Double Fuzzy Aboodh Transform for Solving Fuzzy Partial Differential Equations

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ABSTRACT
Solving partial differential equations, integro-differential equations of various kinds, and ordinary differential equations is a common application for the Aboodh transformation. This effort aims primarily to provide a new double fuzzy transform called the double fuzzy Aboodh transform. Several essential properties of double fuzzy Aboodh transformations are shown. These novel results may be used to solve precisely fuzzy partial differential equations with a generalized Hukuhara partial differentiability. Additionally, an example is provided to show how effective and superiority of the symmetric triangular fuzzy numbers in the double fuzzy Aboodh transform for solving fuzzy partial differential equations.

Keywords: Fuzzy number; double fuzzy Aboodh transform; fuzzy partial differential equations; Strongly generalized differential; Fuzzy valued function.

1. INTRODUCTION

A sophisticated and effective method for resolving fuzzy partial differential equations (FPDEs) is the double fuzzy integral transform. Because imprecise data or beginning circumstances may make standard techniques of solving differential equations difficult or erroneous, these transformations are especially helpful when working with uncertain or fuzzy systems. Double fuzzy integral transforms provide more versatility in addressing fuzzy differential equations by combining two distinct integral transform types into a single framework. With FPDEs, this method enables more precise and reliable. Several double fuzzy integral transformations (Laplace, Natural, Elzaki) have been employed by various researchers [1, 2, 3, 4, 5] in recent years to solve fuzzy partial differential equations. When it comes to solving linear fuzzy partial differential equations, these transformations are very helpful. They translate the fuzzy partial equations to algebraic equations about the unknown function. The aim of this work is to derive the solution Fuzzy Partial Differential Equations under generalized Hukuhara partial derivatives by use of a double fuzzy Aboodh transform. First we define a single fuzzy Aboodh transform [6, 7, 8]. Next, we establish some necessary conditions for its existence and demonstrate some important characteristics of this change. The double fuzzy Aboodh transform (DFAT), a novel double fuzzy integral transformation for multivariate fuzzy functions, is defined by applying the fuzzy Aboodh transform for a one-variable fuzzy function. We provide an overview of the fundamental theorems and characteristics of the DFAT and showcase several findings concerning the generalized Hukuhara partial derivatives. We provide a straightforward method for solving the fuzzy partial differential equations based on the complete DFAT. Lastly, we present examples to show how the given double fuzzy integral transform might be helpful in solving fuzzy partial differential equations utilizing the triangular fuzzy numbers.

2. Fundamental Preliminaries

We review the fundamental ideas that we must use throughout the majority of the text in this section.

(2.1) Definition [9]

The mapping ϯ: ʉ → [0,1]. The fuzzy number is ambiguous if it satisfies the following:

1. ϯis semi continuous in the higher half.
2. ϯis fuzzy convex, i. e. , ϯ(ɠч + (1 − ɠ)ғ) ≥ min{ϯ(ч), ϯ(ғ)},for all ч , ғ ∈ ʉ and ɠ ∈ [0,1].
3. ϯis normal i. e. , ∃ ƥ$_0$ ∈ ʉ for which purposeϯ(ƥ) = 1
4. Supp (ϯ) = { ƥ ∈ ʉ;ϯ(ƥ) > 0 }, and cl(Supp (ϯ)) is compact.

(2.2) Definition [10]
A pair that is sorted parametrically is a fuzzy number, $(\underline{\varrho}, \overline{\varrho})$ of functions $\overline{\varrho}(\mathfrak{g})$, $\underline{\varrho}(\mathfrak{g})$, $\mathfrak{g} \in [0,1]$, which fulfills:
1. $\underline{\varrho}(\mathfrak{g})$is a continuous function with a right function of 0 and a left function of (0,1] that is non-decreasing.
2. $\overline{\varrho}(\mathfrak{g})$is a bounded, non-increasing function with 0 continuous right and (0,1] continuous left.
3. $\underline{\varrho}(\mathfrak{g}) \leq \overline{\varrho}(\mathfrak{g})$, $\mathfrak{g} \in [0,1]$.

(2.3) Definition [11]
Triangular fuzzy number is a fuzzy number represented with three points as follows $\mathcal{F} = (\beta_1, \beta_2, \beta_3)$.Membership functions are applied to this representation.

$$\omega_{\mathcal{F}}(\beta) = \begin{cases} 0 & \beta < \beta_1 \\ \frac{\beta - \beta_1}{\beta_2 - \beta_1} & \beta_1 \leq \beta \leq \beta_2 \\ \frac{\beta_3 - \beta}{\beta_3 - \beta_2} & \beta_2 \leq \beta \leq \beta_3 \\ 0 & \beta > \beta_3 \end{cases}.$$

$\mathfrak{g}$-cut interval for this shape is written $\forall\, \mathfrak{g} \in [0,1], \mathcal{F}_{\mathfrak{g}} = [(\beta_2 - \beta_1)\mathfrak{g} + \beta_1, -(\beta_3 - \beta_2)\mathfrak{g} + \beta_3]$.

(2.4) Definition [12]
Trapezoidal fuzzy number can define $\mathcal{F}$as $\mathcal{F} = (\beta_1, \beta_2, \beta_3, \beta_4)$. Membership functions are applied to this representation.

$$\omega_{\mathcal{F}}(\beta) = \begin{cases} 0 & \beta < \beta_1 \\ \frac{\beta - \beta_1}{\beta_2 - \beta_1} & \beta_1 \leq \beta \leq \beta_2 \\ 1 & \beta_2 \leq \beta \leq \beta_3 \\ \frac{\beta_4 - \beta}{\beta_4 - \beta_3} & \beta_3 \leq \beta \leq \beta_4 \\ 0 & \beta > \beta_4 \end{cases}$$

$\mathfrak{g}$-cut interval for this shape is written $\forall\, \mathfrak{g} \in [0,1], \mathcal{F}_{\mathfrak{g}} = [(\beta_2 - \beta_1)\mathfrak{g} + \beta_1, -(\beta_4 - \beta_3)\mathfrak{g} + \beta_4]$.

(2.5) Definition [10]
Let γ and ι are fuzzy numbers, where $\gamma = (\underline{\gamma}(\mathfrak{g}), \overline{\gamma}(\mathfrak{g})), \iota = (\underline{\iota}(\mathfrak{g}), \overline{\iota}(\mathfrak{g})), 0 \leq \mathfrak{g} \leq 1$ and $\alpha > 0$ we define:
1. Addition $\gamma \oplus \iota = (\underline{\gamma}(\mathfrak{g}) + \underline{\iota}(\mathfrak{g}), \overline{\gamma}(\mathfrak{g}) + \overline{\iota}(\mathfrak{g}))$.
2. Subtraction $\gamma \ominus \iota = (\underline{\gamma}(\mathfrak{g}) - \overline{\iota}(\mathfrak{g}), \overline{\gamma}(\mathfrak{g}) - \underline{\iota}(\mathfrak{g}))$.
3. Multiplication $\gamma \odot \iota =$
$(\min\{\underline{\gamma}(\mathfrak{g})\overline{\iota}(\mathfrak{g}), \underline{\gamma}(\mathfrak{g})\underline{\iota}(\mathfrak{g}), \overline{\gamma}(\mathfrak{g})\overline{\iota}(\mathfrak{g}), \overline{\gamma}(\mathfrak{g})\underline{\iota}(\mathfrak{g})\}, \max\{\underline{\gamma}(\mathfrak{g})\overline{\iota}(\mathfrak{g}), \underline{\gamma}(\mathfrak{g})\underline{\iota}(\mathfrak{g}), \overline{\gamma}(\mathfrak{g})\overline{\iota}(\mathfrak{g}), \overline{\gamma}(\mathfrak{g})\underline{\iota}(\mathfrak{g})\}$
4. Scalar multiplication $\alpha \odot \gamma = \begin{cases} (\alpha\underline{\gamma}, \alpha\overline{\gamma}) & \alpha \geq 0, \\ (\alpha\overline{\gamma}, \alpha\underline{\gamma}) & \alpha < 0. \end{cases}$. If $\alpha = 1$then $\alpha \odot \gamma = -\gamma$.

(2.6) Definition [13]
Let γ and $\iota$are fuzzy numbers, the Hausdorff distance between fuzzy numbers is provided by:
$\mathbb{D}: \vartheta_{\mathcal{F}} \times \vartheta_{\mathcal{F}} \to [0, +\infty]$, where $\vartheta_{\mathcal{F}}$be the set of all fuzzy numbers on ϑ:
$\mathbb{D}(\gamma, \iota) = sup_{\mathfrak{g} \in [0,1]}\, max\{|\underline{\gamma}(\mathfrak{g}) - \underline{\iota}(\mathfrak{g})|, |\overline{\gamma}(\mathfrak{g}) - \overline{\iota}(\mathfrak{g})|\}$,
Where $\gamma = (\underline{\gamma}(\mathfrak{g}), \overline{\gamma}(\mathfrak{g})), \iota = (\underline{\iota}(\mathfrak{g}), \overline{\iota}(\mathfrak{g})) \subset \vartheta$ and following properties are well known:
1.$\mathbb{D}(\gamma \oplus \gamma, \iota \oplus \gamma) = \mathcal{F}(\gamma, \zeta), \forall\, \gamma, \iota, \gamma \in \vartheta_{\mathcal{F}}$.
2.$\mathbb{D}(\mathfrak{g} \odot \gamma, \mathfrak{g} \odot \iota) = |\mathfrak{g}|\, \mathcal{F}(\gamma, \iota), \forall\, \gamma, \iota \in \vartheta_{\mathcal{F}}, \mathfrak{g} \in \vartheta$.
3.$\mathbb{D}(\gamma \oplus \gamma, \iota \oplus \mathfrak{h}) \leq \mathcal{F}(\gamma, \iota) + \Gamma(\gamma, \mathfrak{h}), \forall\, \gamma, \iota, \gamma, \mathfrak{h} \in \vartheta_{\mathcal{F}}$.
4.$(\mathbb{D}, \vartheta_{\mathcal{F}})$is a complete metric space.

(2.7) Definition [14]
Assume that Ɣ, Ɩ ∈ ᵾ_f. There is $\gamma \in$ ᵾ_f such that Ɣ $=$ Ɩ $+ \gamma$ then Ɣ is known the H-differential of Ɣ *and* Ɩ andit is represented by Ɣ ⊖ Ɩ. Where Ɣ ⊖ Ɩ $= \gamma \Leftrightarrow \begin{cases} (i)\ Ɣ = Ɩ + \gamma \\ (ii)\ Ɩ = Ɣ - \gamma \end{cases}$

Note that in this work, the sign ⊖ always meant the Ḩ-difference as well as,
Ɣ ⊖ Ɩ ≠ Ɣ + (−1)Ɩ.

(2.1) Theorem [15]
Let ϯ(ϸ): ᵾ ⟶ ᵾ_f serve as a function and serve as a representation ϯ(ϸ) = ((ϯ̲(ϸ, ɠ), ϯ̄(ϸ, ɠ))in every instance for ɠ ∈ [0,1]. Then:

1. If ϯ(ϸ) is differentiable form i, then (ϯ̲(ϸ, ɠ) and ϯ̄(ϸ, ɠ) functions that are differentiable and ϯ′(ϸ) = (ϯ̲′(ϸ, ɠ), ϯ̄′(ϸ, ɠ)).
2. If ϯ(ϸ) is differentiable form ii, then (ϯ̲(ϸ, ɠ) and ϯ̄(ϸ, ɠ) functions that are differentiable and ϯ′(ϸ) = (ϯ̄′(ϸ, ɠ), ϯ̲′(ϸ, ɠ)).

(2.8) Definition [16]
Let ϯ: ᵾ × ᵾ ⟶ ᵾ_f be a function with fuzzy values.In the event when for a random fixed point (ϸ₀, ʈ₀) ∈ ᵾ × ᵾ and $\epsilon > 0$ there exists $\delta > 0$ such that $|$ϸ − ϸ₀$| + |$ʈ − ʈ₀$| < \delta \Longrightarrow \mathbb{D}\big($ϯ(ϸ, ʈ), ϯ(ϸ₀, ʈ₀)$\big) < \epsilon$, ϯ is said to be a continuous fuzzy-valued function.

(2.9) Definition (2.7) [17]
Fuzzy valued function ϯ: ᵾ × ᵾ ⟶ $\mathcal{R}_{\mathcal{F}}$ and, as we say ϯ is first-order H-differentiable at (ϸ₀, ʈ₀) ∈ ᵾ × ᵾ regarding the variables ϸ and ʈ are the functions ϯ_ϸ′(ϸ₀, ʈ₀) and ϯ′_ʈ(ϸ₀, ʈ₀) defined by

$$ϯ_ϸ'(ϸ_0, ʈ_0) = \lim_{Ŋ \to 0} \frac{ϯ(ϸ_0 + Ŋ, ʈ_0) \ominus ϯ(ϸ_0, ʈ_0)}{Ŋ}$$

$$ϯ'_ʈ(ϸ_0, ʈ_0) = \lim_{Ŋ \to 0} \frac{ϯ(ϸ_0, Ŋ + ʈ_0) \ominus ϯ(ϸ_0, ʈ_0)}{Ŋ}$$

(2.10) Definition [17]
Let ϯ: ᵾ × ᵾ ⟶ ᵾ_f be a function and represents ϯ(ϸ, ʈ) = ((ϯ̲(ϸ, ʈ, ɠ), ϯ̄(ϸ, ʈ, ɠ))is partial differentiable at (ϸ₀, ʈ₀) ∈ ᵾ × ᵾ in each case for ɠ ∈ [0,1], with respect to variable ϸ. We say that

1. If ϯ(ϸ, ʈ) is differentiable form i, then (ϯ̲(ϸ, ʈ, ɠ) and ϯ̄(ϸ, ʈ, ɠ) are differentiable functions and ϯ′(ϸ, ʈ) = (ϯ̲′(ϸ, ʈ, ɠ), ϯ̄′(ϸ, ʈ, ɠ)).
2. If ϯ(ϸ, ʈ) is differentiable form ii, then (ϯ̲(ϸ, ʈ, ɠ) and ϯ̄(ϸ, ʈ, ɠ) are differentiable functions and ϯ′(ϸ, ʈ) = (ϯ̄′(ϸ, ʈ, ɠ), ϯ̲′(ϸ, ʈ, ɠ)).

(2.11) Definition [6]
Let ϯ(ϸ)be a fuzzy-valued continuous function. Suppose that $\frac{1}{℘}$ϯ(ϸ)$e^{-℘ϸ}$is a fuzzy Rimann-integrable that is improper on [0, ∞), then $\frac{1}{℘}\int_0^\infty$ ϯ(ϸ)$e^{-℘ϸ}\, d$ϸis known as the fuzzy Aboodh transform and is also known as

$$\hat{A}[ϯ(ϸ)] = Ş(℘) = \frac{1}{℘}\int_0^\infty ϯ(ϸ)e^{-℘ϸ}\, dϸ, (℘ > 0 \text{ and integer})$$

$$\frac{1}{℘}\int_0^\infty ϯ(ϸ)e^{-℘ϸ}\, dϸ = \left(\frac{1}{℘}\int_0^\infty \underline{ϯ}(ϸ, ɠ)e^{-℘ϸ}\, dϸ, \frac{1}{℘}\int_0^\infty \overline{ϯ}(ϸ, ɠ)e^{-℘ϸ}\, dϸ\right)$$

(2.2) Theorem
Let ϯ(ϸ) is the primitive of ϯ′(ϸ) on [0, ∞) and ϯ(ϸ) be an integrable fuzzy-valued function then:
[a]. $\widehat{A_ϸ}[ϯ'(ϸ)] = ℘\hat{A}[ϯ(ϸ)] \ominus \frac{1}{℘}ϯ(0)$

[b]. $\widehat{A_{þ}}[\dagger^{(2)}(þ)] = \{\wp^2 \hat{A}[\dagger(þ)] \ominus \dagger(0)\} \ominus \frac{1}{\wp}\dagger'(0)$.

Proof (a): For an arbitrarily chosen fixed $\mathcal{g} \in [0,1]$,

$\wp\hat{A}[\dagger(þ)] \ominus \frac{1}{\wp}\dagger(0) = (\wp A[\underline{\dagger}(þ,\mathcal{g})] - \frac{1}{\wp}\underline{\dagger}(0,\mathcal{g}), \wp A[\overline{\dagger}(þ,\mathcal{g})] - \frac{1}{\wp}\overline{\dagger}(0,\mathcal{g}))$.

Since, $A[\underline{\dagger}'(þ,\mathcal{g}))] = \wp A[\underline{\dagger}(þ,\mathcal{g})] - \frac{1}{\wp}\underline{\dagger}(0,\mathcal{g})$ and $A[\overline{\dagger}'(þ,\mathcal{g})] = \wp A[\overline{\dagger}(þ,\mathcal{g})] - \frac{1}{\wp}\overline{\dagger}(0,\mathcal{g})$.

Using **(2.1) Theorem**

$\underline{\dagger}'(þ,\mathcal{g}) = \underline{\dagger}'(þ,\mathcal{g}), \overline{\dagger}'(þ,\mathcal{g}) = \overline{\dagger}'(þ,\mathcal{g})$.

$A[\underline{\dagger}'(þ,\mathcal{g}))] = \wp A[\underline{\dagger}(þ,\mathcal{g})] - \frac{1}{\wp}\underline{\dagger}(0,\mathcal{g})$,

$A[\overline{\dagger}'(þ,\mathcal{g})] = \wp A[\overline{\dagger}(þ,\mathcal{g})] - \frac{1}{\wp}\overline{\dagger}(0,\mathcal{g})$,

$\wp\hat{A}[\dagger(þ)] \ominus \frac{1}{\wp}\dagger(0) = (A[\underline{\dagger}'(þ,\mathcal{g})], A[\overline{\dagger}'(þ,\mathcal{g})])$,

$\widehat{A_{þ}}[\dagger'(þ)] = \wp\hat{A}[\dagger(þ)] \ominus \frac{1}{s}\dagger(0)$.

[b] :Using **(2.11) Definition** and equation [a] of this theorem, we obtain

$$\widehat{A_{þ}}[\dagger^{(2)}(þ)] = \wp\hat{A}[\dagger'(þ)] \ominus \frac{1}{\wp}\dagger'(0) = \wp\left\{\wp\hat{A}[\dagger(þ)] \ominus \frac{1}{\wp}\dagger(0)\right\} \ominus \frac{1}{\wp}\dagger'(0)$$
$$= \{\wp^2\hat{A}[\dagger(þ)] \ominus \dagger(0)\} \ominus \frac{1}{\wp}\dagger'(0).$$

(2.1) Corollary

Let $\dagger(þ,ţ)$ is the primitive of $\dagger'(þ,ţ)$ on $[0,\infty)\times[0,\infty)$ and $\dagger(þ,ţ)$ be an integrable fuzzy-valued function then:

[a]. $\widehat{A_{þ}}[\dagger_{þ}'(þ,ţ)] = \wp\hat{A}[\dagger(þ,ţ)] \ominus \frac{1}{\wp}\dagger(0,ţ)$

[b]. $\widehat{A_{þ}}[\dagger_{þþ}^{(2)}(þ,ţ)] = \{\wp^2\hat{A}[\dagger(þ,ţ)] \ominus \dagger(0,ţ)\} \ominus \frac{1}{\wp}\dagger_{þ}'(0,ţ)$.

3. Double Fuzzy Aboodh Transform

This section defines the double fuzzy Aboodh transform. Additionally, we provide numerous double fuzzy Aboodh transform characteristics and theorems. We also give the double fuzzy Aboodhtransform of several essential functions. We prove further results concerning the new transform's generalized Hukuhara partial differentiability.

(3.1) Definition

Let $\dagger(þ,ţ)$ be a continuous fuzzy-valued function. Assume that $\frac{1}{\wp}\frac{1}{\mathcal{U}}\dagger(þ,ţ)e^{-\wp þ-\mathcal{U}ţ}$ is an improper fuzzy Rimann-integrable on $[0,\infty]\times[0,\infty]$, then $\frac{1}{\wp}\frac{1}{\mathcal{U}}\dagger(þ,ţ)e^{-\wp þ-\mathcal{U}ţ}dþ\, d\,ţ$ is referred to as referred to as double fuzzy Aboodh transform and goes by the name

$\hat{A}_2[\dagger(þ,ţ)] = \S(\wp,\mathcal{U}) = \frac{1}{\wp}\frac{1}{\mathcal{U}}\int_0^\infty\int_0^\infty \dagger(þ,ţ)e^{-\wp þ-\mathcal{U}ţ}dþ\, d\,ţ$, ($\wp,\mathcal{U} > 0$ *and integer*),

$\frac{1}{\wp}\frac{1}{\mathcal{U}}\int_0^\infty\int_0^\infty \dagger(þ,ţ)e^{-\wp þ-\mathcal{U}ţ}d\,þd\,ţ = \left(\frac{1}{\wp}\frac{1}{\mathcal{U}}\int_0^\infty\int_0^\infty \underline{\dagger}(þ,ţ,\mathcal{g})e^{-\wp þ-\mathcal{U}ţ}dþ\,d\,ţ, \frac{1}{\wp}\frac{1}{\mathcal{U}}\int_0^\infty\int_0^\infty \overline{\dagger}(þ,ţ,\mathcal{g})e^{-\wp þ-\mathcal{U}ţ}dþ\,d\,ţ\right)$.

Furthermore possible to display parametrically as follows

$\hat{A}_2[\dagger(þ,ţ)] = \left(A_2\underline{\dagger}(þ,ţ,\mathcal{g}), A_2\overline{\dagger}(þ,ţ,\mathcal{g})\right)$.

Where double fuzzy inverse Aboodh transform can be written as the formula

$\hat{A}_2^{-1}[\S(\wp,\mathcal{U})] = [\dagger(þ,ţ)] = [\left(A_2^{-1}\underline{\dagger}(þ,ţ,\mathcal{g}), A_2^{-1}\overline{\dagger}(þ,ţ,\mathcal{g})\right)$

(3.1) Theorem (linear property)

Let $\dagger(þ,ţ)$,$ᵽ(þ,ţ)$be continuous fuzzy-valued functions, $\mathbb{c}_1$ and $\mathbb{c}_2$ are ʉonstants, then

(1). $\hat{A}_2[\mathbb{c}_1\dagger(þ,ţ)] = \mathbb{c}_1\hat{A}_2[\dagger(þ,ţ)]$.

(2). $\hat{A}_2[\mathbb{c}_1(\dagger(þ,ţ))\oplus\mathbb{c}_2(ᵽ(þ,ţ))] = \mathbb{c}_1\hat{A}_2[\dagger(þ,ţ)]\oplus\mathbb{c}_2\hat{A}_2[ᵽ(þ,ţ)]$.

Proof

(1).$\hat{A}_2[\mathbb{c}_1\dagger(þ,ţ)] = (A_2[\mathbb{c}_1\underline{\dagger}(þ,ţ,\mathcal{g})], A_2[\mathbb{c}_1\overline{\dagger}(þ,ţ,\mathcal{g})]) =$

$(\frac{1}{\wp}\frac{1}{\mathcal{U}}\int_0^\infty\int_0^\infty \mathbb{c}_1\underline{\dagger}(þ,ţ,\mathcal{g})e^{-\wp þ-\mathcal{U}ţ}d\,þd\,ţ, \frac{1}{\wp}\frac{1}{\mathcal{U}}\int_0^\infty\int_0^\infty \mathbb{c}_1\overline{\dagger}(þ,ţ,\mathcal{g})e^{-\wp þ-\mathcal{U}ţ}d\,þd\,ţ)$

$$= \left(\mathbb{c}_1 \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \underline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ, \mathbb{c}_1 \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \overline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ \right)$$

$$= \mathbb{c}_1 (\frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \underline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ, \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \overline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ)$$

$$= \mathbb{c}_1 \left(A_2 \left[\mathbb{c}_1 \underline{\dagger}(\,þ, ţ, ƍ) \right], A_2 \left[\mathbb{c}_1 \overline{\dagger}(\,þ, ţ, ƍ) \right] \right) = \mathbb{c}_1 \hat{A}_2 [\dagger(\,þ, ţ)\,].$$

(2). Suppose $\dagger(\,þ, ţ) = (\underline{\dagger}(\,þ, ţ, ƍ), \overline{\dagger}(\,þ, ţ, ƍ))$, $\mathfrak{p}(\,þ, ţ) = (\underline{\mathfrak{p}}(\,þ, ţ, ƍ), \overline{\mathfrak{p}}(\,þ, ţ, ƍ))$,

$$\hat{A}_2 [\mathbb{c}_1 (\dagger(\,þ, ţ)) \oplus \mathbb{c}_2 (\mathfrak{p}(\,þ\, þ, ţ))] = (A_2 [\mathbb{c}_1 \left(\underline{\dagger}(\,þ, ţ, ƍ) + \mathbb{c}_2 \left(\underline{\mathfrak{p}}(\,þ, ţ, ƍ) \right], A_2 \left[\mathbb{c}_1 \overline{\dagger}(\,þ, ţ, ƍ) + \mathbb{c}_2 \overline{\mathfrak{p}}(\,þ, ţ, ƍ) \right] \right)$$

$$= \left(\frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty (\mathbb{c}_1 \underline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ \right.$$
$$+ \mathbb{c}_2 \underline{\mathfrak{p}}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ), \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty (\mathbb{c}_1 \overline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ$$
$$\left. + \mathbb{c}_2 \overline{\mathfrak{p}}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ) \right)$$

$$= \left(\frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \mathbb{c}_1 \underline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ \right.$$
$$+ \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \mathbb{c}_2 \underline{\mathfrak{p}}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ, \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \mathbb{c}_1 \overline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ$$
$$\left. + \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \mathbb{c}_2 \overline{\mathfrak{p}}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ \right)$$

$$= \left(\frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \mathbb{c}_1 \underline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ, \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \mathbb{c}_1 \overline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ \right)$$
$$+ \left(\frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \mathbb{c}_2 \underline{\mathfrak{p}}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ, \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \mathbb{c}_2 \overline{\mathfrak{p}}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ \right)$$

$=$
$\mathbb{c}_1 \left(\frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \underline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ, \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \overline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ \right) +$
$\mathbb{c}_2 \left(\frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \underline{\mathfrak{p}}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ, \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \overline{\mathfrak{p}}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ \right) = \mathbb{c}_1 A_2 \left(\underline{\dagger}(\,þ, ţ, ƍ), \overline{\dagger}(\,þ, ţ, ƍ) \right) +$
$\mathbb{c}_2 A_2 \left(\underline{\mathfrak{p}}(\,þ, ţ, ƍ), \overline{\mathfrak{p}}(\,þ, ţ, ƍ) \right) = \mathbb{c}_1 \hat{A}_2 [\dagger(\,þ, ţ)] \oplus \mathbb{c}_2 \hat{A}_2\, [\mathfrak{p}(\,þ, ţ)]\,.$

(3.2)Theorem
If $\dagger(\,þ, ţ)$is a continuous fuzzy valued function and$\hat{A}_2[\dagger(\,þ, ţ)] = \mathfrak{S}(\wp, \mathcal{U})$ then:
$\hat{A}\left[\dagger(\,þ, ţ) e^{a\, þ + b\, ţ}\right] = \mathfrak{S}(\wp - a, (\mathcal{U} - b))$, where $e^{a\, þ + b\, ţ}$ is real value function and $(\wp - a, (\mathcal{U} - b) > 0$.
Proof:
$\hat{A}_2[\dagger(\,þ, ţ)] = \mathfrak{S}(\wp, \mathcal{U})$

$$\hat{A}_2[\dagger(\,þ, ţ)] = \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \dagger(\,þ, ţ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ$$

$$\hat{A}_2[\dagger(\,þ, ţ)] = \left(\frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \underline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ, \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \overline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} d\, þ d\, ţ \right)$$

$\hat{A}_2\left[\dagger(\,þ, ţ) e^{a\, þ + b\, ţ}\right]$

$$= \left(\frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \underline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} e^{a\, þ + b\, ţ} d\, þ d\, ţ, \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \overline{\dagger}(\,þ, ţ, ƍ) e^{-\wp\, þ - \mathcal{U}\, ţ} e^{a\, þ + b\, ţ} d\, þ d\, ţ \right)$$

$\hat{A}_2\left[\dagger(\,þ, ţ) e^{a\, þ + b\, ţ}\right]$

$$= \left(\frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \underline{\dagger}(\,þ, ţ, ƍ) e^{-((\wp - a)\, þ + (-\mathcal{U} + b)\, ţ)} d\, þ d\, ţ, \frac{1}{\wp} \frac{1}{\mathcal{U}} \int_0^\infty \int_0^\infty \overline{\dagger}(\,þ, ţ, ƍ) e^{-((\wp - a)\, þ - (\mathcal{U} - b)\, ţ)} d\, þ d\, ţ \right)$$

$\hat{A}_2\left[\dagger(\,þ, ţ) e^{a\, þ + b\, ţ}\right] = \mathfrak{S}(\wp - a, (\mathcal{U} - b))$

(3.3) Theorem (Properties of derivatives):
If $\dagger(\,þ, ţ)$is a continuous fuzzy valued function and$\hat{A}_2[\dagger(\,þ, ţ)] = \mathfrak{S}(\wp, \mathcal{U})$ then:
[a]. $\widehat{A_2}\left[\dagger_{þ}{}'(\,þ, ţ)\,\right] = \wp \widehat{A_2}[\dagger(\,þ, ţ)] \ominus \frac{1}{\wp} \mathfrak{S}(0, \mathcal{U})$,

[b]. $\widehat{A_2}[\dagger_{\mathfrak{t}}{}'(\mathfrak{p},\mathfrak{t})] = \mathcal{U}\widehat{A_2}[\dagger(\mathfrak{p},\mathfrak{t})] \ominus \frac{1}{\mathcal{U}}\mathfrak{S}(\wp,0)$,

[c]. $\widehat{A_2}[\dagger_{\mathfrak{p}\mathfrak{p}}{}^{(2)}(\mathfrak{p},\mathfrak{t})] = \{\wp^2\widehat{A_2}[\dagger(\mathfrak{p},\mathfrak{t})] \ominus \mathfrak{S}(0,\mathcal{U})\} \ominus \frac{1}{\wp}\frac{\partial}{\partial\mathfrak{p}}\mathfrak{S}(0,\mathcal{U})$.

[d]. $\widehat{A_2}[\dagger_{\mathfrak{t}\mathfrak{t}}{}^{(2)}(\mathfrak{p},\mathfrak{t})] = \{\mathcal{U}^2\widehat{A_2}[\dagger(\mathfrak{p},\mathfrak{t})] \ominus \mathfrak{S}(\wp,0)\} \ominus \frac{1}{\mathcal{U}}\frac{\partial}{\partial\mathfrak{t}}\mathfrak{S}(\wp,0)$.

[e]. $\widehat{A_2}[\dagger_{\mathfrak{p}\mathfrak{t}}{}^{(2)}(\mathfrak{p},\mathfrak{t})] = \left\{\wp\mathcal{U}\widehat{A_2}[\dagger(\mathfrak{p},\mathfrak{t})] \ominus \frac{\wp}{\mathcal{U}}\mathfrak{S}(\wp,0)\right\} \ominus \{\frac{\mathcal{U}}{\wp}\mathfrak{S}(0,0) \ominus \frac{1}{\wp\mathcal{U}}\mathfrak{S}(0,0)$,

[f]. $\widehat{A_2}[\dagger_{\mathfrak{t}\mathfrak{p}}{}^{(2)}(\mathfrak{p},\mathfrak{t})] = \left\{\wp\mathcal{U}\widehat{A_2}[\dagger(\mathfrak{p},\mathfrak{t})] \ominus \frac{\mathcal{U}}{\wp}\mathfrak{S}(0,\mathcal{U})\right\} \ominus \{\frac{\wp}{\mathcal{U}}\mathfrak{S}(0,0) \ominus \frac{1}{\wp\mathcal{U}}\mathfrak{S}(0,0)$.

Proof: [a]: For an arbitrarily chosen fixed $\mathfrak{g} \in [0,1]$,

$$\widehat{A_2}[\dagger_{\mathfrak{p}}{}'(\mathfrak{p},\mathfrak{t})] = \left(\frac{1}{\wp}\frac{1}{\mathcal{U}}\int_0^\infty\int_0^\infty \underline{\dagger}_{\mathfrak{p}}(\mathfrak{p},\mathfrak{t},\mathfrak{g})e^{-\wp\mathfrak{p}-\mathcal{U}\mathfrak{t}}d\mathfrak{p}d\mathfrak{t}, \frac{1}{\wp}\frac{1}{\mathcal{U}}\int_0^\infty\int_0^\infty \overline{\dagger}_{\mathfrak{p}}(\mathfrak{p},\mathfrak{t},\mathfrak{g})e^{-\wp\mathfrak{p}-\mathcal{U}\mathfrak{t}}d\mathfrak{p}d\mathfrak{t}\right) =$$

$$\left(\frac{1}{\mathcal{U}}\int_0^\infty e^{-\mathcal{U}\mathfrak{t}}\left(\frac{1}{\wp}\int_0^\infty \underline{\dagger}_{\mathfrak{p}}(\mathfrak{p},\mathfrak{t},\mathfrak{g})e^{-\wp\mathfrak{p}}d\mathfrak{p}\right)d\mathfrak{t}, \frac{1}{\mathcal{U}}\int_0^\infty e^{-\mathcal{U}\mathfrak{t}}\left(\frac{1}{\wp}\int_0^\infty \overline{\dagger}_{\mathfrak{p}}(\mathfrak{p},\mathfrak{t},\mathfrak{g})e^{-\wp\mathfrak{p}}d\mathfrak{p}\right)d\mathfrak{t}\right) =$$

$$\left(\frac{1}{\mathcal{U}}\int_0^\infty e^{-\mathcal{U}\mathfrak{t}}\left(\wp A_2\left[\underline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})\right] - \frac{1}{\wp}\underline{\dagger}(0,\mathfrak{t},\mathfrak{g})\right)d\mathfrak{t}, \frac{1}{\mathcal{U}}\int_0^\infty e^{-\mathcal{U}\mathfrak{t}}\left(\wp A_2[\overline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})] - \frac{1}{\wp}\overline{\dagger}(0,\mathfrak{t},\mathfrak{g})\right)d\mathfrak{t}\right) =$$

$$\left(\wp A_2\left[\underline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})\right] - \frac{1}{\wp}\underline{\mathfrak{S}}(0,\mathcal{U},\mathfrak{g}), \wp A_2[\overline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})] - \frac{1}{\wp}\overline{\mathfrak{S}}(0,\mathcal{U},\mathfrak{g})\right) = \wp\widehat{A_2}[\dagger(\mathfrak{p},\mathfrak{t})] \ominus \frac{1}{\wp}\mathfrak{S}(0,\mathcal{U}).$$

[b] For an arbitrary fixed $\mathfrak{g} \in [0,1]$,

$$\widehat{A_2}[\dagger_{\mathfrak{t}}{}'(\mathfrak{p},\mathfrak{t})] = \left(\frac{1}{\wp}\frac{1}{\mathcal{U}}\int_0^\infty\int_0^\infty \underline{\dagger}_{\mathfrak{p}}(\mathfrak{p},\mathfrak{t},\mathfrak{g})e^{-\wp\mathfrak{p}-\mathcal{U}\mathfrak{t}}d\mathfrak{p}d\mathfrak{t}, \frac{1}{\wp}\frac{1}{\mathcal{U}}\int_0^\infty\int_0^\infty \overline{\dagger}_{\mathfrak{p}}(\mathfrak{p},\mathfrak{t},\mathfrak{g})e^{-\wp\mathfrak{p}-\mathcal{U}\mathfrak{t}}d\mathfrak{p}d\mathfrak{t}\right)$$

$$= \left(\frac{1}{\wp}\int_0^\infty e^{-\wp\mathfrak{p}}\left(\frac{1}{\mathcal{U}}\int_0^\infty \underline{\dagger}_{\mathfrak{t}}(\mathfrak{p},\mathfrak{t},\mathfrak{g})e^{-\mathcal{U}\mathfrak{t}}d\mathfrak{t}\right)d\mathfrak{p}, \frac{1}{\wp}\int_0^\infty e^{-\wp\mathfrak{p}}\left(\frac{1}{\mathcal{U}}\int_0^\infty \overline{\dagger}_{\mathfrak{t}}(\mathfrak{p},\mathfrak{t},\mathfrak{g})e^{-\mathcal{U}\mathfrak{t}}d\mathfrak{t}\right)d\mathfrak{p}\right)$$

$$= \left(\frac{1}{\wp}\int_0^\infty e^{-\wp\mathfrak{p}}\left(\mathcal{U}A_2\left[\underline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})\right] - \frac{1}{\mathcal{U}}\underline{\dagger}(\mathfrak{p},0,\mathfrak{g})\right)d\mathfrak{p}, \frac{1}{\wp}\int_0^\infty e^{-\wp\mathfrak{p}}\left(\mathcal{U}A_2[\overline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})]\right.\right.$$

$$\left.\left. - \frac{1}{\mathcal{U}}\overline{\dagger}(\mathfrak{p},0,\mathfrak{g})d\mathfrak{p}\right)\right) = \left(\mathcal{U}A_2\left[\underline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})\right] - \frac{1}{\mathcal{U}}\underline{\mathfrak{S}}(\wp,0,\mathfrak{g}), \mathcal{U}A_2[\overline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})] - \frac{1}{\mathcal{U}}\overline{\mathfrak{S}}(\wp,0,\mathfrak{g})\right)$$

$$= \mathcal{U}\widehat{A_2}[\dagger(\mathfrak{p},\mathfrak{t})] \ominus \frac{1}{\mathcal{U}}\mathfrak{S}(\wp,0)$$

Same proofs apply to other circumstances.

(3.1) Example: Take into consideration the fuzzy partial differential equation provided by

$\dagger_{\mathfrak{p}}{}'(\mathfrak{p},\mathfrak{t}) = 3\dagger_{\mathfrak{t}}{}'(\mathfrak{p},\mathfrak{t}), \qquad \mathfrak{p} \geq 0, \mathfrak{t} \geq 0$

With initial conditions, $\dagger(\mathfrak{p},0) = e^{-2\mathfrak{p}}(\mathfrak{g}-1, 1-\mathfrak{g}), \dagger(0,\mathfrak{t}) = e^{-6\mathfrak{t}}(\mathfrak{g}-1, 1-\mathfrak{g})$.

Solution: Apply double fuzzy Aboodh transform on both sides, to get

$$\widehat{A_2}[\dagger_{\mathfrak{p}}{}'(\mathfrak{p},\mathfrak{t})] = \widehat{A_2}[3\dagger_{\mathfrak{t}}{}'(\mathfrak{p},\mathfrak{t})]$$

$$\wp\widehat{A_2}[\dagger(\mathfrak{p},\mathfrak{t})] \ominus \frac{1}{\wp}\mathfrak{S}(0,\mathcal{U}) = \mathcal{U}\widehat{A_2}[\dagger(\mathfrak{p},\mathfrak{t})] \ominus \frac{1}{\mathcal{U}}\mathfrak{S}(\wp,0)$$

Using upper and lower functions, to have

$$\left(\wp A_2\left[\underline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})\right] - \frac{1}{\wp}\underline{\mathfrak{S}}(0,\mathcal{U},\mathfrak{g}), \wp A_2[\overline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})] - \frac{1}{\wp}\overline{\mathfrak{S}}(0,\mathcal{U},\mathfrak{g})\right)$$

$$= \left(\mathcal{U}A_2\left[\underline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})\right] - \frac{1}{\mathcal{U}}\underline{\mathfrak{S}}(\wp,0,\mathfrak{g}), \mathcal{U}A_2[\overline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})] - \frac{1}{\mathcal{U}}\overline{\mathfrak{S}}(\wp,0,\mathfrak{g})\right)$$

$$[\wp A\left[\underline{\dagger}(\mathfrak{p},\mathfrak{t})\right] \ominus \frac{1}{\wp}\mathfrak{S}(0,\mathcal{U}) = \mathcal{U}A[\dagger(\mathfrak{p},\mathfrak{t})] \ominus \frac{1}{\mathcal{U}}\mathfrak{S}(\wp,0)$$

$$\wp A_2\left[\underline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})\right] - \frac{1}{\wp}\underline{\mathfrak{S}}(0,\mathcal{U},\mathfrak{g}) = \mathcal{U}A_2\left[\underline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})\right] - \frac{1}{\mathcal{U}}\underline{\mathfrak{S}}(\wp,0,\mathfrak{g})$$

$$\wp A_2[\overline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})] - \frac{1}{\wp}\overline{\mathfrak{S}}(0,\mathcal{U},\mathfrak{g}) = \mathcal{U}A_2[\overline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})] - \frac{1}{\mathcal{U}}\overline{\mathfrak{S}}(\wp,0,\mathfrak{g})$$

$$(\wp-\mathcal{U})A_2\left[\underline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})\right] = \frac{1}{\wp}\underline{\mathfrak{S}}(0,\mathcal{U},\mathfrak{g}) - \frac{1}{\mathcal{U}}\underline{\mathfrak{S}}(\wp,0,\mathfrak{g})$$

$$(\wp-\mathcal{U})A_2[\overline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})] = \frac{1}{\wp}\overline{\mathfrak{S}}(0,\mathcal{U},\mathfrak{g}) - \frac{1}{\mathcal{U}}\overline{\mathfrak{S}}(\wp,0,\mathfrak{g})$$

$$(\wp-\mathcal{U})\mathrm{A}_2\left[\underline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})\right] = \frac{1}{\wp}\mathrm{e}^{-2\mathcal{U}}(\mathfrak{g}-1) - \frac{1}{\mathcal{U}}\mathrm{e}^{-6\wp}(\mathfrak{g}-1)$$

$$(\wp-\mathcal{U})\mathrm{A}_2[\overline{\dagger}(\mathfrak{p},\mathfrak{t},\mathfrak{g})] = \frac{1}{\wp}\mathrm{e}^{-2\mathcal{U}}(1-\mathfrak{g}) - \frac{1}{\mathcal{U}}\mathrm{e}^{-6\wp}(1-\mathfrak{g})$$

$$\mathrm{A}_2\left[\underline{ƭ}(ƀ, ţ, ɠ)\right] = (\wp - \mathcal{U})\left(\frac{1}{\wp}\mathrm{e}^{-2\mathcal{U}}(ɠ - 1) - \frac{1}{\mathcal{U}}\mathrm{e}^{-6\wp}(ɠ - 1)\right)$$

$$\mathrm{A}_2\left[\overline{ƭ}(ƀ, ţ, ɠ)\right] = (\wp - \mathcal{U})\left(\frac{1}{\wp}\mathrm{e}^{-2\mathcal{U}}(1 - ɠ) - \frac{1}{\mathcal{U}}\mathrm{e}^{-6\wp}(1 - ɠ)\right)$$

With simple calculation and applying the inverse double fuzzy Aboodh transform

$$\left[\underline{ƭ}(ƀ, ţ)\right] = \mathrm{e}^{-2ƀ-6ţ}(ɠ - 1), \left[\overline{ƭ}(ƀ, ţ)\right] = \mathrm{e}^{-2ƀ-6ţ}(1 - ɠ)$$

4. CONCLUSIONS

This paper's primary goal is to solve fuzzy partial differential equations under gH-differentiability by developing the double fuzzy Aboodh transform. We present and demonstrate the fundamental characteristics of the novel double fuzzy transformation. We acquire new findings on the double fuzzy Aboodh transform for fuzzy partial gH-derivatives. We offer an example that demonstrates the usefulness and efficacy of the double fuzzy Aboodh transform in solving fuzzy partial differential equations. The study's results show that the double fuzzy Aboodh transform is both effective and simple to apply for solving fuzzy partial differential equations.

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