A comparison between the Maximum likelihood method-and the least squares method for the (Shifted Gompertz) distribution using simulation

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ABSTRACT
Devices suffer during working hours many holidays and sudden stops, and these holidays may be mechanical or electrical failures that may affect the device or the machine, so it was necessary to study the reasons for these stops, as the technological and technical development led to the growing interest in studying the issue of reliability, which is one of the important topics that It is an indicator for expressing the performance of any operating system, and it has received attention in order to determine the efficiency of the machine in terms of filling it without failure for the longest possible period in order to avoid sudden stops for it and to search for the causes of holidays and stops that occur to devices, machines and equipment which in turn leads to maintaining the optimum productivity of the equipment and determining the duration If these devices stop working for a period of time, or their failure to perform their functions for a period of time, or their failure to perform functions during work, this leads to high material losses and a decrease in the level of production.

Keywords: Shifted Gompertz distribution, Reliability, Reliability function, maximum likelihood method, WeiGhted lest squane method

1. INTRODUCTION
In this section, we have studied some basic concepts that will help in understanding the probability distribution that has been made
if it includes the definition of the distribution with two parameters (α.β) and its general form. It also includes studying the characteristics of the distribution, estimating the parameters in different ways, and finding the reliability function for each of the methods used to estimate Shifted Gompertz)).

2. Research problem
The problem of the message lies in several ways that the idea of parameters in the distribution parameters can transform Gompertz and the reliability function, and knowing the most efficient observational methods is a substitute for the known methods.

3. Objective of the message
The methods used for using the methods used for using the methods used for using the methods used for parameter estimation (MSE)

4. Reference review
1- In (2013) a paper by Fernando jimenez torres presented a proposal for non-linear least squares procedures to report the (S.G) distribution. Simulation studies are conducted for the fields of weighted and unweighted least squares methods, the maximum potential method and the moment method. This work concluded that least squares methods using weighting factors to estimate this probability distribution give better performance than unweighted least squares methods, which shows the importance of weighting factors. Besides, the results of this simulation study well show using the maximal possibility method and that the estimators obtained with further improvements are those of the moment method. [9]
2- In (2013) See Angel Molina-García 1, José Carlos Campelo 2, Sara Blanc 2, Juan José Serrano 2, Tania García-Sánchez 3 and María C. Bueso presented a vision and diagnostic presentation study for solar PV modules Hydro is based on a decentralized wireless acquisition system in which all of the DC electrical

variables in the environment are collected at the PV module level with low-cost node devices. Photovoltaic power plants indicate that there is an opportunity for widespread deployment of photovoltaic power plants. Moreover, it can be easily stunned in existing, current PV installations from the presence of additional wires[3]

3- In 2017, the researcher Hassan S. Bakush, Ahmed M.; T. Abdel-Barr, Tanta A study in the use of two sets of real values to evaluate the model, based on some evaluation of the fitness of statistical fitness. As a result, the variance-covariance matrix and its confidence interval for parameters, and some theoretical measures for such a case with suspensions, were calculated. [11].

5. Shifted Gompertz distribution

This distribution is represented by the two largest independent random variables, the first being an exponential distribution with the parameter (b) and the other representing the Gumbel distribution with the two parameters (α and β). In its original formula, the distribution was expressed with reference to the Gompertz distribution rather than the Gumbel distribution, but since the Gompertz distribution is The Gumbel distribution is inverse, the nomenclature can be considered accurate. It was used as a model for the adoption of innovations. It was suggested by Bemmaor [2]. Some of its statistical properties were further studied by Jiménez and Jodrá

[8] and Jiménez Torres [13]. It has been used to predict the growth and decline of social networks and online services and has been shown to be superior to Bass model and Weibull distribution [4]

6. The probability density function of the Shifted Gompertz distribution

(probability density function for Shifted Gompertz)[13]

$f(x;\alpha,\beta) = \beta e^{-\beta x} e^{-\alpha e^{-\beta x}} \left[1 + \alpha\left(1 - e^{-\beta x}\right)\right] \ldots \ldots . . (2-1)$

($\alpha \geq 0$) represents the shape parameter and it is also known as the model parameter and it represents any parameter except the location parameter and the measurement parameter and it is not a function of either of them as this parameter affects the shape of the distribution instead of transferring it as the location parameter does or extending and shrinking as the measurement parameter does.

($\beta \geq 0$) represents the scaling parameter where the larger the size of the scaling parameter, the greater the spread of the distribution.

7. The cumulative function for Shifted Gompertz for the Shifted Gompertz distribution with two parameters is expressed by the formula[13]

$F(x;\alpha,\beta) = \left(1 - e^{-\beta x}\right) e^{-\alpha e^{-\beta x}} \ldots (2-2)$

where ($x \geq 0$)

($\alpha \geq 0$) represents the figure parameter.

($\beta \geq 0$) represents the larger the scale parameter size, the greater the distribution spread

8. Reliability

represents the science that deals with the life of equipment, in particular, the probability of survival and the possibility that the life of the system is greater than a specific time and the average life of the equipment, and before World War II, research was concerned with qualitative control and maintenance of machines and was not dealing with machines as an independent field and after the continued development of the system after World War II All over the world, more complex products have been produced consisting of many components (such as televisions, electrical equipment, etc.) [17].

9. Some basic concepts

Reliability

Reliability represents a measure of the performance of machines, and it is usually used to describe a job or work [13]. As any functional item needs requirements to perform a specific job when it performs, but rather the work required of it to continue to perform the work under normal operating conditions, as it can be said that this item counts. can be defined as follows:

* The probability that the device will perform a specific work under specific conditions and for a specific period of time.[14]

* The possibility of an organization working for a certain period of time and under working conditions designed for it. [19]

Reliability uses two terms, the first deals with the age of machines and equipment (i.e. dealing with machines and their systems) and this is called reliability, and the second deals with humans and survival (survival) that is, the probability of the life of the cell or human organism being greater than a certain time, meaning that they share a measurement The length of life of a machine or a human or animal being [19].

10. Reliability function

It represents the probability that the experimental unit will remain working for at least time t, where (t≤0) that is, if T is the random variable that represents the survival time of the experimental unit, then (R(t)) represents the probability that the unit will work for a future period. [5]
defined mathematically as follows:
$R(t)=p(T< t) \ldots (2-3)$
Where (R(t)) is a positive reliability function for all values
T represents a random variable that indicates the time of operation of the device until the failure, and it is a positive random variable that represents the accumulated time. (t) represents the operating time that is greater or equal to zero (t≥0).
$R(t)=\int_t^\infty f(u)du; t \geq 0 \ldots (2-4)$
It is complementary to the failure function (F(t)), meaning that:
R(t)=1-F(t)...(2-5)

11. Functions related to reliability

Many of the functions are related, but dependency may be related to a direct image or in another way, and these functions through which it is possible to distinguish any of the failure distributions, which are known as the period [0, ∞] of the random variable T, which is often continuous until failure occurs are:- [6][12][19]
$f(t)=\lim_{\Delta t\to 0} \frac{pr(t<T<t+\Delta t)}{\Delta t}, t \geq 0 \ldots (2-6)$
This function is concerned with the properties of the probability density function itself (p.d.f), and the failure density function has an aggregate distribution function (c.d.f) that is:-
1-f(t) is always positive
2- The sum of the area under the curve f(t) is always equal to one, that is:
$$\int_0^\infty f(t)dt = 1 \ldots (2-7)$$
And the probability of failure in the period $[t_2 . t_1 \ ]$ can be expressed mathematically, but the following form:
$P(t_1 \leq T \leq t_2) = \int_{t1}^{t2} f(u)du \ldots (2-8)$

12. Failure rate function

The probability of failure of the system or the device is represented in the period (t, t + △t) provided that the machine remains operating until the time
(t). The function is also called the hazard function, and it is expressed mathematically as follows: [15][16]
$h(t)=\frac{P(t<T\leq t++\triangle t \quad T>T)}{\triangle t}$...(2-17)
And when △t→0 we get the failure rate function, or the so-called risk function (h(t)), as follows:
$h(t)=\lim_{\triangle t\to 0}[\frac{p\{t,t+\triangle t\backslash t\}}{\triangle t}]$
$h(t)=\frac{dF(t)}{dt} . \frac{1}{R(t)} \ldots (2-18)$
$h(t)=\frac{f(t)}{R(t)}$...(2-19)
And because the risk function is the goal of the failure rate when the period approaches zero, the risk function is rather the following formula:
$h(t)=\lim_{\triangle t\to 0}[\frac{R(t)-R(t+\triangle t)}{\triangle t \, R(t)}]$...(2-20)
$h(t)=\frac{1}{R(t)}\left[-\frac{\partial}{\partial t}R(t)\right]$
Likewise, it can be said:
$$h(t)=\begin{cases} \frac{f(t)}{R(t)} \quad , & R(t) > 0 \\ \infty & R(t) = 0 \ldots (2-21) \end{cases}$$ As the hazard rate or the failure rate

13. Reliability of Distribution (shifted Gambertz)

The reliability function for this shifted Gampertz distribution can be found by substitution, rather equation (5-2):
R(t)=1-F(x)

$R(t)=(1-e^{-\beta x})e^{-\alpha e^{-\beta x}} \dots (2-22)$
$= e^{-\beta x-\alpha e^{-\beta x}}$

14. estimation of the parameter for the shifted Gompertz
1. Maximum likelihood method
The best possible method is one of the most common and most important methods for estimating parameters due to its constant and multiple properties. Increase the size of the community.
The estimate can be defined in this way as the parameter values that make the probability function at its maximum, if X_1, X_2,....... X_n are the terms of a random sample of size n taken from a set with a known probability density function, then the function . The probability denoted by the symbol (L) is the density function, and the common probability is:[1][9]
$L(\alpha,\beta)=f(x_1,\alpha,\beta).f(x_2,\alpha,\beta)\dots.f(x_n,\alpha,\beta))$....(2-25)
$L(\alpha,\beta)\prod_{=i=1}^{n} f(x_i,\alpha,\beta) \dots (2-26)$
For the purpose of estimating the possibility function, it must be converted to the linear form by taking the natural logarithm of both sides of the equation:
$L(\alpha,\beta|X_1,X_2,\dots,X_3) = n\ln(\beta) - \beta\sum_{i=1}^{n} x_i - \alpha\sum_{i=1}^{n} e^{-\beta x_i} + \sum_{i=1}^{n} \ln(1+\alpha(1-e^{-\beta x_i})) \dots.(2-27)$
To find the estimator for the parameters α and β, the derivative of the possibility function for the parameters α and β is found and the equation is equal to zero:
$$\frac{\partial L(\alpha,,\beta|X_1,X_2,\dots,X_3)}{\partial\alpha} = -\sum_{i=1}^{n} e^{-\beta x_i} + \sum_{i=1}^{n} \frac{1-e^{-\beta x_i}}{1+\hat{\alpha}(1-e^{-\beta x_i})}$$
$$\frac{\partial L(\alpha,,\beta|X_1,X_2,\dots,X_3)}{\partial\alpha} = 0$$
$$-\sum_{i=1}^{n} e^{-\hat{\beta} x_i} + \sum_{i=1}^{n} \frac{1-e^{-\hat{\beta} x_i}}{1+\hat{\alpha}(1-e^{-\hat{\beta} x_i})} = 0 \dots.(2-28)$$
$$\frac{\partial L(\alpha,,\beta|X_1,X_2,\dots,X_3)}{\partial\beta} = 0$$
$$\frac{\partial \ln L(\alpha,,\beta|X_1,X_2,\dots,X_3)}{\partial\beta} = \frac{n}{\beta} - \sum_{i=1}^{n} x_i(1-\alpha e^{-\beta x_i}) + \alpha\sum_{i=1}^{n} \frac{x_i e^{-\beta x_i}}{1+\alpha(1-e^{-\beta x_i})}$$
Estimates for both parameters and α in equations (28-2) (29-2) cannot be obtained by normal methods because they are non-linear equations, we use Newton-Raphson iterative method to solve the equations.
Since the estimators of the probability function have the property of stability , that is, if α^ and β^ are the possibility estimators of the parameters α and and that h (α, β) is a function in the parameter space Ω, then $h(\hat{\alpha}_{MLE},\hat{\beta}_{MLE})$ are the probability estimators And using this property we get the possibilities of the reliability function, as follows
$\hat{R}(t) \cong e^{-\hat{\beta}t}.e^{-\hat{\alpha}.e^{-\hat{\beta}t}}$

3. WeiGhted lest squane method
It is similar to the ordinary least squares method in that it reduces the sum of squares of error, but it assumes the existence of a weight for each important observation of this weight. Reducing error for each observation. Assuming that ($\hat{F} = \frac{i}{n+i}$) is a non-parametric amount of the cumulative distribution function, the weighted sum of squares of the error will be: [10]
$F(X)=(1-e^{-\hat{\beta}X})e^{-\hat{\alpha}.e^{-\hat{\beta}t}}$
$Q=\sum_{i=1}^{n} w_i\,[(1-e^{-\beta x_i}).e^{-\alpha e^{-\beta x_i}} - \frac{i}{n+i}]^2$
For the purpose of obtaining the value (α,β) that makes the sum of the squares of the weighted error as small as possible, we differentiate Q to α,β and then equate the equation to zero to get the solution:
$\frac{\partial Q}{\partial\alpha} = 2\sum_{i=1}^{n} w_i[(1-e^{-\beta x})e^{-\alpha e^{-\beta x}} - \frac{i}{n+i}][(1-e^{-\beta x_i})e^{-\alpha e^{-\beta x}}.(-e^{-\beta x})]$
$\frac{\partial Q}{\partial\alpha} = 2\sum_{i=1}^{n} w_i[(1-e^{-\beta x})e^{-\alpha e^{-\beta x}} - \frac{i}{n+i}][(1-e^{-\beta x_i})e^{-\alpha e^{-\beta x}}.\alpha e^{-\beta x} + e^{-\alpha e^{-\beta x}}.\ e^{-\beta x}X_i]$
Then we equalize the rates to zero:
$$\sum_{i=1}^{n} w_i[(1-e^{-\beta x})e^{-\alpha e^{-\beta x}} - \frac{i}{n+i}][(1-e^{-\beta x_i})e^{-\alpha e^{-\beta x}}.(-e^{-\beta x}) = 0 \dots (1)$$
$\sum_{i=1}^{n} w_i[(1-e^{-\beta x})e^{-\alpha e^{-\beta x}} - \frac{i}{n+i}][(1-e^{-\beta x_i})e^{-\alpha e^{-\beta x}}.\alpha e^{-\beta x} + e^{-\alpha e^{-\beta x}}.\ e^{-\beta x}X_i] = 0 \dots (2)$

The weight function ($\frac{\alpha}{i^2}$) was chosen as α = 0.90 and because of the fact that Equation (1) and Equation (2) are highly nonlinear, the solution was obtained, i.e. α, β values estimated through numerical analysis methods, Neptune Rafson iterative method.
Assuming that ($\hat{\alpha}_{wls}$, $\hat{\beta}_{wls}$) are the values of the estimators of α and β, respectively, then the estimators of the reliability function are:

$$\hat{R}_{wls} = 1-\left(1-e^{-\hat{\beta}_{wls}\ X}\right).\, e^{-\hat{\alpha}_{wls}\ e^{-\hat{\beta}_{wls}\ X}}$$

The following tables represent the comparison tables between the maximum liklehoodmethod and the weighted least squares method by taking different values for (α) and with constant (β) and then taking different values for (β) with constant(α)

Table 1

Parameter		$\alpha = 0.8$	$\beta = 1$	
N	**t**	**MLE**	**WLS**	**BEST**
10	**0.5**	**0.012910**	**0.014543**	**MLE**
	1.5	**0.008991**	**0.069206**	**MLE**
	2.5	**0.001579**	**0.107308**	**MLE**
	3.5	**0.016087**	**0.061602**	**MLE**
	4.5	**0.010863**	**0.029871**	**MLE**
15	**0.5**	**0.006041**	**0.013645**	**MLE**
	1.5	**0.003176**	**0.005861**	**MLE**
	2.5	**0.001699**	**0.003398**	**MLE**
	3.5	**0.000957**	**0.001949**	**MLE**
	4.5	**0.00057**	**0.000368**	**WLS**
25	**0.5**	**0.000056**	**0.00014**	**MLE**
	1.5	**0.010058**	**0.010777**	**MLE**
	2.5	**0.037041**	**0.045608**	**MLE**
	3.5	**0.048025**	**0.052141**	**MLE**
	4.5	**0.028695**	**0.03308**	**MLE**
50	**0.5**	**0.014804**	**0.01649**	**MLE**
	1.5	**0.006868**	**0.007282**	**MLE**
	2.5	**0.002948**	**0.003003**	**MLE**
	3.5	**0.000324**	**0.001189**	**MLE**
	4.5	**0.000124**	**0.000459**	**MLE**
	0.5	**0.000143**	**0.000174**	**MLE**
	1.5	**0.012910**	**0.054321**	**MLE**
100	**2.5**	**0.012310**	**0.0223134**	**MLE**
	3.5	**0.013980**	**0.0765421**	**MLE**
	4.5	**0.012910**	**0.023098**	**MLE**

It is clear from the above table and by comparing between the least squares method and the maximumlikelihood method using the statistical criterion (MSE) that the percentage of preference for the method of greatest possibility is greater than the method of weighted least squares..

Table 2

Parameter		$\alpha = 1$	$\beta = 1.5$	
N	**t**	**MLE**	**WLS**	**BEST**
10	**0.5**	**0.012320**	**0.015543**	**MLE**
	1.5	**0.006661**	**0.069116**	**MLE**
	2.5	**0.001379**	**0.108308**	**MLE**
	3.5	**0.015487**	**0.062202**	**MLE**
	4.5	**0.010223**	**0.033871**	**MLE**
15	**0.5**	**0.005041**	**0.013445**	**MLE**
	1.5	**0.022176**	**0.001861**	**MLE**
	2.5	**0.001899**	**0.006598**	**MLE**
	3.5	**0.000922**	**0.000449**	**WLS**
	4.5	**0.00111**	**0.000654**	**WLS**
25	**0.5**	**0.000032**	**0.00098**	**MLE**
	1.5	**0.010458**	**0.010875**	**MLE**
	2.5	**0.037431**	**0.054608**	**MLE**
	3.5	**0.041125**	**0.062141**	**MLE**
	4.5	**0.082695**	**0.07608**	**WLS**
50	**0.5**	**0.017604**	**0.01987**	**MLE**
	1.5	**0.006668**	**0.007782**	**MLE**
	2.5	**0.001248**	**0.005003**	**MLE**
	3.5	**0.000224**	**0.001339**	**MLE**
	4.5	**0.000654**	**0.00022**	**WLS**
100	**0.5**	**0.000121**	**0.000156**	**MLE**
	1.5	**0.013310**	**0.089321**	**MLE**
	2.5	**0.022210**	**0.0656134**	**MLE**
	3.5	**0.012777**	**0.0765488**	**MLE**
	4.5	**0.033333**	**0.066098**	**MLE**

The second table also shows that the preference ratio for the(Max mum liklehod), as it is repeated 23 times.

Table 3

Parameter		$\alpha = 1.5$	$\beta = 2$	
N	**t**	**MLE**	**WLS**	**BEST**
10	**0.5**	**0.008991**	**0.069206**	**MLE**
	1.5	**0.01579**	**0.107308**	**MLE**
	2.5	**0.016087**	**0.061602**	**MLE**
	3.5	**0.010863**	**0.029871**	**MLE**
	4.5	**0.006041**	**0.013645**	**MLE**
15	**0.5**	**0.003176**	**0.005861**	**MLE**
	1.5	**0.001699**	**0.002398**	**MLE**
	2.5	**0.000957**	**0.000949**	**MLE**
	3.5	**0.00017**	**0.000368**	**MLE**
	4.5	**0.000356**	**0.00014**	**WLS**

25	0.5	0.010058	0.010777	MLE
	1.5	0.017041	0.045608	MLE
	2.5	0.048025	0.052141	MLE
	3.5	0.028695	0.03308	MLE
	4.5	0.014804	0.01649	MLE
50	0.5	0.006868	0.007282	MLE
	1.5	0.002948	0.003003	MLE
	2.5	0.001128	0.001189	MLE
	3.5	0.000269	0.000459	MLE
	4.5	0.000178	0.000194	MLE
100	0.5	0.018858	0.012777	MLE
	1.5	0.013210	0.076321	MLE
	2.5	0.011110	0.0873134	MLE
	3.5	0.021180	0.0654421	MLE
	4.5	0.012110	0.043098	MLE

Table 4

Parameter		$\alpha = 1$	$\beta = 0.5$	
N	**t**	**MLE**	**WLS**	**BEST**
10	0.5	0.005082	0.012399	MLE
	1.5	0.006808	0.034992	MLE
	2.5	0.006815	0.029727	MLE
	3.5	0.005431	0.017377	MLE
	4.5	0.003524	0.009305	MLE
15	0.5	0.001996	0.004603	MLE
	1.5	0.001041	0.002163	MLE
	2.5	0.000517	0.000998	MLE
	3.5	0.00025	0.000462	MLE
	4.5	0.000119	0.000218	MLE
25	0.5	0.001074	0.018544	MLE
	1.5	0.05213	0.095407	MLE
	2.5	0.031225	0.033568	MLE
	3.5	0.02102	0.028868	MLE
	4.5	0.010886	0.016675	MLE
50	0.5	0.005373	0.007873	MLE
	1.5	0.002545	0.003335	MLE
	2.5	0.00115	0.001327	MLE
	3.5	0.000494	0.000509	MLE
	4.5	0.000103	0.000191	MLE
	0.5	0.000223	0.000344	MLE
	1.5	0.011910	0.02221	MLE

100	2.5	0.014310	0.0276134	MLE
	3.5	0.002980	0.1164421	MLE
	4.5	0.034910	0.088098	MLE

Table 5

Parameter		$\alpha = 1.5$	$\beta = 0.5$	
N	**t**	**MLE**	**WLS**	**BEST**
10	0.5	0.562446	0.584449	MLE
	1.5	0.378496	0.394024	MLE
	2.5	0.244782	0.285574	MLE
	3.5	0.154423	0.184977	MLE
	4.5	0.015937	0.03899	MLE
15	0.5	0.059045	0.066291	MLE
	1.5	0.036132	0.08842	MLE
	2.5	0.022034	0.08758	MLE
	3.5	0.013408	0.018957	MLE
	4.5	0.562446	0.594449	MLE
25	0.5	0.776609	0.789721	MLE
	1.5	0.40041	0.401023	MLE
	2.5	0.088333	0.198952	MLE
	3.5	0.060168	0.089298	MLE
	4.5	0.010839	0.035517	MLE
50	0.5	0.01073	0.011672	MLE
	1.5	0.0000084	0.002237	MLE
	2.5	0.0021031	0.010889	MLE
	3.5	0.000691	0.001524	MLE
	4.5	0.000069	0.001324	MLE
100	0.5	0.000043	0.000324	MLE
	1.5	0.002910	0.066321	MLE
	2.5	0.022910	0.033134	MLE
	3.5	0.000080	0.6565421	MLE
	4.5	0.034910	0.098098	MLE

Table 6

Parameter		$\alpha = 2$	$\beta = 0.5$	
N	**t**	**MLE**	**WLS**	**BEST**
10	0.5	0.00272	0.003343	MLE
	1.5	0.006497	0.016116	MLE
	2.5	0.008475	0.024717	MLE
	3.5	0.006452	0.017574	MLE
	4.5	0.003782	0.009328	MLE

15	**0.5**	**0.001985**	**0.004363**	**MLE**
	1.5	**0.001009**	**0.001932**	**MLE**
	2.5	**0.000518**	**0.000839**	**MLE**
	3.5	**0.000275**	**0.000364**	**MLE**
	4.5	**0.000151**	**0.000149**	**WLS**
25	**0.5**	**0.010544**	**0.013019**	**MLE**
	1.5	**0.027727**	**0.032625**	**MLE**
	2.5	**0.022763**	**0.024829**	**MLE**
	3.5	**0.010145**	**0.011306**	**MLE**
	4.5	**0.001382**	**0.004593**	**MLE**
50	**0.5**	**0.001946**	**0.002186**	**MLE**
	1.5	**0.002449**	**0.003316**	**MLE**
	2.5	**0.000079**	**0.000761**	**MLE**
	3.5	**0.000192**	**0.000394**	**MLE**
	4.5	**0.000254**	**0.000886**	**MLE**
100	**0.5**	**0.000113**	**0.000188**	**MLE**
	1.5	**0.013210**	**0.087321**	**MLE**
	2.5	**0.014400**	**0.0267634**	**MLE**
	3.5	**0.012666**	**0.054321**	**MLE**
	4.5	**0.872910**	**0.893098**	**MLE**

In the previous tables, it was also shown that the method (maximumlikelihood) has the advantage in comparison using the statistical standard(MSE).

15. CONCLUSION

by comparing the method of Max mum liklehood(MLE) and the method of weighted least squares (WLS) using the statistical standard (MSE), we find that the method with the greatest probability achieved priority with (140) times the total sum of values and the amount of (150) units, that is, the greatest probability Possible method achieved the percentage of preference for each of the previous tables. It is also noted that the percentage of preference is achieved by (100%) for the tables when the value of (α) with the coefficient (β)) held constant.

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