Algebraic Structures of ϑ −Translation and ϑ −Multiplication in Doubt Q- Fuzzy Z– Algebra

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Received: 14.04.2024 Revised : 16.05.2024 Accepted: 24.05.2024

ABSTRACT

In this paper, we define the new notation of Algebraic structures of ϑ −Translations and ϑ −Multiplication in Z −Sub Algebra of doubt Q − fuzzy Z −Algebra. And also defined theϑ −translations and ϑ −Multiplications in Z −Ideal of doubt Q-fuzzy Z −algebra and discussed some of their properties in detail by doubt Q − fuzzy$Z$ −algebras.

Keywords: Z–Algebra, Z–Ideal, Z −Sub Algebra, Q −Fuzzy Set, Fuzzy$Z$–Ideal, Z −Algebra, Fuzzy Z −Sub Algebra, and Fuzzy ϑ − Translation, Fuzzy ϑ −Multiplication.

1. INTRODUCTION

In 1965, Zadeh L A [15], initiated by the concept of fuzzy sets. Several researchers explored on the generalization of the notion of fuzzy subset. The study of fuzzy subsets and its applications to various mathematical contents has given rise to what is now commonly called fuzzy mathematics. Iseki K and Tanaka S [3], introduced the concept of an introduction to the theory of BCK-algebras in 1978. In1980, Iseki K [4], first introduced the notation on BCI-algebras. KyoungJa Lee, Young Bae Jun and MyungImDoh [5], introduced the concept of fuzzy translations and fuzzy multiplication of BCK/BCI-algebras in 2009. Abu Ayub Ansari and Chandramouleeswaran M [1], introduced the concept of fuzzy translation of fuzzy β − ideals of β −algebras in 2014. In 2014, Priya and Ramachandran T [11], introduced the new notation of fuzzy translation and multiplication on PS-algebras. Prasanna A, Premkumar M and Ismail Mohideen S [6]& [7], introduced the concept of fuzzy translation and multiplication on B-algebras in 2018 and also derived from Fuzzy Translation and Fuzzy Multiplication in BG – Algebras in 2019. In 2021, Premkumar [8] derived the new notation of Algebraic Properties on Fuzzy Translation and Multiplication in BP–Algebras. Premkumar [9] & [10], introduced the new concept of Algebraic Properties on ω − Fuzzy Translation and Multiplication in BH– Algebras in 2020 and also derived from the concept of Characteristics of $\kappa - Q$ − Fuzzy Translation and Fuzzy Multiplication in T-Ideals in T-Algebra in 2022. Sowmiya[13] & [14]initiated by the concept on Fuzzy Z-ideals in Z-algebras and also Fuzzy Algebraic Structure in Z-Algebras in 2019. In 2009, A new structure and construction of $\bar{Q}$-fuzzy groups developed by Solairaju[12].

We define the new notation of Algebraic structures of ϑ −Translations and ϑ −Multiplication in Z −Sub Algebra of doubt$Q$ − fuzzy$Z$ −algebras. And also defined theϑ −translations and ϑ −Multiplications in Z −Ideal of doubt$Q$ − fuzzy$Z$ −algebrasand discussed some of their properties.

2. Preliminaries

Definition 2.1:

A Z-algebra $(\tilde{\omega},*,0)$ be a Z-algebra. A fuzzy set A in $\tilde{\omega}$ with a membership function $\check{\mathfrak{z}}_A$ is said to be a fuzzy-Z-sub algebra of a Z-algebra $\tilde{\omega}$ if , for all $\acute{r},\check{s}$ in $\tilde{\omega}$ the following condition is satisfied

$\check{\mathfrak{z}}(\acute{r}*\check{s}) \geq \{\check{\mathfrak{z}}(\acute{r}) \wedge \check{\mathfrak{z}}(\check{s})\}$

Definition 2.2:

A Z-algebra $(\tilde{\omega},*,0)$ be a Z-algebra. A fuzzy set V in $\tilde{\omega}$ with a membership function $\check{\mathfrak{z}}_A$ is said to be a fuzzy-Z-sub algebra of a Z-algebra $\tilde{\omega}$ if, for all $\acute{r},\check{s}$ in $\tilde{\omega}$ the following condition is satisfied

(i) $\check{\mathfrak{z}}(0) \geq \check{\mathfrak{z}}(\acute{r})$

(ii) $\check{\mathfrak{z}}(\acute{r}) \geq \{\check{\mathfrak{z}}(\acute{r}*\check{s}) \wedge \check{\mathfrak{z}}(\check{s})\}$

Definition: 2.3

Let Q̄ and G a set and a group respectively. A mapping $\mu: \hat{\mathcal{C}} \times \bar{Q} \to [0,1]$ is called Q̄- FS in G. For any Q̄-$FS\mu$ in G and $t \in [0,1]$ we define the set $U(\mu;t) = \{\mathcal{P} \in \hat{\mathcal{C}}\ /\ \mu(\mathcal{P},q) \geq t, q \in \bar{Q}\}$ which is called an upper cut of "μ" and can be use to the characterization of μ.

3. Algebraic Structures of ϑ −Translation and ϑ −Multiplication in doubt$Q$ −Fuzzy Z-Subalgebra

Let $\tilde{\omega}$ be a Z-algebra. For any fuzzy set $\check{\mathfrak{z}}$ of $\tilde{\omega}$, we define T=1-$sup\{\check{\mathfrak{z}}(\acute{r},q)/\acute{r} \in \tilde{\omega}$ and $q \in Q\}$, unless otherwise we specified.

Definition: 3.1

Let $\check{\mathfrak{z}}$and Q −be two fuzzy subsets of $\tilde{\omega}$and $\vartheta \in [0,T]$. A mapping $\check{\mathfrak{z}}_{\vartheta}^{T}: \tilde{\omega} \times Q \to [0,1]$ is said to be a doubt$Q$ −fuzzy ϑ − translation of $\check{\mathfrak{z}}$ if it satisfies $\check{\mathfrak{z}}_{\vartheta}^{T} = \check{\mathfrak{z}}(\acute{r},q) + \vartheta, \forall\ \acute{r} \in \tilde{\omega}$ and $q \in Q$.

Definition: 3.2

Let $\check{\mathfrak{z}}$and Q −be two fuzzy subsets of $\tilde{\omega}$and $\vartheta \in [0,1]$. A mapping $\check{\mathfrak{z}}_{\vartheta}^{M}: \tilde{\omega} \times Q \to [0,1]$ is said to be a doubt$Q$ −fuzzy ϑ − multiplication of $\check{\mathfrak{z}}$ if it satisfies$\check{\mathfrak{z}}_{\vartheta}^{M} = \vartheta\ \check{\mathfrak{z}}(\acute{r},q)$, $\forall\ \acute{r} \in \tilde{\omega}$ and $q \in Q$.

Example: 3.2.1

Let $\tilde{\omega}$ ={0,1,2,3} be the set with the following table.

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

Then $(\tilde{\omega},*,0)$ is a Z – algebra.

Define doubt$Q$-fuzzy set $\check{\mathfrak{z}}$ is of $\tilde{\omega}$ by $\check{\mathfrak{z}}(\acute{r}) = \begin{Bmatrix} 0.4 & if\ \acute{r} \neq 1 \\ 0.3 & if\ \acute{r} = 1 \end{Bmatrix}$

Thus $\check{\mathfrak{z}}$ is a doubt$Q$-fuzzy Z-sub algebra of X.

Hence $T = 1 - sup\{\check{\mathfrak{z}}(\acute{r},q)/\acute{r} \in \tilde{\omega}\ \ and\ q \in Q\}$ = 1-0.4 = 0.6,

Choose $\vartheta = 0.2 \in [0,1]$ and $\vartheta = 0.3 \in [0,1]$.

Then the mapping $\check{\mathfrak{z}}_{0.2^T} : \tilde{\omega} \to [0,1]$ is defined by

$$\check{\mathfrak{z}}_{0.2^T} = \begin{cases} 0.2 + 0.4 = 0.6 & if\ \acute{r} \neq 1 \\ 0.2 + 0.3 = 0.5 & if\ \acute{r} = 1 \end{cases}$$

Which satisfies $\check{\mathfrak{z}}_{0.2^T}(\acute{r}) = \check{\mathfrak{z}}(\acute{r}) + 0.2,\ \forall\ \acute{r} \in \tilde{\omega}$, is a fuzzy 0.2-translation.

The mapping $\gamma_{0.3^M} : \tilde{\omega} \to [0,1]$ is defined by

$$\check{\mathfrak{z}}_{0.3^M} = \begin{cases} 0.3 * 0.4 = 0.12 & if\ \acute{r} \neq 1 \\ 0.3 * 0.3 = 0.09 & if\ \acute{r} = 1 \end{cases}$$

Which satisfies $\breve{\mathfrak{z}}_{0.3^M}(\acute{r}) = \breve{\mathfrak{z}}(\acute{r})(0.3), \forall\, \acute{r} \in \tilde{\omega}$ and $q \in Q$, is a fuzzy 0.3-Multiplication

Theorem: 3.3
If $\breve{\mathfrak{z}}$ of $\tilde{\omega}$ is a doubt $Q -$ fuzzy Z-sub-algebra and $\vartheta \in [0,1]$, then the doubt $Q -$ fuzzy $\vartheta -$translation. $\breve{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r}, q)$of $\breve{\mathfrak{z}}$ is also a doubt$Q -$ fuzzy Z- sub algebra of $\tilde{\omega}$.
Proof
Let $\acute{r}, \check{s} \in \tilde{\omega}, \vartheta \epsilon [0, T]$ and $q \in Q$
Then, $\breve{\mathfrak{z}}(\acute{r} * \check{s}, q) \le \breve{\mathfrak{z}}(\acute{r}, q) \vee \breve{\mathfrak{z}}(\check{s}, q)$
Now,
$\breve{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r} * \check{s}, q) = \breve{\mathfrak{z}}(\acute{r} * \check{s}, q) + \vartheta$
$\le [\breve{\mathfrak{z}}(\acute{r}, q) \vee \breve{\mathfrak{z}}(\check{s}, q)] + \vartheta$
$=\left[(\breve{\mathfrak{z}}(\acute{r}, q) + \vartheta) \vee (\breve{\mathfrak{z}}(\check{s}, q) + \vartheta)\right]$
$=\left[\breve{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r}, q) \vee \breve{\mathfrak{z}}_{\vartheta}{}^{T}(\check{s}, q)\right]. \forall\, \acute{r}, \check{s} \in \tilde{\omega}$ and $q \in Q$

Theorem: 3.4
Let $\breve{\mathfrak{z}}$and Q be a two fuzzy subset of $\tilde{\omega}$ such that the doubt$Q -$ fuzzy $\vartheta -$translation $\breve{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r}, q)$ of $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy sub algebra of $\tilde{\omega}$, for some $\vartheta \epsilon [0, T]$, then $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z-sub algebra of $\tilde{\omega}$.
Proof
Assume that $\breve{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r}, q)$is a doubt$Q -$ fuzzy sub algebra of $\tilde{\omega}$ for some $\vartheta \epsilon [0, T]$
Let $\acute{r}, \check{s} \in \tilde{\omega}$ and $q \in Q$ we have
$\breve{\mathfrak{z}}(\acute{r} * \check{s}, q) + \vartheta = \breve{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r} * \check{s}, q)$
$\le \left[\breve{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r}, q) \vee \breve{\mathfrak{z}}_{\vartheta}{}^{T}(\check{s}, q)\right]$
$= \left[(\breve{\mathfrak{z}}(\acute{r}, q) + \vartheta) \vee (\breve{\mathfrak{z}}(\check{s}, q) + \vartheta)\right]$
$= [\breve{\mathfrak{z}}(\acute{r}, q) \vee \breve{\mathfrak{z}}(\check{s}, q)] + \vartheta$
$\Longrightarrow \breve{\mathfrak{z}}(\acute{r} * \check{s}, q) \le \left[\breve{\mathfrak{z}}(\acute{r}, q) \vee \breve{\mathfrak{z}}(\check{s}, q)\right], \forall \acute{r}, \check{s} \in \tilde{\omega}$ and $q \in Q$
Hence, $\breve{\mathfrak{z}}$ is doubt$Q -$ fuzzy sub algebra of $\tilde{\omega}$.

Theorem: 3.5
For any doubt$Q -$ fuzzy Z- sub algebra $\breve{\mathfrak{z}}$ of $\tilde{\omega}$ and $\vartheta \epsilon [0,1]$, if the doubt$Q -$ fuzzy $\vartheta -$multiplication$\breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r}, q)$ of $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z-sub algebra of $\tilde{\omega}$.
Proof
Let $\acute{r}, \check{s} \in \tilde{\omega}, \vartheta \epsilon [0, T]$ and $q \in Q$
Then $\breve{\mathfrak{z}}(\acute{r} * \check{s}, q) \le \breve{\mathfrak{z}}(\acute{r}, q) \vee \breve{\mathfrak{z}}(\check{s}, q)$
Now,
$\breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r} * \check{s}, q) = \vartheta \breve{\mathfrak{z}}(\acute{r} * \check{s}, q)$
$\le \vartheta\, [\breve{\mathfrak{z}}(\acute{r}, q) \vee \breve{\mathfrak{z}}(\check{s}, q)]$
$\le \left[\vartheta\, \breve{\mathfrak{z}}(\acute{r}, q) \vee \vartheta\, \breve{\mathfrak{z}}(\check{s}, q)\right]$
$= \left[\breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r}, q) \vee \breve{\mathfrak{z}}_{\vartheta}{}^{M}(\check{s}, q)\right]$
$\Longrightarrow \breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r} * \check{s}, q) \le \left[\breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r}, q) \vee \breve{\mathfrak{z}}_{\vartheta}{}^{M}(\check{s}, q)\right]$
Therefore, $\breve{\mathfrak{z}}_{\vartheta}{}^{M}$ is a doubt$Q -$ fuzzy Z- sub algebra of $\tilde{\omega}$.

Theorem: 3.6
For any fuzzy subset$\breve{\mathfrak{z}}$ of $\tilde{\omega}$, $q \in Q$ and $\vartheta \epsilon [0,1]$, if the doubt$Q -$ fuzzy $\vartheta -$multiplication $\breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r}, q)$ of $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z-sub algebra of $\tilde{\omega}$, then so is $\breve{\mathfrak{z}}$.
Proof
Assume that $\breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r}, q)$ of $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z- sub algebra of $\tilde{\omega}$ for some $\vartheta \epsilon [0, T]$
Let $\acute{r}, \check{s} \in \tilde{\omega}$and $q \in Q$ we have
$\vartheta \breve{\mathfrak{z}}(\acute{r} * \check{s}, q) = \breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r} * \check{s}, q)$
$\le \left[\breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r}, q) \vee \breve{\mathfrak{z}}_{\vartheta}{}^{M}(\check{s}, q)\right]$
$= \left[\vartheta\, \breve{\mathfrak{z}}(\acute{r}, q) \vee \vartheta\, \breve{\mathfrak{z}}(\check{s}, q)\right]$
$= \vartheta\, [\breve{\mathfrak{z}}(\acute{r}, q) \vee \breve{\mathfrak{z}}(\check{s}, q)]$

$\Longrightarrow \check{\mathfrak{z}}(\acute{r} * \check{s}, q) \leq \vartheta \; [\check{\mathfrak{z}}(\acute{r}, q) \vee \check{\mathfrak{z}}(\check{s}, q)]$
Hence, $\check{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z- sub algebra of $\tilde{\omega}$.

4. Algebraic Structures of ϑ −Translation and ϑ −Multiplication in Q − Fuzzy Z-Ideal

Theorem:4.1

If the doubt$Q -$ fuzzy $\vartheta -$ translation ${\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r})$ of $\check{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z-Ideal, then it satisfies the condition${\check{\mathfrak{z}}_{\vartheta}}^{T}(\check{s} * (\acute{r} * \check{s}), q) \leq {\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q)$.

Proof:

${\check{\mathfrak{z}}_{\vartheta}}^{T}(\check{s} * (\acute{r} * \check{s}), q) = \check{\mathfrak{z}}(\check{s} * (\acute{r} * \check{s}), q) + \vartheta$
$\leq \{\check{\mathfrak{z}}(0 * (\check{s} * (\acute{r} * \check{s})), q) + \vartheta \; \vee \; \check{\mathfrak{z}}(0, q) + \vartheta\}$
$\leq \{\check{\mathfrak{z}}(0 * (\check{s} * (\check{s} * \acute{r})), q) + \vartheta \; \vee \; \check{\mathfrak{z}}(0, q) + \vartheta\}$
$= \{\check{\mathfrak{z}}(0 * ((\check{s} * \check{s}) * \acute{r}), q) + \vartheta \; \vee \; \check{\mathfrak{z}}(0, q) + \vartheta\}$
$= \{\check{\mathfrak{z}}(0 * (\check{s} * \acute{r}), q) + \vartheta \; \vee \; \check{\mathfrak{z}}(0, q) + \vartheta\}$
$= \{\check{\mathfrak{z}}((\check{s} * \acute{r}) * 0, q) + \vartheta \; \vee \; \check{\mathfrak{z}}(0, q) + \vartheta\}$
$\leq \{\check{\mathfrak{z}}((\check{s} * \acute{r}) * 0, q) + \vartheta \; \vee \; \check{\mathfrak{z}}(\acute{r}, q) + \vartheta\}$
$\leq \left\{{\check{\mathfrak{z}}_{\vartheta}}^{T}(0, q) \; \vee \; {\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q)\right\}$
$= {\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q)$.
$\Rightarrow {\check{\mathfrak{z}}_{\vartheta}}^{T}(\check{s} * (\acute{r} * \check{s}), q) \leq {\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q) \; \forall \; \acute{r}, \check{s} \in \tilde{\omega}$ and $q \in Q$

Theorem:4.2

If $\check{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z- ideal of $\tilde{\omega}$, then the doubt$Q -$ fuzzy $\vartheta -$translation ${\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q)$ of $\check{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z- ideal of $\tilde{\omega}$, for all $\vartheta \epsilon [0, T]$.

Proof

Let $\check{\mathfrak{z}}$ be a doubt$Q -$ fuzzy Z-ideal of $\tilde{\omega}$ and let $\vartheta \epsilon [0, T]$ and $q \in Q$
Then,(i)${\check{\mathfrak{z}}_{\vartheta}}^{T}(0, q)$= $\check{\mathfrak{z}}(0, q) + \vartheta$
$\leq \check{\mathfrak{z}}(\acute{r}, q) + \vartheta$
=${\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q)$
(ii) ${\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q) = \check{\mathfrak{z}}(\acute{r}, q) + \vartheta$
$\leq \{\check{\mathfrak{z}}(\acute{r} * \check{s}, q) \vee \check{\mathfrak{z}}(\check{s}, q)\} + \vartheta$
$= \{(\check{\mathfrak{z}}(\acute{r} * \check{s}, q) + \vartheta) \vee (\check{\mathfrak{z}}(\check{s}, q) + \vartheta)\}$
$= \left\{{\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r} * \check{s}, q) \vee {\check{\mathfrak{z}}_{\vartheta}}^{T}(\check{s}, q)\right\}$
$\Rightarrow {\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q) \leq \left\{{\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r} * \check{s}, q) \; \vee \; {\check{\mathfrak{z}}_{\vartheta}}^{T}(\check{s}, q)\right\}$
Hence ${\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q)$ of $\check{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z- ideal of $\tilde{\omega}$, $\forall \; \vartheta \epsilon [0, T]$and $q \in Q$

Theorem: 4.3

Let$\check{\mathfrak{z}}$ is a fuzzy subset of $\tilde{\omega}$and $q \in Q$ such that the doubt $Q -$ fuzzy $\vartheta -$translation ${\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q)$ of $\check{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z- ideal of $\tilde{\omega}$, for some $\vartheta \epsilon [0, T]$, then $\check{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z- ideal of $\tilde{\omega}$.

Proof

Assume that ${\check{\mathfrak{z}}_{\vartheta}}^{T}$ is a doubt$Q -$ fuzzy Z- ideal of $\tilde{\omega}$ for some $\vartheta \epsilon [0, T]$.
Let $\acute{r}, \check{s} \in \tilde{\omega}$ and $q \in Q$
Then,
(i) $\check{\mathfrak{z}}(0, q) + \vartheta = {\check{\mathfrak{z}}_{\vartheta}}^{T}(0, q)$
$\leq {\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q)$
$= \check{\mathfrak{z}}(\acute{r}, q) + \vartheta$
And so$\Rightarrow \check{\mathfrak{z}}(0, q) \leq \check{\mathfrak{z}}(\acute{r}, q)$
(ii) $\check{\mathfrak{z}}(\acute{r}, q) + \vartheta = {\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r}, q)$
$\leq \left\{{\check{\mathfrak{z}}_{\vartheta}}^{T}(\acute{r} * \check{s}, q) \; \vee \; {\check{\mathfrak{z}}_{\vartheta}}^{T}(\check{s}, q)\right\}$
$= \{(\check{\mathfrak{z}}(\acute{r} * \check{s}, q) + \vartheta) \vee (\check{\mathfrak{z}}(\check{s}, q) + \vartheta)\}$
$= \{\check{\mathfrak{z}}(\acute{r} * \check{s}, q) \; \vee \; \check{\mathfrak{z}}(\check{s}, q)\} + \vartheta$

and so $\breve{\mathfrak{z}}(\acute{r},q) \leq \{(\acute{r} * \check{s},q) \ \vee \ \breve{\mathfrak{z}}(\check{s},q)\}$
Hence $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z-ideal of $\tilde{\omega}$.

Theorem:4.4
Let $\vartheta\epsilon[0,T], q \in Q$ and let $\breve{\mathfrak{z}}$ be a doubt$Q -$ fuzzy Z-ideal of $\tilde{\omega}$. If $\tilde{\omega}$ is a Z-algebra, then the fuzzy ϑ −translation $\breve{\mathfrak{z}}_{\vartheta}{}^{T}$ of $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z-sub-algebra of $\tilde{\omega}$.
Proof
Let $\acute{r}, \check{s} \in \tilde{\omega}$ and $q \in Q$
Now, we have
$\breve{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r} * \check{s},q) = \breve{\mathfrak{z}}(\acute{r} * \check{s},q) + \vartheta$
$\leq \{\breve{\mathfrak{z}}\big((\acute{r} * \check{s},) * \check{s},q\big) \ \vee \ \breve{\mathfrak{z}}(\check{s},q)\} + \vartheta$
$= \{\breve{\mathfrak{z}}(\check{s} * (\acute{r} * \check{s}),q) \ \ \vee \ \ \breve{\mathfrak{z}}(\check{s},q) \ \} + \vartheta$ by Theorem 3.7
$\leq \{\breve{\mathfrak{z}}(0,q) \ \vee \ \breve{\mathfrak{z}}(\check{s},q)\} + \vartheta$
$\leq \{\breve{\mathfrak{z}}(\acute{r},q) \ \vee \ \breve{\mathfrak{z}}(\check{s},q)\} + \vartheta$
$\leq \{\big(\breve{\mathfrak{z}}(\acute{r},q) + \vartheta\big) \ \vee \ \big(\breve{\mathfrak{z}}(\check{s},q) + \vartheta\big)\}$
$= \left\{\breve{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r},q) \ \vee \ \breve{\mathfrak{z}}_{\vartheta}{}^{T}(\check{s},q)\right\}$
Hence $\breve{\mathfrak{z}}_{\vartheta}{}^{T}$ is a doubt $Q -$ fuzzy Z-sub-algebra of $\tilde{\omega}$.

Theorem:4.5
If the doubt$Q -$ fuzzy ϑ −translation $\breve{\mathfrak{z}}_{\vartheta}{}^{T}$ of $\breve{\mathfrak{z}}$ is a doubt $Q -$ fuzzy Z-sub-algebra of $\tilde{\omega}$, $\vartheta\epsilon[0,T]$, then $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z-sub-algebra of $\tilde{\omega}$.
Proof
Let us assume that $\breve{\mathfrak{z}}_{\vartheta}{}^{T}$ of $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z-ideal of $\tilde{\omega}$ and $q \in Q$
Then
$\breve{\mathfrak{z}}(\acute{r} * \check{s},q) + \vartheta = \breve{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r} * \check{s},q)$
$\leq \left\{\breve{\mathfrak{z}}_{\vartheta}{}^{T}\big((\acute{r} * \check{s}) * \check{s},q\big) \vee \breve{\mathfrak{z}}_{\vartheta}{}^{T}(\check{s},q)\right\}$
$= \left\{\breve{\mathfrak{z}}_{\vartheta}{}^{T}(\check{s} * (\acute{r} * \check{s}),q) \wedge \breve{\mathfrak{z}}_{\vartheta}{}^{T}(\check{s},q)\right\}$ by Theorem 3.7
$\leq \left\{\breve{\mathfrak{z}}_{\vartheta}{}^{T}(0,q) \vee \breve{\mathfrak{z}}_{\vartheta}{}^{T}(\check{s},q)\right\}$
$\leq \left\{\breve{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r},q) \vee \breve{\mathfrak{z}}_{\vartheta}{}^{T}(\check{s},q)\right\}$
$= \{(\breve{\mathfrak{z}}(\acute{r},q) + \vartheta) \vee (\breve{\mathfrak{z}}(\check{s},q) + \vartheta)\}$
$= \{\breve{\mathfrak{z}}(\acute{r},q) \vee \breve{\mathfrak{z}}(\check{s},q)\} + \vartheta$
$\Rightarrow \breve{\mathfrak{z}}(\acute{r} * \check{s},q) \leq \{\breve{\mathfrak{z}}(\acute{r},q) \vee \breve{\mathfrak{z}}(\check{s},q)\}$
Hence $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z-sub algebra of $\tilde{\omega}$.

Theorem:4.6
Let $\breve{\mathfrak{z}}$ is a fuzzy subset of $\tilde{\omega}$and $q \in Q$ such that the doubt$Q -$ fuzzy ϑ −Multiplication$\breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r},q)$ of $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z- ideal of $\tilde{\omega}$, for some $\vartheta\epsilon(0,1]$, then $\breve{\mathfrak{z}}$ is a doubt$Q -$ fuzzy Z- ideal of $\tilde{\omega}$.
Proof
Assume that $\breve{\mathfrak{z}}_{\vartheta}{}^{M}$ is a doubt$Q -$ fuzzy Z- ideal of $\tilde{\omega}$ for some $\vartheta\epsilon[0,T]$.
Let $\acute{r}, \check{s} \in \tilde{\omega}$and $q \in Q$

(i) $\vartheta\breve{\mathfrak{z}}(\acute{r},q) = \breve{\mathfrak{z}}_{\vartheta}{}^{M}(0,q)$
$\leq \breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r},q)$
$= \vartheta \ \breve{\mathfrak{z}}(\acute{r},q)$
And so $\Rightarrow \breve{\mathfrak{z}}(0,q) \leq \breve{\mathfrak{z}}(\acute{r},q)$

(ii) $\vartheta \ \breve{\mathfrak{z}}(\acute{r},q) = \breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r},q)$
$\leq \left\{\breve{\mathfrak{z}}_{\vartheta}{}^{M}(\acute{r} * \check{s},q) \vee \breve{\mathfrak{z}}_{\vartheta}{}^{M}(\check{s},q)\right\}$
$= \left\{\left(\vartheta \ \breve{\mathfrak{z}}(\acute{r} * \check{s},q)\right) \vee (\vartheta \ \breve{\mathfrak{z}}(\check{s},q))\right\}$
$= \vartheta \ \{\breve{\mathfrak{z}}(\acute{r} * \check{s},q) \ \vee \ \breve{\mathfrak{z}}(\check{s},q)\}$
And so$\Rightarrow \breve{\mathfrak{z}}(\acute{r},q) \leq \{\breve{\mathfrak{z}}(\acute{r} * \check{s},q) \ \vee \ \breve{\mathfrak{z}}(\check{s},q)\}$

Hence $\breve{\mathfrak{Z}}$ is a doubt$Q-$ fuzzy Z-ideal of $\tilde{\omega}$.

Theorem: 4.7

If $\breve{\mathfrak{Z}}$ is a doubt$Q-$ fuzzy Z- ideal of $\tilde{\omega}$, then the doubt$Q-$ fuzzy $\vartheta-$multiplication$\breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r},q)$ of $\breve{\mathfrak{Z}}$ is a doubt $Q-$ fuzzy Z- ideal of $\tilde{\omega}$, for all $\vartheta\epsilon(0,1]$.

Proof

Let $\breve{\mathfrak{Z}}$ be a doubt$Q-$ fuzzy Z-ideal of $\tilde{\omega}$ and let $\vartheta\epsilon(0,1]$ and $q \in Q$

Then

(i) $\breve{\mathfrak{Z}}_{\vartheta}{}^{M}(0,q) = \vartheta\, \breve{\mathfrak{Z}}(\acute{r},q)$
$\leq \vartheta\breve{\mathfrak{Z}}(\acute{r},q)$
$= \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r},q)$
$\Rightarrow \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(0,q) \leq \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r},q)$

(ii) $\breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r},q) = \vartheta\, \breve{\mathfrak{Z}}(\acute{r},q)$
$\leq \vartheta \left\{\breve{\mathfrak{Z}}(\acute{r}*\check{s},q) \vee \breve{\mathfrak{Z}}(\check{s},q)\right\}$
$= \vartheta \left\{\breve{\mathfrak{Z}}(\acute{r}*\check{s},q) \vee \breve{\mathfrak{Z}}(\check{s},q)\right\}$
$= \left\{\left(\vartheta\, \breve{\mathfrak{Z}}(\acute{r}*\check{s},q)\right) \vee \left(\vartheta\, \breve{\mathfrak{Z}}(\check{s},q)\right)\right\}$
$\leq \left\{\breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r}*\check{s},q) \vee \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\check{s},q)\right\}$
$\Rightarrow \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r},q) \leq \left\{\breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r}*\check{s},q) \vee \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\check{s},q)\right\}$

Hence $\breve{\mathfrak{Z}}_{\vartheta}{}^{M}$ of $\breve{\mathfrak{Z}}$ is a doubt $Q-$ fuzzy Z-ideal of $\tilde{\omega}$, $\forall \acute{r},\check{s} \in (0,1]$.

Theorem:4.8

Let $\vartheta\epsilon(0,1]$ and let $\breve{\mathfrak{Z}}$ be a doubt$Q-$fuzzy Z-ideal of a Z-algebra $\tilde{\omega}$. Then the doubt $Q-$fuzzy $\vartheta-$multiplication $\breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r})$ of $\breve{\mathfrak{Z}}$ is a doubt$Q-$ fuzzy Z- sub algebra of $\tilde{\omega}$.

Proof

Let $\acute{r},\check{s} \in \tilde{\omega}$ and $q \in Q$

Now, we have

$\breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r}*\check{s},q) = \vartheta\, \breve{\mathfrak{Z}}(\acute{r}*\check{s},q)$
$\leq \vartheta \left\{\breve{\mathfrak{Z}}\left((\acute{r}*\check{s})*\check{s}),q\right) \vee \breve{\mathfrak{Z}}(\check{s},q)\right\}$
$= \left\{\vartheta\, \breve{\mathfrak{Z}}\left(\left(\check{s}*(\acute{r}*\check{s})\right),q\right) \vee \vartheta\breve{\mathfrak{Z}}(\check{s},q)\right\}$
$= \vartheta\left\{\breve{\mathfrak{Z}}(0,q) \vee \breve{\mathfrak{Z}}(\check{s},q)\right\}$
$\leq \vartheta\left\{\breve{\mathfrak{Z}}(\acute{r},,q) \vee \breve{\mathfrak{Z}}(\check{s},q)\right\}$
$\leq \left\{\left(\vartheta\, \breve{\mathfrak{Z}}(\acute{r},q)\right) \vee \left(\vartheta\, \breve{\mathfrak{Z}}(\check{s},q)\right)\right\}$
$= \left\{\breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r},q) \vee \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\check{s},q)\right\}$

Hence $\breve{\mathfrak{Z}}_{\vartheta}{}^{M}$ is a doubt$Q-$ fuzzy Z-sub-algebra of $\tilde{\omega}$, $\forall \acute{r},\check{s} \in (0,1]$and $q \in Q$

Theorem:4.9

If the doubt$Q-$ fuzzy $\vartheta-$translation $\breve{\mathfrak{Z}}_{\vartheta}{}^{M}$ of $\breve{\mathfrak{Z}}$ is a doubt$Q-$ fuzzy Z-sub-algebra of $\tilde{\omega}$, $\vartheta\epsilon(0,1]$, then $\breve{\mathfrak{Z}}$ is a doubt$Q-$ fuzzy Z-sub-algebra of $\tilde{\omega}$.

Proof

Let us assume that $\breve{\mathfrak{Z}}_{\vartheta}{}^{M}$ of $\breve{\mathfrak{Z}}$ is a doubt$Q-$ fuzzy Z-ideal of $\tilde{\omega}$ and $q \in Q$

Then

$\vartheta\, \breve{\mathfrak{Z}}(\acute{r}*\check{s},q) = \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r}*\check{s},q)$
$\leq \left\{\breve{\mathfrak{Z}}_{\vartheta}{}^{M}\left((\acute{r}*\check{s})*\check{s},q\right) \vee \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\check{s},q)\right\}$
$= \left\{\breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\check{s}*(\acute{r}*\check{s}),q) \vee \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\check{s},q)\right\}$
$= \left\{\breve{\mathfrak{Z}}_{\vartheta}{}^{M}(0,q) \vee \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\check{s},q)\right\}$
$\leq \left\{\breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\acute{r},q) \vee \breve{\mathfrak{Z}}_{\vartheta}{}^{M}(\check{s},q)\right\}$

$= \left\{\left(\vartheta\, \check{\mathfrak{z}}(\acute{r}, q)\right) \vee \left(\vartheta\, \check{\mathfrak{z}}(\check{s}, q)\right)\right\}$

$\Rightarrow \check{\mathfrak{z}}(\acute{r} * \check{s}, q) \leq \{\check{\mathfrak{z}}(\acute{r}, q) \vee \check{\mathfrak{z}}(\check{s}, q)\}$

Hence $\check{\mathfrak{z}}$ is a doubt $Q-$ fuzzy Z-sub algebra of $\tilde{\omega}$.

Theorem:4.10

Intersection and union of any twoϑ −translation of a doubt$Q-$ fuzzy Z-ideal of $\check{\mathfrak{z}}$ of $\tilde{\omega}$ is also a doubt $Q-$ fuzzy Z-ideal of $\tilde{\omega}$.

Proof

Let $\check{\mathfrak{z}}_{\vartheta}{}^{T}$ and $\check{\mathfrak{z}}_{\delta}{}^{T}$ be two $\vartheta-$ translations of a doubt$Q-$ fuzzy Z-ideal of $\check{\mathfrak{z}}$ of $\tilde{\omega}$, where $\vartheta, \delta \in [0,1]$ and $q \in Q$

Then by theorem 3.14,$\check{\mathfrak{z}}_{\vartheta}{}^{T}$ and $\check{\mathfrak{z}}_{\delta}{}^{T}$ are doubt $Q-$ fuzzy Z-ideals of $\tilde{\omega}$.

Now, $(\check{\mathfrak{z}}_{\vartheta}{}^{T} \cap \check{\mathfrak{z}}_{\delta}{}^{T})(\acute{r}, q) = \left\{\check{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r}, q) \vee \check{\mathfrak{z}}_{\delta}{}^{T}(\acute{r}, q)\right\}$

$$= \{(\check{\mathfrak{z}}(\acute{r}, q) + \vartheta) \vee (\check{\mathfrak{z}}(\acute{r}, q) + \delta)\}$$
$$= \check{\mathfrak{z}}(\acute{r}, q) + \vartheta$$
$$= \check{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r}, q)$$

And $(\check{\mathfrak{z}}_{\vartheta}{}^{T} \cup \check{\mathfrak{z}}_{\delta}{}^{T})(\acute{r}, q)$=$\left\{\check{\mathfrak{z}}_{\vartheta}{}^{T}(\acute{r}, q) \wedge \check{\mathfrak{z}}_{\delta}{}^{T}(\acute{r}, q)\right\}$

$$= \{(\check{\mathfrak{z}}(\acute{r}, q) + \vartheta) \wedge (\check{\mathfrak{z}}(\acute{r}, q) + \delta)\}$$
$$= \check{\mathfrak{z}}(\acute{r}, q) + \delta$$
$$= \check{\mathfrak{z}}_{\delta}{}^{T}(\acute{r}, q)$$

Hence $\check{\mathfrak{z}}_{\vartheta}{}^{T} \cap \check{\mathfrak{z}}_{\delta}{}^{T}$ and $\check{\mathfrak{z}}_{\vartheta}{}^{T} \cup \check{\mathfrak{z}}_{\delta}{}^{T}$ are doubt $Q-$ fuzzy Z-ideals of $\tilde{\omega}$.

CONCLUSION

In this paper we have discussed ϑ −Translation and ϑ −Multiplication on Z-Algebras through Z- sub algebras anddiscussed with some other properties. And also derived from the ϑ −Translation and ϑ −Multiplication on Z- Ideals of Q −Fuzzy Z-Algebra.

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