M/M/1 Retrial Queueing Model with Server Breakdown and Feedback

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In this article, we present M/M/1 retrial queueing system with feedback and Server breakdown. Arrival follows Poisson process. An arrival finds the system is full, the arrival enters into an orbit of size infinity. From the orbit the customers try their luck. The time between two successive retrials is called retrial time, it follows negative exponential distribution. Service time is exponentially distributed. Once the server experiences an unanticipated failure, it should be repaired and returned to normal functioning. Feedback is when unsatisfied customers join the orbit again for a service. Matrix geometric method is engaged to determined performance measures. Some graphical representations are also acquired.

AMS subject classification number— 90B22, 60K30 and 60K25
Key Words —- Retrial Queue, Arrival Rate, Server Breakdown, Feedback, Matrix Geometric Method (MGM).

1 Introduction

Queueing model can be found in variety of real-life scenarios. Queueing system with feedback have several uses in the manufacturing, computing and telecommunications systems. In queueing theory in which customer arrives who finds the sever and waiting places are engaged, may retry after an irregular measurement of time is known as retrial queue. During the period of getting service the server may get sudden breakdown and send to repair, at that time the customer wait to get complete service. After getting a service the customer has to decide to leave the system or to continue the service. The unsatisfied customer goes to the orbit for another service is called feedback. Artalejo (2012) determined M/M/1 retrial queue with finite population. A survey of retrial queues was explored by Falin (1990). M/M/1 retrial queueing system with variable service rates in priority service was analyzed by Ayyappan Govindan et al (2011). Neuts (1981) discussed several matrix geometric stochastic model solutions. Praveen Deora et al (2021) analyzed the cost analysis and optimization of machine repair model with working vacation and feedback policy.

This model has been investigated by Choi, et al (1998) analyzed multi-server retrial queue with feedback and loss. Choi and Kulkarni (1992) explored feedback retrial queueing model. Chuen-Horng Lin and Jau-Chuan Ke (2011) determined multi server retrial queue with loss and feedback. Retrial queue with server breakdown has been investigated by Kalyanaraman and Seenivasan (2011) analyzed multi-server retrial queue with breakdown and geometric loss. Seenivasan et al investigated different type of queueing models and their characteristics behavior. With the help of that research criteria we developed the concept using in retrial queueing model.

Following is an overview of the remaining sections of this article. Construction of our model is presented in section 2. Section 3 includes some numerical examples. Section 4 describes the system performance measures, as well as the summary follows in the end part of this article. [1]

2 Construction of the model

In this article, we concentrated on retrial queue with server breakdown and Feedback. Arriving customer follows Poisson process with rate λ. Assuming that the server is free, the incoming customer will be served instantly, and if the server is occupied, he will joining the orbit. After certain uneven estimations of time, customers from orbit attempt their luck. In retrial, each customer is viewed as equivalent to a primary customer. The retrial time is exponentially distributed with rate ν. The service time is exponential distributed with service rate μ. Eventually when the server could open to unforeseen breakdown with rate α and after it ought to be fixed and goes to normal service with rate ϕ. Server will wait unless there is no queue at the ending of the vacation. Assuming that the served customer decide to leaves the framework forever with rate $\beta^{'} = (1 - \beta)$ (or) he rejoins the orbit again for another service at a rate β (it is called feedback). Our model's transition diagram is depicted in (Figure. 1).

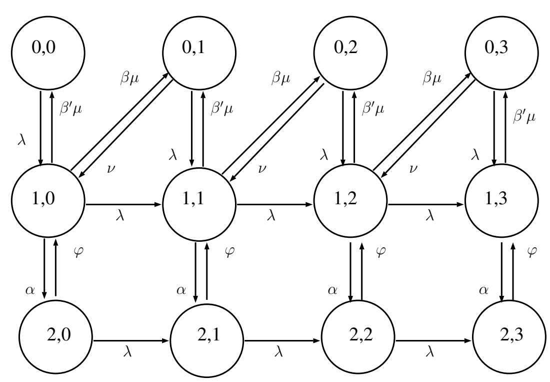

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Figure 1. Transition Diagram

Let $A(t), B(t) : t \geq 0$ be a stochastic process with state space at time t,
A(t) = 0, server is idle,
A(t) = 1, server is working,
A(t) = 2, server gets breakdown.
B(t) indicates no. of customers in the orbit.
Lexicographical series is given by:
$\Omega = (0,0)U(1,0)U(i,j); i = 0,1, j = 1,2,...., n \geq 1$
Infinitesimal generated matrix Q:

$$Q = \begin{pmatrix} K_{00} & L_{00} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ N_{00} & M_{00} & L_{00} & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & N_{00} & M_{00} & L_{00} & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & N_{00} & M_{00} & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & N_{00} & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

Where

$$K_{00} = \begin{pmatrix} -(\lambda) & \lambda & 0 \\ \beta' \mu & -(\lambda+\alpha+\mu) & \alpha \\ 0 & \varphi & -(\lambda+\varphi) \end{pmatrix}; \; L_{00} = \begin{pmatrix} 0 & 0 & 0 \\ \beta\mu & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix};$$

$$N_{00} = \begin{pmatrix} 0 & \nu & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \; M_{00} = \begin{pmatrix} -(\lambda+\nu) & \lambda & 0 \\ \beta' \mu & -(\lambda+\alpha+\mu) & \alpha \\ 0 & \varphi & -(\lambda+\varphi) \end{pmatrix};$$

We define $\pi_{ij} = \{A = i, B = j\} = \lim_{t\to\infty} Pr\{A(t) = i, B(t) = j\}$, where j indicates no. of customers in the orbit & i indicates the server state.
From the balance equation $\Pi Q = 0$. (1)
$\pi_0 K_{00} + \pi_1 N_{00} = 0$ (2)
$\pi_0 L_{00} + \pi_1 M_{00} + \pi_2 N_{00} = 0$ (3)
$\pi_1 L_{00} + \pi_2 M_{00} + \pi_3 N_{00} = 0$ (4)
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$\pi_i L_{00} + \pi_{i+1} M_{00} + \pi_{i+2} N_{00} = 0$ (5)
And $\pi_j = \pi_0 R^j$ for$j \geq 1$. (6)
We can assuming that R is a rate matrix.
$\pi_0[K_{00} + RN_{00}] = 0$ (7)
$\pi_0[R^2 N_{00} + RM_{00} + L_{00}] = 0$ (8)
The normalizing condition is
$\Pi_0 [I - R]^{-1} e = 1$ (9)
'e' is a column vector with all elements equal to 1.
Π partitioned as $\Pi = (\Pi_0, \Pi_1, \Pi_2)$ is a static prob. vector of the (reducible) generator matrix is $D = L_{00} + M_{00} + N_{00}$.

$$D = \begin{pmatrix} -(\lambda+\nu) & (\lambda+\nu) & 0 \\ \mu & -(\mu+\alpha) & \alpha \\ 0 & \varphi & -\varphi \end{pmatrix} \tag{10}$$

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And Π could be displayed to be stationary in order that $\Pi D = 0$ & $\Pi e = 1$.

$$\Pi_0 = [1 + \frac{\lambda+\nu}{\mu} + \frac{\alpha(\lambda+\nu)}{\varphi\mu}]^{-1}; \Pi_1 = \frac{\lambda+\nu}{\mu}\Pi_0; \Pi_2 = \frac{\alpha(\lambda+\nu)}{\varphi\mu}\Pi_0.$$

The static condition adopts the format actually determined by the drift condition. $\Pi L_{00} e < \Pi N_{00} e$. Equation (10) determines D's static probability. After obtaining rate matrix R, our probability vectors Πj's ($j \geq 1$) are calculated using Eqs. (6) and (9).

3 Numerical Study

By changing the values of the parameter λ & fixing all other parameters

Case i

If $\lambda = 0.10$, $\mu = 2.0$, $\beta = 0.4$,$\beta^{'} = 0.6$,$\alpha = 0.30$, $\varphi = 0.50$, $\nu = 0.05$ & $R = \begin{pmatrix} 0.3950 & 0.2226 & 0.0247 \\ 0.5926 & 0.1838 & 0.0370 \\ 0.4938 & 0.1868 & 0.1975 \end{pmatrix}$

Table 1. Probability vectors

Π_j	π_{0j}	π_{1j}	π_{2j}	Total
π_0	0.2436	0.0203	0.0361	0.3000
π_1	0.1261	0.0647	0.0139	0.2047
π_2	0.0950	0.0426	0.0083	0.1459
π_3	0.0668	0.0305	0.0056	0.1029
π_4	0.0472	0.0215	0.0039	0.0726
π_5	0.0333	0.0152	0.0027	0.0512
π_6	0.0235	0.0107	0.0019	0.0316
π_7	0.0166	0.0076	0.0014	0.0256
π_8	0.0117	0.0053	0.0010	0.0180
π_9	0.0083	0.0038	0.0007	0.0128
π_{10}	0.0058	0.0027	0.0005	0.0090
π_{11}	0.0041	0.0012	0.0003	0.0063
π_{12}	0.0029	0.0008	0.0002	0.0044
π_{13}	0.0020	0.0006	0.0002	0.0031
π_{14}	0.0014	0.0004	0.0001	0.0022
π_{15}	0.0010	0.0003	0.0001	0.0016
π_{16}	0.0007	0.0002	0.0001	0.0011

π_{17}	0.0005	0.0001	0.0000	0.0007
π_{18}	0.0004	0.0001	0.0000	0.0006
π_{19}	0.0003	0.0001	0.0000	0.0004
π_{20}	0.0002	0.0001	0.0000	0.0003
Total				0.9999

The prob. vectors in table 1 were calculated by using the matrix R in Equation (7) and Equation (9), we get the vector $\Pi_0 = (0.2436\ 0.0203\ 0.0361)$. Utilizing Π_0 in Equation (6), the rest of the vectors are obtained. Hence the sum of the probability is affirmed to be $0.9999 \approx 1$.

Case ii

If $\lambda = 0.15$, $\mu = 2.0$, $\beta = 0.4$, $\beta' = 0.6$, $\alpha = 0.30$, $\varphi = 0.50$, $\nu = 0.05$ & $R = \begin{pmatrix} 0.4548 & 0.2665 & 0.0394 \\ 0.6064 & 0.2264 & 0.0525 \\ 0.4665 & 0.2100 & 0.2711 \end{pmatrix}$

Table 2. Probability vectors

Π_j	π_{0j}	π_{1j}	π_{2j}	Total
π_0	0.1364	0.0171	0.0338	0.1873
π_1	0.0882	0.0473	0.0154	0.1509
π_2	0.0760	0.0375	0.0101	0.1236
π_3	0.0621	0.0309	0.0077	0.1006
π_4	0.0505	0.0251	0.0062	0.0818
π_5	0.0411	0.0204	0.0050	0.0665
π_6	0.0334	0.0166	0.0040	0.0540
π_7	0.0272	0.0135	0.0033	0.0440
π_8	0.0221	0.0110	0.0027	0.0358
π_9	0.0179	0.0089	0.0022	0.0290
π_{10}	0.0146	0.0073	0.0018	0.0237
π_{11}	0.0119	0.0059	0.0014	0.0192
π_{12}	0.0096	0.0048	0.0012	0.0156
π_{13}	0.0078	0.0039	0.0009	0.0126
π_{14}	0.0064	0.0032	0.0008	0.0104
π_{15}	0.0052	0.0026	0.0006	0.0084
π_{16}	0.0042	0.0021	0.0005	0.0068
π_{17}	0.0034	0.0017	0.0004	0.0055
π_{18}	0.0028	0.0014	0.0003	0.0045

π_{19}	0.0023	0.0011	0.0003	0.0037
π_{20}	0.0018	0.0009	0.0002	0.0029
π_{21}	0.0015	0.0007	0.0002	0.0024
π_{22}	0.0012	0.0006	0.0001	0.0019
π_{23}	0.0010	0.0005	0.0001	0.0021
π_{24}	0.0008	0.0004	0.0001	0.0013
π_{25}	0.0007	0.0003	0.0001	0.0011
π_{26}	0.0005	0.0003	0.0001	0.0009
π_{27}	0.0004	0.0002	0.0001	0.0007
π_{28}	0.0004	0.0002	0.0000	0.0006
π_{29}	0.0003	0.0001	0.0000	0.0004
π_{30}	0.0002	0.0001	0.0000	0.0003
Total				0.9998

The prob. vectors in table 2 were calculated by using the matrix R in Equation (7) and Equation (9), we get the vector $\Pi_0 = (0.1364\ 0.0171\ 0.0338)$. Utilizing Π_0 in Equation (6), the rest of the vectors are obtained. Hence the sum of the probability is affirmed to be $0.9998 \approx 1$.

Case iii

If $\lambda = 0.20$, $\mu = 2.0$, $\beta = 0.4$,$\beta' = 0.6$, $\alpha = 0.30$, $\varphi = 0.50$, $\nu = 0.05$ & $R = \begin{pmatrix} 0.4827 & 0.2894 & 0.0517 \\ 0.6034 & 0.2543 & 0.0647 \\ 0.4310 & 0.2160 & 0.3319 \end{pmatrix}$

Table 3. Probability vectors

Π_j	π_{0j}	π_{1j}	π_{2j}	Total
π_0	0.0849	0.0142	0.0305	0.1296
π_1	0.0627	0.0348	0.0154	0.1129
π_2	0.0579	0.0303	0.0106	0.0988
π_3	0.0508	0.0268	0.0085	0.0861
π_4	0.0443	0.0233	0.0072	0.0748
π_5	0.0386	0.0203	0.0062	0.0651
π_6	0.0335	0.0177	0.0054	0.0566
π_7	0.0292	0.0154	0.0047	0.0493
π_8	0.0253	0.0133	0.0040	0.0416
π_9	0.0220	0.0116	0.0035	0.0371
π_{10}	0.0192	0.0101	0.0031	0.0324
π_{11}	0.0166	0.0088	0.0027	0.0281
π_{12}	0.0145	0.0076	0.0023	0.0244

π_{13}	0.0126	0.0066	0.0020	0.0212
π_{14}	0.0109	0.0058	0.0017	0.0184
π_{15}	0.0095	0.0050	0.0015	0.0160
π_{16}	0.0083	0.0044	0.0013	0.0140
π_{17}	0.0072	0.0038	0.0011	0.0121
π_{18}	0.0062	0.0033	0.0010	0.0105
π_{19}	0.0054	0.0029	0.0009	0.0092
π_{20}	0.0047	0.0025	0.0008	0.0080
π_{21}	0.0041	0.0022	0.0007	0.0070
π_{22}	0.0036	0.0019	0.0006	0.0061
π_{23}	0.0031	0.0016	0.0005	0.0052
π_{24}	0.0027	0.0014	0.0004	0.0045
π_{25}	0.0023	0.0012	0.0004	0.0039
π_{26}	0.0020	0.0011	0.0003	0.0034
π_{27}	0.0018	0.0009	0.0003	0.0030
π_{28}	0.0015	0.0008	0.0002	0.0025
π_{29}	0.0013	0.0007	0.0002	0.0022
π_{30}	0.0012	0.0006	0.0002	0.0020
π_{31}	0.0010	0.0005	0.0002	0.0017
π_{32}	0.0009	0.0005	0.0001	0.0015
π_{33}	0.0008	0.0004	0.0001	0.0013
π_{34}	0.0007	0.0003	0.0001	0.0011
π_{35}	0.0006	0.0003	0.0001	0.0010
π_{36}	0.0005	0.0003	0.0001	0.0009
π_{37}	0.0004	0.0002	0.0001	0.0007
π_{38}	0.0003	0.0002	0.0001	0.0006
π_{39}	0.0003	0.0002	0.0000	0.0005
π_{40}	0.0002	0.0001	0.0000	0.0003
Total				0.9980

The prob. vectors in table 3 were calculated by using the matrix R in Equation (7) and Equation (9), we get the vector $\Pi_0 = (0.0849\ 0.0142\ 0.0305)$. Utilizing Π_0 in Equation (6), the rest of the vectors are obtained. Hence the sum of the probability is affirmed to be $0.9980 \approx 1$.

Case iv

If $\lambda = 0.25$, $\mu = 2.0$,$\beta = 0.4$,$\beta^{'} = 0.6$, $\alpha = 0.30$, $\varphi = 0.50$, $\nu = 0.05$ & $R = \begin{pmatrix} 0.4938 & 0.3055 & 0.0617 \\ 0.5926 & 0.2768 & 0.0741 \\ 0.3950 & 0.2161 & 0.3827 \end{pmatrix}$

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Table 4. Probability vectors

Π_j	π_{0j}	π_{1j}	π_{2j}	Total
π_0	0.0570	0.0119	0.0275	0.0964
π_1	0.0416	0.0267	0.0149	0.0877
π_2	0.0444	0.0247	0.0105	0.0796
π_3	0.0407	0.0227	0.0086	0.0720
π_4	0.0369	0.0206	0.0075	0.0650
π_5	0.0334	0.0186	0.0067	0.0587
π_6	0.0301	0.0168	0.0060	0.0529
π_7	0.0272	0.0152	0.0054	0.0478
π_8	0.0245	0.0137	0.0049	0.0431
π_9	0.0221	0.0123	0.0044	0.0388
π_{10}	0.0200	0.0111	0.0040	0.0351
π_{11}	0.0180	0.0100	0.0036	0.0316
π_{12}	0.0163	0.0091	0.0032	0.0286
π_{13}	0.0147	0.0082	0.0029	0.0258
π_{14}	0.0132	0.0074	0.0026	0.0232
π_{15}	0.0119	0.0067	0.0024	0.0210
π_{16}	0.0108	0.0060	0.0021	0.0189
π_{17}	0.0097	0.0054	0.0019	0.0170
π_{18}	0.0088	0.0049	0.0017	0.0154
π_{19}	0.0079	0.0044	0.0016	0.0139
π_{20}	0.0071	0.0040	0.0014	0.0125
π_{21}	0.0064	0.0036	0.0013	0.0113
π_{22}	0.0058	0.0032	0.0012	0.0102
π_{23}	0.0052	0.0029	0.0010	0.0091
π_{24}	0.0047	0.0026	0.0009	0.0082
π_{25}	0.0043	0.0024	0.0008	0.0075
π_{26}	0.0038	0.0021	0.0008	0.0067
π_{27}	0.0035	0.0019	0.0007	0.0061
π_{28}	0.0033	0.0017	0.0006	0.0054
π_{29}	0.0028	0.0016	0.0006	0.0050
π_{30}	0.0025	0.0014	0.0005	0.0044
π_{31}	0.0023	0.0013	0.0005	0.0041
π_{32}	0.0021	0.0012	0.0004	0.0037

π_{33}	0.0019	0.0010	0.0004	0.0033
π_{34}	0.0017	0.0009	0.0003	0.0029
π_{35}	0.0015	0.0008	0.0003	0.0026
π_{36}	0.0014	0.0008	0.0003	0.0025
π_{37}	0.0012	0.0007	0.0002	0.0021
π_{38}	0.0011	0.0006	0.0002	0.0018
π_{39}	0.0010	0.0006	0.0002	0.0018
π_{40}	0.0009	0.0005	0.0002	0.0016
π_{41}	0.0008	0.0005	0.0002	0.0015
π_{42}	0.0007	0.0004	0.0001	0.0012
π_{43}	0.0007	0.0004	0.0001	0.0012
π_{44}	0.0006	0.0003	0.0001	0.0010
π_{45}	0.0005	0.0003	0.0001	0.0009
π_{46}	0.0005	0.0003	0.0001	0.0009
π_{47}	0.0004	0.0002	0.0001	0.0007
Total				0.9990

The prob. vectors in table 4 were calculated by using the matrix R in Equation (7) and Equation (9), we get the vector $\Pi_0 = (0.0570\ 0.0119\ 0.0275)$. Utilizing Π_0 in Equation (6), the rest of the vectors are obtained. Hence the sum of the probability is affirmed to be $0.9990 \approx 1$.

4 Performance Measures

The following performance measures were discovered using steady-state probabilities.

- Pr{server is in idle} $E(I) = \Pi_0$ (11)
- Pr{server is on busy period} $E(B) = \sum_{j=1}^{\infty} j\pi_{1j}$ (12)
- Pr{server gets breakdown} $E(BD) = \sum_{j=1}^{\infty} j\pi_{2j}$ (13)
- Pr{Total no. of customers in the system} $E(N) = E(I) + E(B) + E(BD)$ (14)
- Pr{No customer in the orbit} $PNCO = \sum_{i=0}^{2} \pi_{i0}$ (15)

Table 5. Performance Measures

λ	0.1	0.15	0.2	0.25
E(I)	0.6917	0.6327	0.5954	0.5660
E(B)	0.7077	1.3304	2.0656	3.6330
E(BD)	0.1297	0.3255	0.6596	1.0317
E(N)	2.3764	4.3319	6.6537	9.8917
PNCO	0.3000	0.1873	0.1296	0.0964

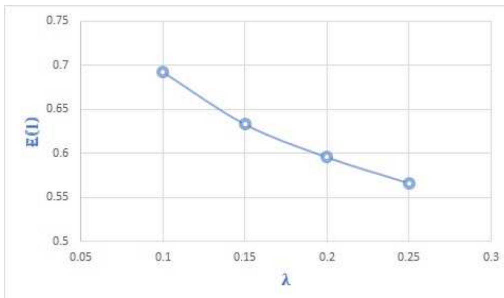

Figure 2. Arrival rate versus E(I)

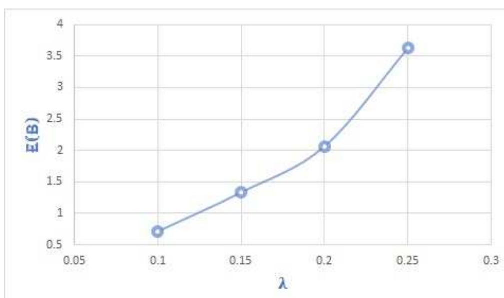

Figure 3. Arrival rate versus E(B)

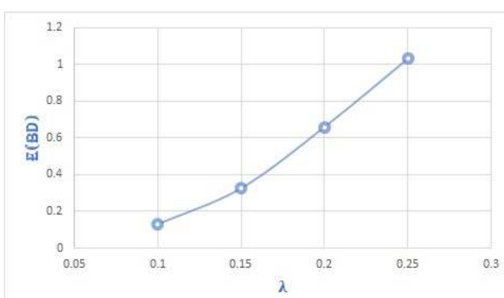

Figure 4. Arrival rate versus E(BD)

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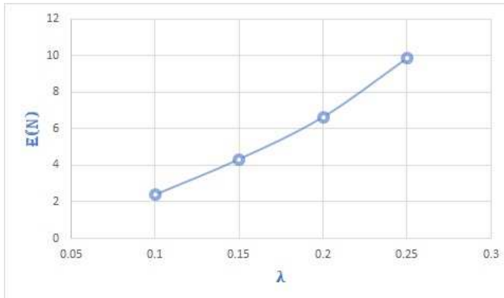

Figure 5. Arrival rate versus E(N)

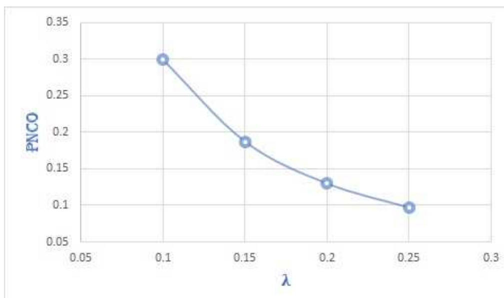

Figure 6. Arrival rate versus PNCO

The values of arrival rate have been varied from 0.1 to 2.5 As the arrival increases, Prob. that server is on idle and Prob. that orbit has no customer are decreases(refer Fig. 2 & Fig. 6). Similarly, if arrival rate increases, then Prob. that server is on busy period, Prob. that server gets breakdown and Prob. that total customers in the system are gradually increases (refer Fig. 3, Fig. 4 & Fig. 5).

5 Summary

This article focused on M/M/1 retrial queue with breakdown & feedback by utilizing Matrix geometric method. Using this type of model we can able to manage the time during the server breakdown and customer who is not satisfied are also able to get a servers again without any issues. By this producing this method the steady state probability vectors are obtained. From that some system performance measures are also determined with graphical representations.

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