac{8k^2\delta_1\left(1-2m^2\right)+\delta_2}{2\delta_2} + \frac{4k^2\delta_1}{\delta_2}\ \frac{sn(z)dn(z)}{cn(z)} + \frac{4k^2\delta_1}{2\delta_2}\left\{\frac{sn(z)dn(z)}{cn(z)}\right\}^2$$

6 Discussion and Numerical Sketch

It should be noted that, although many exact solutions are obtained in this work, it has been proved that some solutions in applications 3.1, 3.2 and 3.3 are equivalent to the solution of in the literature. like solution for 2.1 of application 3.2 [[26]] and solution for 3.2 of application 3.3 [[27]].

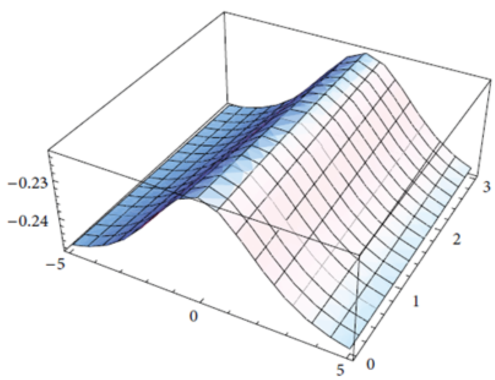

Figure 1: Travelling waves solution for 2 of 3.1 are plotted: bright solitary waves $m \to 1$

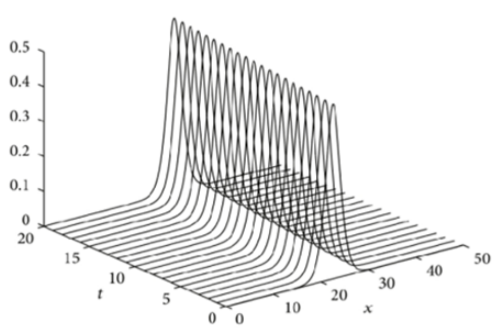

Figure 2: Soliton solution for 2.1 of 3.2 are plotted: solitary waves,$m \to 1$

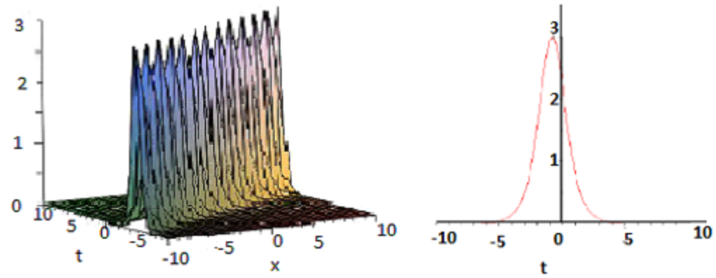

Figure 3: Soliton solution for 3.1 of 3.3 are plotted: solitary waves,$m \to 1$

7 Result and Conclusions

The Extended Jacobi Elliptic Function Technique has been successfully applied to obtain exact solution for three nonlinear wave equations. Moreover, the soliton-like solutions and trigonometric-function solutions have been also obtained as limiting cases on Jacobi Elliptic Function as $m \to 1$ and $m \to 0$.All solutions were verified by Maple package program and fig. (1), fig. (2) and fig. (3) are also new solitary wave solution for eq. (6), eq. (9) and eq. (11) respectively.
The main advantage of this method over other methods is that it provides exact solutions for all types, including Jacobian-elliptic functions. Finally, it is pertinent to mention that the proposed method is also a straightforward, short, promising and powerful method for other nonlinear evolution equations in mathematical physics. The algorithm of the method is very applicative and influential to investigate many solutions.

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