Investigating Bipolar Controlled Neutrosophic Metric Spaces In Automobile Suspension

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Received: 17.04.2024 Revised : 12.05.2024 Accepted: 18.05.2024

ABSTRACT
As an extension of Sezen's result, we convey the notion of bipolar controlled neutrosophic metric spaces. The research topic is about our ongoing efforts to bring forth controlled metric spaces for neutrosophic theory.We looked at and illustrated a few axioms of (Bipolar controlled Neutrosophic metric space) $\mathfrak{BCNMS}$ in this article. As a way to generalise the Banach contraction principle in the earlier mentioned spaces, we employed $\mathfrak{BCNMS}$ .For the purpose of reviewing what we discovered, we graphically validated several examples and supported some findings. Furthermore, we provide evidence of usage and implemented it by proving their presence with a distinctive and integrative solution.

Keywords: Fixed point, Bipolar controlled neutrosophic metric space, Integral equation.

1. INTRODUCTION
In his 1906 dissertation, Fretchet pioneered the idea of metric space. Later, in his doctoral dissertation, Banach [3] demonstrated the Banach contraction principle in 1922. Numerous researchers have tried out this idea in various circumstances since then. It is regarded as the most essential non-linear analysis tool. It explains why every contractive mapping in whole metric spaces has a single fixed point. It is a generalisation and extension of many kinds of metric spaces. In 1965, Zadeh [17] developed his concept of a fuzzy set, characterised by a modified version of a conventional set in which each element bears a membership value within an acceptable range. The term "neutrosophic set" was originated by Smarandache in 1998, and it was demonstrated beside Sowndrarajan [9]. As a team, they illustrated several significant discoveries from neutrosophic metric space. In 2019, Kirisci and Simsek [11] relocated out with a proposal of neutrosophic metric space. Sowdrarajan and Jeyaraman et al. [9] confirmed significant fixed point results in neutrosophic metric space in 2020.
The concept of bipolar metric spaces was introduced very recently, in 2016 by Mutlu and Gurdal [2]. Additionally, they looked into a few linked and fixed point outcomes on this space (see to [1] and [2] for details). We shall carry on our investigation of fixed points in the bipolar metric-space frame in this paper. More specifically, a few shared fixed-point outcomes for a pair of covariant and contra variant.
This topic was used to obtain different structures and to generalise the outcome in different spaces. Some basic results on this subject may be retained, notably controlled metric type spaces and the related contraction principle in [15], controlled neutrosophic metric spaces and some related fixed point results in [5], and, more recently, the novel aspects of metric spaces in [15]. We show the efficacy and validity of the results' hypotheses. The ideas in the paper [15] in several recent literatures are enhanced and elaborated upon by the current findings.

2. Preliminaries
The definitions of a neutrosophic metric space that we begin with are as follows.

Definition 2.1[8]
An ordered 6-tuple ($\mathfrak{K}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond$) is called $\mathfrak{NMS}$ if $\mathfrak{K}$ is an arbitrary non empty set, $\star$neutrosophic CTN , $\diamond$neutrosophic CTC and $\mathfrak{A}, \mathfrak{I}, \mathfrak{D}$ are neutrosophic sets on $\mathfrak{K} \times \mathfrak{K} \times (0,\infty)$ satisfying the following condition: For all $\mathfrak{k}, \mathfrak{g}, \mathfrak{h} \in \mathfrak{K}$, $\Theta, \mathfrak{x} > 0$.

a) $0 \leq \mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \Theta) \leq 1; 0 \leq \mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta) \leq 1; 0 \leq \mathfrak{D}(\mathfrak{k}, \mathfrak{g}, \Theta) \leq 1;$
b) $\mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \Theta)) + \mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta) + \mathfrak{D}(\mathfrak{k}, \mathfrak{g}, \Theta) \leq 3;$
c) $\mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \Theta) = 1, \forall \Theta > 0, \Leftrightarrow \mathfrak{k} = \mathfrak{g};$
d) $\mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \Theta) = \mathfrak{A}(\mathfrak{g}, \mathfrak{k}, \Theta)$; for $\Theta > 0$
e) $\mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \Theta) \star \mathfrak{A}(\mathfrak{g}, \mathfrak{h}, \mathfrak{x}) \geq \mathfrak{A}(\mathfrak{k}, \mathfrak{h}, \Theta + \mathfrak{x})\ \forall \Theta, \mathfrak{x} > 0$
f) $\mathfrak{A}(\mathfrak{k}, \mathfrak{g}, .) : (0,+\infty) \to [0,1]$ is neutrosophic CTS and $\lim_{\mathfrak{z} \to +\infty} \mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \Theta) = 1$
g) $\mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta) = 0, \forall \Theta > 0, \Leftrightarrow \mathfrak{k} = \mathfrak{g};;$
h) $\mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta) = \mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta)$; for $\Theta > 0$
i) $\mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta) \diamond \mathfrak{I}(\mathfrak{g}, \mathfrak{h}, \mathfrak{x}) \leq \mathfrak{I}(\mathfrak{k}, \mathfrak{h}, \Theta + \mathfrak{x})\ \forall \Theta, \mathfrak{x} > 0$
j) $\mathfrak{I}(\mathfrak{k}, \mathfrak{g}, .) : (0,+\infty) \to [0,1]$ is neutrosophic CTS and $\lim_{\mathfrak{z} \to +\infty} \mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta) = 0$
k) $\mathfrak{D}(\mathfrak{k}, \mathfrak{g}, \Theta) = 0, \forall \Theta > 0, \Leftrightarrow \mathfrak{k} = \mathfrak{g};;$
l) $\mathfrak{D}(\mathfrak{k}, \mathfrak{g}, \Theta) = \mathfrak{D}(\Theta, \varsigma, \mathfrak{z})$; for$\Theta > 0$
m) $\mathfrak{D}(\mathfrak{k}, \mathfrak{g}, \Theta) \diamond \mathfrak{D}(\mathfrak{g}, \mathfrak{h}, \mathfrak{x}) \leq \mathfrak{D}(\mathfrak{k}, \mathfrak{h}, \Theta + \mathfrak{x})\ \forall \Theta, \mathfrak{x} > 0$
n) $\mathfrak{D}(\mathfrak{k}, \mathfrak{g}, .) : (0,+\infty) \to [0,1]$ is neutrosophic CTS and$\lim_{\mathfrak{z} \to +\infty} \mathfrak{D}(\mathfrak{k}, \mathfrak{g}, \Theta) = 0$

Then, ($\mathfrak{K}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond$) is called a $\mathfrak{NMS}$.

Definition 2.2 [5]
Let $\mathfrak{K}$ be a non empty set and $\beth : \mathfrak{K} \times \mathfrak{K} \to [1,+\infty)$, $\star$ neutrosophic CTN , $\diamond$ neutrosophic CTC and $\mathfrak{A}, \mathfrak{I}, \mathfrak{D}$ are neutrosophic sets on $\mathfrak{K} \times \mathfrak{K} \times (0,\infty)$ satisfying the following condition: For all $\mathfrak{k}, \mathfrak{g}, \mathfrak{h} \in \mathfrak{K}$,$\Theta,\mathfrak{x} > 0$

a. $0 \leq \mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \Theta) \leq 1; 0 \leq \mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta) \leq 1; 0 \leq \mathfrak{D}(\mathfrak{k}, \mathfrak{g}, \Theta) \leq 1;$
b. $\mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \Theta) + \mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta) + \mathfrak{D}(\mathfrak{k}, \mathfrak{g}, \Theta) \leq 3;$
c. $\mathfrak{A}(\mathfrak{k}, \mathfrak{g}, 0) = 0$
d. $\mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \Theta) = 1, \forall \Theta > 0, \Leftrightarrow \mathfrak{k} = \mathfrak{g};$
e. $\mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \Theta) = \mathfrak{A}(\mathfrak{g}, \mathfrak{k}, \Theta);$
f. $\mathfrak{A}(\mathfrak{k}, \mathfrak{h}, \Theta + \mathfrak{x}) \geq \mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \frac{\Theta}{\beth(\mathfrak{k},\mathfrak{g})}) \star \mathfrak{A}(\mathfrak{g}, \mathfrak{h}, \frac{\mathfrak{x}}{\beth(\mathfrak{g},\mathfrak{h})});$
g. $\mathfrak{A}(\mathfrak{k}, \mathfrak{g}, .) : (0,+\infty) \to [0,1]$ is CTS and $\lim_{\mathfrak{z} \to +\infty} \mathfrak{A}(\mathfrak{k}, \mathfrak{g}, \Theta) = 1$;
h. $\mathfrak{I}(\mathfrak{k}, \mathfrak{g}, 0) = 1$
i. $\mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta) = 0, \forall \Theta > 0 \Leftrightarrow \mathfrak{k} = \mathfrak{g};$
j. $\mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta) .= \mathfrak{I}(\mathfrak{g}, \mathfrak{k}, \Theta);$
k. $\mathfrak{I}(\mathfrak{k}, \mathfrak{h}, \Theta + \mathfrak{x}) \leq \mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \frac{\Theta}{\beth(\mathfrak{k},\mathfrak{g})}) \diamond \mathfrak{I}(\mathfrak{g}, \mathfrak{h}, \frac{\mathfrak{x}}{\beth(\mathfrak{g},\mathfrak{h})});$
l. $\mathfrak{I}(\mathfrak{k}, \mathfrak{g}, .) : (0,+\infty) \to [0,1]$ is CTS and $\lim_{\mathfrak{z} \to +\infty} \mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \Theta) = 0$;
m. $\mathfrak{D}(\mathfrak{k}, \mathfrak{g}, 0) = 1$
n. $\mathfrak{D}(\mathfrak{k}, \mathfrak{g}, \Theta) = 0, \forall \Theta > 0 \Leftrightarrow \mathfrak{k} = \mathfrak{g};$
o. $\mathfrak{D}(\mathfrak{k}, \mathfrak{g}, \Theta) = \mathfrak{I}(\mathfrak{g}, \mathfrak{k}, \Theta);$
p. $\mathfrak{D}(\mathfrak{k}, \mathfrak{h}, \Theta + \mathfrak{x}) \leq \mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \frac{\theta}{\beth(\mathfrak{k},\mathfrak{g})}) \diamond \mathfrak{I}(\mathfrak{g}, \mathfrak{h}, \frac{\mathfrak{x}}{\beth(\mathfrak{g},\mathfrak{h})});$
q. $\mathfrak{D}(\mathfrak{k}, \mathfrak{g}, .) : (0, +\infty) \to [0,1]$ is CTS and $lim_{\mathfrak{z} \to +\infty} \mathfrak{I}(\mathfrak{k}, \mathfrak{g}, \theta) = 0$;

Then, ($\mathfrak{K}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond$) is called a $\mathfrak{NCMS}$.

Definition 2.3 [8]
Suppose $\mathfrak{K}, \mathfrak{L} \neq \emptyset$ and $\beth : \mathfrak{K} \times \mathfrak{K} \to [1,+\infty)$ are considered as a incompetent mappings, $\star$ as t-norm defined as $r \star s = \min\{r,s\}$ and $\diamond$as t-conorm defined as $r \diamond s = \max\{r,s\}$, and $\mathfrak{A}, \mathfrak{I}, \mathfrak{D}$ are neutrosophic sets on $\mathfrak{K} \times \mathfrak{L} \times (0, +\infty)$ is characterized $\mathfrak{NBMS}$, if for each one ($\mathfrak{K}, \mathfrak{L}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond$) fulfills all$\mathfrak{k} \in \mathfrak{K}$, $\mathfrak{l} \in \mathfrak{L}$ and $\theta, \mathfrak{x}, \mathfrak{z} > 0$ holds the following:

a. $0 \leq \mathfrak{A}(\mathfrak{k}, \mathfrak{l}, \theta) \leq 1; 0 \leq \mathfrak{I}(\mathfrak{k}, \mathfrak{l}, \theta) \leq 1; 0 \leq \mathfrak{D}(\mathfrak{k}, \mathfrak{l}, \theta) \leq 1;$
b. $\mathfrak{A}(\mathfrak{k}, \mathfrak{l}, \theta) + \mathfrak{I}(\mathfrak{k}, \mathfrak{l}, \theta) + \mathfrak{D}(\mathfrak{k}, \mathfrak{l}, \theta) \leq 3;$
c. $\mathfrak{A}(\mathfrak{k}, \mathfrak{l}, 0) > 0$ for all $\mathfrak{k}, \mathfrak{l} \in \mathfrak{K} \times \mathfrak{L}$
d. $\mathfrak{A}(\mathfrak{k}, \mathfrak{l}, \theta) = 1$iff $\mathfrak{k} = \mathfrak{l}\ for\ \mathfrak{k} \in \mathfrak{K}$, $\mathfrak{l} \in \mathfrak{L}$;
e. $\mathfrak{A}(\mathfrak{k}, \mathfrak{l}, \theta) = \mathfrak{A}(\mathfrak{l}, \mathfrak{k}, \theta)$for all $\mathfrak{k}, \mathfrak{l} \in \mathfrak{K} \cap \mathfrak{L}$
f. $\mathfrak{A}(\mathfrak{k}_1, \mathfrak{l}_2, \theta + \mathfrak{x} + \mathfrak{z}) \geq \mathfrak{A}(\mathfrak{k}_1, \mathfrak{l}_1, \theta) \star \mathfrak{A}(\mathfrak{k}_2, \mathfrak{l}_1, \mathfrak{x}) \star \mathfrak{A}(\mathfrak{k}_2, \mathfrak{l}_2, \mathfrak{z}),\ for\ \mathfrak{k}_1, \mathfrak{k}_2 \in \mathfrak{K}$ and $\mathfrak{l}_1, \mathfrak{l}_2 \in \mathfrak{L}\ \forall \theta, \mathfrak{x}, \mathfrak{z} > 0$;
g. $\mathfrak{A}(\mathfrak{k}, \mathfrak{l}, .) : [0,+\infty) \to [0,1]$ is CTS
h. $\mathfrak{A}(\mathfrak{k}, \mathfrak{l}, .)$ is non decreasing for all $\mathfrak{k} \in \mathfrak{K}$, $\mathfrak{l} \in \mathfrak{L}$
i. $\mathfrak{I}(\mathfrak{k}, \mathfrak{l}, 0) < 1$ for all $\mathfrak{k}, \mathfrak{l} \in \mathfrak{K} \times \mathfrak{L}$
j. $\mathfrak{I}(\mathfrak{k}, \mathfrak{l}, \theta) = 1$iff $\mathfrak{k} = \mathfrak{l}\ for\ \mathfrak{k} \in \mathfrak{K}$, $\mathfrak{l} \in \mathfrak{L}$;

k. $\mathfrak{I}(\mathfrak{k},\mathfrak{l},\Theta)=\mathfrak{I}(\mathfrak{l},\mathfrak{k},\Theta)$ for all $\mathfrak{k},\mathfrak{l}\in\mathfrak{K}\cap\mathfrak{L}$
l. $\mathfrak{I}(\mathfrak{k}_1,\mathfrak{l}_2,\Theta+\mathfrak{x}+\mathfrak{z})\geq\mathfrak{I}(\mathfrak{k}_1,\mathfrak{l}_1,\Theta)\diamond\mathfrak{I}(\mathfrak{k}_2,\mathfrak{l}_1,\mathfrak{x})\diamond\mathfrak{I}(\mathfrak{k}_2,\mathfrak{l}_2,\mathfrak{z}),\ for\ \mathfrak{k}_1,\mathfrak{k}_2\in\mathfrak{K}$ and $\mathfrak{l}_1,\mathfrak{l}_2\in\mathfrak{L}\forall\Theta,\mathfrak{x},\mathfrak{z}>0$;
m. $\mathfrak{I}(\mathfrak{k},\mathfrak{l},.):[0,+\infty)\to[0,1]$ is CTS
n. $\mathfrak{I}(\mathfrak{k},\mathfrak{l},.)$ is non decreasing for all $\mathfrak{k}\in\mathfrak{K}$, $\mathfrak{l}\in\mathfrak{L}$
o. $\mathfrak{D}(\mathfrak{k},\mathfrak{l},0)<1$ for all $\mathfrak{k},\mathfrak{l}\in\mathfrak{K}\times\mathfrak{L}$.
p. $\mathfrak{D}(\mathfrak{k},\mathfrak{l},\Theta)=1$ iff $\mathfrak{k}=\mathfrak{l}\ for\ \mathfrak{k}\in\mathfrak{K}$, $\mathfrak{l}\in\mathfrak{L}$;
q. $\mathfrak{D}(\mathfrak{k},\mathfrak{l},\Theta)=\mathfrak{I}(\mathfrak{l},\mathfrak{k},\Theta)$ for all $\mathfrak{k},\mathfrak{l}\in\mathfrak{K}\cap\mathfrak{L}$.
r. $\mathfrak{D}(\mathfrak{k}_1,\mathfrak{l}_2,\Theta+\mathfrak{x}+\mathfrak{z})\geq\mathfrak{D}(\mathfrak{k}_1,\mathfrak{l}_1,\Theta)\diamond\mathfrak{D}(\mathfrak{k}_2,\mathfrak{l}_1,\mathfrak{x})\diamond\mathfrak{D}(\mathfrak{k}_2,\mathfrak{l}_2,\mathfrak{z}),\ for\ \mathfrak{k}_1,\mathfrak{k}_2\in\mathfrak{K}$ and $\mathfrak{l}_1,\mathfrak{l}_2\in\mathfrak{L}$, $\forall\,\Theta,\mathfrak{x},\mathfrak{z}>0$;
s. $\mathfrak{D}(\mathfrak{k},\mathfrak{l},.):[0,+\infty)\to[0,1]$ is CTS.
t. $\mathfrak{D}(\mathfrak{k},\mathfrak{l},.)$ is non decreasing for all $\mathfrak{k}\in\mathfrak{K}$, $\mathfrak{l}\in\mathfrak{L}$.

Then,($\mathfrak{K},\mathfrak{L},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond$) is called a $\mathfrak{NBMS}$.

3. Main results

Definition 3.1

$\mathfrak{K},\mathfrak{L}\neq\emptyset$ and $\beth:\mathfrak{K}\times\mathfrak{K}\to[1,+\infty)$ are considered as a incompetent mappings, $\star$ as t-norm defined as $r\star s=\min\{r,s\}$ and $\diamond$ as t-conorm defined as $r\diamond s=\max\{r,s\}$, and $\mathfrak{A},\mathfrak{I},\mathfrak{D}$ are neutrosophic sets on $\mathfrak{K}\times\mathfrak{K}\times(0,+\infty)$ is characterized $\mathfrak{BCNMS}$ on $\mathfrak{K}$, if for each one ($\mathfrak{K},\mathfrak{L},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond$) fulfills all $\mathfrak{k}\in\mathfrak{K}$, $\mathfrak{l}\in\mathfrak{L}$ and $\mathfrak{x},\Theta,\mathfrak{z}>0$ holds the following:

a. $0\leq\mathfrak{A}(\mathfrak{k},\mathfrak{l},\Theta)\leq1$; $0\leq\mathfrak{I}(\mathfrak{k},\mathfrak{l},\Theta)\leq1$; $0\leq\mathfrak{D}(\mathfrak{k},\mathfrak{l},\Theta)\leq1$;
b. $\mathfrak{A}(\mathfrak{k},\mathfrak{l},\Theta)+\mathfrak{I}(\mathfrak{k},\mathfrak{l},\Theta)+\mathfrak{D}(\mathfrak{k},\mathfrak{l},\Theta)\leq3$;
c. $\mathfrak{A}(\mathfrak{k},\mathfrak{l},0)=0$;
d. $\mathfrak{A}(\mathfrak{k},\mathfrak{l},\Theta)=1$ iff $\mathfrak{k}=\mathfrak{l}\ for\ \mathfrak{k}\in\mathfrak{K}$, $\mathfrak{l}\in\mathfrak{L}$;
e. $\mathfrak{A}(\mathfrak{k},\mathfrak{l},\Theta)=\mathfrak{A}(\mathfrak{l},\mathfrak{k},\Theta)$ for all $\mathfrak{k},\mathfrak{l}\in\mathfrak{K}\cap\mathfrak{L}$;
f. $\mathfrak{A}(\mathfrak{k}_1,\mathfrak{l}_2,\Theta+\mathfrak{x}+\mathfrak{z})\geq\mathfrak{A}(\mathfrak{k}_1,\mathfrak{l}_1,\frac{\Theta}{\beth(\mathfrak{k}_1,\mathfrak{l}_1)})\star\mathfrak{A}(\mathfrak{k}_2,\mathfrak{l}_1,\frac{\mathfrak{x}}{\beth(\mathfrak{k}_2,\mathfrak{l}_1)})\star\mathfrak{A}(\mathfrak{k}_2,\mathfrak{l}_2,\frac{\mathfrak{z}}{\beth(\mathfrak{k}_2,\mathfrak{l}_2)}),\ for\ \mathfrak{k}_1,\mathfrak{k}_2\in\mathfrak{K}$ and $\mathfrak{l}_1,\mathfrak{l}_2\in\mathfrak{L}$;
g. $\mathfrak{A}(\mathfrak{k},\mathfrak{l},.):[0,+\infty)\to[0,1]$ is CTS;
h. $\mathfrak{A}(\mathfrak{k},\mathfrak{l},.)$ is non decreasing for all $\mathfrak{k}\in\mathfrak{K}$, $\mathfrak{l}\in\mathfrak{L}$;
i. $\mathfrak{I}(\mathfrak{k},\mathfrak{l},0)=1$
j. $\mathfrak{I}(\mathfrak{k},\mathfrak{l},\Theta)=0$ iff $\mathfrak{k}=\mathfrak{l}\ for\ \mathfrak{k}\in\mathfrak{K}$, $\mathfrak{l}\in\mathfrak{L}$;
k. $\mathfrak{I}(\mathfrak{k},\mathfrak{l},\Theta)=\mathfrak{I}(\mathfrak{l},\mathfrak{k},\Theta)$ for all $\mathfrak{k},\mathfrak{l}\in\mathfrak{K}\cap\mathfrak{L}$;
l. $\mathfrak{I}(\mathfrak{k}_1,\mathfrak{l}_2,\Theta+\mathfrak{x}+\mathfrak{z})\leq\mathfrak{I}(\mathfrak{k}_1,\mathfrak{l}_1,\frac{\Theta}{\beth(\mathfrak{k}_1,\mathfrak{l}_1)})\diamond\mathfrak{I}(\mathfrak{k}_2,\mathfrak{l}_1,\frac{\mathfrak{x}}{\beth(\mathfrak{k}_2,\mathfrak{l}_1)})\diamond\mathfrak{I}(\mathfrak{k}_2,\mathfrak{l}_2,\frac{\mathfrak{z}}{\beth(\mathfrak{k}_2,\mathfrak{l}_2)}),\ for\ \mathfrak{k}_1,\mathfrak{k}_2\in\mathfrak{K}$ and $\mathfrak{l}_1,\mathfrak{l}_2\in\mathfrak{L}$;
m. $\mathfrak{I}(\mathfrak{k},\mathfrak{l},.):[0,+\infty)\to[0,1]$ is CTS;
n. $\mathfrak{I}(\mathfrak{k},\mathfrak{l},.)$ is non increasing for all $\mathfrak{k}\in\mathfrak{K}$, $\mathfrak{l}\in\mathfrak{L}$;
o. $\mathfrak{D}(\mathfrak{k},\mathfrak{l},0)=1$;
p. $\mathfrak{D}(\mathfrak{k},\mathfrak{l},\Theta)=0$ iff $\mathfrak{k}=\mathfrak{l}\ for\ \mathfrak{k}\in\mathfrak{K}$, $\mathfrak{l}\in\mathfrak{L}$;
q. $\mathfrak{D}(\mathfrak{k},\mathfrak{l},\Theta)=\mathfrak{D}(\mathfrak{l},\mathfrak{k},\Theta)$ for all $\mathfrak{k},\mathfrak{l}\in\mathfrak{K}\cap\mathfrak{L}$;
r. $\mathfrak{D}(\mathfrak{k}_1,\mathfrak{l}_2,\Theta+\mathfrak{x}+\mathfrak{z})\leq\mathfrak{D}(\mathfrak{k}_1,\mathfrak{l}_1,\frac{\Theta}{\beth(\mathfrak{k}_1,\mathfrak{l}_1)})\diamond\mathfrak{D}(\mathfrak{k}_2,\mathfrak{l}_1,\frac{\mathfrak{x}}{\beth(\mathfrak{k}_2,\mathfrak{l}_1)})\diamond\mathfrak{D}(\mathfrak{k}_2,\mathfrak{l}_2,\frac{\mathfrak{z}}{\beth(\mathfrak{k}_2,\mathfrak{l}_2)}),\ for\ \mathfrak{k}_1,\mathfrak{k}_2\in\mathfrak{K}$ and $\mathfrak{l}_1,\mathfrak{l}_2\in\mathfrak{L}$;
s. $\mathfrak{D}(\mathfrak{k},\mathfrak{l},.):[0,+\infty)\to[0,1]$ is CTS
t. $\mathfrak{D}(\mathfrak{k},\mathfrak{l},.)$ is non increasing for all $\mathfrak{k}\in\mathfrak{K}$, $\mathfrak{l}\in\mathfrak{L}$

Then, ($\mathfrak{K},\mathfrak{L},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond$) is called a $\mathfrak{BCNMS}$.

Example 3.2

Let $\mathfrak{K}=(0,3]$, $\mathfrak{L}=[3,\infty)$. Define $\mathfrak{A}$ is a fuzzy set on $\mathfrak{K}\times\mathfrak{K}\times(0,+\infty)$, as

$$\mathfrak{A}(\mathfrak{k},\mathfrak{l},\Theta)=\begin{cases}1 & if\ \mathfrak{k}=\mathfrak{l}\\ \mathfrak{k}\mathfrak{l}\Theta & if\ \mathfrak{k}\neq\mathfrak{l}\ and\ \Theta\geq0,\end{cases}$$

$$\mathfrak{I}(\mathfrak{k},\mathfrak{l},\Theta)=\begin{cases}0 & if\ \mathfrak{k}=\mathfrak{l}\\ 1-\mathfrak{k}\mathfrak{l}\Theta & if\ \mathfrak{k}\neq\mathfrak{l}\ and\ \Theta\geq0,\end{cases}$$

$$\mathfrak{D}(\mathfrak{k},\mathfrak{l},\Theta)=\begin{cases}0 & if\ \mathfrak{k}=\mathfrak{l}\\ \frac{1-\mathfrak{k}\mathfrak{l}\Theta}{\mathfrak{k}\mathfrak{l}\Theta} & if\ \mathfrak{k}\neq\mathfrak{l}\ and\ \Theta\geq0,\end{cases}$$

With the continuous t-norm * such that $\Theta_1*\Theta_2=min\{\Theta_1,\Theta_2\}$.

Define $\beth:\mathfrak{K}\times\mathfrak{K}\to[1,+\infty)$ as $\beth(\mathfrak{k},\mathfrak{l})=\begin{cases}1 & if\ \mathfrak{k}\in\mathfrak{K}\ and\ \mathfrak{l}\in\mathfrak{L}\\ 1+\frac{1}{r} & otherwise,\end{cases}$

Now, ($\mathfrak{K},\mathfrak{L},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond$) is a $\mathfrak{BCNMS}$. It is easy to prove the first three conditions. To prove the fourth condition.

For $\mathfrak{k}\neq\mathfrak{l}$ and $\Theta\geq0$, By assuming $\mathfrak{k}_1=3$, $\mathfrak{l}_1=4$, $\mathfrak{k}_2=2$, $\mathfrak{l}_2=5$, we obtain a non-trivial sequence as $(\mathfrak{k}_n,\mathfrak{l}_n)$ = $\left\{(3,4),(1,4),\left(\frac{1}{2},5\right),\ldots\right\}$ and taking $\Theta=1$, $\mathfrak{x}=2$, $\mathfrak{z}=3$.

$\mathfrak{A}(\mathfrak{k}_1, \mathfrak{l}_2, \Theta + \mathfrak{x} + \mathfrak{z}) = \mathfrak{A}(3,5,6) \geq \mathfrak{A}(3,4,\frac{1}{\beth(2,3)}) \star \mathfrak{A}(2,4,\frac{2}{\beth(2,4)}) \star \mathfrak{A}(2,5,\frac{3}{\beth(2,5)})$
$= \mathfrak{A}(\mathfrak{k}_1, \mathfrak{l}_1, \frac{\Theta}{\beth(\mathfrak{k}_1,\mathfrak{l}_1)}) \star \mathfrak{A}(\mathfrak{k}_2, \mathfrak{l}_1, \frac{\mathfrak{x}}{\beth(\mathfrak{k}_2,\mathfrak{l}_1)}) \star \mathfrak{A}(\mathfrak{k}_2, \mathfrak{l}_2, \frac{\mathfrak{z}}{\beth(\mathfrak{k}_2,\mathfrak{l}_2)})$
$\mathfrak{I}(\mathfrak{k}_1, \mathfrak{l}_2, \Theta + \mathfrak{x} + \mathfrak{z}) = \mathfrak{I}(3,5,6) \leq \mathfrak{I}(3,4,\frac{1}{\beth(2,3)}) \diamond \mathfrak{I}(2,4,\frac{2}{\beth(2,4)}) \diamond \mathfrak{I}(2,5,\frac{3}{\beth(2,5)})$
$= \mathfrak{I}(\mathfrak{k}_1, \mathfrak{l}_1, \frac{\Theta}{\beth(\mathfrak{k}_1,\mathfrak{l}_1)}) \diamond \mathfrak{I}(\mathfrak{k}_2, \mathfrak{l}_1, \frac{\mathfrak{x}}{\beth(\mathfrak{k}_2,\mathfrak{l}_1)}) \diamond \mathfrak{I}(\mathfrak{k}_2, \mathfrak{l}_2, \frac{\mathfrak{z}}{\beth(\mathfrak{k}_2,\mathfrak{l}_2)})$
$(\mathfrak{k}_1, \mathfrak{l}_2, \Theta + \mathfrak{x} + \mathfrak{z}) = (3,5,6) \leq \mathfrak{D}(3,4,\frac{1}{\beth(2,3)}) \diamond \mathfrak{D}(2,4,\frac{2}{\beth(2,4)}) \diamond \mathfrak{D}(2,5,\frac{3}{\beth(2,5)})$
$= \mathfrak{D}(\mathfrak{k}_1, \mathfrak{l}_1, \frac{\Theta}{\beth(\mathfrak{k}_1,\mathfrak{l}_1)}) \diamond \mathfrak{D}(\mathfrak{k}_2, \mathfrak{l}_1, \frac{\mathfrak{x}}{\beth(\mathfrak{k}_2,\mathfrak{l}_1)}) \diamond \mathfrak{D}(\mathfrak{k}_2, \mathfrak{l}_2, \frac{\mathfrak{z}}{\beth(\mathfrak{k}_2,\mathfrak{l}_2)})$.

By computing the above which satisfies condition of $\mathfrak{BCNMS}$. But, if we take some other values which does not satisfies the condition of $\mathfrak{CFMS}$.

Example 3.3

Let $\mathfrak{K}$=[0,1), $\mathfrak{L}$=[1,∞). Define $\mathfrak{A}$, $\mathfrak{I}$, $\mathfrak{D}$ are neutrosophic sets on $\mathfrak{K} \times \mathfrak{K} \times (0, +\infty)$ as $\mathfrak{A}(\mathfrak{k}, \mathfrak{l}, \Theta) = \frac{\Theta}{\Theta + d(\mathfrak{k},\mathfrak{l})}$, $\mathfrak{I}(\mathfrak{k}, \mathfrak{l}, \Theta) = \frac{d(\mathfrak{k},\mathfrak{l})}{\Theta + d(\mathfrak{k},\mathfrak{l})}$, $\mathfrak{D}(\mathfrak{k}, \mathfrak{l}, \Theta) = \frac{d(\mathfrak{k},\mathfrak{l})}{\Theta}$with the continuous t-norm * such that $\Theta_1 \star \Theta_2 = \min\{\Theta_1, \Theta_2\}$ and $\diamond$as t-conorm defined as $\Theta_1 \diamond \Theta_2 = \max\{\Theta_1, \Theta_2\}$.

Define $\beth : \mathfrak{K} \times \mathfrak{K} \to [1,+\infty)$ as $\beth(\mathfrak{k}, \mathfrak{l}) = \begin{cases} 1 & if\ \mathfrak{k} \in \mathfrak{K}\ and\ \mathfrak{l} \in \mathfrak{L} \\ max\{\mathfrak{k}, \mathfrak{l}\} & otherwise, \end{cases}$

Then $(\mathfrak{K}, \mathfrak{L}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond)$ be a $\mathfrak{BCNMS}$.

Theorem 3.4

Let $(\mathfrak{K}, \mathfrak{L}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond)$ be a $\mathfrak{BCNMS}$ with $\beth : \mathfrak{K} \times \mathfrak{K} \to [1,+\infty)$ and suppose that

$$lim_{n\to\infty} \mathfrak{A}(\mathfrak{k}, \mathfrak{l}, \Theta) = 1, lim_{n\to\infty} \mathfrak{I}(\mathfrak{k}, \mathfrak{l}, \Theta) = 0, lim_{n\to\infty} \mathfrak{D}(\mathfrak{k}, \mathfrak{l}, \Theta) = 0 \quad (3.4.1)$$

for all $\mathfrak{k} \in \mathfrak{K}\ and\ \mathfrak{l} \in \mathfrak{L}$. If $\pi : \mathfrak{K} \cup \mathfrak{L} \to \mathfrak{K} \cup \mathfrak{L}$ satisfies that:

a. $\pi(\mathfrak{K}) \subseteq \mathfrak{K}$ and $\pi(\mathfrak{L}) \subseteq \mathfrak{L}$,
b. $\mathfrak{A}(\pi\mathfrak{k}, \pi\mathfrak{l}, i\Theta) \geq \mathfrak{A}(\mathfrak{k}, \mathfrak{l}, \Theta)$, $\mathfrak{I}(\pi\mathfrak{k}, \pi\mathfrak{l}, i\Theta) \leq \mathfrak{I}(\mathfrak{k}, \mathfrak{l}, \Theta)$, $\mathfrak{D}(\pi\mathfrak{k}, \pi\mathfrak{l}, i\Theta) \leq \mathfrak{D}(\mathfrak{k}, \mathfrak{l}, \Theta)$ (3.4.2)

where $i \in (0,1)$. Also, we assume that for every $\mathfrak{k}_n \in$, $lim_{n\to\infty} \beth(\mathfrak{k}_n, \mathfrak{l})$
Exist and are finite.
Then $\beth$ has a unique fixed point.

Proof

Let $\mathfrak{k}_0 \in \mathfrak{K}\ and\ \mathfrak{l}_0 \in \mathfrak{L}\ and\ define\ (\mathfrak{k}_n, \mathfrak{l}_n)$ as a sequence by $\mathfrak{k}_n = \pi\mathfrak{k}_{n-1}$ and $\mathfrak{l}_n = \pi l_{n-1}$ for all $n \in \mathbb{N}$ on $\mathfrak{BCNMS}$ $(\mathfrak{K}, \mathfrak{L}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond)$. If $\mathfrak{k}_n = \mathfrak{k}_{n-1}$ then $\mathfrak{k}_n$ is a fixed point of T. Suppose that $\mathfrak{k}_n \neq \mathfrak{k}_{n-1}$ for all $\Theta > 0$ and $n \in \mathbb{N}$.

Successively applying inequality (3.4.2), we get

$$\mathfrak{A}(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \Theta) = \mathfrak{A}(\pi\mathfrak{k}_{n-1}, \pi\mathfrak{l}_n, \Theta) \geq \mathfrak{A}\left(\mathfrak{k}_{n-2}, \mathfrak{l}_{n-1}, \frac{\Theta}{i}\right) \ldots \geq \mathfrak{A}\left(\mathfrak{k}_0, \mathfrak{k}_1, \frac{\Theta}{i^{n-1}}\right)$$
$$\mathfrak{I}(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \Theta) = \mathfrak{I}(\pi\mathfrak{k}_{n-1}, \pi\mathfrak{l}_n, \Theta) \leq \mathfrak{I}\left(\mathfrak{k}_{n-2}, \mathfrak{l}_{n-1}, \frac{\Theta}{i}\right) \ldots \leq \mathfrak{I}\left(\mathfrak{k}_0, \mathfrak{k}_1, \frac{\Theta}{i^{n-1}}\right)$$
$$\mathfrak{D}(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \Theta) = \mathfrak{D}(\pi\mathfrak{k}_{n-1}, \pi\mathfrak{l}_n, \Theta) \leq \mathfrak{I}\left(\mathfrak{k}_{n-2}, \mathfrak{l}_{n-1}, \frac{\Theta}{i}\right) \ldots \leq \mathfrak{I}\left(\mathfrak{k}_0, \mathfrak{k}_1, \frac{\Theta}{i^{n-1}}\right) \quad (3.4.4)$$

Now, using the condition (iv), we have

$$\mathfrak{A}(\mathfrak{k}_n, \mathfrak{l}_{n+m}, \Theta) \geq \mathfrak{A}\left(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\mathfrak{k}_n, \mathfrak{l}_{n+1})}\right) \star \mathfrak{A}\left(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2})}\right) \star \mathfrak{A}\left(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})}\right)$$
$$\geq \mathfrak{A}\left(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\mathfrak{k}_n, \mathfrak{l}_{n+1})}\right) \star \mathfrak{A}\left(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2})}\right) \star \mathfrak{A}\left(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3})}\right)$$
$$\star \mathfrak{A}\left(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4})}\right)$$
$$\star \mathfrak{A}\left(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+m}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+m})}\right)$$

$$\geq \mathfrak{A}\left(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\mathfrak{k}_n, \mathfrak{l}_{n+1})}\right) \star \mathfrak{A}\left(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2})}\right) \star \mathfrak{A}\left(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3})}\right)$$
$$\star \mathfrak{A}\left(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4})}\right)$$
$$\star \mathfrak{A}\left(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+5}, \frac{\Theta}{(3)^3\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+5})}\right)$$
$$\star \mathfrak{A}\left(\mathfrak{k}_{n+5}, \mathfrak{l}_{n+6}, \frac{\Theta}{(3)^3\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+5}, \mathfrak{l}_{n+6})}\right)$$
$$\star \mathfrak{A}\left(\mathfrak{k}_{n+6}, \mathfrak{l}_{n+7}, \frac{\Theta}{(3)^3\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+6}, \mathfrak{l}_{n+7})}\right)$$

...

...

$$\mathfrak{A}(\mathfrak{k}_n, \mathfrak{l}_{n+m}, \Theta) \geq \mathfrak{A}\left(\mathfrak{k}_0, \mathfrak{k}_1, \frac{\Theta}{3\imath^{n-1}\beth(\mathfrak{k}_n,\mathfrak{l}_{n+1})}\right) \star \left[\prod_{i=n+1}^{n+m+2} \mathfrak{A}\left(\mathfrak{k}_0, \mathfrak{l}_1, \frac{\Theta}{(3)^{m-1}\imath^{i-1}\prod_{j=n+1}^{i}\beth(\mathfrak{k}_j,\mathfrak{l}_{n+m})\beth(\mathfrak{k}_i,\mathfrak{l}_{i+1})}\right)\right] \star$$
$$\left[\mathfrak{A}\left(\mathfrak{k}_0, \mathfrak{l}_1, \frac{\Theta}{(3)^{m-1}\imath^{n+m-1}\left(\prod_{j=n+1}^{n+m-1}\beth(\mathfrak{k}_j,\mathfrak{l}_{n+m})\right)}\right)\right] \quad (3.4.5)$$

$$\mathfrak{I}(\mathfrak{k}_n, \mathfrak{l}_{n+m}, \Theta) \leq \mathfrak{I}\left(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\mathfrak{k}_n,\mathfrak{l}_{n+1})}\right) \diamond$$
$$\mathfrak{I}\left(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2})}\right) \diamond \mathfrak{I}\left(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})}\right)$$
$$\leq \mathfrak{I}\left(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\mathfrak{k}_n, \mathfrak{l}_{n+1})}\right) \diamond \mathfrak{I}\left(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2})}\right) \diamond \mathfrak{I}\left(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3})}\right)$$
$$\diamond \mathfrak{I}\left(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4})}\right)$$
$$\diamond \mathfrak{I}\left(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+m}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+m})}\right)$$
$$\leq \mathfrak{I}\left(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\mathfrak{k}_n, \mathfrak{l}_{n+1})}\right) \diamond \mathfrak{I}\left(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2})}\right) \diamond \mathfrak{I}\left(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3})}\right)$$
$$\diamond \mathfrak{I}\left(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4})}\right)$$
$$\diamond \mathfrak{I}\left(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+5}, \frac{\Theta}{(3)^3\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+5})}\right)$$
$$\diamond \mathfrak{I}\left(\mathfrak{k}_{n+5}, \mathfrak{l}_{n+6}, \frac{\Theta}{(3)^3\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+5}, \mathfrak{l}_{n+6})}\right)$$
$$\diamond \mathfrak{I}\left(\mathfrak{k}_{n+6}, \mathfrak{l}_{n+7}, \frac{\Theta}{(3)^3\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+6}, \mathfrak{l}_{n+7})}\right)$$

...

$$\mathfrak{I}(\mathfrak{k}_n, \mathfrak{l}_{n+m}, \Theta) \leq \mathfrak{I}\left(\mathfrak{k}_0, \mathfrak{k}_1, \frac{\Theta}{3\imath^{n-1}\beth(\mathfrak{k}_n,\mathfrak{l}_{n+1})}\right) \diamond \left[\prod_{i=n+1}^{n+m+2} \mathfrak{I}\left(\mathfrak{k}_0, \mathfrak{l}_1, \frac{\Theta}{(3)^{m-1}\imath^{i-1}\prod_{j=n+1}^{i}\beth(\mathfrak{k}_j,\mathfrak{l}_{n+m})\beth(\mathfrak{k}_i,\mathfrak{l}_{i+1})}\right)\right] \diamond$$
$$\left[\mathfrak{I}\left(\mathfrak{k}_0, \mathfrak{l}_1, \frac{\Theta}{(3)^{m-1}\imath^{n+m-1}\left(\prod_{j=n+1}^{n+m-1}\beth(\mathfrak{k}_j,\mathfrak{l}_{n+m})\right)}\right)\right] \quad (3.4.6)$$

$$\mathfrak{D}(\mathfrak{k}_n, \mathfrak{l}_{n+m}, \Theta) \leq \mathfrak{D}\left(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\mathfrak{k}_n,\mathfrak{l}_{n+1})}\right) \diamond$$
$$\mathfrak{D}\left(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+1},\mathfrak{l}_{n+2})}\right) \diamond \mathfrak{D}\left(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+2},\mathfrak{l}_{n+m})}\right)$$
$$\leq \mathfrak{D}\left(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\mathfrak{k}_n, \mathfrak{l}_{n+1})}\right) \diamond \mathfrak{D}\left(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2})}\right) \diamond \mathfrak{D}\left(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\pi(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3})}\right)$$
$$\diamond \mathfrak{D}\left(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4})}\right)$$
$$\diamond \mathfrak{D}\left(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+m}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+m})}\right)$$

$$\leq \mathfrak{D}\left(\mathfrak{k}_n, \mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\mathfrak{k}_n, \mathfrak{l}_{n+1})}\right) \diamond \mathfrak{D}\left(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2}, \frac{\Theta}{3\beth(\mathfrak{k}_{n+1}, \mathfrak{l}_{n+2})}\right)$$
$$\diamond \mathfrak{D}\left(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+3})}\right)$$
$$\diamond \mathfrak{D}\left(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4}, \frac{\Theta}{(3)^2\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+3}, \mathfrak{l}_{n+4})}\right)$$
$$\diamond \mathfrak{D}\left(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+5}, \frac{\Theta}{(3)^3\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+4}, \mathfrak{l}_{n+5})}\right)$$
$$\diamond \mathfrak{D}\left(\mathfrak{k}_{n+5}, \mathfrak{l}_{n+6}, \frac{\Theta}{(3)^3\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+5}, \mathfrak{l}_{n+6})}\right)$$
$$\diamond \mathfrak{D}\left(\mathfrak{k}_{n+6}, \mathfrak{l}_{n+7}, \frac{\Theta}{(3)^3\beth(\mathfrak{k}_{n+2}, \mathfrak{l}_{n+m})\beth(\mathfrak{k}_{n+6}, \mathfrak{l}_{n+7})}\right)$$

$$\mathfrak{D}(\mathfrak{k}_n, \mathfrak{l}_{n+m}, \Theta) \leq \mathfrak{D}\left(\mathfrak{k}_0, \mathfrak{k}_1, \frac{\Theta}{3i^{n-1}\beth(\mathfrak{k}_n, \mathfrak{l}_{n+1})}\right) \diamond$$
$$\left[\prod_{i=n+1}^{n+m+2} \mathfrak{D}\left(\mathfrak{k}_0, \mathfrak{l}_1, \frac{\Theta}{(3)^{m-1} i^{i-1} \prod_{j=n+1}^{i} \beth(\mathfrak{k}_j, \mathfrak{l}_{n+m}) \beth(\mathfrak{k}_i, \mathfrak{l}_{i+1})}\right)\right] \diamond$$
$$\left[\mathfrak{D}\left(\mathfrak{k}_0, \mathfrak{l}_1, \frac{\Theta}{(3)^{m-1} i^{n+m-1}\left(\prod_{j=n+1}^{n+m-1} \beth(\mathfrak{k}_j, \mathfrak{l}_{n+m})\right)}\right)\right] \quad (3.4.7)$$

Therefore, by taking limit as $n \to \infty$ in (3.4.5),(3.4.6),(3.4.7) , from (3.4.4) together with (3.4.1),we have $lim_{n\to\infty} \mathfrak{A}(\mathfrak{k}_n, \mathfrak{l}_{n+m}, \Theta) \geq 1 \star 1 \star 1 \star. . . \star 1 = 1$,
$lim_{n\to\infty}\mathfrak{J}(\mathfrak{k}_n, \mathfrak{l}_{n+m}, \Theta) \leq 0 \diamond 0 \diamond 0 \diamond. . . \diamond 0 = 0$ and
$lim_{n\to\infty}\mathfrak{D}(\mathfrak{k}_n, \mathfrak{l}_{n+m}, \Theta) \leq 0 \diamond 0 \diamond 0 \diamond. . . \diamond 0 = 0$, for all $\Theta > 0, n < m$ *and* $n, m \in \mathbb{N}$.
Thus, $(\mathfrak{k}_n, \mathfrak{l}_n)$ is a BPC- (Bipolar controlled) Cauchy sequence in X. From the completeness of$(\mathfrak{K}, \mathfrak{L},\mathfrak{A}, \mathfrak{J}, \mathfrak{D}, \star, \diamond)$there exits $\mathfrak{u} \in \mathfrak{K}\cap \mathfrak{L}$ which is a limit of the both sequences $\{\mathfrak{k}_n\}$*and* $\{\mathfrak{l}_n\}$ such that,
$\lim_{n\to\infty}\mathfrak{A}(\pi\mathfrak{u}, \mathfrak{u}, \Theta) = 1$, $\lim_{n\to\infty}\mathfrak{J}(\pi\mathfrak{u}, \mathfrak{u}, \Theta) = 0$, $\lim_{n\to\infty}\mathfrak{D}(\pi\mathfrak{u}, \mathfrak{u}, \Theta) = 0$ for all $\Theta > 0$.
Now, we show that Θ is a fixed point of π. For any $\Theta > 0$ and from the condition (iv) we have,

$$\mathfrak{A}(\pi\mathfrak{u}, \mathfrak{u}, \Theta) \geq \mathfrak{A}\left(\pi\mathfrak{u}, \pi\mathfrak{l}_n, \frac{\Theta}{3\beth(\pi\mathfrak{u}, \pi\mathfrak{l}_n)}\right) \star \mathfrak{A}\left(\pi\mathfrak{k}_n, \pi\mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\pi\mathfrak{k}_n, \pi\mathfrak{l}_{n+1})}\right) \star \mathfrak{A}\left(\pi\mathfrak{k}_{n+1}, \mathfrak{u}, \frac{\Theta}{3\beth(\pi\mathfrak{k}_{n+1}, \mathfrak{u})}\right)$$
$$\mathfrak{J}(\pi\mathfrak{u}, \mathfrak{u}, \Theta) \leq \mathfrak{J}\left(\pi\mathfrak{u}, \pi\mathfrak{l}_n, \frac{\Theta}{3\beth(\pi\mathfrak{u}, \pi\mathfrak{l}_n)}\right) \diamond \mathfrak{J}\left(\pi\mathfrak{k}_n, \pi\mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\pi\mathfrak{k}_n, \pi\mathfrak{l}_{n+1})}\right) \diamond \mathfrak{J}\left(\pi\mathfrak{k}_{n+1}, \mathfrak{u}, \frac{\Theta}{3\beth(\pi\mathfrak{k}_{n+1}, \mathfrak{u})}\right)$$
$$\mathfrak{D}(\pi\mathfrak{u}, \mathfrak{u}, \Theta \leq \mathfrak{D}\left(\pi\mathfrak{u}, \pi\mathfrak{l}_n, \frac{\Theta}{3\beth(\pi\mathfrak{u}, \pi\mathfrak{l}_n)}\right) \diamond \mathfrak{D}\left(\pi\mathfrak{k}_n, \pi\mathfrak{l}_{n+1}, \frac{\Theta}{3\beth(\pi\mathfrak{k}_n, \pi\mathfrak{l}_{n+1})}\right) \diamond \mathfrak{D}\left(\pi\mathfrak{k}_{n+1}, \mathfrak{u}, \frac{\Theta}{3\beth(\pi\mathfrak{k}_{n+1}, \mathfrak{u})}\right)$$

By taking limit as $n \to \infty$ in above equation and using (3.4.5),(3.4.6),(3.4.7) we get$\lim_{n\to\infty}\mathfrak{A}(\pi\mathfrak{u}, \mathfrak{u}, \Theta) = 1$, $\lim_{n\to\infty}\mathfrak{J}(\pi\mathfrak{u}, \mathfrak{u}, \Theta) = 0$, $\lim_{n\to\infty}\mathfrak{D}(\pi\mathfrak{u}, \mathfrak{u}, \Theta) = 0$ for all $\Theta > 0$ that is $\pi\mathfrak{u} = \mathfrak{u}$.
For uniqueness, let $\mathfrak{w} \in \mathfrak{K} \cap \mathfrak{L}$ is another fixed point of π and there exists $\Theta > 0$ such that $\mathfrak{A}(\mathfrak{u}, \mathfrak{w}, \Theta) \neq 1, \mathfrak{J}(\mathfrak{u}, \mathfrak{w}, \Theta) \neq 0, \mathfrak{D}(\mathfrak{u}, \mathfrak{w}, \Theta) \neq 0$, then it follows from (3.4.2) that

$$\mathfrak{A}(\mathfrak{u}, \mathfrak{w}, \Theta) = \mathfrak{A}(\pi\mathfrak{u}, \pi\mathfrak{w}, \Theta) \geq \mathfrak{A}\left(\mathfrak{u}, \mathfrak{w}, \frac{\Theta}{i}\right) \geq \mathfrak{A}\left(\mathfrak{u}, \mathfrak{w}, \frac{\Theta}{i^2}\right) \dots \geq \mathfrak{A}\left(\mathfrak{u}, \mathfrak{w}, \frac{\Theta}{i^n}\right)$$
$$\mathfrak{J}(\mathfrak{u}, \mathfrak{w}, \Theta) = \mathfrak{J}(\pi\mathfrak{u}, \pi\mathfrak{w}, \Theta) \leq \mathfrak{J}\left(\mathfrak{u}, \mathfrak{w}, \frac{\Theta}{i}\right) \leq \mathfrak{J}\left(\mathfrak{u}, \mathfrak{w}, \frac{\Theta}{i^2}\right) \dots \leq \mathfrak{J}\left(\mathfrak{u}, \mathfrak{w}, \frac{\Theta}{i^n}\right)$$
$$\mathfrak{D}(\mathfrak{u}, \mathfrak{w}, \Theta) = \mathfrak{D}(\pi\mathfrak{u}, \pi\mathfrak{w}, \Theta) \leq \mathfrak{D}\left(\mathfrak{u}, \mathfrak{w}, \frac{\Theta}{i}\right) \leq \mathfrak{D}\left(\mathfrak{u}, \mathfrak{w}, \frac{\Theta}{i^2}\right) \dots \leq \mathfrak{D}\left(\mathfrak{u}, \mathfrak{w}, \frac{\Theta}{i^n}\right) \quad (3.4.8)$$

for all $n \in \mathbb{N}$. By taking limit as $n \to \infty$ in (3.8) , $\mathfrak{A}(\mathfrak{u}, \mathfrak{w}, \Theta) = 1$ for all $\Theta > 0$ that is
$\mathfrak{u} = \mathfrak{w}$. This completes the proof.

Example 3.5
Let $\mathfrak{K} = [0,3)$ and $\mathfrak{L} = [3, \infty)$. Define $\mathfrak{A}, \mathfrak{J}, \mathfrak{D}: \mathfrak{K} \times \mathfrak{K} \times [0, \infty) \to [0,1]$ as

$$\mathfrak{A}(\mathfrak{k}, \mathfrak{l}, \Theta) = \begin{cases} 1 & if\ \mathfrak{k} = \mathfrak{l} \\ \dfrac{\Theta}{\Theta + \frac{2}{\mathfrak{l}}} & if\ \mathfrak{k} \in \mathfrak{K}\ and\ \mathfrak{l} \in \mathfrak{L} \\ \dfrac{\Theta}{\Theta + \frac{2}{\mathfrak{k}}} & if\ \mathfrak{k} \in \mathfrak{K}\ and\ \mathfrak{l} \in \mathfrak{L} \\ \dfrac{1}{\Theta + 1} & otherwise \end{cases}$$

$$\mathfrak{I}(\mathfrak{k},\mathfrak{l},\Theta)=\begin{cases}0 & if\ \ \mathfrak{k}=\mathfrak{l}\\ \frac{\frac{2}{\mathfrak{l}}}{\Theta+\frac{2}{\mathfrak{l}}} & if\ \mathfrak{k}\in\mathfrak{K}\ and\ \mathfrak{l}\in\mathfrak{L}\\ \frac{\frac{2}{\mathfrak{k}}}{\Theta+\frac{2}{\mathfrak{k}}} & if\ \mathfrak{k}\in\mathfrak{K}\ and\ \mathfrak{l}\in\mathfrak{L}\\ \frac{\Theta}{\Theta+1} & otherwise\end{cases}$$

$$\mathfrak{D}(\mathfrak{k},\mathfrak{l},\Theta)=\begin{cases}0 & if\ \ \mathfrak{k}=\mathfrak{l}\\ \frac{2}{\mathfrak{l}\Theta} & if\ \mathfrak{k}\in\mathfrak{K}\ and\ \mathfrak{l}\in\mathfrak{L}\\ \frac{2}{\mathfrak{k}\Theta} & if\ \mathfrak{k}\in\mathfrak{K}\ and\ \mathfrak{l}\in\mathfrak{L}\\ \Theta & otherwise\end{cases}$$

For CTS product t-norm and t-conorm.

Consider $\beth:\mathfrak{K}\times\mathfrak{L}\to[1,\frac{1}{\mathcal{B}^n})$ where$\mathcal{B}\in(0,1)\ and\ n\in\mathbb{N}$ as

$$\beth(\mathfrak{k},\mathfrak{l})=\begin{cases}1 & if\ \ \mathfrak{k},\mathfrak{l}\in\mathfrak{M}\\ max\{\mathfrak{k},\mathfrak{l}\} & otherwise\end{cases}$$

Clearly, $(\mathfrak{K},\mathfrak{L},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond)$ be a $\mathfrak{BCNMS}$. Consider$\pi:\mathfrak{K}\cup\mathfrak{L}\to\mathfrak{K}\cup\mathfrak{L}$by

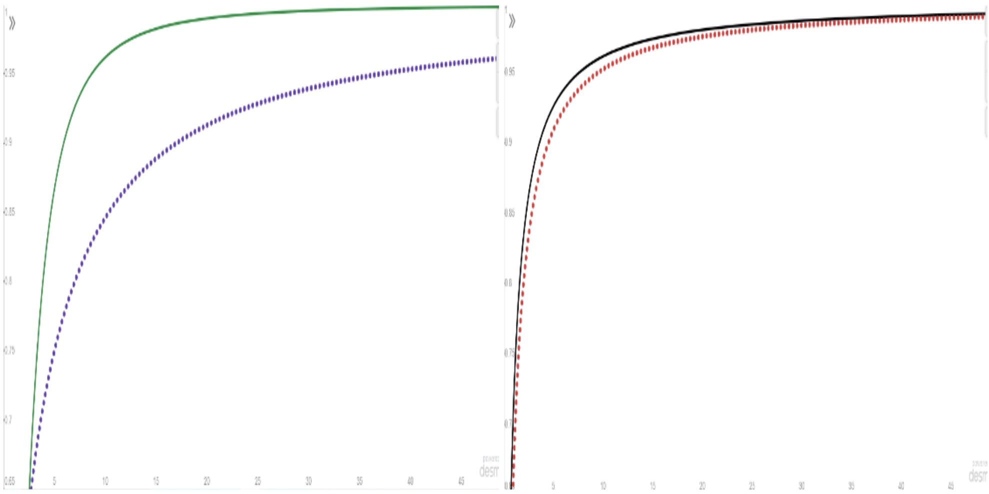
$$\pi(v)=\begin{cases}v & if\ v\in\mathfrak{M}\\ v^2+1 & if\ v\in\mathfrak{W}\end{cases}$$

For all $v\in\mathfrak{K}\ and\ \mathcal{B}=0.5$.For each of the four scenarios listed below, the disparity needs to be confirmed.

Case I: If$\mathfrak{k}=\mathfrak{l}$ then there's $\pi\mathfrak{k}=\pi\mathfrak{l}$. In the present case:$\mathfrak{A}(\pi\mathfrak{k},\pi\mathfrak{l},i\Theta)=1=\mathfrak{A}(\mathfrak{k},\mathfrak{l},\Theta)$,$\mathfrak{I}(\pi\mathfrak{k},\pi\mathfrak{l},i\Theta)=0=\mathfrak{I}(\mathfrak{k},\mathfrak{l},\Theta)$, $\mathfrak{D}(\pi\mathfrak{k},\pi\mathfrak{l},i\Theta)=0=\mathfrak{D}(\mathfrak{k},\mathfrak{l},\Theta)$ (3.4.9)

Case II: If$\mathfrak{k}\in\mathfrak{K}\ and\ \mathfrak{l}\in\mathfrak{L}$, we have $\pi\mathfrak{k}\in\mathfrak{K},\pi\mathfrak{l}\in\mathfrak{L}$.

$$\mathfrak{A}(\pi\mathfrak{k},\pi\mathfrak{l},i\Theta)=\frac{i\Theta}{i\Theta+\frac{2}{\pi\mathfrak{l}}}=\frac{0.5\Theta}{0.5\Theta+\frac{2}{c^2+1}}\geq\frac{\Theta}{\Theta+\frac{2}{\mathfrak{l}+1}}=\mathfrak{A}(\mathfrak{k},\mathfrak{l},\Theta) \quad (3.4.10)$$

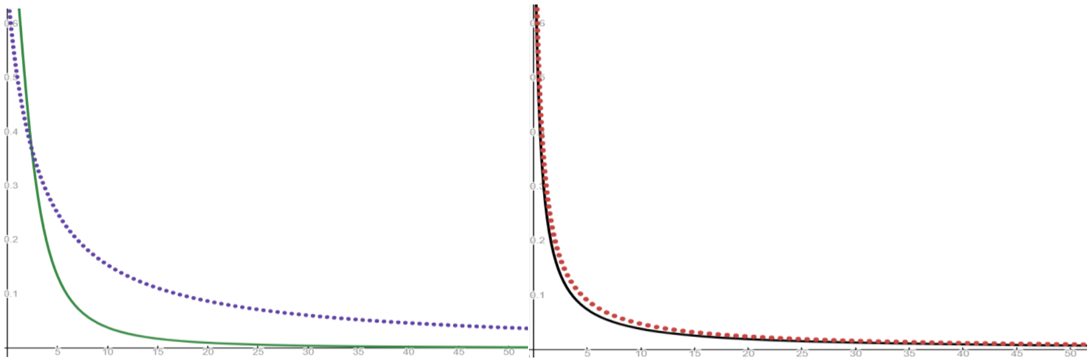


value of $\mathfrak{l}$ in $\mathfrak{A}$ value of Θ in $\mathfrak{A}$

(a) (b)

Figure 1. Fluctuation of $\mathfrak{A}(\pi\mathfrak{k},\pi\mathfrak{l},i\Theta)$ with $\mathfrak{A}(\mathfrak{k},\mathfrak{l},\Theta)$ of case II on 2D view, for:
(a) $\mathfrak{A}(\pi\mathfrak{k},\pi\mathfrak{l},i\Theta)$blue curve vs$\mathfrak{A}(\mathfrak{k},\mathfrak{l},\Theta)$ green dotted curve at$\Theta=1$ and $\mathfrak{l}\in(3,50)$.
(b) $\mathfrak{A}(\pi\mathfrak{k},\pi\mathfrak{l},i\Theta)$violet curve vs$\mathfrak{A}(\mathfrak{k},\mathfrak{l},\Theta)$black dotted curve at $\Theta\in(1,50)$and $\mathfrak{l}=3$.

$$\mathfrak{I}(\pi\mathfrak{k},\pi\mathfrak{l},i\Theta)=\frac{\frac{2}{q\mathfrak{l}}}{i\Theta+\frac{2}{\pi\mathfrak{l}}}=\frac{\frac{2}{\mathfrak{l}^2+1}}{0.5\Theta+\frac{2}{\mathfrak{l}^2+1}}\leq\frac{\frac{2}{\mathfrak{l}+1}}{\Theta+\frac{2}{\mathfrak{l}+1}}=\mathfrak{I}(\mathfrak{k},\mathfrak{l},\Theta) \quad (3.4.11)$$

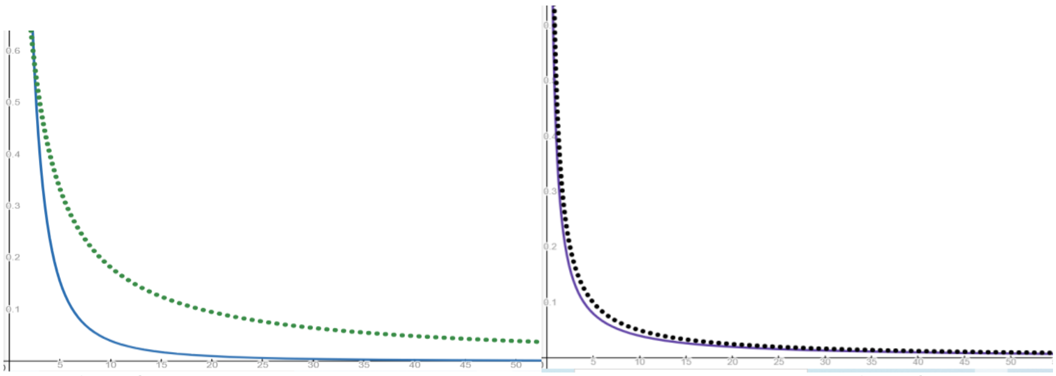


value of ɭ in $\mathfrak{I}$ value of Θ in $\mathfrak{I}$

(a) (b)

Figure 2. Fluctuation of $\mathfrak{I}(\pi ƚ, \pi ɭ, i\Theta)$ with $\mathfrak{I}(ƚ, ɭ, \Theta)$ of case II on 2D view, for:
(a) $\mathfrak{I}(\pi ƚ, \pi ɭ, i\Theta)$blue curve vs$\mathfrak{I}(ƚ, ɭ, \Theta)$ green dotted curve at$\Theta = 1$ and $ɭ \in (3,50)$.
(b) $\mathfrak{I}(\pi ƚ, \pi ɭ, i\Theta)$violet curve vs$\mathfrak{I}(ƚ, ɭ, \Theta)$black dotted curve at $\Theta \in (1,50)$and $ɭ = 3$.

$$\mathfrak{D}(\pi ƚ, \pi ɭ, i\Theta) = \frac{2}{\pi ɭ \mathcal{B}\Theta} = \frac{2}{0.5\Theta(ɭ^2+1)} \leq \frac{2}{(ɭ+1)\Theta} = \mathfrak{D}(ƚ, ɭ, \Theta) (3.4.12)$$



value of ɭ in $\mathfrak{D}$ value of Θ in $\mathfrak{D}$

(a) (b)

Figure 3. Fluctuation of$\mathfrak{D}(\pi ƚ, \pi ɭ, i\Theta)$ with $\mathfrak{D}(ƚ, ɭ, \Theta)$ of case II on 2D view, for:
(a) $\mathfrak{D}(\pi ƚ, \pi ɭ, i\Theta)$blue curve vs$\mathfrak{D}(ƚ, ɭ, \Theta)$ green dotted curve at$\Theta = 1$ and $ɭ \in (3,50)$.
$(b)$$\mathfrak{D}(\pi ƚ, \pi ɭ, i\Theta)$violet curve vs $(ƚ, ɭ, \Theta)$black dotted curve at $\Theta \in (1,50)$and $ɭ = 3$.

Table 1 and 2 shows the fluctuation between $\mathfrak{A}(\pi ƚ, \pi ɭ, i\Theta) and\ \mathfrak{A}(ƚ, ɭ, \Theta), \mathfrak{I}(\pi ƚ, \pi ɭ, i\Theta)\ and\ \mathfrak{I}(ƚ, ɭ, \Theta), \mathfrak{D}(\pi ƚ, \pi ɭ, \mathcal{B}\Theta)\ and\ \mathfrak{D}(ƚ, ɭ, \Theta)$ as a mapping of ɭ with relative to Θ.The contour for the estimation of Θ is towering to 50 as a mapping of ɭ.
At $\Theta = 70$,
$\mathfrak{A}(\pi ƚ, \pi ɭ, i\Theta)$changed to 1, and after higher values of Θ , it changeless, $(\Theta = 1)$.
$\mathfrak{A}(ƚ, ɭ, \Theta)$doesn't change till $\Theta = 100$, but it arrived nearer to 1.
$\mathfrak{I}(\pi ƚ, \pi ɭ, i\Theta)$changed 0, and after higher values of Θ , it changeless, $(\Theta = 0)$. $\mathfrak{I}(ƚ, ɭ, \Theta)$doesn't change till $\Theta = 100$, but it arrived nearer to 0.
$\mathfrak{D}(\pi ƚ, \pi ɭ, i\Theta)$changed 0, and after higher values of Θ , it changeless, $(\Theta = 0)$. $\mathfrak{D}(ƚ, ɭ, \Theta)$doesn't change till $\Theta = 100$, but it arrived nearer to 0.

Table 1. Fluctuation between
$\mathfrak{A}(\pi ƚ, \pi ɭ, i\Theta) and\ \mathfrak{A}(ƚ, ɭ, \Theta), \mathfrak{I}(\pi ƚ, \pi ɭ, \mathcal{B}\Theta)\ and\ \mathfrak{I}(ƚ, ɭ, \Theta), \mathfrak{D}(\pi ƚ, \pi ɭ, i\Theta)\ and\ \mathfrak{D}(ƚ, ɭ, \Theta)$ as a mapping of ɭ with unchanged estimation of $\Theta = 1\ and\ \Theta = 50$.

Value of Θ	Value of ɭ	$\mathfrak{A}(\pi ƚ, \pi ɭ, i\Theta)$	$\mathfrak{A}(ƚ, ɭ, \Theta)$	$\mathfrak{I}(\pi ƚ, \pi ɭ, i\Theta)$	$\mathfrak{I}(ƚ, ɭ, \Theta)$	$\mathfrak{D}(\pi ƚ, \pi ɭ, i\Theta)$	$\mathfrak{D}(ƚ, ɭ, \Theta)$
1	2	0.5556	0.5000	0.2857	0.4444	0.8000	0.4000
	20	0.9901	0.9091	0.0050	0.0099	0.0100	0.0050
	50	0.9984	0.9615	0.0008	0.0016	0.0016	0.0008
	100	0.9996	0.9804	0.0002	0.0004	0.0004	0.0002
50	2	0.9843	0.9804	0.0079	0.0157	0.0160	0.0080
	20	0.9998	0.9980	0.0000	0.0002	0.0002	0.0001

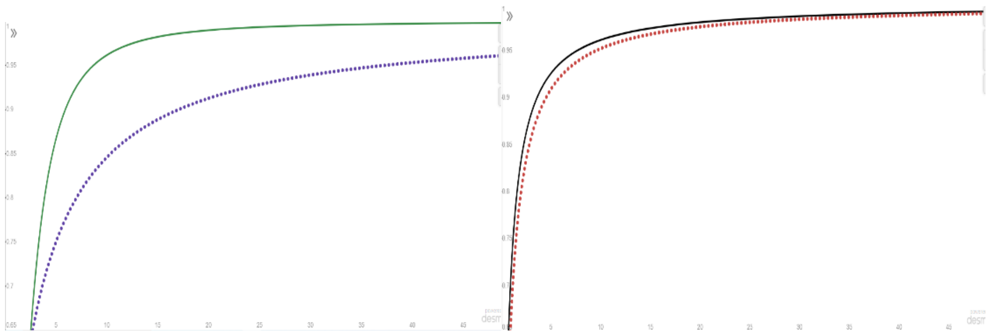
	50 100	1.0000 1.0000	0.9992 0.9996	0.0000 0.0000	0.0000 0.0000	0.0000 0.0000	0.0000 0.0000

Table 2. Fluctuation between $\mathfrak{A}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)\, and\ \mathfrak{A}(\mathfrak{t},\mathfrak{l},\Theta)$, $\mathfrak{I}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)\ and\ \mathfrak{I}(\mathfrak{t},\mathfrak{l},\Theta)$, $\mathfrak{D}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)\ and\ \mathfrak{D}(\mathfrak{t},\mathfrak{l},\Theta)$ as a mapping of Θ with unchanged value of $\mathfrak{l} = 3\ and\ \mathfrak{l} = 50$.

Value of $\mathfrak{l}$	Value of Θ	$\mathfrak{A}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)$	$\mathfrak{A}(\mathfrak{t},\mathfrak{l},\Theta)$	$\mathfrak{I}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)$	$\mathfrak{I}(\mathfrak{t},\mathfrak{l},\Theta)$	$\mathfrak{D}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)$	$\mathfrak{D}(\mathfrak{t},\mathfrak{l},\Theta)$
2	1 20 50 100	0.7143 0.9615 0.9843 0.9921	0.6667 0.9524 0.9804 0.9901	0.4444 0.0385 0.0157 0.0079	0.2857 0.0196 0.0079 0.0040	0.8000 0.0400 0.0160 0.0080	0.4000 0.0200 0.0080 0.0040
50	1 20 50 100	0.9984 0.9999 1.0000 1.0000	0.9615 0.9980 0.9992 0.9996	0.0016 0.0000 0.0000 0.0000	0.0008 0.0000 0.0000 0.0000	0.0016 0.0000 0.0000 0.0000	0.0008 0.0000 0.0000 0.0000

Case III: If $\mathfrak{t} \in \mathfrak{W}, \mathfrak{l} \in \mathfrak{M}$, we have $\pi\mathfrak{t} \in \mathfrak{W}, \pi\mathfrak{l} \in \mathfrak{M}$.

$$\mathfrak{A}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta) = \frac{\iota\Theta}{\iota\Theta+\frac{2}{q\,\mathfrak{t}}} = \frac{0.5\Theta}{0.5\Theta+\frac{2}{\mathfrak{t}^2+1}} \geq \frac{\Theta}{\Theta+\frac{2}{\mathfrak{t}+1}} = \mathfrak{A}(\mathfrak{t},\mathfrak{l},\Theta)(3.4.13)$$

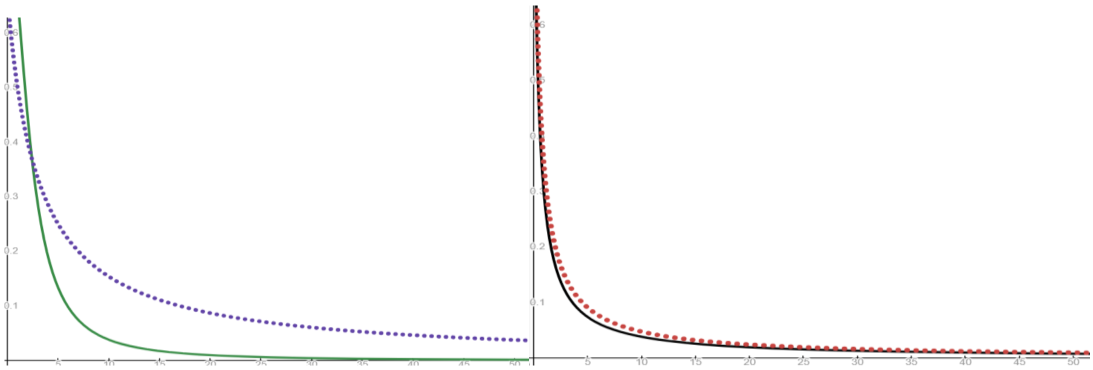


value of $\mathfrak{t}$in $\mathfrak{A}$ value of Θ in $\mathfrak{A}$

(a) (b)

Figure 4. Fluctuation of$\mathfrak{A}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)$ with $\mathfrak{A}(\mathfrak{t},\mathfrak{l},\Theta)$ of case III on 2D view, for:
(a) $\mathfrak{A}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)$bluevs$\mathfrak{A}(\mathfrak{t},\mathfrak{l},\Theta)$green dotted at $\Theta = 1$and $\mathfrak{t} \in (3{,}50)$.
$(b)$$\mathfrak{A}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)$violetvs$\mathfrak{A}(\mathfrak{t},\mathfrak{l},\Theta)$ black dotted at $\Theta \in (1{,}50)$ and $\mathfrak{t} = 3$.

$$\mathfrak{I}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta) = \frac{\frac{2}{\pi\mathfrak{t}}}{\iota\Theta+\frac{2}{\pi\,\mathfrak{t}}} = \frac{\frac{2}{\mathfrak{t}^2+1}}{0.5\Theta+\frac{2}{\mathfrak{t}^2+1}} \leq \frac{\frac{2}{\mathfrak{t}+1}}{\Theta+\frac{2}{\mathfrak{t}+1}} = \mathfrak{I}(\mathfrak{t},\mathfrak{l},\Theta) \qquad (3.4.14)$$

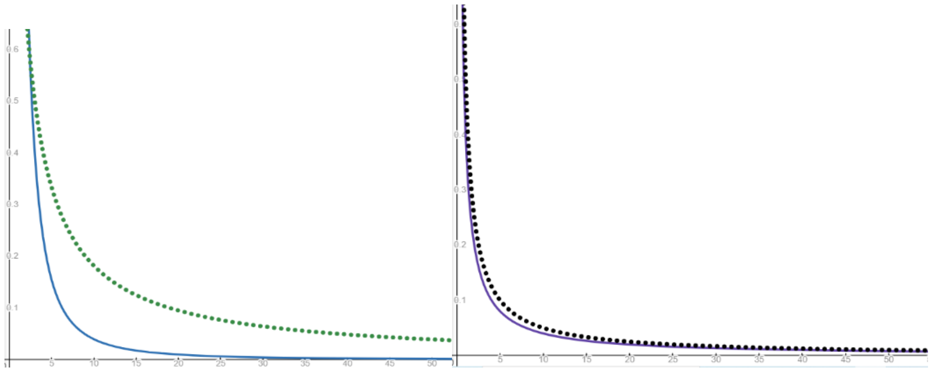


value of $\mathfrak{t}$in $\mathfrak{I}$ value of Θin $\mathfrak{I}$

(a) (b)

Figure 5. Fluctuation of$\mathfrak{I}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)$ with $\mathfrak{I}(\mathfrak{t},\mathfrak{l},\Theta)$ of case III on 2D view, for:
(a) $\mathfrak{I}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)$bluevs$\mathfrak{I}(\mathfrak{t},\mathfrak{l},\Theta)$green dotted at $\Theta = 1$and $\mathfrak{t} \in (3{,}50)$.
(b) $\mathfrak{I}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta)$violetvs$\mathfrak{I}(\mathfrak{t},\mathfrak{l},\Theta)$ black dotted at $\Theta \in (1{,}50)$ and $\mathfrak{t} = 3$.

$$\mathfrak{D}(\pi\mathfrak{t},\pi\mathfrak{l},\iota\Theta) = \frac{2}{\pi\mathfrak{t}B\Theta} = \frac{2}{0.5\Theta(\mathfrak{t}^2+1)} \leq \frac{2}{(\mathfrak{t}+1)\Theta} = \mathfrak{D}(\mathfrak{t},\mathfrak{l},\Theta)(3.4.15)$$



value of $\mathfrak{t} in \mathfrak{D}$ value of Θ in $\mathfrak{D}$
(a) (b)

Figure 6. Fluctuation of $\mathfrak{D}(\pi\mathfrak{t},\pi\mathfrak{l},\imath\Theta)$ with $\mathfrak{D}(\mathfrak{t},\mathfrak{l},\Theta)$ of case III on 2D view, for:
(a) $\mathfrak{D}(\pi\mathfrak{t},\pi\mathfrak{l},\imath\Theta)$bluevs$\mathfrak{D}(\mathfrak{t},\mathfrak{l},\Theta)$green dotted at $\Theta = 1$and $\mathfrak{t} \in (3,50)$.
(b) $\mathfrak{D}(\pi\mathfrak{t},\pi\mathfrak{l},\imath\Theta)$violetvs$\mathfrak{D}(\mathfrak{t},\mathfrak{l},\Theta)$ black dotted at $\Theta \in (1,50)$ and $\mathfrak{t} = 3$.

Case IV

If $\mathfrak{t},\mathfrak{l}$ does not belongs to above any cases, then $\mathfrak{A}(q\mathfrak{t},q\mathfrak{l},\mathcal{B}\Theta) and\ \mathfrak{A}(\mathfrak{t},\mathfrak{l},\Theta), \mathfrak{I}(\pi\mathfrak{t},\pi\mathfrak{l},\imath\Theta)\ and\ \mathfrak{I}(\mathfrak{t},\mathfrak{l},\Theta), \mathfrak{D}(\pi\mathfrak{t},\pi\mathfrak{l},\mathcal{B}\Theta)\ and\ \mathfrak{D}(\mathfrak{t},\mathfrak{l},\Theta)$ depends on only Θ, we have $\mathfrak{A}(\pi\mathfrak{t},\pi\mathfrak{l},\imath\Theta) = \frac{1}{\imath\Theta+1} = \frac{1}{0.5\Theta+1} \geq \frac{1}{\Theta+1} = \mathfrak{A}(\mathfrak{t},\mathfrak{l},\Theta)$(3.4.16)

$\mathfrak{I}(\pi\mathfrak{t},\pi\mathfrak{l},\imath\Theta) = \frac{\mathfrak{z}}{\imath\Theta+1} = \frac{\Theta}{0.5\Theta+1} \leq \frac{\Theta}{\Theta+1} = \mathfrak{I}(\mathfrak{t},\mathfrak{l},\Theta)$(3.4.17)

$\mathfrak{D}(\pi\mathfrak{t},\pi\mathfrak{l},\imath\Theta) = \imath\Theta = 0.5\Theta \leq \Theta = \mathfrak{D}(\mathfrak{t},\mathfrak{l},\Theta)$(3.4.18)

Which implies that all the condition of theorem 3.5 hold and π has a unique fixed point $\mathfrak{t} = 1$.

Theorem 3.6

Let $(\mathfrak{K},\mathfrak{L},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond)$ be a $\mathfrak{BCNMS}$and $\pi : \mathfrak{K} \cup \mathfrak{L} \to \mathfrak{K} \cup \mathfrak{L}$ be a mapping satisfying $lim_{n\to\infty}\,\mathfrak{A}(\mathfrak{t},\mathfrak{l},\Theta) = 1$, $lim_{n\to\infty}\mathfrak{I}(\mathfrak{t},\mathfrak{l},\Theta) = 0$, $lim_{n\to\infty}\mathfrak{D}(\mathfrak{t},\mathfrak{l},\Theta) = 0$

Suppose there exist a constant $\imath \in (0,1)$ such that $\int_0^{\mathfrak{A}(\pi\mathfrak{t},\pi\mathfrak{l},\imath\Theta)} \varphi(t)dt \geq \int_0^{\mathfrak{A}(\mathfrak{t},\mathfrak{l},\Theta)} \varphi(t)dt$,
$\int_0^{\mathfrak{I}(\pi\mathfrak{t},\pi\mathfrak{l},\imath\Theta)} \varphi(t)dt \leq \int_0^{\mathfrak{I}(\mathfrak{t},\mathfrak{l},\Theta)} \varphi(t)dt$, $\int_0^{\mathfrak{D}(\pi\mathfrak{t},\pi\mathfrak{l},\imath\Theta)} \varphi(t)dt \leq \int_0^{\mathfrak{D}(\mathfrak{t},\mathfrak{l},\Theta)} \varphi(t)dt$ (3.4.19)

For all $\mathfrak{t} \in \mathfrak{K}, \mathfrak{l} \in \mathfrak{L}$. Then π has a fixed point.

Proof

By taking $\varphi(t) = 1$ in equation (3.4.19) we obtain the above theorem (3.5).

4. Application for the spring mass system

It is commonly recognised that an automobile suspension system is a practical application for the spring mass system in engineering challenges. Examine the motion of an automobile spring on a rough, pitted road, where the road acts as a forcing term and shock absorbers act as a dampening agent. The system may be affected by tension force, earthquakes, ground vibrations, and gravity, among other external influences.

The following initial value issue governs the critically damped motion of this system when it is subjected to an external force F. Let m be the mass of the spring and F be the external force acting on it.

$$\begin{cases} m\frac{d^2q}{d\Theta^2} + l\frac{dq}{d\Theta} - mF(\Theta, q(\Theta)) = 0 \\ \qquad q(0) = 0 \\ \qquad q'(0) = 0 \end{cases} \tag{4.1}$$

Where $l > 0$ is a continuous function that represents the damping constant. It is simple to demonstrate that the integral equation and problem (4.1) are equivalent.

$$q(\Theta) = \int_0^T \Upsilon(\Theta,\gamma)F(\Theta,\gamma,u(\gamma))d\gamma. \tag{4.2}$$

Where $\Upsilon(\Theta,\gamma)$ is Green's function given by

$$\Upsilon(\Theta,\gamma) = \begin{cases} \frac{1-e^{\mu(\Theta-\gamma)}}{\mu} & if\ 0 \leq \gamma \leq \Theta \leq T \\ 0 & if\ 0 \leq \Theta \leq \gamma \leq T \end{cases} \tag{4.3}$$

Where $\mu = l/m$keeps constant. This section goes over the existence of q as a solution to the integral problem using theorem (3.5)

$$q(\Theta) = \int_0^T G(\Theta,\gamma,u(\gamma))d\gamma. \tag{4.4}$$

Let $\mathfrak{K}$ = C([0,T]) be the set of real continuous functions defined on [0,T].
For $k \in (0,1)$ we define

$$\mathfrak{A}(\mathfrak{k}, \mathfrak{l}, \Theta) = sup_{\Theta \in [0,T]} \left(\frac{\Theta}{\Theta + (|x(\Theta) - y(\Theta)|)} \right)$$

$\mathfrak{I}(\mathfrak{k}, \mathfrak{l}, \Theta) = Inf_{\Theta \in [0,T]} \left(\frac{|x(\Theta)-y(\Theta)|}{\Theta + (|x(\Theta)-y(\Theta)|)} \right)$, $\mathfrak{D}(\mathfrak{k}, \mathfrak{l}, \Theta) = Inf_{\Theta \in [0,T]} \left(\frac{(|x(\Theta)-y(\Theta)|)}{\Theta} \right)$, For all $\mathfrak{k} \in \mathfrak{K}$ and $\mathfrak{l} \in \mathfrak{L}$. Define $\beth : \mathfrak{K} \times \mathfrak{K} \to [1,+\infty)$ as

$$\beth(\mathfrak{k}, \mathfrak{l}) = \begin{cases} 1 & if \ \mathfrak{k} \in \mathfrak{K} \ and \ \mathfrak{l} \in \mathfrak{L} \\ max\{\mathfrak{k}, \mathfrak{l}\} & otherwise \end{cases}$$

It is easy to prove that $(\mathfrak{K}, \mathfrak{L}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond)$ is a $\mathfrak{BCNMS}$.
consider the mapping $\omega : \mathfrak{K} \cup \mathfrak{L} \to \mathfrak{K} \cup \mathfrak{L}$ defined by $\omega\mathfrak{k}(\Theta) = \int_0^T G(\Theta, \gamma, q(\gamma)) d\gamma$.

Theorem 4.1
Suppose that
1. There exist a continuous function $\Upsilon : [0,T] \times [0,T] \to \mathbb{R}^+$ such that $sup_{t \in [0,T]} \int_0^T \Upsilon(t,\gamma) d\gamma \le 1$
2. $\Delta : [0,T] \times [0,T] \times \mathbb{R} \to \mathbb{R}^+$ is continuous such that $|\Delta(\Theta, \gamma, \mathfrak{k}(\gamma)) - \Delta(\Theta, \gamma, \mathfrak{l}(\gamma))| \ge |\mathfrak{k}(\gamma) - \mathfrak{l}(\gamma)|$, for all $i \in (0,1)$. Then, the integral equation (4.4) has a unique solution.

Proof
Let $\mathfrak{k} \in \mathfrak{K}$ and $\in \mathfrak{L}$, by using of assumptions (1) – (2), we have $\mathfrak{A}(\omega\mathfrak{k}, \omega\mathfrak{l}, i\Theta) = \sup_{\Theta \in [0,T]} \frac{i\Theta}{i\Theta + (|\omega x(\Theta) - \omega y(\Theta)|)}$

$$= \sup_{\Theta \in [0,T]} \frac{i\Theta}{i\Theta + (\left|\int_0^T \Delta(\Theta,\gamma,\mathfrak{k}(\gamma))d\gamma - \Delta(\Theta,\gamma,\mathfrak{l}(\gamma))d\gamma\right|)}$$

$$= \sup_{\Theta \in [0,T]} \frac{i\Theta}{i\Theta + (\int_0^T |\Delta(\Theta,\gamma,\mathfrak{k}(\gamma))d\gamma - \Delta(\Theta,\gamma,\mathfrak{l}(\gamma))d\gamma|)} \ge \sup_{\Theta \in [0,T]} \frac{i\Theta}{i\Theta + \left(\int_0^T |\mathfrak{k}(\gamma) - \mathfrak{l}(\gamma)|\right) d\gamma} \ge$$

$$\sup_{\Theta \in [0,T]} \frac{\Theta}{\Theta + \left(\int_0^T |\mathfrak{k}(\gamma) - \mathfrak{l}(\gamma)|\right) d\gamma} \ge \mathfrak{A}(\mathfrak{k}, \mathfrak{l}, \Theta)$$

$$\mathfrak{I}(\omega\mathfrak{k}, \omega\mathfrak{l}, i\Theta) = \mathrm{Inf}_{\Theta \in [0,T]} \left(1 - \frac{i\Theta}{i\Theta + (|\omega x(\Theta) - \omega y(\Theta)|)} \right)$$

$$= \mathrm{Inf}_{\Theta \in [0,T]} \left(1 - \frac{i\Theta}{i\Theta + (\left|\int_0^T \Delta(\Theta,\gamma,\mathfrak{k}(\gamma))d\gamma - \Delta(\Theta,\gamma,\mathfrak{l}(r))d\gamma\right|)} \right)$$

$$= \mathrm{Inf}_{\Theta \in [0,T]} \left(1 - \frac{i\Theta}{i\Theta + (\int_0^T |\Delta(\Theta,\gamma,\mathfrak{k}(\gamma))d\gamma - \Delta(\Theta,\gamma,\mathfrak{l}(\gamma))d\gamma|)} \right)$$

$$\le \mathrm{Inf}_{\Theta \in [0,T]} \left(1 - \frac{i\Theta}{i\Theta + \left(\int_0^T |\mathfrak{k}(\gamma) - \mathfrak{l}(\gamma)|\right) d\gamma} \right)$$

$$\le \mathrm{Inf}_{\Theta \in [0,T]} \left(1 - \frac{\Theta}{\Theta + \left(\int_0^T |\mathfrak{k}(\gamma) - \mathfrak{l}(\gamma)|\right) d\gamma} \right) \le \mathfrak{I}(\mathfrak{k}, \mathfrak{l}, \Theta)$$

$$\mathfrak{D}(\omega\mathfrak{k}, \omega\mathfrak{l}, k\Theta) = \mathrm{Inf}_{\Theta \in [0,T]} \left(\frac{(|\omega x(\Theta) - \omega y(\Theta)|)}{i\Theta} \right)$$

$$= \mathrm{Inf}_{\Theta \in [0,T]} \left(\frac{(\left|\int_0^T \Delta(\Theta,\gamma,\mathfrak{k}(\gamma))d\gamma - \Delta(\Theta,\gamma,\mathfrak{l}(\gamma))d\gamma\right|)}{i\Theta} \right) = \mathrm{Inf}_{\Theta \in [0,T]} \left(\frac{(\int_0^T |\Delta(\Theta,\gamma,\mathfrak{k}(\gamma))d\gamma - \Delta(\Theta,\gamma,\mathfrak{l}(\gamma))d\gamma|)}{i\Theta} \right)$$

$$\le \mathrm{Inf}_{\Theta \in [0,T]} \left(\frac{\left(\int_0^T |\mathfrak{k}(\gamma) - \mathfrak{l}(\gamma)|\right) d\gamma}{i\Theta} \right) \le \mathrm{Inf}_{\Theta \in [0,T]} \left(\frac{\left(\int_0^T |\mathfrak{k}(\gamma) - \mathfrak{l}(\gamma)|\right) d\gamma}{\Theta} \right) \le \mathfrak{D}(\mathfrak{k}, \mathfrak{l}, \Theta).$$

Therefore, all the conditions of (theorem 3.5) are satisfied. As a result, mapping S has a unique fixed point $\mathfrak{k} \in \mathfrak{K} \cup \mathfrak{L}$, which is a solution of the integral equation(4.4).

CONCLUSION
By introducing $\mathfrak{BCNMS}$ and a number of new verifiable $\mathbb{FP}$ theorems, we enhance Sezen's $\mathfrak{CFMS}$ in the current investigation. We additionally addressed a few complex cases. Due to the fact that our structure is more generic than a category of fuzzy and $\mathfrak{BCNMS}$, our verdict and notions add to a number of previously reported findings that have been more specifically applied.

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