Some New Results for the $\mathcal{M}$-Transform Involving the Incomplete H- and $\overline{H}$-Functions

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Abstract

In this paper, we construct some new image formulas for the incomplete H-and $\overline{H}$-functions under the Akel's $\mathcal{M}$-transform. We also provide image formulas for the incomplete Meijer's G-functions, incomplete Fox-Wright functions and Fox's H-function, as special cases of our main findings in corollaries.

Key Words and Phrases. Incomplete gamma function; $\mathcal{M}$-transform; Incomplete H-functions; Incomplete $\overline{H}$-functions; Mellin-Barnes type contour; Incomplete Fox-Wright generalized hypergeometric functions.
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1 Introduction and Preliminaries

Integral transforms have been useful in solving numerous differential and integral problems for many years. It is possible to convert differential and integral operators from one domain under consideration into multiplication operators in another domain by using the right integral transform.

The Laplace transform, the Fourier integral transform, the Mellin transform are the classical integral transforms used to solve differential equations, integral equations, and in analysis and the theory of functions. For further information, see the research papers [5, 10, 12, 13].

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Akel's $\mathcal{M}$-transform

Akel in [1] recently, introduced the following $\mathcal{M}$-transform in this sequence:

$$\mathcal{M}_{\rho,m}[f(x)](u,v,w) = \int_0^\infty \frac{e^{-ux-\frac{v}{x}}}{(x^m+w^m)^\rho} f(wx)\,dx, \tag{1}$$

with $\rho \in \mathbb{C}, \mathfrak{Re}(\rho) > 0, m \in \mathbb{N}$ and $u, v \in \mathbb{C},\ w \in \mathbb{R}^+$ are called the transform variables.

The $\mathcal{M}$-transform given by (1), depends on a number of parameters, so that it covers many known integral transforms as its special cases. This transform has the duality relations with well-known transforms such as the Laplace transform, the natural transform and the Srivastava-Luo-Raina $\mathbb{M}$-transform.
This transform is a precious tool for solving certain initial and boundary value problems with certain variable coefficients. Additional ionformation on this transform, may be found in [1].

The incomplete H-and $\overline{H}$-functions

The incomplete H-functions $\gamma^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}$ and $\Gamma^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}$ have studied and defined by Srivastava et al. [13] in the form of Mellin-Barness contour integral as follow:

$$\gamma^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}(z) = \gamma^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[z \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1,y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix} \right] = (2\pi\iota)^{-1} \int_{\pounds} g(\vartheta,y)\; z^{-\vartheta}\; d\vartheta, \tag{2}$$

and

$$\Gamma^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}(z) = \Gamma^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[z \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1,y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix} \right] = (2\pi\iota)^{-1} \int_{\pounds} G(\vartheta,y)\; z^{-\vartheta}\; d\vartheta, \tag{3}$$

where,

$$g(\vartheta,y) = \frac{\gamma(1-\mathfrak{g}_1-\nu_1\vartheta,y) \prod\limits_{\mathsf{j}=1}^{\mathsf{m}} \Gamma(\mathfrak{h}_\mathsf{j}+\omega_\mathsf{j}\vartheta) \prod\limits_{\mathsf{j}=2}^{\mathsf{n}} \Gamma(1-\mathfrak{g}_\mathsf{j}-\nu_\mathsf{j}\vartheta)}{\prod\limits_{\mathsf{j}=\mathsf{m}+1}^{\mathsf{q}} \Gamma(1-\mathfrak{h}_\mathsf{j}-\omega_\mathsf{j}\vartheta) \prod\limits_{\mathsf{j}=\mathsf{n}+1}^{\mathsf{p}} \Gamma(\mathfrak{g}_\mathsf{j}+\nu_\mathsf{j}\vartheta)}, \tag{4}$$

and

$$G(\vartheta,y) = \frac{\Gamma(1-\mathfrak{g}_1-\nu_1\vartheta,y) \prod\limits_{\mathsf{j}=1}^{\mathsf{m}} \Gamma(\mathfrak{h}_\mathsf{j}+\omega_\mathsf{j}\vartheta) \prod\limits_{\mathsf{j}=2}^{\mathsf{n}} \Gamma(1-\mathfrak{g}_\mathsf{j}-\nu_\mathsf{j}\vartheta)}{\prod\limits_{\mathsf{j}=\mathsf{m}+1}^{\mathsf{q}} \Gamma(1-\mathfrak{h}_\mathsf{j}-\omega_\mathsf{j}\vartheta) \prod\limits_{\mathsf{j}=\mathsf{n}+1}^{\mathsf{p}} \Gamma(\mathfrak{g}_\mathsf{j}+\nu_\mathsf{j}\vartheta)}. \tag{5}$$

This family of incomplete H-functions characterized as (2) and (3) exist for $x \geq 0$, according to the conditions specified by Srivastava [13].

Srivastava in [13] developed a generalisation for the family of incomplete H-functions, referred as the incomplete $\overline{H}$-functions, which is described by:

$$\overline{\gamma}^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}(z) = \overline{\gamma}^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[z \,\middle|\, \begin{array}{c} (\mathfrak{g}_1, \nu_1; \mathfrak{G}_1; y),\, (\mathfrak{g}_\mathsf{j}, \nu_\mathsf{j}; \mathfrak{G}_\mathsf{j})_{2,\,\mathsf{n}},\, (\mathfrak{g}_\mathsf{j}, \nu_\mathsf{j})_{\mathsf{n}+1,\,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j}, \omega_\mathsf{j})_{1,\mathsf{m}},\, (\mathfrak{h}_\mathsf{j}, \omega_\mathsf{j}; \mathfrak{H}_\mathsf{j})_{\mathsf{m}+1,\,\mathsf{q}} \end{array} \right]$$
$$= (2\pi\iota)^{-1} \int_{\pounds} \overline{g}(\vartheta, y)\, z^{-\vartheta}\, d\vartheta, \tag{6}$$

and

$$\overline{\Gamma}^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}(z) = \overline{\Gamma}^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[z \,\middle|\, \begin{array}{c} (\mathfrak{g}_1, \nu_1; \mathfrak{G}_1; y),\, (\mathfrak{g}_\mathsf{j}, \nu_\mathsf{j}; \mathfrak{G}_\mathsf{j})_{2,\,\mathsf{n}},\, (\mathfrak{g}_\mathsf{j}, \nu_\mathsf{j})_{\mathsf{n}+1,\,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j}, \omega_\mathsf{j})_{1,\mathsf{m}},\, (\mathfrak{h}_\mathsf{j}, \omega_\mathsf{j}; \mathfrak{H}_\mathsf{j})_{\mathsf{m}+1,\,\mathsf{q}} \end{array} \right]$$
$$= (2\pi\iota)^{-1} \int_{\pounds} \overline{G}(\vartheta, y)\, z^{-\vartheta}\, d\vartheta, \tag{7}$$

where

$$\overline{g}(\vartheta, y) = \frac{[\gamma(1-\mathfrak{g}_1-\nu_1\vartheta, y)]^{\mathfrak{G}_1} \prod_{\mathsf{j}=1}^{\mathsf{m}} \Gamma(\mathfrak{h}_\mathsf{j}+\omega_\mathsf{j}\vartheta) \prod_{\mathsf{j}=2}^{\mathsf{n}} [\Gamma(1-\mathfrak{g}_\mathsf{j}-\nu_\mathsf{j}\vartheta)]^{\mathfrak{G}_\mathsf{j}}}{\prod_{\mathsf{j}=\mathsf{m}+1}^{\mathsf{q}} [\Gamma(1-\mathfrak{h}_\mathsf{j}-\omega_\mathsf{j}\vartheta)]^{\mathfrak{H}_\mathsf{j}} \prod_{\mathsf{j}=\mathsf{n}+1}^{\mathsf{p}} \Gamma(\mathfrak{g}_\mathsf{j}+\nu_\mathsf{j}\vartheta)}, \tag{8}$$

and

$$\overline{G}(\vartheta, y) = \frac{[\Gamma(1-\mathfrak{g}_1-\nu_1\vartheta, y)]^{\mathfrak{G}_1} \prod_{\mathsf{j}=1}^{\mathsf{m}} \Gamma(\mathfrak{h}_\mathsf{j}+\omega_\mathsf{j}\vartheta) \prod_{\mathsf{j}=2}^{\mathsf{n}} [\Gamma(1-\mathfrak{g}_\mathsf{j}-\nu_\mathsf{j}\vartheta)]^{\mathfrak{G}_\mathsf{j}}}{\prod_{\mathsf{j}=\mathsf{m}+1}^{\mathsf{q}} [\Gamma(1-\mathfrak{h}_\mathsf{j}-\omega_\mathsf{j}\vartheta)]^{\mathfrak{H}_\mathsf{j}} \prod_{\mathsf{j}=\mathsf{n}+1}^{\mathsf{p}} \Gamma(\mathfrak{g}_\mathsf{j}+\nu_\mathsf{j}\vartheta)}. \tag{9}$$

Numerous authors are actively working on the development and wide variety of implications for these incomplete functions, such as in [3, 15], authors established modified saigo fractional integral operators involving the product of a general class of multivariable polynomials and the multivariable H-function and an integral operator involving the family of incomplete H-function in its kernel, respectively. The authors of [11] investigated applications of the incomplete H-function on the influence of environmental pollution on the occurrence of biological populations, whereas the authors of [6, 7] developed an equation of internal blood pressure involving incomplete $\overline{H}$-functions and specific expansion formulae for the incomplete H-functions.
The main purpose of this paper is to give new image formulas for incomplete H- and $\overline{H}$-functions under Akel's $\mathcal{M}$-transform. And by giving suitable values to the involved parameters, we also present some special cases of our main findings.

The paper is organized in the following way. In Section 2, we establish the Akel's $\mathcal{M}$-transform image formulae for the incomplete H- and $\overline{H}$-functions. In Section 3 , we derive some interesting and important special cases of our main findings. Finally, a brief conclusion in Section 4.

2 The $\mathcal{M}$-Transform of Incomplete H- and $\overline{H}$-Functions

In this segment, we establish new image formulas for the incomplete H- and $\overline{H}$-functions under the Akel's $\mathcal{M}$-transform.

Theorem 1. *If $\rho \in \mathbb{C}, \mathfrak{Re}(\rho) > 0, m \in \mathbb{N}$ and $u, v \in \mathbb{C}$, $w \in \mathbb{R}^+$, then the following image formula exists for $\gamma_{\mathsf{p},\,\mathsf{q}}^{\mathsf{m},\,\mathsf{n}}[z]$:*

$$\mathcal{M}_{\rho,m}\left\{\gamma_{\mathsf{p},\,\mathsf{q}}^{\mathsf{m},\,\mathsf{n}}\left[zx \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1,y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix}\right]\right\}(u,v,w) = \frac{w^{-m\rho}}{u\,m}\,\frac{1}{2\pi\iota}\int_{\pounds}\mathbb{B}\left(\rho-\frac{\xi}{m},\frac{\xi}{m}\right)(uw)^{\xi}$$
$$\gamma_{\mathsf{p}+1,\,\mathsf{q}}^{\mathsf{m},\,\mathsf{n}+1}\left[z\frac{w}{u} \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1,y),(\xi,1)_{uv},(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix}\right]d\xi. \tag{10}$$

Here, $\mathbb{B}(x,y)$ represents the classical Euler-Beta function.

Proof. To get the result (1), first we take the L.H.S of (10) and use the definition (1), we have

$$\mathcal{M}_{\rho,m}\left\{\gamma_{\mathsf{p},\,\mathsf{q}}^{\mathsf{m},\,\mathsf{n}}\left[zx \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1,y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix}\right]\right\}(u,v,w) = \int_0^\infty \frac{e^{-ux-\frac{v}{x}}}{(x^m+w^m)^\rho}\,\gamma_{\mathsf{p},\,\mathsf{q}}^{\mathsf{m},\,\mathsf{n}}(zw\,x)\,dx$$
$$= \int_0^\infty \frac{e^{-ux-\frac{v}{x}}}{(x^m+w^m)^\rho}\,\frac{1}{2\pi\iota}\int_{\pounds} g(\vartheta,y)z^{-\vartheta}(wx)^{-\vartheta}\,d\vartheta\,dx$$

On interchanging the orders of the integration

$$= \frac{1}{2\pi\iota}\int_{\pounds} g(\vartheta,y)z^{-\vartheta}\int_0^\infty \frac{e^{-ux-\frac{v}{x}}}{(x^m+w^m)^\rho}(wx)^{-\vartheta}\,dx\,d\vartheta$$

Now, on utilizing [1, pg. 6, Eqn. (2.11)], we get

$$= \frac{1}{2\pi\iota}\int_{\pounds} g(\vartheta,y)z^{-\vartheta}\,\frac{w^{-\vartheta-m\rho}\,u^{\vartheta-1}}{m\,\Gamma(\rho)}\,H_{1,\,2}^{2,\,1}\left[u\,w \,\middle|\, \begin{matrix} (1,\frac{1}{m}) \\ (1-\vartheta,1)_{uv},(\rho,\frac{1}{m}) \end{matrix}\right]d\vartheta$$
$$= \frac{w^{-m\rho}}{um\,\Gamma(\rho)}\frac{1}{2\pi\iota}\int_{\pounds} g(\vartheta,y)\left(z\frac{w}{u}\right)^{-\vartheta}\frac{1}{2\pi\iota}\int_{-\iota\infty}^{+\iota\infty}(u\,w)^{\xi}\Gamma(\frac{\xi}{m})\Gamma_{uv}(1-\xi-\vartheta)\Gamma(\rho-\frac{\xi}{m})d\xi\,d\vartheta$$

On changing the order of the integrations and after some adjustment of terms

$$= \frac{w^{-m\rho}}{u\,m}\,\frac{1}{2\pi\iota}\int_{\pounds}\frac{\Gamma(\frac{\xi}{m})\,\Gamma(\rho-\frac{\xi}{m})}{\Gamma(\rho)}(uw)^{\xi}\,\frac{1}{2\pi\iota}\int_{\pounds} g(\vartheta,y)\,\Gamma_{uv}(1-\xi-\vartheta)\left(z\frac{w}{u}\right)^{-\vartheta}d\vartheta\,d\xi, \tag{11}$$

using (2), we obtain the required R.H.S of (10). □

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Theorem 2. *If $\rho \in \mathbb{C}, \mathfrak{Re}(\rho) > 0, m \in \mathbb{N}$ and $u, v \in \mathbb{C}$, $w \in \mathbb{R}^+$, then the following image formula exists for $\Gamma^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}[z]$:*

$$\mathcal{M}_{\rho,m}\left\{\Gamma^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[zx \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1,y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix}\right]\right\}(u,v,w) = \frac{w^{-m\rho}}{u\,m}\frac{1}{2\pi\iota}\int_{\pounds}\mathbb{B}\left(\rho-\frac{\xi}{m},\frac{\xi}{m}\right)(uw)^{\xi}$$
$$\Gamma^{\mathsf{m},\,\mathsf{n}+1}_{\mathsf{p}+1,\,\mathsf{q}}\left[z\frac{w}{u} \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1,y),(\xi,1)_{uv},(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix}\right]d\xi. \tag{12}$$

Here, $\mathbb{B}(x, y)$ represents the classical Euler-Beta function.

Proof. To get the result (12), we take the Akel's $\mathcal{M}$-transform presented in (1) of (3), then on interchanging the order of the integrations and making use of the known result given in [1, p. 6, Eq. (2.11)], and after some small arrangements of the terms, we easily get the right hand side assertion of (12). ☐

Theorem 3. *If $\rho \in \mathbb{C}, \mathfrak{Re}(\rho) > 0, m \in \mathbb{N}$ and $u, v \in \mathbb{C}$, $w \in \mathbb{R}^+$, then the following image formula exists for $\overline{\gamma}^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}[z]$:*

$$\mathcal{M}_{\rho,m}\left\{\overline{\gamma}^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[zx \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1;\mathfrak{G},y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j};\mathfrak{G}_\mathsf{j})_{2,\mathsf{n}},(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{\mathsf{n}+1,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{m}},(\mathfrak{h}_\mathsf{j},\omega_\mathsf{j};\mathfrak{H}_\mathsf{j})_{\mathsf{m}+1,\mathsf{q}} \end{matrix}\right]\right\}(u,v,w)$$
$$= \frac{w^{-m\rho}}{u\,m}\frac{1}{2\pi\iota}\int_{\pounds}\mathbb{B}\left(\rho-\frac{\xi}{m},\frac{\xi}{m}\right)(uw)^{\xi}$$
$$\overline{\gamma}^{\mathsf{m},\,\mathsf{n}+1}_{\mathsf{p}+1,\,\mathsf{q}}\left[z\frac{w}{u} \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1;\mathfrak{G},y),(\xi,1;1)_{uv},(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j};\mathfrak{G}_\mathsf{j})_{2,\mathsf{n}},(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{\mathsf{n}+1,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{m}},(\mathfrak{h}_\mathsf{j},\omega_\mathsf{j};\mathfrak{H}_\mathsf{j})_{\mathsf{m}+1,\mathsf{q}} \end{matrix}\right]d\xi. \tag{13}$$

Here, $\mathbb{B}(x, y)$ represents the classical Euler-Beta function.

Proof. To get the result (3), first we take the L.H.S of (13) and use the definition (1), we have

$$\mathcal{M}_{\rho,m}\left\{\overline{\gamma}^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[zx \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1;\mathfrak{G}_1;y),\ (\mathfrak{g}_\mathsf{j},\nu_\mathsf{j};\mathfrak{G}_\mathsf{j})_{2,\,\mathsf{n}},\ (\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{\mathsf{n}+1,\,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{m}},\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j};\mathfrak{H}_\mathsf{j})_{\mathsf{m}+1,\,\mathsf{q}} \end{matrix}\right]\right\}(u,v,w)$$
$$= \int_0^\infty \frac{e^{-ux-\frac{v}{x}}}{(x^m+w^m)^\rho}\,\overline{\gamma}^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}(zw\,x)\,dx$$
$$= \int_0^\infty \frac{e^{-ux-\frac{v}{x}}}{(x^m+w^m)^\rho}\,\frac{1}{2\pi\iota}\int_{\pounds}\overline{g}(\vartheta,y)z^{-\vartheta}(wx)^{-\vartheta}\,d\vartheta\,dx$$

On interchanging the orders of the integration

$$= \frac{1}{2\pi\iota}\int_{\pounds}\overline{g}(\vartheta,y)z^{-\vartheta}\int_0^\infty \frac{e^{-ux-\frac{v}{x}}}{(x^m+w^m)^\rho}(wx)^{-\vartheta}\,dx\,d\vartheta$$

Now, on utilizing [1, pg. 6, Eqn. (2.11)], we get

$$= \frac{1}{2\pi\iota}\int_{\pounds} \overline{g}(\vartheta, y) z^{-\vartheta} \frac{w^{-\vartheta-m\rho}\, u^{\vartheta-1}}{m\,\Gamma(\rho)} H^{2,\,1}_{1,\,2}\left[u\,w \;\middle|\; \begin{matrix} (1, \frac{1}{m}) \\ (1-\vartheta, 1)_{uv}, (\rho, \frac{1}{m}) \end{matrix}\right] d\vartheta$$

$$= \frac{w^{-m\rho}}{um\,\Gamma(\rho)} \frac{1}{2\pi\iota}\int_{\pounds} \overline{g}(\vartheta, y) \left(z\frac{w}{u}\right)^{-\vartheta} \frac{1}{2\pi\iota}\int_{-\iota\infty}^{+\iota\infty} (u\,w)^{\xi}\Gamma(\frac{\xi}{m})\Gamma_{uv}(1-\xi-\vartheta)\Gamma(\rho-\frac{\xi}{m}) d\xi\, d\vartheta$$

On changing the order of the integrations and after some adjustment of terms

$$= \frac{w^{-m\rho}}{u\,m}\,\frac{1}{2\pi\iota}\int_{\pounds} \frac{\Gamma(\frac{\xi}{m})\,\Gamma(\rho-\frac{\xi}{m})}{\Gamma(\rho)} (uw)^{\xi}\, \frac{1}{2\pi\iota}\int_{\pounds} \overline{g}(\vartheta, y)\,\Gamma_{uv}(1-\xi-\vartheta) \left(z\frac{w}{u}\right)^{-\vartheta} d\vartheta\, d\xi, \tag{14}$$

using (6), we obtain the required R.H.S of (13). □

Theorem 4. *If $\rho \in \mathbb{C}, \Re\mathfrak{e}(\rho) > 0, m \in \mathbb{N}$ and $u, v \in \mathbb{C}$, $w \in \mathbb{R}^+$, then the following image formula exists for $\overline{\Gamma}^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}[z]$:*

$$\mathcal{M}_{\rho,m}\left\{\overline{\Gamma}^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[zx \;\middle|\; \begin{matrix} (\mathfrak{g}_1, \nu_1; \mathfrak{G}, y), (\mathfrak{g}_{\mathsf{j}}, \nu_{\mathsf{j}}; \mathfrak{G}_{\mathsf{j}})_{2,\mathsf{n}}, (\mathfrak{g}_{\mathsf{j}}, \nu_{\mathsf{j}})_{\mathsf{n}+1,\mathsf{p}} \\ (\mathfrak{h}_{\mathsf{j}}, \omega_{\mathsf{j}})_{1,\mathsf{m}}, (\mathfrak{h}_{\mathsf{j}}, \omega_{\mathsf{j}}; \mathfrak{H}_{\mathsf{j}})_{\mathsf{m}+1,\mathsf{q}} \end{matrix}\right]\right\}(u, v, w)$$
$$= \frac{w^{-m\rho}}{u\,m}\,\frac{1}{2\pi\iota}\int_{\pounds} \mathbb{B}\left(\rho-\frac{\xi}{m}, \frac{\xi}{m}\right)(uw)^{\xi}$$
$$\overline{\Gamma}^{\mathsf{m},\,\mathsf{n}+1}_{\mathsf{p}+1,\,\mathsf{q}}\left[z\frac{w}{u} \;\middle|\; \begin{matrix} (\mathfrak{g}_1, \nu_1; \mathfrak{G}, y), (\xi, 1; 1)_{uv}, (\mathfrak{g}_{\mathsf{j}}, \nu_{\mathsf{j}}; \mathfrak{G}_{\mathsf{j}})_{2,\mathsf{n}}, (\mathfrak{g}_{\mathsf{j}}, \nu_{\mathsf{j}})_{\mathsf{n}+1,\mathsf{p}} \\ (\mathfrak{h}_{\mathsf{j}}, \omega_{\mathsf{j}})_{1,\mathsf{m}}, (\mathfrak{h}_{\mathsf{j}}, \omega_{\mathsf{j}}; \mathfrak{H}_{\mathsf{j}})_{\mathsf{m}+1,\mathsf{q}} \end{matrix}\right] d\xi. \tag{15}$$

Here, $\mathbb{B}(x, y)$ represents the classical Euler-Beta function.

Proof. To get the result (15), we take the Akel's $\mathcal{M}$-transform presented in (1) of (7), then on interchanging the order of the integrations and making use of the known result given in [1, p. 6, Eq. (2.11)], and after some small arrangements of the terms, we easily get the right hand side assertion of (15). □

3 Special Cases

In this section, we derive some interesting and important special cases of our main findings by giving some particular values to the parameters involved in the definitions of $\mathcal{M}$-transform (1) and incomplete H-functions (2) and (3).
(1) Taking $\mathsf{n} = \mathsf{p}$, $\mathsf{m} = 1$, substitute q with $\mathsf{q}+1$ and choosing appropriate parameters such as $z = -z$, $\mathfrak{g}_{\mathsf{j}} \to (1-\mathfrak{g}_{\mathsf{j}})$ $(\mathsf{j} = 1, \cdots, \mathsf{p})$, and $\mathfrak{h}_{\mathsf{j}} \to (1-\mathfrak{h}_{\mathsf{j}})$ $(\mathsf{j} = 1, \cdots, \mathsf{q})$, the incomplete H-functions (2) and (3) convert, respectively, to the incomplete Fox-Wright ${}_{\mathsf{p}}\Psi^{(\gamma)}_{\mathsf{q}}$- and ${}_{\mathsf{p}}\Psi^{(\Gamma)}_{\mathsf{q}}$-functions (see [13, Eqs. (6.3)

and (6.4)]):

$$\gamma^{1,\,\mathsf{p}}_{\mathsf{p},\,\mathsf{q}+1}\left[-z\;\middle|\;\begin{matrix}(1-\mathfrak{g}_1,\nu_1,y),(1-\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}}\\(0,1),(1-\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}}\end{matrix}\right]={}_{\mathsf{p}}\Psi^{(\gamma)}_{\mathsf{q}}\left[\begin{matrix}(\mathfrak{g}_1,\nu_1,y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}}\,;\\(\mathfrak{g}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}}\,;\end{matrix}\;z\right] \tag{16}$$

and

$$\Gamma^{1,\,\mathsf{p}}_{\mathsf{p},\,\mathsf{q}+1}\left[-z\;\middle|\;\begin{matrix}(1-\mathfrak{g}_1,\nu_1,y),(1-\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}}\\(0,1),(1-\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}}\end{matrix}\right]={}_{\mathsf{p}}\Psi^{(\Gamma)}_{\mathsf{q}}\left[\begin{matrix}(\mathfrak{g}_1,\nu_1,y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}}\,;\\(\mathfrak{g}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}}\,;\end{matrix}\;z\right]. \tag{17}$$

Using above relations (16) and (17), in (10) and (12), respectively, we will get the following corollaries.

Corollary 1. *If $\rho\in\mathbb{C},\mathfrak{Re}(\rho)>0, m\in\mathbb{N}$ and $u,v\in\mathbb{C}$, $w\in\mathbb{R}^+$, then the following image formulae exist for ${}_{\mathsf{p}}\Psi^{(\gamma)}_{\mathsf{q}}[z]$ and ${}_{\mathsf{p}}\Psi^{(\Gamma)}_{\mathsf{q}}[z]$:*

$$\mathcal{M}_{\rho,m}\left\{{}_{\mathsf{p}}\Psi^{(\gamma)}_{\mathsf{q}}\left[zx\;\middle|\;\begin{matrix}(\mathfrak{g}_1,\nu_1,y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}}\\(\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}}\end{matrix}\right]\right\}(u,v,w)=\frac{w^{-m\rho}}{u\,m}\frac{1}{2\pi\iota}\int_{\pounds}\mathbb{B}\left(\rho-\frac{\xi}{m},\frac{\xi}{m}\right)(uw)^{\xi}$$
$${}_{\mathsf{p}+1}\Psi^{(\gamma)}_{\mathsf{q}}\left[z\frac{w}{u}\;\middle|\;\begin{matrix}(\mathfrak{g}_1,\nu_1,y),(\xi,1)_{uv},(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}}\\(\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}}\end{matrix}\right]d\xi \tag{18}$$

and

$$\mathcal{M}_{\rho,m}\left\{{}_{\mathsf{p}}\Psi^{(\Gamma)}_{\mathsf{q}}\left[zx\;\middle|\;\begin{matrix}(\mathfrak{g}_1,\nu_1,y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}}\\(\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}}\end{matrix}\right]\right\}(u,v,w)=\frac{w^{-m\rho}}{u\,m}\frac{1}{2\pi\iota}\int_{\pounds}\mathbb{B}\left(\rho-\frac{\xi}{m},\frac{\xi}{m}\right)(uw)^{\xi}$$
$${}_{\mathsf{p}+1}\Psi^{(\Gamma)}_{\mathsf{q}}\left[z\frac{w}{u}\;\middle|\;\begin{matrix}(\mathfrak{g}_1,\nu_1,y),(\xi,1)_{uv},(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}}\\(\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}}\end{matrix}\right]d\xi. \tag{19}$$

Here, $\mathbb{B}(x,y)$ indicates the classical Euler-Beta function.

(2) Letting $(\nu_\mathsf{j})_{1,\mathsf{p}}=1=(\omega_\mathsf{j})_{1,\mathsf{q}}$, the functions (2) and (3) convert into Meijer's incomplete ${}^{(\gamma)}G^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}$- and ${}^{(\Gamma)}G^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}$- functions:

$$\gamma^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}(z)\left[z\;\middle|\;\begin{matrix}(\mathfrak{g}_1,1,y),(\mathfrak{g}_\mathsf{j},1)_{2,\mathsf{p}}\\(\mathfrak{h}_\mathsf{j},1)_{1,\mathsf{q}}\end{matrix}\right]={}^{(\gamma)}G^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[z\;\middle|\;\begin{matrix}(\mathfrak{g}_1,y),(\mathfrak{g}_\mathsf{j})_{2,\mathsf{p}}\\(\mathfrak{h}_\mathsf{j})_{1,\mathsf{q}}\end{matrix}\right] \tag{20}$$

and

$$\Gamma^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}(z)\left[z\;\middle|\;\begin{matrix}(\mathfrak{g}_1,1,y),(\mathfrak{g}_\mathsf{j},1)_{2,\mathsf{p}}\\(\mathfrak{h}_\mathsf{j},1)_{1,\mathsf{q}}\end{matrix}\right]={}^{(\Gamma)}G^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[z\;\middle|\;\begin{matrix}(\mathfrak{g}_1,y),(\mathfrak{g}_\mathsf{j})_{2,\mathsf{p}}\\(\mathfrak{h}_\mathsf{j})_{1,\mathsf{q}}\end{matrix}\right]. \tag{21}$$

Using above relations (20) and (21) in (10) and (12), respectively, we get the following corollaries.

Corollary 2. *If $\rho \in \mathbb{C}, \mathfrak{Re}(\rho) > 0, m \in \mathbb{N}$ and $u, v \in \mathbb{C}$, $w \in \mathbb{R}^+$, then the following image formulae exist for ${}^{(\gamma)}G^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}[z]$ and ${}^{(\Gamma)}G^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}[z]$, respectively:*

$$\mathcal{M}_{\rho,m}\left\{ {}^{(\gamma)}G^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[zx \,\middle|\, \begin{matrix} (\mathfrak{g}_1, 1, y), (\mathfrak{g}_\mathsf{j}, 1)_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j}, 1)_{1,\mathsf{q}} \end{matrix} \right] \right\}(u,v,w) = \frac{w^{-m\rho}}{u\,m}\,\frac{1}{2\pi\iota}\int_{\pounds} \mathbb{B}\left(\rho - \frac{\xi}{m}, \frac{\xi}{m}\right)(uw)^{\xi} \, {}^{(\gamma)}G^{\mathsf{m},\,\mathsf{n}+1}_{\mathsf{p}+1,\,\mathsf{q}}\left[z\frac{w}{u} \,\middle|\, \begin{matrix} (\mathfrak{g}_1, 1, y), (\xi, 1)_{uv}, (\mathfrak{g}_\mathsf{j}, 1)_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j}, 1)_{1,\mathsf{q}} \end{matrix} \right] d\xi \tag{22}$$

and

$$\mathcal{M}_{\rho,m}\left\{ {}^{(\Gamma)}G^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[zx \,\middle|\, \begin{matrix} (\mathfrak{g}_1, 1, y), (\mathfrak{g}_\mathsf{j}, 1)_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j}, 1)_{1,\mathsf{q}} \end{matrix} \right] \right\}(u,v,w) = \frac{w^{-m\rho}}{u\,m}\,\frac{1}{2\pi\iota}\int_{\pounds} \mathbb{B}\left(\rho - \frac{\xi}{m}, \frac{\xi}{m}\right)(uw)^{\xi} \, {}^{(\Gamma)}G^{\mathsf{m},\,\mathsf{n}+1}_{\mathsf{p}+1,\,\mathsf{q}}\left[z\frac{w}{u} \,\middle|\, \begin{matrix} (\mathfrak{g}_1, 1, y), (\xi, 1)_{uv}, (\mathfrak{g}_\mathsf{j}, 1)_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j}, 1)_{1,\mathsf{q}} \end{matrix} \right] d\xi. \tag{23}$$

Here, $\mathbb{B}(x, y)$ indicates the classical Euler-Beta function.

(3) If we put $y = 0$ in (3), we get the Fox's H-function

$$\Gamma^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[z \,\middle|\, \begin{matrix} (\mathfrak{g}_1, \nu_1, 0), (\mathfrak{g}_\mathsf{j}, \nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j}, \omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix} \right] = H^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[z \,\middle|\, \begin{matrix} (\mathfrak{g}_\mathsf{j}, \nu_\mathsf{j})_{1,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j}, \omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix} \right]. \tag{24}$$

Using relation (24), we obtain the subsequent corollaries.

Corollary 3. *If $\rho \in \mathbb{C}, \mathfrak{Re}(\rho) > 0, m \in \mathbb{N}$ and $u, v \in \mathbb{C}$, $w \in \mathbb{R}^+$, then the following image formula exists for $H^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}[z]$:*

$$\mathcal{M}_{\rho,m}\left\{ H^{\mathsf{m},\,\mathsf{n}}_{\mathsf{p},\,\mathsf{q}}\left[zx \,\middle|\, \begin{matrix} (\mathfrak{g}_\mathsf{j}, \nu_\mathsf{j})_{1,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j}, \omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix} \right] \right\}(u,v,w) = \frac{w^{-m\rho}}{u\,m}\,\frac{1}{2\pi\iota}\int_{\pounds} \mathbb{B}\left(\rho - \frac{\xi}{m}, \frac{\xi}{m}\right)(uw)^{\xi} \, H^{\mathsf{m},\,\mathsf{n}+1}_{\mathsf{p}+1,\,\mathsf{q}}\left[z\frac{w}{u} \,\middle|\, \begin{matrix} (\xi, 1)_{uv}, (\mathfrak{g}_\mathsf{j}, \nu_\mathsf{j})_{1,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j}, \omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix} \right] d\xi. \tag{25}$$

Here, $\mathbb{B}(x, y)$ indicates the classical Euler-Beta function.

(4) If we put $v = 0$ in (1), then the Akel's $\mathcal{M}$-transform converts into the Srivastava-Luo-Raina $\mathbb{M}$-transform (see [1]):

$$\mathcal{M}_{\rho,m}[f(x)](u, 0, w) = \mathbb{M}_{\rho,m}[f(x)](u, w), \tag{26}$$

here, $\mathbb{M}_{\rho,m}[f(x)](u, w)$ is the Srivastava-Luo-Raina $\mathbb{M}$-transform, defined in [14]. Using relation (26) in (10) and (12), we obtain the results derived by Bansal et al. [2, p. 720, Eqs. (2.1) and (2.2)],

$$\mathbb{M}_{\rho,m}\left\{\gamma_{\mathsf{p},\,\mathsf{q}}^{\mathsf{m},\,\mathsf{n}}\left[zx \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1,y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix}\right]\right\}(u,w) = \frac{w^{-m\rho}}{u\,m}\frac{1}{2\pi\iota}\int_{\pounds}\mathbb{B}\left(\rho-\frac{\xi}{m},\frac{\xi}{m}\right)(uw)^{\xi} \gamma_{\mathsf{p}+1,\,\mathsf{q}}^{\mathsf{m},\,\mathsf{n}+1}\left[z\frac{w}{u} \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1,y),(\xi,1),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix}\right] d\xi \tag{27}$$

and

$$\mathbb{M}_{\rho,m}\left\{\Gamma_{\mathsf{p},\,\mathsf{q}}^{\mathsf{m},\,\mathsf{n}}\left[zx \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1,y),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix}\right]\right\}(u,w) = \frac{w^{-m\rho}}{u\,m}\frac{1}{2\pi\iota}\int_{\pounds}\mathbb{B}\left(\rho-\frac{\xi}{m},\frac{\xi}{m}\right)(uw)^{\xi} \Gamma_{\mathsf{p}+1,\,\mathsf{q}}^{\mathsf{m},\,\mathsf{n}+1}\left[z\frac{w}{u} \,\middle|\, \begin{matrix} (\mathfrak{g}_1,\nu_1,y),(\xi,1),(\mathfrak{g}_\mathsf{j},\nu_\mathsf{j})_{2,\mathsf{p}} \\ (\mathfrak{h}_\mathsf{j},\omega_\mathsf{j})_{1,\mathsf{q}} \end{matrix}\right] d\xi. \tag{28}$$

Remark. *If we take $v = 0$ in* (18) *and* (19)*, then we get the known results obtained by Bansal et al. [2, p. 720-721, Eqs. (2.3) and (2.4)].*

4 Conclusion

In this paper, we have derived image formulas for the incomplete H- and $\overline{H}$-functions under the Akel's $\mathcal{M}$-transform. Furthermore, from our key findings various special cases can be evaluated by giving suitable values to the involved parameters and variables with applications in engineering and science, some of which are clearly indicated in section 3.

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