# Analysing the Possibility of Having Heavy Metal Content in Cosmetic Products Using Linear Regression with Fuzz Model

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Received: 08.04.2024 Revised : 06.05.2024 Accepted: 04.05.2024

**ABSTRACT**
Linear Regression with FUZZ analysis was first introduced by Tanaka et al. [4]. It is a variation on traditional regression analysis where any kind of fuzzy number can be used to represent some model components. Fuzzy regression models are regression models that capture the imprecision of the system they are modeling. The indefinite nature of the fuzzy regression model is reflected in the derived fuzzy output value and fuzzy regression coefficients. This study examines the possibility that cosmetic products include heavy metals using a fuzzy linear regression model.

**Keywords:** Regression using fuzzy logic, Possibilistic odds, Cosmetic products, Heavy metals, Minimization of fuzziness.

## 1. Overview

Lotfi Zadeh proposed fuzzy sets in 1965 as a way to represent vagueness and ambiguity. Any realistic process is not perfect and ambiguity may arise from the interpretation of inputs or in the formulation of relationship between various attributes. One method that can be utilized to connect knowledge-based systems and human reasoning abilities is fuzzy sets. A mathematical foundation for the transformation of specific linguistic and perceptual characteristics for subsequent computation and decision-making is provided by fuzzy logic.

Pourahmad et al.'s fuzzy logistic regression method used Tanaka's fuzzy lin-ear regression model (FLR) to estimate the model parameters. [6]. Tanaka H. used possibilistic linear model to study fuzzy data analysis; his findings were published in the journal Fuzzy Sets and Systems [8]. For the purpose of the study, the theoretical framework of fuzzy linear regression based on revised Tanaka FLR and its applications are covered.

## 2. Initials

**Definition 2.1 [1]**

Consider a non-empty set X = $\{x_1, x_2, \ldots x_n\}$ as the discourse universe's representation.
The definition of a fuzzy set B in X is so as follows:
$B = \{x, \mu_B(x)/x \in X\}$
where the membership degree is denoted by $\mu B(x)$: X → [0, 1].

**Definition 2.2 [7]**

If a fuzzy number B is called LR-type if it has the following membership function,
follows

$$B(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), x \le m \quad \forall \quad x \in \Re \\ R\left(\frac{x-m}{\beta}\right), x > m \end{cases}$$

Similar rules apply to R and L, which in this case are decreasing functions from R+ to [0, 1]. L(0) = 1, L(x) < 1 for all x > 0, L(x) > 0 for all x < 1, and L(1) = 0. The spreads on the left and right are represented by α and β, respectively, and the mean value of B is the actual number, m. When L(x) = R(x), the triangular fuzzy number, B, is represented by$(m, \alpha, \beta)_{LR}$ in the above case. Its function inside the membership function is

$$B(x) = \begin{cases} 1 - \left(\frac{m-x}{\alpha}\right), m - \alpha \leq x \leq m \\ 1 - \left(\frac{x-m}{\beta}\right), m < x < m + \beta \end{cases}$$

If in Addition, α = β. Then B is denoted by $(m, \alpha)_T$ stands for the sym-metric triangular fuzzy number,

**Definition 2.3 [5]**

Take a look at the subsequent problem with linear programming.:

$$\text{Maximum } \tilde{z} = k\tilde{x}$$
$$\text{subject to } \begin{cases} A\tilde{x} = \tilde{d} \\ \tilde{x} \geq 0 \end{cases}$$

A fuzzy linear programming (FLP) issue is one in which x˜ = x˜j, d⊦ = (d˜i) are nonnegative fuzzy vectors such that the coefficient matrix A =$[a_{i,j}]_{n*m}$ and the vector k = $(k_1, k, ... k_n)$ are nonnegative crisp matrix and vector, respectively. x˜j, d˜i ∈ F (IR) for all 1 ≤ j ≤ m, 1 ≤ i ≤ n.

**Definition 2.4 [10]**

A fuzzy set F is also convex if and only if the sets $F_\alpha$, which are defined as Fα = {x/F (x) ≥ α}, If all α in the interval [0, 1] are convex.

## 3. Assessing possible existence of heavy metals content in cosmetic products

### 3.1.Fuzzy Linear Regression Model [7]

**Step 1**

This is how the fuzzy linear regression is displayed.

$\tilde{X} = g(y, X) = B_1, k, ... k_n \tilde{B}_0 + 1y_1 + ... + \tilde{B}ny_n$

The fuzzy output is represented by X˜. $\tilde{B} = \{\tilde{B}_0, \tilde{B}_1, ... \tilde{B}_n\}$ is a collection of fuzzy numbers, and y = $\{y_1, y_2, ... y_n\}$T is the real valued input vector.

**Step 2**

Based on expert consultation and considering the defined attributes of category (1 = high and 0 = low) within the response variable, a real number is assigned to each fuzzy case within range [0, 1]. Alternatively, based in- sights and the established characteristic of category (1 = high and 0 = low) a real valued number is determined for each fuzzy case within the the interval [0,1].

**Step 3**

Identifying the parameter B that makes With non-fuzzy data, the fuzzy output set {xi} associated with a membership value greater than k is the goal of the fuzzy regression approach.

$\mu_{\tilde{X}}(x_i) \geq k \qquad i = 1, 2, \ldots n.$

where the best-fitting model is produced by selecting a value for k.

Minimize $Z = g^c(x_1) + g^c(x_2) + \cdots + g^c(x_m)$ subject to the constraints

$z_j \in [g(x_j)]_k$

**Step 4**

The parameters of the model are symmetric triangular fuzzy integers, $\tilde{d}j = (b^c, t_j)_T$, where $j = 0, 1, 2, \ldots n$, based on fuzzy arithmetic, it is obvious that $\tilde{X}i$ will be a symmetric triangular fuzzy number.

$$\tilde{X}= (g_i^c, g_i^s)_T,$$

where

$$g_i^c(x) = b_0^c + b_1^c x_1 + \cdots + b_n^c x_n$$

$$g_i^s(x) = t_0^s + t_1^s x_1 + \cdots + t_n^s x_n$$

From the expression, the constraints of the regression as

$$(1-k)g_i^c + (1-k)\sum_i g_i^s|x_i| + \sum_i g_0^s \geq x$$

$$(1-k)g_i^s + (1-k)\sum_i g_i^s|x_i| + \sum_i g_i xi - g_0 \geq -x$$

Using the software ”Lingo”.

**Step 5**

The extension principle states that if g(x) = exp(x) and B˴ is a fuzzy number, then f (B˴) = exp(B˴) is an indeterminate number with the subsequent membership feature.

$$exp(\tilde{B}(x)) = \begin{cases} (\tilde{B} \text{ In } \quad \text{x}) \text{ , } x > 0 \\ 0 \qquad \text{,otherwise} \end{cases}$$

The following represents the X˜i membership function.

$$\tilde{X}_i(x_i) = \begin{cases} 1 - \dfrac{g_i^c(x) - x_i}{g_i^s(x)}, & g_i^c(x) - g_i^s \leq x_i \leq g_i^c(x) \\ \\ 1 - \dfrac{x_i - g_i^c(x)}{g_i^s(x)}, & g_i^c(x) \leq x_i \leq g_i^c(x) + g_i^s(x) \\ \\ 0, & otherwise. \end{cases}$$

### 3.2 Evaluating heavy metals content in cosmetic product using fuzzy linear regression

**Table 3.1.** The Data Set of cosmetic products

| **Cosmetics** | **Sb** | **As** | **Cd** | **Pb** | **Hg** | **X** |
|---|---|---|---|---|---|---|
| Eyeliner | 0.11 | 0.353 | 0.031 | 0.038 | 0.11 | 1 |
| EyeShadow | 0.194 | 0.418 | 0.004 | 0.275 | 0.007 | 1 |
| Foundation | 0.031 | 0.292 | 0.012 | 0.369 | 0.009 | 0 |
| LipLiner | 0.038 | 0.505 | 0.026 | 0.017 | 0.064 | 1 |
| Lipstick | 0.275 | 0.078 | 0.014 | 0.014 | 0.011 | 1 |
| Mouthwash | 0.029 | 0.007 | 0.001 | 0.007 | 0.002 | 0 |
| Sunprotect | 0.047 | 0.087 | 0.002 | 0.168 | 0.008 | 0 |
| ToothPaste | 0.034 | 0.101 | 0.038 | 0.382 | 0.014 | 0 |
| Mascara | 0.015 | 0.049 | 0.002 | 0.021 | 0.005 | 0 |
| Lipgloss | 0.028 | 0.09 | 0.012 | 0.13 | 0.101 | 1 |

| | df | SS | MS | F | Significance F |
|---|---|---|---|---|---|
| **Regression** | 5 | 2.269852857 | 0.45397057 | 7.890092659 | 0.033723755 |
| **Residual** | 4 | 0.230147143 | 0.05753679 | | |
| **Total** | 9 | 2.5 | | | |

**Table 3.2.** Statistical regression results

| | **Coefficients** | **S.Error** | **t Stat** | **P value** | **Lower 95%** | **Upper 95%** |
|---|---|---|---|---|---|---|
| Intercept | -0.10910 | 0.172046 | -0.63417 | 0.56043 | -0.586780 | 0.368570 |
| Sb | 3.62830 | 0.970711 | 3.73777 | 0.020160 | 0.933167 | 6.323419 |
| As | 0.81806 | 0.519913 | 1.57346 | 0.190723 | -0.62545 | 2.26157 |
| Cd | -5.1193 | 7.6806430 | -0.666520 | 0.541555 | -26.44418 | 16.20558 |
| Pb | -0.2684 | 0.639344 | -0.41980 | 0.69620 | -2.043503 | 1.506704 |
| Hg | 8.07661 | 2.618523 | 3.084413 | 0.03677 | 0.80642 | 15.34679 |

**Table 3.3.** Heavy metals and cosmetic product given data

| **H** | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ | $y_7$ | $y_8$ | $y_9$ | $y10$ | $X$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Sb | 0.11 | 0.194 | 0.031 | 0.038 | 0.275 | 0.029 | 0.047 | 0.034 | 0.015 | 0.028 | 3.62830 |
| As | 0.353 | 0.418 | 0.292 | 0.505 | 0.078 | 0.007 | 0.087 | 0.101 | 0.049 | 0.09 | 0.81806 |
| Cd | 0.031 | 0.004 | 0.012 | 0.026 | 0.014 | 0.001 | 0.002 | 0.038 | 0.002 | 0.012 | -5.1193 |
| Pb | 0.038 | 0.275 | 0.369 | 0.017 | 0.013 | 0.007 | 0.168 | 0.382 | 0.021 | 0.13 | -0.2684 |
| Hg | 0.11 | 0.007 | 0.009 | 0.064 | 0.011 | 0.002 | 0.008 | 0.014 | 0.005 | 0.101 | 8.07661 |

$$\text{Min } Z = g_0^c + 0.642 * g_1{}^c + 0.898 * g_2{}^c + 0.713 * g_3{}^c + 0.65 * g_4{}^c + 0.391 * g_5{}^c + 0.046 * g_6{}^c + 0.312 * g_7{}^c + 0.569 * g_8{}^c + 0.092 * g_9{}^c + 0.361 * g_{10}{}^c$$

If $k = 0.5$, then minimization of LPP within the limitations of $(0.5)g_0{}^c + (0.5)g_1{}^c + (0.5)g_2{}^c + (0.5)g_3{}^c + (0.5)g_4{}^c + (0.5)g_5{}^c + (0.5)g_6{}^c + (0.5)g_7{}^c + (0.5)g_8{}^c + (0.5)g_9{}^c + (0.5)g_{10}{}^c - g_0{}^s - y_1g_1{}^s - y_2g_2{}^s - y_3g_3{}^s - y_4g_4{}^s - y_5g_5{}^s - y_6g_6{}^s - y_7g_7{}^s - y_8g_8{}^s - y_9g_9{}^s - y_{10}g_{10}{}^s \geq X;$

$$(0.5)g_0{}^c + (0.5)g_1{}^c + (0.5)g_2{}^c + (0.5)g_3{}^c + (0.5)g_4{}^c + (0.5)g_5{}^c + (0.5)g_6{}^c + (0.5)g_7{}^c + (0.5)g_8{}^c + (0.5)g_9{}^c + (0.5)g_{10}{}^c - g_0{}^s - y_1g_1{}^s - y_2g_2{}^s - y_3g_3{}^s - y_4g_4{}^s - y_5g_5{}^s - y_6g_6{}^s - y_7g_7{}^s - y_8g_8{}^s - y_9g_9{}^s - y_{10}g_{10}{}^s \geq -X;$$

Thus, in the $n^{th}$ data set, $y_n$ stands for the $i^{th}$ component of x. The quantity of data sets that are accessible determines the precise of functional constraints;
A data collection with five different sets of values for {$\tilde{X}$, y1, y2, y3, y4, y5, y6, y7, y8, y9, y10} will result in ten functional constraints.
When the values of the above data are replaced, The structure of the LPP for the given problem is as follows.
Min Z = $g_0{}^c + 0.642 *g_1{}^c + 0.898 *g_2{}^c + 0.713 *g_3{}^c + 0.65 *g_4{}^c + 0.391 *g_5{}^c +$
$0.046 *g_6{}^c + 0.312 *g_7{}^c + 0.569 *g_8{}^c + 0.092 *g_9{}^c + 0.361 *g_{10}{}^c$
Subject to

$(0.5)g_0^c+(0.055)g_1^c+(0.097)g_2^c+(0.0155)g_3^c+(0.019)g_4^c+(0.1375)g_5^c+(0.0145)g_6^c+(0.0235)g_7^c+(0.017)g_8^c+(0.0075)g_9^c+(0.014)g_{10}^c+g_0^s+(0.11)g_1^s+(0.194)g_2^s+(0.031)g_3^s+(0.038)g_4^s+(0.275)g_5^s+(0.029)g_6^s+(0.047)g_7^s+(0.034)g_8^s+(0.015)g_9^s+(0.028)g_{10}^s \geq 3.6283;$

$(0.5)g_0^c+(0.055)g_1^c+(0.097)g_2^c+(0.0155)g_3^c+(0.019)g_4^c+(0.1375)g_5^c+(0.0145)g_6^c+(0.0235)g_7^c + (0.017)g_8^c+(0.0075)g_9^c+(0.014)g_{10}^c-g_0^s-(0.11)g_1^s-(0.194)g_2^s-(0.031)g_3^s-(0.038)g_4^s-(0.275)g_5^s-(0.029)g_6^s-(0.047)g_7^s-(0.034)g_8^s-(0.015)g_9^s-$

$(0.028)g_{10}^8 \geq -3.6823;\ and$
$g_0^c \geq 0,\quad g_1^c \geq 0, g_2^c \geq 0, g_3^c \geq 0, g_4^c \geq 0, g_5^c \geq 0, g_6^c \geq 0. g_7^c \geq 0, g_8^c \geq 0, g_9^c \geq 0, g_{10}^c \geq 0,$
$g_0^8 \geq 0, g_1^8 \geq 0, g_2^8 \geq 0, g_3^8 \geq 0, g_4^8 \geq 0, g_5^8 \geq 0, g_6^8 \geq 0, g_7^8 \geq 0, g_8^8 \geq 0, g_9^c \geq 0, g_{10}^c \geq 0$

One way to find the value of k is to fit the model using several values of k.
As a result, we obtain the following fuzzy number coefficients:
$B_0 = (7.6826, 4.0434), B_1 = (0, 1.7451), B_2 = (0, 0), B_3 = (0, 0), B_4 = (0.0),$
$B_5 = (0, 0), B_6 = (0, 0), B_7 = (0, 0), B_8 = (0.0), B_9 = (0, 0), B_{10} = (0, 0).$
Prediction pertaining to a heavy metal in cosmetic products:

$$\tilde{X}_i(x_i) = \begin{cases} 1\text{-}\dfrac{g_i^c(x)-x_i}{g_i^s(x)}, & g_i^c(x)-g_i^s \leq x_i \leq g_i^c(x) \\ 1-\dfrac{x_i-g_i^c(x)}{g_i^s(x)}, & g_i^c(x) \leq x_i \leq g_i^c(x)+g_i^s(x) \\ 0, & otherwise. \end{cases}$$

If the following from the data:

$$Sb = exp(\tilde{X}_i(x)) = \tilde{X}_i(In(x))=$$
$$\begin{cases} 1\text{-}\dfrac{7.6826-In(x)}{4.2351}, & 3.4475 \leq In(x) \leq 7.6826 \\ 1\text{-}\dfrac{In(x)-7.6826}{4.2351}, & 7.6826 < In(x) \leq 11.9177 \end{cases}$$

The Possibility of having heavy metal Antimony based on cosmetic product is $(7.6826, 4.2351)_T$.

$$As = exp(\tilde{X}_i(x)) = \tilde{X}_i(In(x))=$$
$$\begin{cases} 1\text{-}\dfrac{7.6826-In(x)}{4.6591}, & 3.0235 \leq In(x) \leq 7.6826 \\ 1\text{-}\dfrac{In(x)-7.6826}{4.6591}, & 7.6826 < In(x) \leq 12.3417 \end{cases}$$

The Possibility of having heavy metal Arsenic based on cosmestic product is $(7.6826,4.6591)_T$

$$Cd = exp(\tilde{X}_i(x)) = \tilde{X}_i(In(x))=$$
$$\begin{cases} 1\text{-}\dfrac{7.6826-In(x)}{4.082}, & 3.6006 \leq In(x) \leq 7.6826 \\ 1\text{-}\dfrac{In(x)-7.6826}{4.082}, & 7.6826 < In(x) \leq 11.7646 \end{cases}$$

The Possibility of having heavy metal cadmium based on cosmetic product is $(7.6826, 4.082)_T$.

$$Pb = exp(\tilde{X}_i(x)) = \tilde{X}_i(In(x))=$$
$$\begin{cases} 1\text{-}\dfrac{7.6826-In(x)}{4.1094}, & 3.5732 \leq In(x) \leq 7.6826 \\ 1\text{-}\dfrac{In(x)-7.6826}{4.1094}, & 7.6826 < In(x) \leq 11.792 \end{cases}$$

The Possibility of having heavy metal cadmium based on cosmetic product is $(7.6826, 4.1094)_T$

$$Hg = exp(\tilde{X}_i(x)) = \tilde{X}_i(In(x))=$$

$$\begin{cases} 1\text{-}\ \dfrac{7.6826 - In(z)}{4.2351}, & 3.4475 \leq In(x) \leq 7.6826 \\ 1\text{-}\ \dfrac{In(x) - 7.6826}{4.2351}, & 7.6826 < In(x) \leq 11.9177 \end{cases}$$

The Possibility of having heavy metal mercury based on cosmetic product is $(7.6826, 4.2351)_T$.

**CONCLUSION**

The Fuzzy Linear Regression method was used in this study to estimate the model parameters. The fuzzy regression model was constructed and the relationship to the fuzzy coefficients was assessed using Tanaka's possibility technique. When there is ambiguity in the data set, the suggested model can be applied. This method was described to investigate the heavy metal concentration in cosmetic items.

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