Application Of Multi Fuzzy Graph In Medical Diagnosis By using Various Distance Measures

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ABSTRACT
Nowadays, similarity measures play an essential role in many areas, such as medical diagnosis, pattern recognition and clustering etc.,Most mathematicians and researchers worked on numerous decision making problems in intuitionistic fuzzy sets and the intuitionistic fuzzy multi sets by operating similarity measures.This motivates us to involve various distance measures in multi fuzzy graph.In this paper, the definition of bipartite and complete bipartite multi fuzzy graph, the strength of a vertex and the strength of an edge in the multifuzzy graph are defined.Using various distance measures, we have examined the application of multi fuzzy graph in medical diagnosis.

KeyWords: Multi fuzzy graph, strength of a vertex, strength of an edge, Proposed distance measures.

1. INTRODUCTION

In 1965, L. A. Zadeh [13] introduced the concept of fuzzy set and fuzzy relations have been widely used to model uncertainty present in real-world applications. The concept of the fuzzy graph was first introduced by Kauffman [3] from the concept of fuzzy relation introduced by L.A. Zadeh in 1973. Rosenfeld [10] developed the theory of fuzzy graphs in 1975 and proved many results on fuzzy graphs. In 1987, Bhattacharya[1] defined some remarks on fuzzy graphs. Sebu Sebastian, T.V. Ramakrishnan[12] defined Multi fuzzy set in 2010. In 2020, R.Muthuraj and S.Revathi [4] introduced the concept of multi fuzzy graph which is the extension of a fuzzy graph with a single phenomenon into a multi phenomenon that suits to describe real-life problems in a better manner than a fuzzy graph.

Measures of similarity between fuzzy sets and intuitionistic fuzzy sets have attained attention from many researchers for their broad applications in various fields. Many measures of similarity between fuzzy sets and intuitionistic fuzzy sets have been introduced by different authors. In this paper, we proposed new distance measures and discussed an application of a multi fuzzy graph in medical diagnosis by using new distance measures.

2. Preliminaries

Definition 2.1

A fuzzy graph $G=(\sigma,\mu)$ is defined on the underlying crisp graph $G^{*}=(V,E)$ where $E\subseteq V\times V$ is a pair of functions $\sigma:V\rightarrow[0,1]$ and $\mu:V\times V\rightarrow[0,1]$, μ is a symmetric fuzzy relation on σ such that $\mu(u,v)\leq\min\{\sigma(u),\sigma(v)\}$ for $u,v\in V$

Definition 2.2

Let X be a non-empty set. A Multi Fuzzy set A in X is defined as a set of ordered sequences: $A=\{(x,\mu_1(x),\mu_2(x),\ldots\ldots\mu_i(x)\ldots):x\in X\}$ where $\mu_i:X\rightarrow[0,1]$ for all i. If the sequences of the membership function have only k-terms (finite number of terms), k is called the dimension of A.

Definition 2.3

A Multi fuzzy Graph (MFG) of dimension m defined on the underlying crisp graph $G^* = (V, E)$ where $E \subseteq V \times V$, is denoted as $G = ((\sigma_1, \sigma_2, ...\sigma_m), (\mu_1, \mu_2, ...\mu_m))$ and $\sigma_i : V \to [0,1]$ and $\mu_i : V \times V \to [0,1]$, μ_i is a symmetric fuzzy relation on σ_i such that $\mu_i(u, v) \le \min\{\sigma_i(u), \sigma_i(v)\}$ for all $i = 1,2,3....m$ where $u, v \in V$ and $uv \in E$.

Definition 2.4

Let G be a multi fuzzy graph of dimension m and let 'v' be a vertex in G. The strength of a vertex (cardinality of a vertex) is denoted as |v| and it is defined as $|v| = \frac{1 + \sigma_1(v) + \sigma_2(v) + ... + \sigma_m(v)}{m}$. The strength of an edge (cardinality of an edge) is denoted as |uv| and it is defined as $|uv| = \frac{1 + \mu_1(uv) + \mu_2(uv) + ... + \mu_m(uv)}{m}$ where $uv \in E$.

Definition2.5

A Multi fuzzy Graph (MFG) of dimension m is called the bipartite MFG of dimension m if there are two non-empty vertex set V_1 *and* V_2 such that $\mu_i(u, v) = 0$ *if* $u, v \in V_1$ *or* $u, v \in V_2$ for all $i = 1,2,3....m$.

Definition2.6

A Multi fuzzy Graph (MFG) of dimension m is called the **Complete bipartite MFG**of dimension mif there are two non-empty vertex set V_1 *and* V_2 such that $\mu_i(u, v) = 0$ *if* $u, v \in V_1$ *or* $u, v \in V_2$ and $\mu_i(u, v) = \sigma_i(u) \wedge \sigma_i(v)$ *for all* $u \in V_1$ *and* $v \in V_2$,for all $i = 1,2,3....m$.

3. Various Distance Measures

Similarity and distance measures are effective tools for determining the degree of similarity between two objects in many fields, such as pattern recognition, medical diagnosis, and so on. Nowadays, different authors have proposed different similarity and distance measures. In this paper, we are using some popular distance measures such as Hamming distance, Euclidean distance, Correlation distance,Cosine distance and Canberra distance.

- The Hamming distance is defined as $d_H(A, B) = \sum_{i=1}^{n} \frac{|x_i - y_i|}{2}$ where $A = (x_1, x_2, ...x_n)$ and $B = (y_1, y_2, ...y_n)$
- Euclidean distance is defined as $d_E(A, B) = \left(\sum_{i=1}^{n} \frac{|x_i - y_i|^2}{2} \right)^{\frac{1}{2}}$ where $A = (x_1, x_2, ...x_n)$ and $B = (y_1, y_2, ...y_n)$
- Correlation distance is defined as $d_{Cor}(A, B) = 1 - \frac{\sum_{i=1}^{n} x_i . y_i - \frac{\sum_{i=1}^{n} x_i . \sum_{i=1}^{n} y_i}{n}}{\sqrt{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i \right)^2}{n}} . \sqrt{\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i \right)^2}{n}}}$ where $A = (x_1, x_2, ...x_n)$ and $B = (y_1, y_2, ...y_n)$

- Cosine distance is defined as $d_{Cos}(A,B) = 1 - \frac{\sum_{i=1}^{n} x_i . y_i}{\sqrt{\sum_{i=1}^{n} {x_i}^2} \sqrt{\sum_{i=1}^{n} {y_i}^2}}$ where $A = (x_1, x_2, \ldots x_n)$ and $B = (y_1, y_2, \ldots y_n)$
- Canberra distance is defined as $d_{Canb}(A,B) = \sum_{i=1}^{n} \frac{|x_i - y_i|}{|x_i + y_i|}$ where $A = (x_1, x_2, \ldots x_n)$ and $B = (y_1, y_2, \ldots y_n)$

4. Proposed Distance Measure

In many real-life applications, firstly situation can be modelled graphically then it will be easy to find the solutions. In the Fuzzy graph theory, many researchers develop many applications in diverse fields with a single phenomenon. But in the Multi Fuzzy Graph, each vertex and edge has multi phenomena. Hence any problem can be analyzed with the multi phenomena so that we can get better solutions to the problems. We have proposed our research work to make a decision in medical diagnosis using various distance measures in the Multi fuzzy graph. In our proposed distance measure, we are considering the strength of the edge as one of the factors. So, it gives more accurate results.

Let $G = ((\sigma_1, \sigma_2, \ldots \sigma_n), (\mu_1, \mu_2, \ldots \mu_n))$ be a multi fuzzy graph of dimension n and 'u' and 'v' be two vertices in a multi fuzzy graph G with dimension n. Let us define new distance measures between two vertices with n dimension as follows.

- The Hamming distance between two vertices u and v of dimension n is $d_H{}^1(u,v) = \sum_{i=1}^{n} \frac{|\sigma_i(u) - \sigma_i(v)|}{2} + \frac{1 + \sum_{i=1}^{n} \mu_i(uv)}{n}$ where $u, v \in V$ and $uv \in E$
- Euclidean distance between two vertices u and vof dimension nis defined as $d_E{}^1(u,v) = \left(\sum_{i=1}^{n} \frac{|\sigma_i(u) - \sigma_i(v)|^2}{2} \right)^{\frac{1}{2}} + \frac{1 + \sum_{i=1}^{n} \mu_i(uv)}{n}$
- Correlation distance between two vertices u and v is defined as

$$d_{Cor}{}^1(u,v) = 1 - \frac{\sum_{i=1}^{n} \sigma_i(u).\sigma_i(v) - \frac{\sum_{i=1}^{n} \sigma_i(u) \sum_{i=1}^{n} \sigma_i(v)}{n}}{\sqrt{\sum_{i=1}^{n} (\sigma_i(u))^2 - \frac{\left(\sum_{i=1}^{n} \sigma_i(u)\right)^2}{n}} \; . \; \sqrt{\sum_{i=1}^{n} (\sigma_i(v))^2 - \frac{\left(\sum_{i=1}^{n} \sigma_i(v)\right)^2}{n}}} + \frac{1 + \sum_{i=1}^{n} \mu_i(uv)}{n}$$

- Cosine distance between two vertices u and v is defined as

$$d_{Cos}{}^1(u,v) = 1 - \frac{\sum_{i=1}^{n} \sigma_i(u).\sigma_i(v)}{\sqrt{\sum_{i=1}^{n} (\sigma_i(u))^2} \sqrt{\sum_{i=1}^{n} (\sigma_i(v))^2}} + \frac{1 + \sum_{i=1}^{n} \mu_i(uv)}{n}$$

- Canberra distance between two vertices u and v is defined as

$$d_C{}^1(u,v) = \sum_{i=1}^{n} \frac{|\sigma_i(u) - \sigma_i(v)|}{|\sigma_i(u) + \sigma_i(v)|} + \frac{1 + \sum_{i=1}^{n} \mu_i(uv)}{n}$$

4. Application Of Multi Fuzzy Graph In Medical Diagnosis By Using Various Distance Measures

In recent days, many health problems cannot be diagnosed immediately. This situation can be simplified by using the multi fuzzy graph model and using the proposed distance measures we can identify the disease without any confusion.

Let us consider two vertex sets V_1 and V_2. This set consists of four patients namely P_1, P_2, P_3 and P_4. Every patient suffered from some disease. These patients are considered as vertices and the proportions of symptoms are considered as vertex membership values. Then the set of all diseases is considered as vertex set V_2. Based on the symptoms of the disease, the proportions of symptoms are considered as vertex membership values. The edges between these two vertex sets are defined as the connectivity of these two vertex sets.

Let $V_1 = \{P_1, P_2, P_3, P_4\}$ be the vertex set of dimension 5 and vertex membership values are defined based on their proportions of symptoms of the patients like Temperature, Head ache, Vomiting, Cough, and Muscles pain.

Let $V_2 = \{Typhoid,\ Dengue,\ Pneumonia,\ Malaria,\ Tuberculosis\}$ beanother vertex set of diseases of dimension 5 and vertex membership values of these set is defined based on the proportions of symptoms of the disease like Temperature, Head ache, Vomiting, Cough, Muscles pain.

Let the edges denotes the common symptoms between the patients and the disease and edge membership values are equal to the minimum of patients and diseases.

The vertex membership values of the patientset $V_1 = \{P_1, P_2, P_3, P_4\}$ are as follows

Patient	Temperature	Head ache	Vomiting	Cough	Muscles pain
P_1	0.8	0.8	0.2	0.3	0.1
P_2	0.7	0.4	0.6	0.1	0.1
P_3	0.8	0.8	0.1	0.2	0.7
P_4	0.6	0.5	0.3	0.7	0.3

The vertex membership values of the disease set
$V_2 = \{Typhoid,\ Dengue,\ Pneumonia,\ Malaria,\ Tuberculosis\}$ are as follows

Diseases	Temperature	Head ache	Vomiting	Cough	Muscles pain
Typhoid (D1)	0.9	0.2	0.8	0.2	0.1
Dengue (D2)	0.8	0.8	0.4	0.2	0.7
Pneumonia (D3)	0.7	0.2	0.3	0.9	0.1
Malaria (D4)	0.8	0.8	0.1	0.1	0.2
Tuberculosis (D5)	0.3	0.1	0.2	0.9	0.3

The edge membership values are the minimum of patients' symptoms and disease symptoms. So, the common symptoms are not below the minimum of two. Therefore this graph structure can be modeled as a complete bipartite multi fuzzy graph.

The edge membership values are as follows:

	Typhoid (D1)	Dengue (D2)	Pneumonia (D3)	Malaria (D4)	Tuberculosis (D5)
P_1	P1D1 (0.8,0.2,0.2,0.2,0.1)	P1D2 (0.8,0.8,0.2,0.2,0.1)	P1D3 (0.7,0.2,0.2,0.3,0.1)	P1D4 (0.8,0.8,0.1,0.1,0.1)	P1D5 (0.3,0.1,0.2,0.3,0.1)
P_2	P2D1 (0.7,0.2,0.6,0.1,0.1)	P2D2 (0.7,0.4,0.4,0.1,0.1)	P2D3 (0.7,0.2,0.3,0.1,0.1)	P2D4 (0.7,0.4,0.1,0.1,0.1)	P2D5 (0.3,0.1,0.2,0.1,0.1)
P_3	P3D1 (0.8,0.2,0.1,0.2,0.1)	P3D2 (0.8,0.8,0.1,0.2,0.7)	P3D3 (0.7,0.2,0.1,0.2,0.1)	P3D4 (0.8,0.8,0.1,0.1,0.2)	P3D5 (0.3,0.1,0.1,0.2,0.3)
P_4	P4D1 (0.6,0.2,0.3,0.2,0.1)	P4D2 (0.6,0.5,0.3,0.2,0.3)	P4D3 (0.6,0.2,0.3,0.7,0.1)	P4D4 (0.6,0.5,0.1,0.1,0.2)	P4D5 (0.3,0.1,0.2,0.7,0.3)

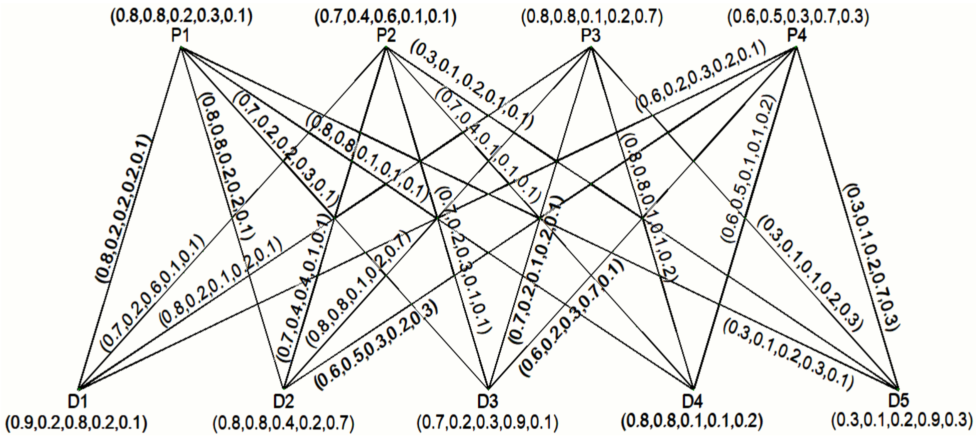

Figure 1. Complete bipartite MFGof dimension 5

Significance Of The Proposed Hamming Distance

The Hamming distance measures the similarity between two sets of values. The smallest Hamming distance between two sets of values indicates the measure of similarity. In our Hamming distance proposal, we looked at the relationship between two sets of values as a factor. So we can make the right decision.

Table 1. Hamming Distance

Patient	Diseases				
	Typhoid (D1)	Dengue (D2)	Pneumonia (D3)	Malaria (D4)	Tuberculosis (D5)
P_1	0.7	0.45	0.7	**0.2**	1
P_2	**0.35**	0.7	0.65	0.55	1.05
P_3	1	**0.15**	1.1	0.3	1.2
P_4	0.9	0.75	**0.4**	0.7	0.5

Table 2. Proposed Hamming Distance

Patient	Diseases				
	Typhoid (D1)	Dengue (D2)	Pneumonia (D3)	Malaria (D4)	Tuberculosis (D5)
P_1	1.2	1.07	1.2	**0.78**	1.4
P_2	**0.89**	1.24	1.13	1.03	1.41
P_3	1.48	**0.87**	2.66	0.9	1.6
P_4	1.38	1.33	**0.98**	1.2	1.02

Based on Tables 1 and 2, we can identify each patient affected by the disease using the Hamming distance and the proposed Hamming distance with the minimum Hamming distance between patients and diseases.Patient P_1 suffers from Malaria, P_2 suffers from Typhoid, P_3 suffers from Dengue and P_4 suffers from Pneumonia. The same disease confirms all patients using the Hamming distance and the proposed Hamming distance. So we can confirm our decision by using the proposed Hamming distance.

Significance Of The Proposed Euclidean Distance

The Euclidean distance between two sets of values indicates the distance between the given set of values. The smallest Euclidean distance between two sets of values indicates the measure of similarity. In our proposed Euclidean distance, we considered the relationship between two sets of values as a factor. So it will help us to make the correct decision.

Table 3. Euclidean Distance

Patient	Diseases				
	Typhoid (D1)	Dengue (D2)	Pneumonia (D3)	Malaria (D4)	Tuberculosis (D5)
P_1	0.61	0.45	0.61	**0.17**	0.75
P_2	**0.25**	0.54	0.62	0.46	0.74
P_3	0.78	**0.21**	0.79	0.36	0.84
P_4	0.6	0.52	**0.3**	0.52	0.39

Table 4. Proposed Euclidean Distance

Patient	Diseases				
	Typhoid (D1)	Dengue (D2)	Pneumonia (D3)	Malaria (D4)	Tuberculosis (D5)
P_1	1.11	1.07	1.11	**0.75**	1.15
P_2	**0.79**	1.08	1.10	0.94	1.1
P_3	1.26	**0.93**	1.25	0.96	1.24
P_4	1.08	1.1	**0.88**	1.02	0.91

Tables 3 and 4 indicateseach patient suffered from the disease by using minimumEuclidean distance and Proposed Euclidean distancebetween patients and diseases. Patient P_1 suffers from Malaria, P_2 suffers from Typhoid, P_3 suffers from Dengue and P_4 suffers from Pneumonia. The same disease confirms all patients by using Euclidean distance and Proposed Euclidean distance.

Significance Of The Proposed Correlation Distance

The Correlation similarity provides a value which indicates the strength of association between the set of values. The minimum correlation distance between two sets of values indicates the measure of similarity. In our proposed correlation distance, the relationship between two sets of values is a factor. So we can make the right decision.

Table 5. Correlation Distance

Patient	Diseases				
	Typhoid (D1)	Dengue (D2)	Pneumonia (D3)	Malaria (D4)	Tuberculosis (D5)
P_1	0.72	0.41	0.80	**0.05**	1.31
P_2	**0.09**	0.64	0.96	0.49	1.56
P_3	1.14	**0.07**	1.31	0.18	1.48
P_4	1.02	1.22	**0.13**	0.70	0.35

Table 6. Proposed Correlation Distance

Patient	Diseases				
	Typhoid (D1)	Dengue (D2)	Pneumonia (D3)	Malaria (D4)	Tuberculosis (D5)
P_1	1.22	1.03	1.30	**0.63**	1.71
P_2	**0.63**	1.18	1.44	0.97	1.92
P_3	1.62	0.79	1.77	**0.78**	1.88
P_4	1.50	1.80	**0.71**	1.20	0.87

From Table5, each patient suffered from the disease which has the minimum correlation distance between patients and diseases. Patient P_1 suffers from Malaria, P_2 suffers from Typhoid, P_3 suffers from Dengue and P_4 suffers from Pneumonia. From Table 6, the proposed correlation distance indicates the distance between patients and diseases. That is, Patient P_1 suffers from Malaria, P_2 suffers from Typhoid,

P_3 suffers from Malaria and P_4 suffers from Pneumonia. All patients confirmed the same disease except P_3. In the proposed correlation distance includes the strength of an edge as a factor.Hence, we conclude that Patient P_3 suffers from Malaria.

Significance Of The Proposed Cosine Distance

The Cosine similarity measure gives the degree of similarity between two sets of values and the cosine distance indicates the degree of difference between the set of values. The minimum cosine distance provides the degree of similarity. In our Cosine distance proposal, we looked at the relationship between two sets of values as a factor. So we can make the right decision.

Table 7. Cosine Distance

Patient	Diseases				
	Typhoid (D1)	Dengue (D2)	Pneumonia (D3)	Malaria (D4)	Tuberculosis (D5)
P_1	0.25	0.11	0.26	**0.02**	0.46
P_2	**0.03**	0.15	0.30	0.17	0.53
P_3	0.36	**0.02**	0.38	0.07	0.47
P_4	0.25	0.15	**0.06**	0.21	0.12

Table 8. Proposed Cosine Distance

Patient	Diseases				
	Typhoid (D1)	Dengue (D2)	Pneumonia (D3)	Malaria (D4)	Tuberculosis (D5)
P_1	0.75	0.73	0.76	**0.60**	0.86
P_2	**0.57**	0.69	0.78	0.65	0.89
P_3	0.84	0.74	0.84	**0.67**	0.87
P_4	0.73	0.73	**0.6445**	0.71	0.6446

From Table7, each patient suffered from the disease which has the minimum cosine distance between patients and diseases. Patient P_1 suffers fromMalaria, P_2 suffers from Typhoid, P_3 suffers from Dengue and P_4 suffers from Pneumonia.From Table 8, the proposed cosine distance indicates the distance between patients and diseases. That is, Patient P_1 suffers from Malaria, P_2 suffers from Typhoid, P_3 suffers from Malaria and P_4 suffers from Pneumonia. All patients confirmed the same disease except P_3. As our proposed cosine distance is more significant, we conclude that the Patient P_3 suffers from Malaria.

Significance Of The Proposed Canberra Distance

The Canberra distance provides a value which indicates the distance between the set of values. The minimum Canberra distance between two sets of values gives the degree of similarity. In our proposed Canberra distance, the relationship between two sets of values is a factor. So it will help us to make the correct decision.

Table 9. Canberra Distance

Patient	Diseases				
	Typhoid (D1)	Dengue (D2)	Pneumonia (D3)	Malaria (D4)	Tuberculosis (D5)
P_1	1.46	1.28	1.37	**1.17**	2.23
P_2	1.67	1.68	1.47	**1.45**	2.80
P_3	2.19	**0.6**	2.55	0.89	2.60
P_4	2.14	1.47	**1.13**	1.82	1.33

Table 10. Proposed Canberra Distance

Patient	Diseases				
	Typhoid (D1)	Dengue (D2)	Pneumonia (D3)	Malaria (D4)	Tuberculosis (D5)
P_1	1.96	1.90	1.87	**1.75**	2.63
P_2	2.21	2.22	1.95	**1.93**	3.16
P_3	2.67	**1.32**	3.01	1.49	3.00
P_4	2.62	2.05	**1.71**	2.32	1.85

From Table 9& 10, each patient suffered from the disease which has the minimum Canberra distance between patients and diseases. Patient $P_1 \& P_2$ suffers from Malaria , P_3 suffers from Dengue and P_4 suffers from Pneumonia. The same disease confirms all patients by using Canberra distance and Proposed Canberra distance.

CONCLUSION

In this paper, the definition of bipartite and complete bipartite multi fuzzy graph are defined. We have defined a new distance measure that includes the strength of an edge. We have discussed the application of multi fuzzy graph in medical diagnosis by using various distance measures. Even though most of the results are the same while using the general and proposed distance measure, in our proposed distance measure, we have considered the strength of an edge which strengthens the decision. Thus, without any confusion, we can make the decision easily.

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