

Spherical Fuzzy Cycle and Spherical Fuzzy Tree

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ABSTRACT

Spherical fuzzy end vertices and spherical fuzzy cut vertices is presented in this work along with an example and some of its properties. Spherical fuzzy cycle characterized by spherical fuzzy multim and spherical fuzzy locamin. The spherical fuzzy tree initiated and investigated by the maximum spherical fuzzy spanning subgraph extensively.

Keywords: Spherical fuzzy cut vertices, spherical fuzzy end vertices, spherical fuzzy cycle.

1. INTRODUCTION

Kirchhoff fostered the hypothesis of trees, in 1847 to tackle an arrangement of simultaneous linear equations. After that trees are used in many areas particularly in the computer directories to the great extent. The conceptional idea of fuzzy graph was instigated by Azriel Rosenfeld[1]. He deliberately explained about cutnodes and bridges in the fuzzy environment with the use of weakest and strongest edge. Besides he elucidate about fuzzy trees using fuzzy spanning sugraph. Kiran R and et al., [2] discussed and fuzzy end nodes in fuzzy graphs by strong neighbor. To boot about Multim and Locamin F-cycle. They explained deeply about fuzzy end nodes into the fuzzy trees. Sunitha and Vijayakumar[3] characterized the fuzzy trees and they portrayed fuzzy trees utilizing its unique maximum spanning fuzzy tree. Akram et al.,[4] introduced the notion of spherical fuzzy graphs (SFG). Lavanya T and Amsaveni D[9,10] studied about the primitive concepts of SFG enormously. As well they introduced the concept of connectedness in SFG. Also the spherical fuzzy bridges are established by the level sets in SFG and discussed about their properties. Basic terminologies and definitions are referred for this work from [5,6,7,8,11]. In this connection, the concept of spherical fuzzy cut vertices is introduced and discussed with some of its properties. Additionally, we pioneered spherical fuzzy cycle (SF-cycle) and spherical fuzzy tree (SF-tree) with the use of strongest edge, weakest edge and connectedness. It plays the vital role when charactedrizing the SF-cycle and SF-tree

2. Spherical Fuzzy Cut Vertices

According to crisp graph theory, when a vertex is removed, the graph becomes detached, and that type of vertex is then studied as a cut vertex. But this concept is not suitable for spherical fuzzy graph. In SFG, the eviction of a vertex diminishes the strength of connectedness of G. The spherical fuzzy cut vertex is defined in this section and illustrated by an appropriate examples. Also, the connection between spherical fuzzy bridge and spherical fuzzy cut vertex is discussed.

Definition 2.1

Let $G=(\Psi,\Omega)$ be a SFG. Any two vertices, let m and n, are considered neighbors if they meet one or both of the following requirements.:

$$\alpha_{\Omega}(m, n) > 0; \gamma_{\Omega}(m, n) > 0; \beta_{\Omega}(m, n) > 0,$$

$$\alpha_{\Omega}(m, n) = 0; \gamma_{\Omega}(m, n) \geq 0; \beta_{\Omega}(m, n) > 0,$$

$$\alpha_{\Omega}(m, n) > 0; \gamma_{\Omega}(m, n) = 0; \beta_{\Omega}(m, n) \geq 0,$$

$$\alpha_{\Omega}(m, n) \geq 0; \gamma_{\Omega}(m, n) > 0; \beta_{\Omega}(m, n) = 0$$

for some m, n in V^* .

Definition 2.2

Let $m \in V^*$ then m is called spherical fuzzy cut vertex if $SFCON_{\alpha_{\Omega}}(G) - m(p, q) <$

$SFCON_{\alpha\Omega}(G)(p, q)$, $SFCON_{\gamma\Omega}(G)-m(p, q) < SFCON_{\gamma\Omega}(G)(p, q)$ and $SFCON_{\beta\Omega}(G)-m(p, q) > SFCON_{\beta\Omega}(G)(p, q)$ for all $p, q \in V^*$ where the points p, q and m are all distinct.

Example 2.1

$G^* = (V^*, E^*)$ represents a graph with $V^* = \{u, v, w, x\}$ and $E^* = \{(u, v), (v, w), (w, x), (x, y), (y, z), (z, u)\}$. G is thus defined on G^* as an SFG. Given a spherical fuzzy set $\Psi = (\alpha\Psi, \gamma\Psi, \beta\Psi)$, a spherical fuzzy edge set $\Omega = (\alpha\Omega, \gamma\Omega, \beta\Omega)$ and $\Omega \subseteq \Psi \times \Psi$ defined by

$$\begin{aligned} (\alpha_{\Omega}(u, v), \gamma_{\Omega}(u, v), \beta_{\Omega}(u, v)) &= (0.5, 0.4, 0.2), & (\alpha_{\Omega}(v, w), \gamma_{\Omega}(v, w), \beta_{\Omega}(v, w)) &= (0.7, 0.4, 0.1), \\ (\alpha_{\Omega}(w, x), \gamma_{\Omega}(w, x), \beta_{\Omega}(w, x)) &= (0.2, 0.3, 0.5), & (\alpha_{\Omega}(x, y), \gamma_{\Omega}(x, y), \beta_{\Omega}(x, y)) &= (0.6, 0.7, 0.3), \\ (\alpha_{\Omega}(y, z), \gamma_{\Omega}(y, z), \beta_{\Omega}(y, z)) &= (0.7, 0.2, 0.6), & (\alpha_{\Omega}(z, u), \gamma_{\Omega}(z, u), \beta_{\Omega}(z, u)) &= (0.1, 0.1, 0.7). \end{aligned}$$

Thus, w is the spherical fuzzy cut vertex in u - z path. Since $SFCON_{\alpha\Omega}(G)-w(u, z) = 0.1 < SFCON_{\alpha\Omega}(G)(u, z) = 0.2$, $SFCON_{\gamma\Omega}(G)-w(u, z) = 0.1 < SFCON_{\gamma\Omega}(G)(u, z) = 0.2$ and $SFCON_{\beta\Omega}(G)-w(u, z) = 0.7 > SFCON_{\beta\Omega}(G)(u, z) = 0.6$.

Definition 2.3

Any edge in SFG is said to be a SF-strong edge if $\alpha_{\Omega}(p, q) \geq SFCON_{\alpha\Omega}G(p, q)$; $\gamma_{\Omega}(p, q) \geq SFCON_{\gamma\Omega}G(p, q)$; $\beta_{\Omega}(p, q) \leq SFCON_{\beta\Omega}G(p, q)$.

Definition 2.4

Any edge in SFG is said to be a SF-weakest edge if $\alpha_{\Omega}(p, q) < SFCON_{\alpha\Omega}G(p, q)$; $\gamma_{\Omega}(p, q) < SFCON_{\gamma\Omega}G(p, q)$; $\beta_{\Omega}(p, q) > SFCON_{\beta\Omega}G(p, q)$.

Definition 2.5

Let $G = (\Psi, \Omega)$ be a SFG and consider a path P from any vertex p to any vertex q in G accommodates only SF-strong edges then the path P is called SF-strong- $\#$ -path.

Definition 2.6

The vertices p and q in G are called strong neighbors each other if it satisfies the condition $(\alpha_{\Omega}(p, q) \geq SFCON_{\alpha}G(p, q), \gamma_{\Omega}(p, q) \geq SFCON_{\gamma}G(p, q), \beta_{\Omega}(p, q) \leq SFCON_{\beta}G(p, q))$.

Proposition 2.1

Let $G = (\Psi, \Omega)$ be a SFG. If (e, f) is a spherical fuzzy bridge of G , then $SFCON_{\Omega}(e, f) = (\alpha_{\Omega}(e, f), \gamma_{\Omega}(e, f), \beta_{\Omega}(e, f))$.

Proof. Let the edge (e, f) be a spherical fuzzy bridge of G . Suppose its connectedness $SFCON_{\Omega}(e, f)$ exceeds the edge $(\alpha_{\Omega}(e, f), \gamma_{\Omega}(e, f), \beta_{\Omega}(e, f))$ value. Then there exists a SF-strongest path from e to f whose α -strength is greater than $\alpha_{\Omega}(e, f)$, γ -strength is greater than $\gamma_{\Omega}(e, f)$ and β -strength is less than $\beta_{\Omega}(e, f)$. Also, the strength of all edge of this -strongest- $\#$ -path is greater than $(\alpha_{\Omega}(e, f), \gamma_{\Omega}(e, f), \beta_{\Omega}(e, f))$. Hence, the spherical fuzzy path along side the edge Ω forms a cycle for which the (e, f) is SF-weakest edge. It is the contradiction. Therefore, $SFCON_{\Omega}(e, f)$ is equal to $(\alpha_{\Omega}(e, f), \gamma_{\Omega}(e, f), \beta_{\Omega}(e, f))$.

Proposition 2.2

Let $G = (\Psi, \Omega)$ be a SFG. A vertex in spherical fuzzy cut vertex if it is the common spherical fuzzy vertex of two neighboring spherical fuzzy bridges.

Proof. Choose the spherical fuzzy cut vertex u in G which is adjacent to the edges (p, u) and (u, q) . While u is a spherical fuzzy cut vertex then, at that point, its evacuation decreases the spherical fuzzy strength of connectedness. Suppose, (p, u) is not a spherical fuzzy bridge if in either one of the edges of (p, u) and (u, q) . Then that edge may be the SF- weakest edge of G . Subsequently, on the off chance that the vertex u is dispensed with, the strength of connectedness doesn't decrease. In consequence, both the edges (p, u) and (u, q) must be spherical fuzzy bridges. Conversely, Assume that u be the common vertex of two spherical fuzzy bridges (p, u) and (u, q) . Then the removal of both spherical fuzzy bridges (p, u) and (u, q) implies the reduction in spherical fuzzy strength of connectedness. Hence u is a spherical cut vertices.

Proposition 2.3

Let $G = (\Psi, \Omega)$ be a SFG. In the event where vertex G is spherical fuzzy cut, then m has more than two

strong neighbors.

Proof. Let m be a spherical fuzzy cut vertex in G . Thus, the eviction of spherical fuzzy cut vertex m diminishes the connectedness between p and q . Then, there is a SF-strongest- $\#$ - path $P : (u, \dots, l, m, n, \dots, v)$ exists from u to v which passes through m . If (l, m) is not strong, clearly $\alpha_\Omega(l, m) < \text{SFCON}_G(l, m)$, $\gamma_\Omega(l, m) < \text{SFCON}_G(l, m)$ and $\beta_\Omega(l, m) > \text{SFCON}_G(l, m)$ assuming the edge (l, m) is removed. Then, there subsist a path Q from u to m , not including the edge (l, m) , which is stronger than (l, m) . Choose k just proceeding m on Q . Since the strength of the path Q is at most the spherical fuzzy edge $(\alpha_\Omega(k, m), \gamma_\Omega(k, m), \beta_\Omega(k, m))$, it must be $\alpha_\Omega(k, m) > \alpha_\Omega(l, m)$; $\gamma_\Omega(k, m) > \gamma_\Omega(l, m)$; $\beta_\Omega(k, m) < \beta_\Omega(l, m)$. If (k, m) is not $\#$ -strong, then this contention can be rehashed. Since the graph is confined, after limited number of reiteration, the process ends. Then there exist saver tex k such that (k, m) is SF-strong. Like wise, there exists a vertex 'r' such that (m, r) is SF-strong. Assumethat $k = r$, there is a path from u to $k = r$ to v which is stronger than Q . Thus, $\text{SFCON}_G(u, v)$ is not diminished after expulsion of m . It is a contradiction. Thus, the vertex m has atleast two SF-strong neighbours.

3. SF-Multimin and SF-Locamin

In this section, the spherical fuzzy end vertex is defined with an example and thesalient concepts of spherical fuzzy multimin and spherical fuzzy locamin is defined by the use of spherical fuzzy weakest edge. Also the characterizations of SF-locamin and SF-multimin are established.

Definition 3.1

Let $G = (\Psi, \Omega)$ be a SFG. Any vertex $(\alpha_\Psi(p), \gamma_\Psi(p), \beta_\Psi(p))$ in Ψ is spherical fuzzy end vertex if it has no more than one SF-strong neighbor.

Example 3.1

Consider a graph $G^* = (V^*, E^*)$ such that $V^* = \{u, v, w, x\}$ and $E^* = \{(u, v), (v, w), (w, x), (x, y), (y, z), (z, u)\}$. Then G be a SFG defined on G^* . Let $\Psi = (\alpha_\Psi, \gamma_\Psi, \beta_\Psi)$ be a spherical fuzzy vertex set and let $\Omega = (\alpha_\Omega, \gamma_\Omega, \beta_\Omega)$ be a spherical fuzzy edge set and $\Omega \subseteq \Psi \times \Psi$ defined by
 $(\alpha_\Omega(u, v), \gamma_\Omega(u, v), \beta_\Omega(u, v)) = (0.1, 0.1, 0.3)$, $(\alpha_\Omega(v, w), \gamma_\Omega(v, w), \beta_\Omega(v, w)) = (0.1, 0.1, 0.3)$,
 $(\alpha_\Omega(w, x), \gamma_\Omega(w, x), \beta_\Omega(w, x)) = (0.5, 0.6, 0.1)$, $(\alpha_\Omega(x, u), \gamma_\Omega(x, u), \beta_\Omega(x, u)) = (0.7, 0.4, 0.2)$,
 $(\alpha_\Omega(u, w), \gamma_\Omega(u, w), \beta_\Omega(u, w)) = (0.6, 0.3, 0.3)$, $(\alpha_\Omega(x, v), \gamma_\Omega(x, v), \beta_\Omega(x, v)) = (0.9, 0.2, 0.01)$.
 The vertex v is a spherical fuzzy end vertex since it has only SF- strong neighbor x in G . From this, $\alpha_\Omega(x, v) = 0.9 > 0.1 = \text{SFCON}_{G-\alpha_\Omega(x, v)}(x, v)$; $\gamma_\Omega(x, v) = 0.2 > 0.1 = \text{SFCON}_{G-\gamma_\Omega(x, v)}(x, v)$; $\beta_\Omega(x, v) = 0.01 < 0.3 = \text{SFCON}_{G-\beta_\Omega(x, v)}(x, v)$. But the remaining two vertices u and w are not SF- strong neighbors of v .

Definition 3.2

Let $G = (\Psi, \Omega)$ be a SFG. Any cycle $C : p_0, p_1, \dots, p_i, i \geq 3$ is said to be a spherical fuzzy cycle if $p_0 = p_i$.

Definition 3.3

Let $G = (\Psi, \Omega)$ be a SFG. Any spherical fuzzy subgraph H of G is said to be a full spherical fuzzy subgraph of G if $(\alpha_\Psi(p), \gamma_\Psi(p), \beta_\Psi(p)) > 0 \forall p \in V^*$ & $(\alpha_\Omega(p, q), \gamma_\Omega(p, q), \beta_\Omega(p, q)) > 0 \forall (p, q) \in E^*$.

Definition 3.4

Let $G = (\Psi, \Omega)$ be a SFG.

- (a) Any FSF-cycle, C , is said to be SF-multimin if C has more than one spherical fuzzy weakest edge
- (b) Any FSF-cycle, C , is said to be SF-locamin if every vertex of C lies on the spherical fuzzy weakest edge.

Proposition 3.1

Any SF-cycle is SF-multimin iff C has no spherical fuzzy cut vertices.

Proof. Suppose that there is a spherical fuzzy strong edge in a SF-multimin. Then, for any vertex $m \in V^*$, both edges incident at m are spherical fuzzy strong. This implies that m is not a spherical fuzzy end vertex. On the other hand, assume that there is one spherical fuzzy weakest edge (p, q) on the SF-cycle. Then, q is not a spherical fuzzy strong neighbour of p or p is not a spherical fuzzy strong neighbour of q . This implies that the other neighbours of p and q are spherical fuzzy strong neighbour. Therefore, the vertices p and q are spherical fuzzy end vertices.

Proposition 3.2

Any SF-multimin cycle C is a SF-locamin iff C has no spherical fuzzy cut vertices.

Proof. Suppose that the edge (p, q) is the spherical fuzzy weakest edge of spherical fuzzy cycle C , then the vertices p and q are not spherical fuzzy cut vertices of C . Thus, if C is SF-locamin then each vertex of C lies

on the spherical fuzzy weakest edge. Therefore none of the vertex of C is a spherical fuzzy cut vertex. Conversely, assume that C is not SF-locamin. Consider any three consecutive vertices p, q and r of C such that neither (p, q) nor (q, r) is the SF-weakest edge. Therefore eliminating the vertex q reduces the spherical fuzzy connectedness $SFCON_{\Omega(G)}(q, r)$ and hence q is a spherical fuzzy cut vertex.

Proposition 3.3

Let $G=(\Psi, \Omega)$ be a SFG.. Any SF- cycle C is SF- multimin and only if it has atleast one vertex which is neither a spherical fuzzy cut vertex nor a spherical fuzzy end vertex.

Proof. Let C be a SF-multimin cycle. Then by Proposition 3.1, C has no spherical fuzzy end vertex. This implies that a vertex on the spherical fuzzy weakest edge of C cannot be a spherical fuzzy cut vertex.

On the otherhand, assume that C is not a SF-multimin cycle, it has the spherical fuzzy weakest edge. Hence, the vertices that lie on this spherical fuzzy weakest edgemust be spherical fuzzy end vertices. Then, all other vertices of C must be spherical fuzzy cut vertices. Hence, each vertex of C is either a spherical fuzzy end vertex or a spherical fuzzy cut vertex.

4. Spherical Fuzzy Trees

Crisp trees and spherical fuzzy trees are quite different. but no cycle in a crisp graph will be a tree. This section introduces spherical fuzzy spanning subgraph and spherical fuzzy tree, establishing some of their qualities with an example.

Definition 4.1

Let $G=(\Psi, \Omega)$ and $T=(\Psi^S, \Omega^S)$ be any two SFG. .Then T is said to be a spherical fuzzy spanning subgraph of G , if $\alpha_{\Psi}(u)=\alpha^S(u); \gamma_{\Psi}(u)=\gamma^S(u); \beta_{\Psi}(u)=\beta^S(u)$ for all $u \in V^*$ and $\alpha_{\Psi}(u,v)<\alpha^S(u,v); \gamma^S(u,v)<\gamma^S(u,v); \beta_{\Psi}(u,v)<\beta^S(u,v)$ for all $(u,v) \in E^*$.

Definition 4.2

The SFG, $G=(\Psi, \Omega)$ is said to be a spherical fuzzy tree if it has a spherical fuzzy spanning subgraph $T = (\Psi^S, \Omega^S)$ which is an full spherical fuzzy sub-graph of G if an edge (u, v) in G but not in T , then $\alpha_{\Omega(G)}(u, v) < SFCON_{\Omega^S(T)}(u, v); \gamma_{\Omega(G)}(u,v) < SFCON_{\Omega^S(T)}(u,v); \beta_{\Omega(G)}(u,v) > SFCON_{\Omega^S(T)}(u,v)$ for all $(u,v) \in E^*$.

Example 4.1.

Consider a graph $G^*=(V^*, E^*)$ such that $V^*=\{a, b, c, d\}$ and $E^*=\{(a, b),(b, d),(c, d),(a, c)\}$. Then G be a SFG defined on G^* . Let $\Psi=(\alpha_{\Psi}, \gamma_{\Psi}, \beta_{\Psi})$ be a spherical fuzzy set and let $\Omega= (\alpha_{\Omega}, \gamma_{\Omega}, \beta_{\Omega})$ be a spherical fuzzy edge set and $\Omega \subseteq \Psi \times \Psi$ defined by

$$(\alpha_{\Omega}(a, b), \gamma_{\Omega}(a, b), \beta_{\Omega}(a, b)) = (0.1, 0.1, 0.6),$$

$$(\alpha_{\Omega}(b, d), \gamma_{\Omega}(b, d), \beta_{\Omega}(b, d)) = (0.4, 0.5, 0.3),$$

$$(\alpha_{\Omega}(c, d), \gamma_{\Omega}(c, d), \beta_{\Omega}(c, d)) = (0.5, 0.6, 0.1),$$

$$(\alpha_{\Omega}(a, c), \gamma_{\Omega}(a, c), \beta_{\Omega}(a, c)) = (0.7, 0.4, 0.2)$$

The spherical fuzzy spanning subgraph $T=(\Psi^S, \Omega^S)$ which has the edge set

$T_E=\{(b, d), (c, d), (a, c)\}$ with the spherical fuzzy values as in Ω . Also, this spherical fuzzy spanning subgraph is maximum spherical fuzzy spanning tree in theSFG.

Definition 4.3

A maximum spherical fuzzy spanning tree of a connected SFG, $G = (\Psi, \Omega)$ is a spanning sub graph $T = (\Psi', \Omega')$ of G , which is a tree such that $SFCON_G(u, v)$ is the strength of the unique strongest $u-v$ path in T for all u, v in G . Some significant properties identified with the spherical fuzzy tree are presented here. Every internal vertex (other than the end vertex)is a cut vertex.

Proposition 4.1

Let T be a spherical fuzzy tree. If every internal vertex of T is a spherical fuzzy cut vertex.

Proof. Let us assume that any of the internal vertices in a spherical fuzzy tree is removed means the incident edges are evicted automatically. We know that the spherical fuzzy trees are spherical fuzzy graphs with a spherical fuzzy spanning tree such that all the edges which are not in the spherical fuzzy spanning tree have less connectivity than the strength of spherical fuzzy graphs. Consequently, the evacuation of such interior vertices, certainly eliminates a few spherical fuzzy bridges. Thus, the inner vertices of a spherical fuzzy tree are spherical fuzzy cut vertices.

Proposition 4.2

If (u, v) is a spherical fuzzy bridge in a spherical fuzzy tree $T = (\Psi, \Omega)$, then $SFCON_T(u, v) = \alpha_\Omega(u, v)$, $SFCON_T(u, v) = \gamma_\Omega(u, v)$ and $SFCON_T(u, v) = \beta_\Omega(u, v)$

Proof. Let T be a spherical fuzzy tree and (u, v) be a spherical fuzzy bridge in T . Then, there is a spherical fuzzy spanning tree such that all the edges which are not in spherical fuzzy spanning tree have less connectivity than the strength of spherical fuzzy graph. Assume that, $SFCON_T(\alpha_\Omega(u, v)) < \alpha_\Omega(u, v)$, $SFCON_T(\gamma_\Omega(u, v)) < \gamma_\Omega(u, v)$ and $SFCON_T(\beta_\Omega(u, v)) < \beta_\Omega(u, v)$. Then, $\alpha_\Omega(u, v) < SFCON_G(u, v)$, $\gamma_\Omega(u, v) < SFCON_G(u, v)$ and $\beta_\Omega(u, v) > SFCON_G(u, v)$. Therefore, (u, v) is not a spherical fuzzy bridge, a contradiction, Hence, $SFCON_T(u, v) = \alpha_\Omega(u, v)$, $SFCON_T(u, v) = \gamma_\Omega(u, v)$ and $SFCON_T(u, v) = \beta_\Omega(u, v)$.

Remark 4.1

Let $G = (\Psi, \Omega)$ be a SFG. Then G is a spherical fuzzy tree if and only if the following conditions are equivalent for all $p, q \in V^*$.

- (a) $(\alpha_\Omega(p, q), \gamma_\Omega(p, q), \beta_\Omega(p, q))$ is a spherical fuzzy bridge.
- (b) $SFCON_G(u, v) = (\alpha_\Omega(p, q), \gamma_\Omega(p, q), \beta_\Omega(p, q))$

Remark 4.2

If $G = (\Psi, \Omega)$ be a SFG and it is spherical fuzzy tree, then the edges of its spherical fuzzy maximum spanning tree $F = (\Psi', \Omega')$ are just spherical fuzzy bridges of G .

Proposition 4.3

A SFG G is a spherical fuzzy tree iff there is a unique strong path in G between any two vertices of G .

Proof. Suppose that, there is a strong path P between any two vertices u and v in G . Let T be a spanning spherical fuzzy tree in G . Then the spherical fuzzy path P lies completely in T . Since T is a spherical fuzzy tree, there is a unique spherical fuzzy path in T from u to v . Hence, the spherical fuzzy path P is unique. Conversely, take G to be not a spherical fuzzy tree. There is a spherical fuzzy cycle C in G such that $\alpha_\Omega(u, v) \geq SFCON_{G-(u,v)}(u, v)$, $\gamma_\Omega(u, v) \geq SFCON_{G-(u,v)}(u, v)$ and $\beta_\Omega(u, v) \geq SFCON_{G-(u,v)}(u, v)$ for any spherical fuzzy edge of C . Specifically to say each edge on spherical fuzzy cycle C is strong. As a result, for any two vertices u and v on C , there are two strong spherical fuzzy paths between the vertices u and v . This is a contradiction. Hence, G is a spherical fuzzy tree.

Proposition 4.4

Let G be a spherical fuzzy tree. In G , there is a strong u - v spherical fuzzy path is a path of maximum strength joining u and v .

Proof. Let T be a spherical fuzzy tree and let P be the unique spherical fuzzy strong u - v path. We know that T contains every edge of P is strong. Assume that, the path P is not a maximum strength. So that take another P' having maximum strength. Then the paths P and P' are distinct. Thus P and the swap of P form a spherical fuzzy cycle. In the spherical fuzzy tree, there is none of the spherical fuzzy cycle as T . Consequently, some spherical fuzzy edges (u', v') of Ω' must not satisfy to be in spherical fuzzy tree T . By definition of T , it shows authentically that $\alpha_\Omega(u', v') < SFCON_G(u', v')$, $\gamma_\Omega(u', v') < SFCON_G(u', v')$ and $\beta_\Omega(u', v') > SFCON_G(u', v')$. Thus, there exists a spherical fuzzy path R in T . Therefore each edge (u', v') of P' that declines to be in spherical fuzzy tree T can be restored by a path in T and this implicit that the u - v path R in T . Significantly, it was established by restoring the edge (u', v') of P' by paths stronger than these edges, R is at least as strong as P' . Then, R is not as like as P , so P and the restoration of R form a spherical fuzzy cycle, and this cycle is in spherical fuzzy tree T , which is not correct. Hence P is a path of maximum strength joining the vertices u and v in G .

5. Application

All things considered in data organizations, electric circuit, etc., the decrease of stream between sets of vertices is more significant and may regularly happen than the absolute disturbance of the stream or the disengagement of the whole organization. Considerably more work should be possible to explore the structure of . It would be valuable since it has applications in decision making, pattern recognising and network analysis.

6. CONCLUSION

This paper focused ultimately spherical fuzzy tree with the use of spherical fuzzy cut vertices, spherical fuzzy bridges, spherical fuzzy end vertices. SF-multimin and SF-locamin cycles are also discussed. This research can be further extend to devise the SF-vertex connectivity and SF-edge connectivity. These connectivity parameters are related with the sets disconnecting the SFG.

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