Dynamical analysis of fractional yellow fever virus model with efficient numerical approach

Chandrali Baishya[1,*], Sindhu J. Achar[1], P. Veeresha[2], Devendra Kumar[3]

December 28, 2022

[1]Department of Studies and Research in Mathematics, Tumkur University, Tumkur-572103, Karnataka, India
Email: baishyachandrali@gmail.com, sindhuachar6@gmail.com
[2] Department of Mathematics, CHRIST (Deemed to be University), Bengaluru-560029, India
Email: pveeresha.maths@gmail.com & pundikala.veeresha@christuniversity.in
[3] Department of Mathematics, University of Rajasthan, Jaipur-302004, Rajasthan, India
Email: devendra.maths@gmail.com

Abstract

In this paper, we have projected the theoretical and numerical investigation of the mathematical model representing the yellow fever virus transmission from infected mosquitoes to humans or vise-versa through mosquito bites in the framework of the Caputo derivative. Theoretical aspects of the dynamics of susceptible individuals, exposed individuals, infected individuals, toxic infected individuals, recovered and immune individuals, and susceptible mosquitoes and infected mosquitoes have been analyzed by using the theory of fractional calculus such as boundedness, uniqueness and existence of the solutions. Sufficient conditions for the global stability of the virus-free point of equilibrium are inspected. T validate the theoretical results numerical analysis is performed using the generalized Adams-Bashforth-Moultan method.

Keywords: Predator-Prey, Yellow Fever Virus, Predictor-Corrector Method.
2000 Mathematics Subject Classification: 34A34, 92D30.

---

* Corresponding Author

1 Introduction

Infectious illness outbreaks have become the greatest threat to mankind over time, resulting in the loss of many lives. They may also bring economic and political upheaval if they are not handled properly. Yellow fever (YF) virus which belongs to a family of about 70 viruses was the first human virus discovered. YF is an intense viral disease spread by infected female 'Aedes aegypti' mosquitoes. These mosquitoes are also the vector of Zika virus, dengue and chikungunya [1, 2]. In Africa, sylvatic and peridomestic Aedes species transmit rural and intermediate YF. The incubation period of the virus on the infected individuals is generally 3 to 6 days [3]. Vomiting, nausea, lack of appetite, muscle pain with backache, slight fever, headache, jaundice, and weariness are some of the symptoms that patients experience [4]. Nonetheless, these symptoms fade after four or five days, while other individuals may continue to the infection's toxicity phase, in which 50 percent of instances lead to death within eight to ten days [5]. Although YF is compartmentalized as viral hemorrhagic fever, it causes 1000 times more risk to death than the virus like Ebola [1].

Among the scientific community, the study of disease dynamics has remained a popular issue [6, 7]. To help humankind in fighting against YF by understanding its dynamics mathematically, a few mathematicians have contributed their expertise in modeling this infection [8, 9]. In the recent times, fractional derivatives namely the Caputo, Riemann-Liouville, Gr*ü*nwald Letnikov, Jumarie, and Caputo-Fabrizio are investigated by the researchers in search of new behavioral findings while representing real world problems using such derivatives. Notably, many results associated with memory, hereditary, longrange memory, random walk, anomalous diffusion, non-Markovian processes, and others made the concept of fractional derivatives a highly significant to take into account [10–14]. Over the years, theories of these derivatives have also been developed to a great extent [15–18]. Many phenomena related to mathematical biology and their interdisciplinary fields [18, 19] have been studied using these fractional derivatives [20, 21]. They have been used to model many complex phenomena of disease dynamics [22–24].

This work aims to examine the qualitative nature of the yellow fever virus mathematical model with interaction of seven categories of the population namely susceptible , YF exposed, YF infected, toxic-infected individuals, recovered and immune individuals, susceptible mosquitoes, and infected mosquitoes incorporating the Caputo fractional derivative. Adams-Bashforth-Moulton method has been used to perform the numerical simulation [25–29]. The rest of the paper is structured as follows: in Section 2, we provide some elementary definitions, theorems and lemmas of fractional calculus which is followed by the formulation of the model in Section 3. Sections 4, 5, 6

dispense the existence and uniqueness, boundedness, the existence of various points of equilibrium and their local stability respectively. Sections 7 depicts the numerical method and simulation in detail. Finally, we discuss the concluding remarks in Section 8.

2 Some Essential Theorems

In the present work, we have used the Caputo fractional derivatives because it supports the integer order initial condition. In this section, we have presented certain theorems those have been applied to determine the theoretical results corresponding to the solution of the projected model. The Caputo fractional derivative is denoted by ${}^{C}D$.

Definition 2.1. [15] (Caputo Fractional Derivative) Suppose $g(\mathrm{t})$ is k times continuously differentiable function and $g^{(k)}(\mathrm{t})$ is integrable in $[\mathrm{t}_0, T]$. The fractional derivative of the order α established by Caputo sense for $g(\mathrm{t})$,is

$$ {}^{C}_{\mathrm{t}_0}D^{\alpha}_{\mathrm{t}}\, g(\mathrm{t}) = \frac{1}{\Gamma(k-\alpha)} \int_{\mathrm{t}_0}^{\mathrm{t}} \frac{g^{(k)}(\tau)}{(\mathrm{t}-\tau)^{\alpha+1-k}} d\tau $$

where $\Gamma(\cdot)$ refers to Gamma function, $\mathrm{t} > a$ and k is a positive integer with the property that $k-1 < \alpha < k$.

Lemma 1. [17] Consider the system

$$ {}^{C}_{\mathrm{t}_0}D^{\alpha}_{\mathrm{t}}\, v(\mathrm{t}) = g(\mathrm{t}, v),\ \mathrm{t} > \mathrm{t}_0, \tag{1} $$

choosing the initial condition as $v(\mathrm{t}_0)$, where $0 < \alpha \leq 1$ and $g : [\mathrm{t}_0, \infty) \times \Omega \to \mathbb{R}^n, \Omega \in \mathbb{R}^n$. When $g(\mathrm{t}, v)$ holds the locally Lipchitz conditions concerning to v, Eq.1 has a unique solution on $[\mathrm{t}_0, \infty) \times \Omega$.

Lemma 2. [18] We assume that $g(\mathrm{t})$ is a continuous function on $[\mathrm{t}_0, +\infty)$ satisfying

$$ {}^{C}_{\mathrm{t}_0}D^{\alpha}_{\mathrm{t}}\, g(\mathrm{t}) \leq -\lambda g(\mathrm{t}) + \xi,\ g(\mathrm{t}_0) = g_0, $$

where $\mathrm{t}_0 \geq 0$ is the initial time, $0 < \alpha \leq 1,\ \lambda \neq 0,\ \ (\lambda, \xi) \in \mathbb{R}^2$. Then,

$$ g(\mathrm{t}) \leq (g(\mathrm{t}_0) - \frac{\xi}{\lambda}) E_{\alpha}[-\lambda(\mathrm{t}-\mathrm{t}_0)^{\alpha}] + \frac{\xi}{\lambda} $$

Lemma 3. [18] Let $v(\mathrm{t}) \in \mathbb{R}_+$ be a derivable and continuous function. Then, at any time $\mathrm{t} > \mathrm{t}_0$,

$$ {}^{C}_{\mathrm{t}_0}D^{\alpha}_{\mathrm{t}}(v(\mathrm{t}) - v^* - v^* ln\frac{v(\mathrm{t})}{v^*}) \leq (1 - \frac{v^*}{v}) {}^{C}_{\mathrm{t}_0}D^{\alpha}_{\mathrm{t}}\, v(t), \quad v^* \in \mathbb{R}_+, \quad \forall \alpha \in (0,1). $$

3 Model Formulation

Yusuf and Daniel's [9] work has inspired the mathematical model described in this study. We observe that fractional derivatives influence coexistence. When a new virus emerges, it is never completely eradicated from the world. A fraction of humans will always be infected by that virus in some part of the universe. This nature of the virus's existence prompts us to model YF infection by incorporating fractional derivatives. In this paper, the YF virus mathematical model has been proposed within the population of humans and mosquitoes. It has been assumed that both populations mix freely without any barriers. Since there is a high risk of YF transmission from travelers, their vaccination is essential. It is assumed that some portion of the travelers is vaccinated. Again, a part of the infected population may become toxic. It is also assumed that once an individual becomes toxic, he or she does not recover. In the present model, the human population has been subdivided into five different compartments namely: $S_H(t), E_H(t), I_H(t), T_H(t), R_H(t)$ which represents the density of susceptible, YF exposed, YF infected, toxic-infected individuals, and recovered and immune individuals respectively. In the same way, the mosquito population is divided into two categories: $S_V(t), I_V(t)$ represent the density of susceptible mosquitoes and infected mosquitoes respectively. We have considered the Caputo sense fractional derivative to represent the projected model. The present model is as follows:

$$
\begin{aligned}
{}^{C}_{t_0}D^{\alpha}_{t} S_H &= r + (1-\sigma)\lambda - \theta\beta_1 S_H I_V - \rho S_H - d S_H, \\
{}^{C}_{t_0}D^{\alpha}_{t} E_H &= \theta\beta_1 S_H I_V - \mu E_H - d E_H, \\
{}^{C}_{t_0}D^{\alpha}_{t} I_H &= (1-\xi)\mu E_H - \psi I_H - (d+\Lambda) I_H, \\
{}^{C}_{t_0}D^{\alpha}_{t} T_H &= \xi\mu E_H - (d+\Lambda) T_H \\
{}^{C}_{t_0}D^{\alpha}_{t} R_H &= \psi I_H + \sigma\lambda + \rho S_H - d R_H, \\
{}^{C}_{t_0}D^{\alpha}_{t} S_V &= \nu - \theta\beta_2 S_V I_H - \phi S_V, \\
{}^{C}_{t_0}D^{\alpha}_{t} I_V &= \theta\beta_2 S_V I_H - \phi I_V.
\end{aligned}
\tag{2}
$$

with initial condition $S_H(t_0) = S_H(0)$, $E_H(t_0) = E_H(0)$, $I_H(t_0) = I_H(0)$, $T_H(t_0) = T_H(0)$, $R_H(t_0) = R_H(0)$, where t_0 is the initial time. All the parameters $r, \sigma, \lambda, \theta, \beta_1, \rho, d, \mu, \xi$, $\psi, \Lambda, \nu, \beta_2, \phi$ are non-negative. Where, r is the birth rate of human, σ is the vaccinated proportion of immigrants, λ is the arrival rate of immigrants per individual per time, and θ is the daily biting rate. β_1 and β_2 represent the transmission probability of YF from mosquitoes to human and from human to mosquitoes respectively. ρ is the effective vaccination rate of susceptible humans, d is the natural death rate of human, μ is the rate at which E_H progresses to I_H, ξ is the proportion of E_H which converts to the toxic case, ψ is the recovery rate of human, Λ is the death rate of human-induced due

to YF, ν is the birth rate of mosquitoes, and ϕ is the natural death rate of the vectors.

4 Existence of the solutions

The existence of the solution of the model 2 is demonstrated using the Fixed-Point Theorem. Due to the complex and non-local nature of the system 2, there are no precise algorithms or approaches for evaluating the exact solutions. However, the existence of the solution is assured if certain conditions are met. To initiate the process of establishing the existence of the solution, the system 2 is rewritten as:

$$\begin{aligned}
&{}^{C}_{t_0}D^{\alpha}_{t}[S_H(t)] = \mathfrak{P}_1(t,S_H),\ {}^{C}_{t_0}D^{\alpha}_{t}[E_H(t)] = \mathfrak{P}_2(t,E_H),\ {}^{C}_{t_0}D^{\alpha}_{t}[I_H(t)] = \mathfrak{P}_3(t,I_H),\\
&{}^{C}_{t_0}D^{\alpha}_{t}[T_H(t)] = \mathfrak{P}_4(t,T_H),\ {}^{C}_{t_0}D^{\alpha}_{t}[R_H(t)] = \mathfrak{P}_5(t,R_H),\ {}^{C}_{t_0}D^{\alpha}_{t}[S_V(t)] = \mathfrak{P}_6(t,S_V),\\
&{}^{C}_{t_0}D^{\alpha}_{t}[I_V(t)] = \mathfrak{P}_7(t,I_V).
\end{aligned} \tag{3}$$

The above system can be transformed into Volterra type integral equation as:

$$\begin{aligned}
S_H(t) - S_H(0) &= \frac{1}{\Gamma(\alpha)}\int_0^t \mathfrak{P}_1(\tau,S_H(\tau))(t-\tau)^{\alpha-1}d\tau,\\
E_H(t) - E_H(0) &= \frac{1}{\Gamma(\alpha)}\int_0^t \mathfrak{P}_2(\tau,E_H(\tau))(t-\tau)^{\alpha-1}d\tau,\\
I_H(t) - I_H(0) &= \frac{1}{\Gamma(\alpha)}\int_0^t \mathfrak{P}_3(\tau,I_H(\tau))(t-\tau)^{\alpha-1}d\tau,\\
T_H(t) - T_H(0) &= \frac{1}{\Gamma(\alpha)}\int_0^t \mathfrak{P}_4(\tau,T_H(\tau))(t-\tau)^{\alpha-1}d\tau,\\
R_H(t) - R_H(0) &= \frac{1}{\Gamma(\alpha)}\int_0^t \mathfrak{P}_5(\tau,R_H(\tau))(t-\tau)^{\alpha-1}d\tau,\\
S_V(t) - S_V(0) &= \frac{1}{\Gamma(\alpha)}\int_0^t \mathfrak{P}_6(\tau,S_V(\tau))(t-\tau)^{\alpha-1}d\tau,\\
I_V(t) - I_V(0) &= \frac{1}{\Gamma(\alpha)}\int_0^t \mathfrak{P}_7(\tau,I_V(\tau))(t-\tau)^{\alpha-1}d\tau.
\end{aligned} \tag{4}$$

Theorem 4.1. In the region $\Omega \times [t_0, T]$, where

$$\Omega = \{(S_H, E_H, I_H, T_H, R_H, S_V, I_V) \in \mathbb{R}^7 : max\{|S_H|, |E_H|, |I_H|, |T_H|, |R_H|, |S_V|, |I_V|\} \leq \mathfrak{M}\},$$

and $T < +\infty$, the Lipschitz condition is satisfied and contraction occurs by the kernel $\mathfrak{P}_1$ if $0 \leq \theta\beta_1\mathfrak{M} + \rho + d < 1$.

Proof: We consider the two functions S_H and $\bar{S}_H$ such as:

$$||\mathfrak{P}_1(t,S_H) - \mathfrak{P}_1(t,\bar{S}_H)|| = ||(r + (1-\sigma)\lambda - \theta\beta_1 S_H I_V - \rho S_H - dS_H)$$

$$
\begin{aligned}
&-(r+(1-\sigma)\lambda-\theta\beta_1\bar{S}_H I_V-\rho\bar{S}_H-d\bar{S}_H)||.\\
&\leq(\theta\beta_1\mathfrak{M}+\rho+d)||S_H(t)-\bar{S}_H(t)||\\
&=\zeta_1||S_H(t)-\bar{S}_H(t)||, \qquad (5)
\end{aligned}
$$

where $\zeta_1=\theta\beta_1\mathfrak{M}+\rho+d$. As a result, the Lipschitz condition is met for $\mathfrak{P}_1$ and if $0\leq\zeta_1<1$, then $\mathfrak{P}_1$ follows contraction. Similarly, it can be shown and illustrated in case of the other equations as follows:

$$
\begin{aligned}
&||\mathfrak{P}_2(t,E_H)-\mathfrak{P}_2(t,\bar{E}_H)||\leq\zeta_2||E_H(t)-\bar{E}_H(t)||,\ ||\mathfrak{P}_3(t,I_H)-\mathfrak{P}_3(t,\bar{I}_H)||\leq\zeta_3||I_H(t)-\bar{I}_H(t)||,\\
&||\mathfrak{P}_4(t,T_H)-\mathfrak{P}_4(t,\bar{T}_H)||\leq\zeta_4||T_H(t)-\bar{T}_H(t)||,\ ||\mathfrak{P}_5(t,R_H)-\mathfrak{P}_5(t,\bar{R}_H)||\leq\zeta_5||R_H(t)-\bar{R}_H(t)||,\\
&||\mathfrak{P}_6(t,S_V)-\mathfrak{P}_6(t,\bar{S}_V)||\leq\zeta_6||S_V(t)-\bar{S}_V(t)||,\ ||\mathfrak{P}_7(t,I_V)-\mathfrak{P}_7(t,\bar{I}_V)||\leq\zeta_7||I_V(t)-\bar{I}_V(t)||, \qquad (6)
\end{aligned}
$$

where $\zeta_2=(\mu+d)$, $\zeta_3=(\psi+d+\Lambda)$, $\zeta_4=(d+\Lambda)$, $\zeta_5=(\psi+d+\Lambda)$, $\zeta_6=(\theta\beta_2\mathfrak{M}+\phi)$ and $\zeta_7=\phi$. $\mathfrak{P}_i$, $i=2,3,4,5,6,7$ are the contraction if $0<\zeta_i<1$, $i=2,3,4,5,6,7$. Using system 4, the recursive form can now be written as follows:

$$
\begin{aligned}
\kappa_{1,n}(t)&=S_{H_n}(t)-S_{H_{n-1}}(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(\mathfrak{P}_1(\tau,S_{H_{n-1}})-\mathfrak{P}_1(\tau,S_{H_{n-2}}))(t-\tau)^{\alpha-1}d\tau,\\
\kappa_{2,n}(t)&=E_{H_n}(t)-E_{H_{n-1}}(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(\mathfrak{P}_2(\tau,E_{H_{n-1}})-\mathfrak{P}_2(\tau,E_{H_{n-2}}))(t-\tau)^{\alpha-1}d\tau,\\
\kappa_{3,n}(t)&=I_{H_n}(t)-I_{H_{n-1}}(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(\mathfrak{P}_3(\tau,I_{H_{n-1}})-\mathfrak{P}_3(\tau,I_{H_{n-2}}))(t-\tau)^{\alpha-1}d\tau,\\
\kappa_{4,n}(t)&=T_{H_n}(t)-T_{H_{n-1}}(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(\mathfrak{P}_4(\tau,T_{H_{n-1}})-\mathfrak{P}_4(\tau,T_{H_{n-2}}))(t-\tau)^{\alpha-1}d\tau,\\
\kappa_{5,n}(t)&=R_{H_n}(t)-R_{H_{n-1}}(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(\mathfrak{P}_5(\tau,R_{H_{n-1}})-\mathfrak{P}_5(\tau,R_{H_{n-2}}))(t-\tau)^{\alpha-1}d\tau,\\
\kappa_{6,n}(t)&=S_{V_n}(t)-S_{V_{n-1}}(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(\mathfrak{P}_6(\tau,S_{V_{n-1}})-\mathfrak{P}_6(\tau,S_{V_{n-2}}))(t-\tau)^{\alpha-1}d\tau,\\
\kappa_{7,n}(t)&=I_{V_n}(t)-I_{V_{n-1}}(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(\mathfrak{P}_7(\tau,I_{V_{n-1}})-\mathfrak{P}_7(\tau,I_{V_{n-2}}))(t-\tau)^{\alpha-1}d\tau. \qquad (7)
\end{aligned}
$$

The prerequisites are: $S_{H_0}(t)=S_H(0)$, $E_{H_0}(t)=E_H(0)$, $I_{H_0}(t)=I_H(0)$, $T_{H_0}(t)=T_H(0)$, $R_{H_0}(t)=R_H(0)$, $S_{V_0}(t)=S_V(0)$, $I_{V_0}(t)=I_V(0)$.
By applying the norm to the first equation of the system 7, we obtained

$$
\begin{aligned}
||\kappa_{1,n}(t)||&=||S_{H_n}(t)-S_{H_{n-1}}(t)||\\
&=||\frac{1}{\Gamma(\alpha)}\int_0^t(\mathfrak{P}_1(\tau,S_{H_{n-1}})-\mathfrak{P}_1(\tau,S_{H_{n-2}}))(t-\tau)^{\alpha-1}d\tau||\\
&\leq\frac{1}{\Gamma(\alpha)}\int_0^t||(\mathfrak{P}_1(\tau,S_{H_{n-1}})-\mathfrak{P}_1(\tau,S_{H_{n-2}}))(t-\tau)^{\alpha-1}d\tau||. \qquad (8)
\end{aligned}
$$

Using Lipchitz condition 5, we get

$$||\kappa_{1,n}(t)|| \leq \frac{1}{\Gamma(\alpha)}\zeta_1 \int_0^t ||\kappa_{1,n-1}(\tau)d\tau||. \tag{9}$$

Similarly,

$$\begin{aligned}
&||\kappa_{2,n}(t)|| \leq \frac{1}{\Gamma(\alpha)}\zeta_2 \int_0^t ||\kappa_{2,n-1}(\tau)d\tau||,\ ||\kappa_{3,n}(t)|| \leq \frac{1}{\Gamma(\alpha)}\zeta_3 \int_0^t ||\kappa_{3,n-1}(\tau)d\tau||,\\
&||\kappa_{4,n}(t)|| \leq \frac{1}{\Gamma(\alpha)}\zeta_4 \int_0^t ||\kappa_{4,n-1}(\tau)d\tau||,\ ||\kappa_{5,n}(t)|| \leq \frac{1}{\Gamma(\alpha)}\zeta_5 \int_0^t ||\kappa_{5,n-1}(\tau)d\tau||,\\
&||\kappa_{6,n}(t)|| \leq \frac{1}{\Gamma(\alpha)}\zeta_6 \int_0^t ||\kappa_{6,n-1}(\tau)d\tau||,\ ||\kappa_{7,n}(t)|| \leq \frac{1}{\Gamma(\alpha)}\zeta_7 \int_0^t ||\kappa_{7,n-1}(\tau)d\tau||.
\end{aligned} \tag{10}$$

As a result, it yields
$S_{H_n}(t) = \sum_{i=1}^n \kappa_{1,i}, \quad E_{H_n}(t) = \sum_{i=1}^n \kappa_{2,i}, \quad I_{H_n}(t) = \sum_{i=1}^n \kappa_{3,i}, \quad T_{H_n}(t) = \sum_{i=1}^n \kappa_{4,i},$
$R_{H_n}(t) = \sum_{i=1}^n \kappa_{5,i}, \quad S_{V_n}(t) = \sum_{i=1}^n \kappa_{6,i}, \quad I_{V_n}(t) = \sum_{i=1}^n \kappa_{7,i}.$
This theorem will be used to illustrate the next theorem.

Theorem 4.2. The solution of the fractional model 2 exists and will be unique, if we acquire some t_α such that

$$\frac{1}{\Gamma(\alpha)}\zeta_i t_\alpha < 1,\ \ i = 1,2,3,...,7.$$

Proof: Applying equations 9 and 10 recursively, we have

$$\begin{aligned}
&||\kappa_{1,n}(t)|| \leq ||S_{H_n}(0)||\left[\frac{1}{\Gamma(\alpha)}\zeta_1 t\right]^n,\ ||\kappa_{2,n}(t)|| \leq ||E_{H_n}(0)||\left[\frac{1}{\Gamma(\alpha)}\zeta_2 t\right]^n,\\
&||\kappa_{3,n}(t)|| \leq ||I_{H_n}(0)||\left[\frac{1}{\Gamma(\alpha)}\zeta_3 t\right]^n,\ ||\kappa_{4,n}(t)|| \leq ||T_{H_n}(0)||\left[\frac{1}{\Gamma(\alpha)}\zeta_4 t\right]^n,\\
&||\kappa_{5,n}(t)|| \leq ||R_{H_n}(0)||\left[\frac{1}{\Gamma(\alpha)}\zeta_5 t\right]^n,\ ||\kappa_{6,n}(t)|| \leq ||S_{V_n}(0)||\left[\frac{1}{\Gamma(\alpha)}\zeta_6 t\right]^n,\\
&||\kappa_{7,n}(t)|| \leq ||I_{V_n}(0)||\left[\frac{1}{\Gamma(\alpha)}\zeta_7 t\right]^n.
\end{aligned} \tag{11}$$

As a result, the existence and continuity are established. To illustrate that the above relations formulate the solution of the model 2, we assume the following:

$$S_H(t) - S_{H_0}(t) = S_{H_n}(t) - \Delta_{1n}(t),\ E_H(t) - E_{H_0}(t) = E_{H_n}(t) - \Delta_{2n}(t), \tag{12}$$
$$I_H(t) - I_{H_0}(t) = I_{H_n}(t) - \Delta_{3n}(t),\ T_H(t) - T_{H_0}(t) = T_{H_n}(t) - \Delta_{4n}(t), \tag{13}$$
$$R_H(t) - R_{H_0}(t) = R_{H_n}(t) - \Delta_{5n}(t),\ S_V(t) - S_{V_0}(t) = S_{V_n}(t) - \Delta_{6n}(t), \tag{14}$$

$$I_V(t) - I_{V_0}(t) = I_{V_n}(t) - \Delta_{7n}(t). \tag{15}$$

In order to achieve the desired outcomes, set that

$$||\Delta_{1n}(t)|| = ||\frac{1}{\Gamma(\alpha)} \int_0^t (\mathfrak{P}_1(\tau, S_H) - \mathfrak{P}_1(\tau, S_{H_{n-1}}))d\tau|| \tag{16}$$

This implies,

$$||\Delta_{1n}(t)|| \leq \frac{1}{\Gamma(\alpha)} \zeta_1 ||S_H - S_{H_{n-1}}||t. \tag{17}$$

Continuing the same procedure recursively, we get

$$||\Delta_{1n}(t)|| \leq \left(\frac{1}{\Gamma(\alpha)} \zeta_1 t\right)^{n+1} \mathfrak{M}. \tag{18}$$

At certain t_α, we have

$$||\Delta_{1n}(t)|| \leq \left(\frac{1}{\Gamma(\alpha)} \zeta_1 t_\alpha\right)^{n+1} \mathfrak{M}. \tag{19}$$

From equation [**?**], we observe that $||\Delta_{1n}(t)||$ approaches to 0 as n tends to ∞, provided $\left(\frac{1}{\Gamma(\alpha)} \zeta_1 t_\alpha\right) < 1$. Similarly, it may be demonstrated that $||\Delta_{2n}(t)||$, $||\Delta_{3n}(t)||$, $||\Delta_{4n}(t)||$, $||\Delta_{5n}(t)||$, $||\Delta_{6n}(t)||$, $||\Delta_{7n}(t)||$ tends to 0. Hence the proof. We shall now demonstrate the uniqueness for the solution of the system 2. Let us assume that there is a different set of solutions, namely $\hat{S}_H, \hat{E}H, \hat{I}_H, \hat{T}_H, \hat{R}_H, \hat{S}_V, \hat{I}_V$ for the system 2. Then, as a result of the first equation, we have

$$S_H(t) - \hat{S}_H(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (\mathfrak{P}_1(t, S_H) - \mathfrak{P}_1(t, \hat{S}_H))d\tau. \tag{20}$$

Using the norm, the equation above becomes:

$$||S_H(t) - \hat{S}_H(t)|| = \frac{1}{\Gamma(\alpha)} \int_0^t ||(\mathfrak{P}_1(t, S_H) - \mathfrak{P}_1(t, \hat{S}_H))d\tau||. \tag{21}$$

By applying the Lipschitz condition,

$$||S_H(t) - \hat{S}_H(t)|| \leq \frac{1}{\Gamma(\alpha)} \zeta t ||S_H - \hat{S}_H||.$$

This results in,

$$||S_H(t) - \hat{S}_H(t)|| \left(1 - \frac{(1-\alpha)}{\Gamma(\alpha)} \zeta t\right) \leq 0.$$

Since $\left(1 - \frac{1}{\Gamma(\alpha)} \zeta_1 t\right) > 0$, we much have $||S_H(t) - \hat{S}_H(t)|| = 0$. This implies $S_H(t) = \hat{S}_H(t)$.

5 Boundedness

In this Section, we have established the boundedness of the solution of the system 2.

Theorem 5.1. The solution of the system 2 is uniformly bounded.

Proof. Considering the function, $\mathfrak{L}(t) = S_H(t)+E_H(t)+I_H(t)+T_H(t)+R_H(t)+S_V(t)+I_V(t)$.
and applying fractional derivative on it, we get

$$\begin{aligned} {}^{C}_{t_0}D^{\alpha}_{t}\mathfrak{L}(t)+d\mathfrak{L}(t) &= {}^{C}_{t_0}D^{\alpha}_{t}[S_H(t)+E_H(t)+I_H(t)+T_H(t)+R_H(t)+S_V(t)+I_V(t)] \\ &\quad +d[S_H(t)+E_H(t)+I_H(t)+T_H(t)+R_H(t)+S_V(t)+I_V(t)] \\ &= r+\lambda-\Lambda(I_H+T_H)+\nu-\phi(S_V+I_V)+d(S_V+I_V) \\ &\leq r+\lambda+\nu+dS_V+dI_V. \end{aligned} \tag{22}$$

The solution exists and is unique in
$\mho = \{(S_H,E_H,I_H,T_H,R_H,S_V,I_V)/max\{|S_H|,|E_H|,|I_H|,|T_H|,|R_H|,|S_V|,|I_V|\} \leq \mathfrak{M}\}$.
The above inequality yields,

$${}^{C}_{t_0}D^{\alpha}_{t}\mathfrak{L}(t)+d\mathfrak{L}(t) \leq r+\lambda+\nu+2d\mathfrak{M}.$$

By the Lemma 2, we get

$${}^{C}_{t_0}D^{\alpha}_{t}\mathfrak{L}(t) \leq (\mathfrak{L}(t_0)-\frac{1}{d}(r+\lambda+\nu+2d\mathfrak{M})E_{\alpha}[-\eta(t-t_0)^{\alpha}]+\frac{1}{d}(r+\lambda+\nu+2d\mathfrak{M}) \rightarrow r+\lambda+\nu+2d\mathfrak{M}$$

as $t \rightarrow \infty$. Therefore, all the solution of the system 2 that initiates in $\mho$ remained bounded in
$\Theta = \{(S_H,E_H,I_H,T_H,R_H,S_V,I_V) \in \mho_+ | \mathfrak{L}(t) \leq r+\lambda+\nu+2d\mathfrak{M}+\varepsilon, \quad \varepsilon > 0\}$. □

6 Existence of points of equilibrium

In this section, we find the points of equilibrium of the system 2. We have the following points of equilibrium for the fractional order system 2:

1. The disease-free equilibrium point is $\bar{\mathfrak{I}} = (\frac{r+\lambda(1-\sigma)}{d+\rho},0,0,0,\frac{\rho(r+\lambda)+d\lambda\sigma}{d^2+d\rho},\frac{\nu}{\phi},0)$ and it always exists.

2. The endemic equilibrium point $\tilde{\mathfrak{I}} = (\tilde{S_H},\tilde{E_H},\tilde{I_H},\tilde{T_H},\tilde{R_H},\tilde{S_V},\tilde{I_V})$ exists if $\nu\theta^2\mu\beta_1\beta_2(1-\xi)(r+\lambda(1-\sigma)) > \phi^2(d+\mu)(d+\rho)(d+\Lambda+\psi)$. Coexistence equilibrium point can be obtained by solving the algebraic equations given below:

$$r+(1-\sigma)\lambda-\theta\beta_1\tilde{S_H}\tilde{I_V}-\rho\tilde{S_H}-d\tilde{S_H}=0,\ \theta\beta_1\tilde{S_H}\tilde{I_V}-\mu\tilde{E_H}-d\tilde{E_H}=0,$$

$$(1-\xi)\mu\tilde{E_H} - \psi\tilde{I_H} - (d+\Lambda)\tilde{I_H} = 0,\ \xi\mu\tilde{E_H} - (d+\Lambda)\tilde{T_H} = 0,$$
$$\psi\tilde{I_H} + \sigma\lambda + \rho\tilde{S_H} - d\tilde{R_H} = 0,\ \nu - \theta\beta_2\tilde{S_V}\tilde{I_H} - \phi\tilde{S_V} = 0,$$
$$\theta\beta_2\tilde{S_V}\tilde{I_H} - \phi\tilde{I_V} = 0.$$

Solving these equations we obtain,

$$\tilde{S_H} = \frac{r+\lambda(1-\sigma)}{d+\rho+\tilde{I_V}\theta\beta_1},$$
$$\tilde{E_H} = \frac{\tilde{I_V}\theta\beta_1(r+\lambda(1-\sigma))}{(d+\mu)(d+\rho+\tilde{I_V}\theta\beta_1)},$$
$$\tilde{T_H} = \frac{\tilde{I_V}\theta\mu\xi\beta_1(r+\lambda(1-\sigma))}{(d+\mu)(d+\Lambda+\psi)(d+\rho+\tilde{I_V}\theta\beta_1)},$$
$$\tilde{R_H} = \frac{1}{d}\left(\lambda\sigma + \frac{\rho(r+\lambda(1-\sigma))}{d+\rho+\tilde{I_V}\theta\beta_1} + \frac{\tilde{I_V}\phi^2\psi}{\theta\beta_2(\nu-\tilde{I_V}\phi)}\right),$$
$$\tilde{I_H} = \frac{\tilde{I_V}\phi^2}{\theta\beta_2(\nu-\tilde{I_V}\phi)},$$
$$\tilde{S_V} = \frac{\nu(\nu-\tilde{I_V}\phi)}{\phi(\nu-\tilde{I_V}\phi)+\tilde{I_V}\phi^2},$$
$$\tilde{I_V} = \frac{-\phi^2(d+\mu)(d+\rho)(d+\Lambda+\psi)+\nu\theta^2\mu\beta_1\beta_2(1-\xi)(r+\lambda(1-\sigma))}{\theta\phi^2\beta_1(d+\mu)(d+\Lambda+\psi)+\theta\mu\beta_2(1-\xi)(r+\lambda(1-\sigma))}.$$

Clearly, $\tilde{I_V} > 0$ if $\nu\theta^2\mu\beta_1\beta_2(1-\xi)(r+\lambda(1-\sigma)) > \phi^2(d+\mu)(d+\rho)(d+\Lambda+\psi)$ and hence the endemic equilibrium point exists if this condition is satisfied.

7 Numerical Simulation

Here, we have evaluated the model 2 numerically taking into consideration the influences of various parameters on the dynamics of YF transmission. We have considered the initial values as: $S_{H_0}(t) = 0.62$, $E_{H_0}(t) = 0.23$, $I_{H_0}(t) = 0.1$, $T_{H_0}(t) = 0.05$, $R_{H_0}(t) = 0$, $S_{V_0}(t) = 0.9$, $I_{V_0}(t) = 0.1$. Values of the parameters are considered as:$r = \frac{4.94}{10^5}$, $\theta = 3$, $\beta_1 = 0.6$, $\beta_2 = 0.5$, $\sigma = \frac{0.5}{10^6}$, $\lambda = \frac{1}{10^6}$, $\rho = 0.01$, $d = \frac{4.94}{10^5}$, $\mu = 0.31$, $\xi = 0.15$, $\psi = 0.143$, $\Lambda = \frac{3.5}{10}$, $\nu = 0.051$, $\phi = 0.051$. It is assumed that 50 of the immigrants are vaccinated. From the Figures 1 it is visible that, the daily biting rate θ influences the infected human and infected mosquito population. For $\alpha = 1$, these populations tends to grow and reaches the peak, and thereafter they start to decrease and tend to extinction. As the fractional values are incorporated, it is notable that, there is a delay in extinction of the I_H and I_V population. As the value of α further decreases, we notice that infections in human and mosquitoes are never eradicated. In fact a small portion of population are always with infection.

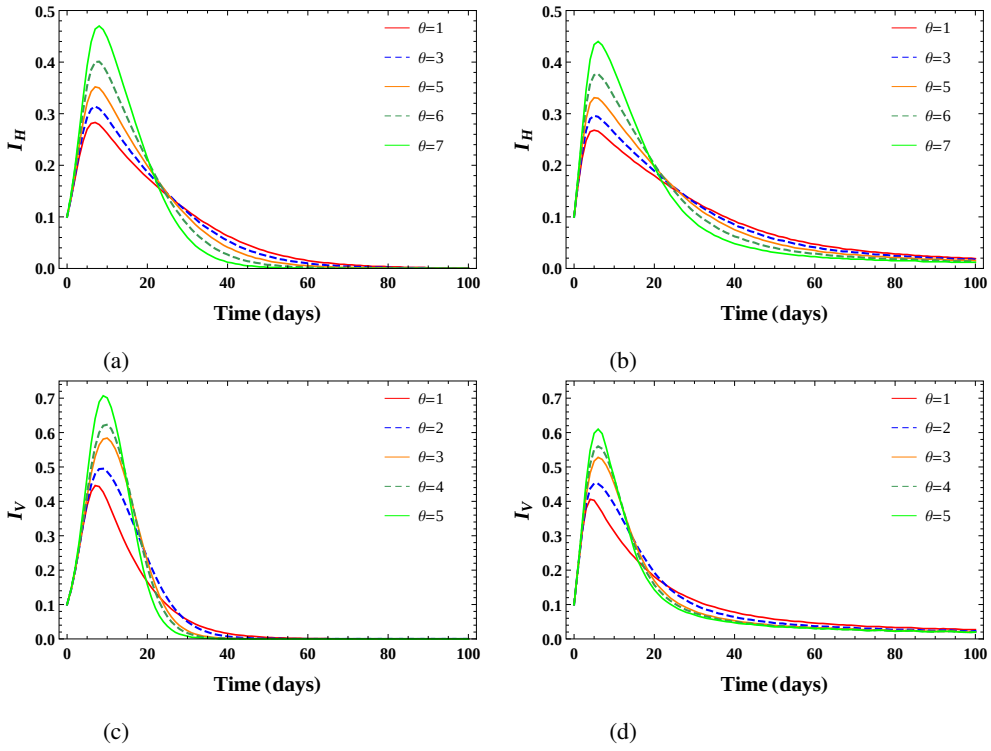

Figure 1: Profile of I_H and I_V for distinct values of θ for (A) $\alpha = 1$, (B) $\alpha = 0.65$, (C) $\alpha = 1$, (D) $\alpha = 0.65$

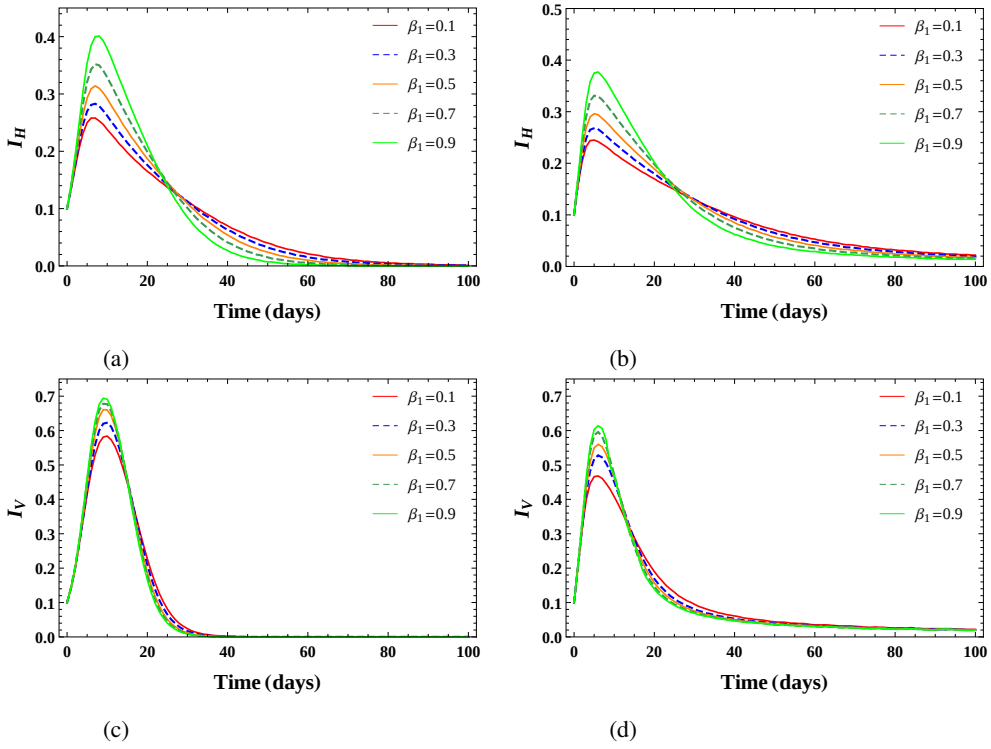

Figure 2: Profile of I_H and I_V for distinct values of β_1 for (A) $\alpha = 1$, (B) $\alpha = 0.65$, (C) $\alpha = 1$, (D) $\alpha = 0.65$

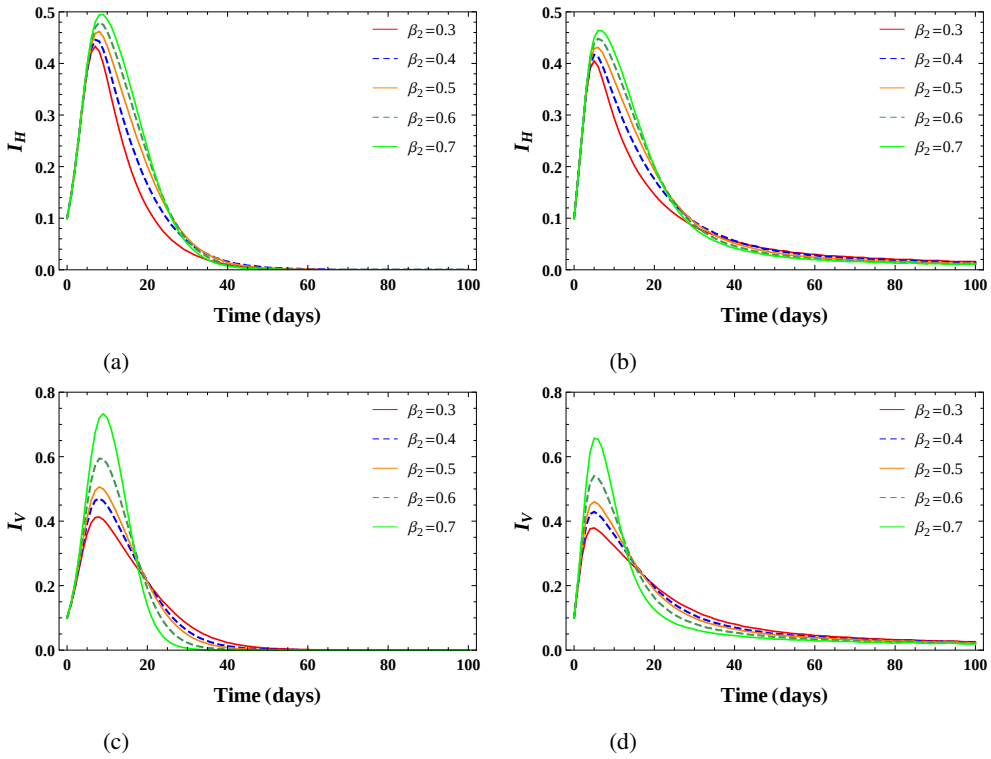

Figure 3: Profile of I_H and I_V for distinct values of β_2 for (A) $\alpha = 1$, (B) $\alpha = 0.65$, (C) $\alpha = 1$, (D) $\alpha = 0.65$

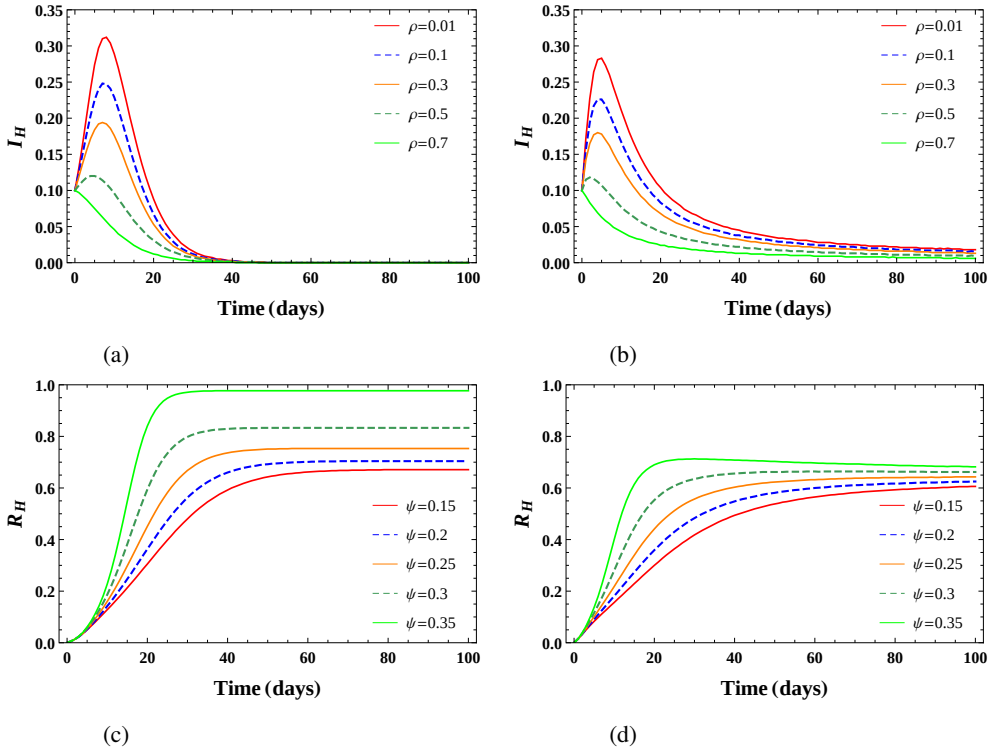

Figure 4: Profile of I_H and R_H for distinct values of (A) ρ and $\alpha = 1$, (B) ρ and $\alpha = 0.65$, (C) ψ and $\alpha = 1$, (D)ψ and $\alpha = 0.65$

The variations in the profiles of I_H and I_V for different values of β_1 are depicted in Figures 2. Figures 3 shows the variation in the profile of I_H and I_V for distinct values of β_2. From the above graphs it is visible that, as the value of β_1 and β_2 increases, the I_H and I_V population tends to grow and attains maximum value. Subsequently, they start decreasing and reach nullity which results in extinction of the I_H and I_V population. But decrease in the value of fractional derivative results in existence of the infection among the populations for longer duration. Figure 4 represents the varied profile of I_H for discrete values of effective vaccination rate of susceptible population. From the figure it is notable that, as the vaccination rate increases, the infected host population keeps on decreasing. Further, as the time progress, the infection extincts due to the influence of vaccination. As the fractional values are introduced, a fall in the infected population peak is notable. Figure 4 also depicts the profile of R_H for different recovery rate ψ and various fractional values. It may be observed that, as the value of recovery rate ψ is increased, the total recovery population also increases and leads to reduction of the

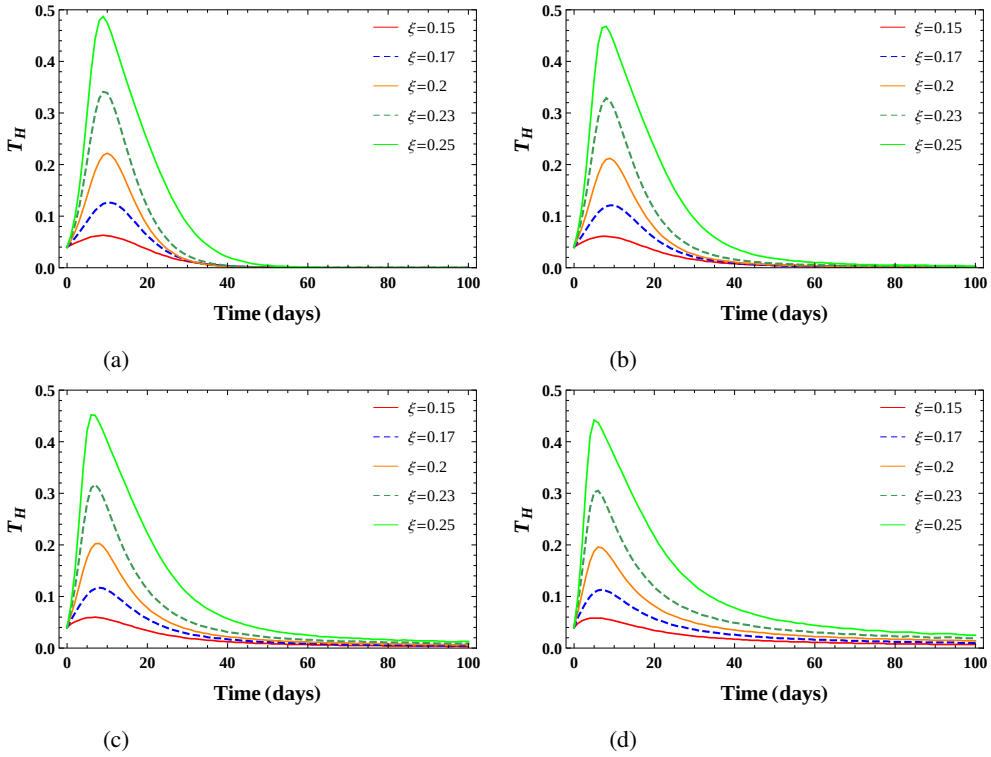

Figure 5: Profile of T_H for distinct values of ξ for (A) $\alpha = 1$, (B) $\alpha = 0.95$, (C) $\alpha = 0.85$, (D) $\alpha = 0.65$

epidemic in the host population.Figure 5 presents the dynamics of the toxic population as the proportion of exposed population deteriorates into toxic case at the rate ξ. As the value of ξ increases, the population of toxic ones grows. Further, since there is no recovery in toxic population, they die and hence population tends to extinction.

8 Conclusion

The fractional dynamics of the YF model is investigated in the present work in Caputo sense. Boundedness, existence, continuity, and uniqueness of the solution have been established. In the figures we have demonstrated the profile of the infected human and infected vectors under the influence of the biting rate, transmission rate from mosquitoes to human and human to mosquitoes, vaccination rate of susceptible population, recovery rate, and toxicity rate in presence of the Caputo fractional derivatives. We have observed that the Caputo derivative provides more realistic information than

that of the classical derivative. The reseason behind this claim is that it does not show the extinction of the infection from the environment. Graphical representations establish that the Adams-Bashfort-Moulton predictor-corrector method gives expected depiction of the results for analyzing the dynamics of the projected model. Numerical analysis of disease dynamics in the framework of various fractional derivatives can enrich the applications of mathematics for betterment of humankind as a future direction of studies of fractional calculus.

References

[1] T. P. Monath, "Dengue and yellow fever-challenges for the development and use of vaccines," *New England Journal of Medicine*, vol. 357, no. 22, pp. 2222–2225, 2007.

[2] T. P. Monath and P. F. Vasconcelos, "Yellow fever," *Journal of clinical virology*, vol. 64, pp. 160–173, 2015.

[3] M. Kung'aro, L. S. Luboobi, and F. Shahada, "Reproduction number for yellow fever dynamics between primates and human beings," *Commun. Math. Biol. Neurosci.*, 2014.

[4] T. P. Monath, "Treatment of yellow fever," *Antiviral research*, vol. 78, no. 1, pp. 116–124, 2008.

[5] T. P. Monath and A. D. Barrett, "Pathogenesis and pathophysiology of yellow fever," *Advances in virus research*, vol. 60, pp. 343–397, 2003.

[6] W. O. Kermack and A. G. McKendrick, "Contributions to the mathematical theory of epidemics-I," *Bulletin of mathematical biology*, vol. 53, no. 1-2, pp. 33–55, 1991.

[7] A. S. Quintero and R. E. Gutierrez-Carvajal, "Modeling the Evolution of SARS-CoV-2 Using a Fractional-Order SIR Approach," *TecnoLogicas*, vol. 24, no. 51, 2021.

[8] L. Esteva, C. Vargas, and H. M. Yang, "A model for yellow fever with migration," vol. 1, no. 6, 2019.

[9] T. T. Yusuf and D. O. Daniel, "Mathematical modeling of yellow fever transmission dynamics with multiple control measures," *Asian Research Journal of Mathematics*, pp. 1–15, 2019.

[10] C. Baishya, "An operational matrix based on the Independence polynomial of a complete bipartite graph for the Caputo fractional derivative," *SeMA Journal*, pp. 1–19, 2021.

[11] M. R. Ali and R. Sadat, "Construction of Lump and optical solitons solutions for (3+ 1) model for the propagation of nonlinear dispersive waves in inhomogeneous media," *Optical and Quantum Electronics*, vol. 53, no. 5, pp. 1–13, 2021.

[12] C. Baishya and P. Veeresha, "Laguerre polynomial-based operational matrix of integration for solving fractional differential equations with non-singular kernel," *Proceedings of the Royal Society A*, vol. 477, 2021.

[13] P. Veeresha, "A numerical approach to the coupled atmospheric ocean model using a fractional operator," *Mathematical Modelling and Numerical Simulation with Applications (MMNSA)*, vol. 1, no. 1, pp. 1–10, 2021.

[14] M. R. Ali, W.-X. Ma, and R. Sadat, "Lie symmetry analysis and invariant solutions for (2+ 1) dimensional Bogoyavlensky-Konopelchenko equation with variable-coefficient in wave propagation," *Journal of Ocean Engineering and Science*, 2021.

[15] I. Podlubny, *Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications*. Elsevier, 1998.

[16] J. Losada and J. J. Nieto, "Properties of a new fractional derivative without singular kernel," *Progr. Fract. Differ. Appl*, vol. 1, no. 2, pp. 87–92, 2015.

[17] B. Wang and L.-Q. Chen, "Asymptotic stability analysis with numerical confirmation of an axially accelerating beam constituted by the standard linear solid model," *Journal of Sound and Vibration*, vol. 328, no. 4-5, pp. 456–466, 2009.

[18] H. L. Li, L. Zhang, C. Hu, Y.-L. Jiang, and Z. Teng, "Dynamical analysis of a fractional-order predator-prey model incorporating a prey refuge," *Journal of Applied Mathematics and Computing*, vol. 54, no. 1-2, pp. 435–449, 2017.

[19] C. Baishya, S. J. Achar, P. Veeresha, and D. G. Prakasha, "Dynamics of a fractional epidemiological model with disease infection in both the populations," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 31, no. 4, 2021.

[20] M. A. Dokuyucu, E. Celik, H. Bulut, and H. M. Baskonus, "Cancer treatment model with the Caputo-Fabrizio fractional derivative," *The European Physical Journal Plus*, vol. 133, no. 3, pp. 1–6, 2018.

[21] M. A. Khan, Z. Hammouch, and D. Baleanu, "Modeling the dynamics of hepatitis E via the Caputo–Fabrizio derivative," *Mathematical Modelling of Natural Phenomena*, vol. 14, no. 3, 2019.

[22] E. Ahmed and H. A. El-Saka, "On fractional order models for Hepatitis C," *Nonlinear biomedical physics*, vol. 4, no. 1, pp. 1–3, 2010.

[23] P. Veeresha, D. Kumar, "Analysis and dynamics of the Ivancevic option pricing model with a novel fractional calculus approach," *Waves in Random and Complex Media*, pp. 1-18, 2022.

[24] S. J. Achar, C. Baishya, P. Veeresha, and L. Akinyemi, "Dynamics of fractional model of biological pest control in tea plants with Beddington-DeAngelis functional response," *Fractal and Fractional*, vol. 6, no. 1, 2022.

[25] K. Diethelm, N. J. Ford, and A. D. Freed, "A predictor-corrector approach for the numerical solution of fractional differential equations," *Nonlinear Dynamics*, vol. 29, no. 1, pp. 3–22, 2001.

[26] K. Diethelm and N. J. Ford, "Analysis of fractional differential equations," *Journal of Mathematical Analysis and Applications*, vol. 265, no. 2, pp. 229–248, 2002.

[27] C. Baishya, "Dynamics of fractional stage structured predator prey model with prey refuge," *Indian Journal of Ecology*, vol. 47, no. 4, pp. 1118–1124, 2020.

[28] S. J. Achar, C. Baishya, and M. K. Kaabar, "Dynamics of the worm transmission in wireless sensor network in the framework of fractional derivatives," *Mathematical Methods in the Applied Sciences*, 2021.

[29] C. Baishya, "Dynamics of fractional holling type-ii predator-prey model with prey refuge and additional food to predator," *Journal of Applied Nonlinear Dynamics*, vol. 10, no. 2, pp. 315–328, 2020.