Picture Fuzzy Graceful Labeling For Some Graphs

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ABSTRACT
In uncertain real-world circumstances, when simple fuzzy graphs and intuitionistic fuzzy graphs might not be sufficient, the picture of the fuzzy graph is a recently developed fuzzy graph model. This work explores picture fuzzy graceful labeling for some graphs like Path graph, Star graph and circular graph providing with illustrative examples.

Keywords: Picture fuzzy, Graceful labeling, path graph, star graph, circular graph

1. INTRODUCTION

A fuzzy subset μ of a set X is defined as a function from X to the interval [0,1], as originally proposed by L.A. Zadeh [1]. The fuzzy set theory has been utilized across nearly every field of mathematics since it was first introduced. Later, Atanassov extended this notion by introducing the notion of an intuitionistic fuzzy set (IFS), which is a fuzzy set concept generalized. Intuitionistic fuzzy sets introduce an additional membership degree called the hesitation margin. In 1736, Euler developed the concept of graph theory. A collection of edges E (G) and a set of vertices V (G) form a graph (G). If an edge (vi, vj) in E (G) connect two vertices (vi) and (vj) in a simple graph, then the graph is said to be connected. Rosa [3] introduced the concept of graph labeling. Fuzzy graphs, derived from fuzzy relations, were introduced by Kauffmann [5] and further developed by Rosenfeld [3] in 1975. Fuzzy graph labeling was investigated and the characteristics of fuzzy labeled graphs (FLG) were covered by A.N. Gani et al. [8]. Various types of fuzzy labeling exist, including cordial, magic, and graceful labeling. Fuzzy magic labeling has been applied to fuzzy graphs, intuitionistic fuzzy graphs [9], neutrosophic path and star graphs [10]. With the emergence of intuitionistic fuzzy graphs, neutrosophic fuzzy graphs, hesitant fuzzy graphs, and more, the evolution of fuzzy graphs has proceeded. From picture fuzzy relations, the picture fuzzy graph [11] was developed. In 1967, Rosa introduced a concept in graph theory where he defined a labeling function f as a β-labeling for a graph G with n edges, which Golomb later termed "graceful." This labeling involves assigning a unique label from the set {0, 1... n} to each vertex of G such that the labels on the edges, defined as the absolute differences between the labels of their endpoints, are all distinct. This method ensures a one-to-one labeling of edges from 1 up to the total

2. Preliminaries

Definition 2.1: Fuzzy labeling graph

A fuzzy graph G = (V, σ, μ) is defined with a membership function for vertices (σ: V → [0, 1]) and a membership function for edges (μ: V x V → [0, 1]). For any vertices a and b in V, the condition $\boldsymbol{\mu}(\mathbf{a},\mathbf{b}) \leq \boldsymbol{\sigma}(\mathbf{a}) \wedge \boldsymbol{\sigma}(\mathbf{b})$ must hold. If $\boldsymbol{\mu}(\mathbf{a},\mathbf{b}) \leq \boldsymbol{\sigma}(\mathbf{a}) \wedge \boldsymbol{\sigma}(\mathbf{b})$ for all a, b in V, the fuzzy graph G is referred to as a fuzzy labeling graph.

Definition 2.2: Graceful labeling

A graph G = (V, E) with m vertices and n edges has a graceful labeling (also known as β-valuation) when each vertex set V(G) has a unique assignment Ψ to a different number from the set {0,1,2,..., n }.For each edge e ={u , v} the label Ψ*(e)calculated as the absolute difference |Ψ(u)-Ψ(v) | then Ψ* uniquely corresponds to each integer in the set { 1,2, ... ,n}. A graph is considered graceful if it can be assigned such a labeling.

Definition 2.3: Picture fuzzy Graceful labeling

A Picture fuzzy graph G = (V, μ, σ, η) consists of a set of vertices $V = \{a_1, a_2, a_3 \dots \dots a_n\}$ and is characterized by three types of membership functions for both vertices and edges: positive, negative, and hesitant. Define the same types of memberships for the edges as $\alpha(v, v_i)$, $\beta(v, v_i)$ and $\gamma(v, v_i)$ respectively. Satisfying the conditions

$$\alpha(v, v_i) \leq \mu(v) \wedge \mu(v_i)$$
$$\beta(v, v_i) \leq \sigma(v) \wedge \sigma(v_i)$$
$$\gamma(v, v_i) \leq \eta(v) \wedge \eta(v_i)$$

$0 \leq \mu(v, v_i) + \sigma(v, v_i) + \eta(v, v_i) \leq 1$ for every $v, v_i \in V$ is said to be a Picture fuzzy graph under the condition. The graph features edge labeling that is called graceful if the labels $(v, v_i) = |v - v_i|$ for each edge are distinct.

Definition 2.4: Path Graph

A path graph is a graph whose edges are $\{v_i, v_{i+1}\}$, where i = 1, 2, …, n − 1. Its vertices can be expressed as a succession of $v_1, v_2, \dots, v_n$. Conversely, a path with two vertices or more is connected, has two terminal vertices, and is represented by P_n. All other paths, on the other hand, have degree 2.

Definition 2.5: Circular Graph

A Circle (or) Circular graph is a graph that consists of a single cycle (or) a finite number of vertices connected in a closed chain, is called a circular graph. C_n is the symbol for the cycle graph with n vertices. Every vertex in C_n has degree 2, and the number of vertices is equal to the number of edges.

Definition 2.6: Star Graph

One central vertex has a degree of n-1, and there are n-1 vertices overall on a star graph, which is a special kind of graph. This configuration is similar to having n-1 vertices connected to one central vertex. A star graph with a total of n vertices is denoted as S_n.

3. Main Results

Theorem 3.1

A path graph P_n which consists of n vertices can be assigned to picture fuzzy graceful labeling.

Proof: Let P_n be a path graph with n vertices and (n-1) edges. The general path graph is shown in the figure 3.1.

Let w → (0, 1]. Assume w=0.01

The membership functions for the vertices are defined as follows:

True membership function, μ: V→ [0, 1]

Indeterminacy membership function σ: V→ [0, 1],

False membership function, η: V→ [0, 1]

Similarly, the membership functions for the edges are defined as:

True membership function α: E→ [0, 1],

Indeterminacy membership function β: E→ [0, 1],

False membership function γ: E→ [0, 1].

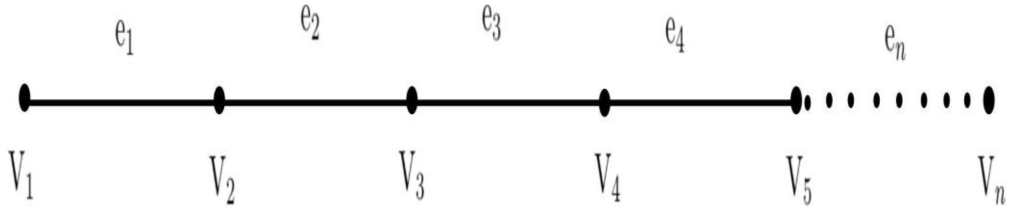

Figure 3.1. Path Graph (P_n)

True function

Vertex Labels: $\mu(v) = (2n+3)w$

$$\mu(v_i) = \left(2n + \frac{i}{2} + 3.5\right) w \quad \text{for all } 1 \leq i \leq n$$

Edge labels: $\alpha(e_i) = \left(2n + \frac{i}{2} - 0.5\right)$ wmembership of the edges for all i (1≤ i ≤ n-1) edge labels

A picture Fuzzy path graph P_n verifies the condition for picture Fuzzy graceful labeling by ensuring that each vertex and edge has a distinct true membership value (μ)

$\alpha(v_i, v_j) \leq \mu(v_i) \wedge \mu(v_j)$ and $\alpha(v, v_i) \leq \mu(v) \wedge \mu(v_i)$

i.e., $\mu(v) \wedge \mu(v_i)=\left(2n + \frac{i}{2} + 3.5\right)$ and $\alpha(v, v_i) = \left(2n + \frac{i}{2} - 0.5\right)$
Therefore, $2n + \frac{i}{2} - 0.5 < 2n + \frac{i}{2} + 3.5$ which holds true.
This relationship verifies that $\alpha(v, v_i) \leq \mu(v) \wedge \mu(v_i)$ for all i.
As a result, the true membership value of μ assigned to the vertices and edges in the P_n path graph under picture fuzzy graceful labeling must be distinct for all elements (vertices& edges) in the graph.

Indeterminacy function
Vertex Labels: σ (v) = (2n+2) w
$\sigma(v_i) = \left(2n + \frac{i}{2} + 2.5\right)$ wfor all 1≤ i ≤ n
Edge labels: $\beta(e_i) = \left(2n + \frac{i}{2} - 1.5\right)$ wmembership of the edges for all i
(1≤ i ≤ n-1) edge labels
A picture Fuzzy graph verified the condition
$\beta(v_i, v_j) \leq \sigma(v_i) \wedge \sigma(v_j)$ and $\beta(v, v_i) \leq \sigma(v) \wedge \sigma(v_i)$
i.e., $\sigma(v) \wedge \sigma(v_i) = 2n + \frac{i}{2} + 2.5$ and $\beta(v, v_i) = 2n + \frac{i}{2} - 1.5$
Therefore, $2n + \frac{i}{2} - 1.5 < 2n + \frac{i}{2} + 2.5$ which holds true.
$\beta(v, v_i) \leq \sigma(v) \wedge \sigma(v_i)$ for all i.
Therefore, under the picture fuzzy graceful labeling of the P_n path graph, each vertex and each edge must be assigned a unique membership value σ. This means that no two vertices or edges can share the same membership value, ensuring that every element (vertex or edge) in the graph has a distinct label.

False function
Vertex Labels: η (v) = (2n+1) w
$\eta(v_i) = (2n + \frac{i}{2} - 1.5)$wfor all 1≤ i ≤ n
Edge labels: $\gamma(e_i) = (2n - \frac{i}{2} - 0.5)$ w membership of the edges for all i (1≤ i ≤ n-1) edge labels
A picture Fuzzy graph verified the condition
$\gamma(v_i, v_j) \leq \eta(v_i) \wedge \eta(v_j)$ and $\gamma(v, v_i) \leq \eta(v) \wedge \eta(v_i)$
$\gamma(v, v_i) \leq \eta(v) \wedge \eta(v_i)$
i.e., $\eta(v) \wedge \eta(v_i) = 2n + \frac{i}{2} - 1.5$ and $\gamma(v_i, v_j) = 2n - \frac{i}{2} - 0.5$
Therefore, $2n - \frac{i}{2} - 0.5 < 2n + \frac{i}{2} - 1.5$ which holds true.
This relationship verifies that $\gamma(v, v_i) \leq \eta(v) \wedge \eta(v_i)$ for all i.
Thus, the path graph P_n satisfies the conditions of a fuzzy graph, exhibiting clear and valid false membership values for both vertices and edges. For n=6 path graph **(P_6)** as shown in figure 3.6. In essence, these memberships align with the principles of fuzzy graceful labeling, thereby confirming the uniqueness and validity of the graph's arrangement.
Therefore, all the membership values of vertices and edges are distinct. So, each value given to the points and lines is different, proving the picture fuzzy graceful labeling on the path graph.

Example 3.2
Let's consider the path graph P_6, which consists of 6 vertices and 5 edges. The vertices are labeled v_1, v_2, v3, v_4, v_5, v_6 and the edges are labeled e_1= (v_1, v_2), e2= (v_2, v_3), e_3= (v_3, v_4), e_4= (v_4, v_5), e_5= (v_5, v_6)
Under picture fuzzy graceful labeling, we need to assign unique true membership values, indeterminacy values and false membership values to each vertex and edge such that no two elements share the same set of values.
Here's an example of such labeling:

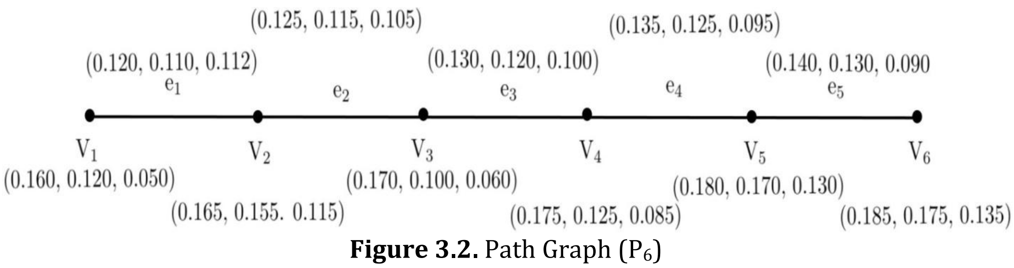

Figure 3.2. Path Graph (P_6)

In this example, each vertex and edge in the P_3 path graph has distinct true membership, indeterminacy, and false membership values, satisfying the requirements of picture fuzzy graceful labeling.

Theorem: 3.3
A circular graph C_n with n vertices can be labeled in a way that satisfies the condition for a picture fuzzy graceful labeling.
Proof
Let C_n be a circular graph with n vertices and (n-1) edges as shown in figure 3.3

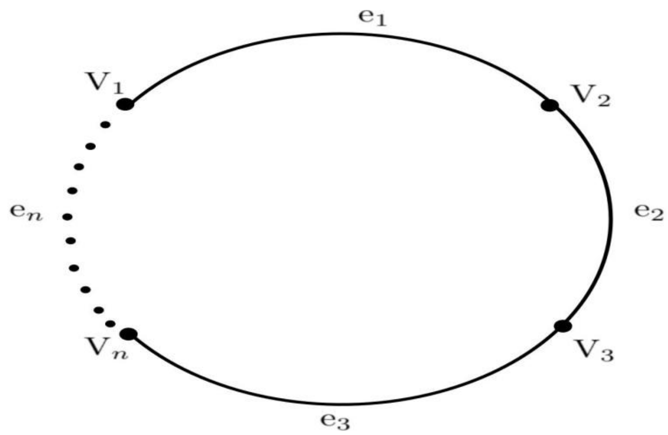

Figure 3.3. Circular Graph C_n

Define functions for true membership (T), false membership (F), and hesitancy (H) that can be applied to both vertex labels and edge labels.
These functions provide a comprehensive approach to assigning membership values to both vertex labels and edge labels, satisfying the requirements of picture fuzzy graceful labeling.
Assume z = 0.01, Consider the membership function of the circular graph.

True function
Vertex Labels: μ (v) = (3n+4) z
$\mu (v_i) = (3n + \frac{i}{3} - 4)z$ for all $1 \leq i \leq n$
Edge labels: Edge labels are defined from the concept of graceful labeling
i.e., $\alpha(e_i) = |\mu(v_i) - \mu(v_j)|$
$\alpha(e_i) = n * |\mu(v_i) - \mu(v_j)|$
Membership of the edges for all i and j $(1 \leq i \leq n) \& (1 \leq j \leq n-1)$edge labels
A picture Fuzzy circular graph C_n verifies the condition for picture fuzzy graceful labeling by ensuring that each vertex and edge has a distinct true membership value (μ)
$\alpha(v_i, v_j) \leq \mu(v_i) \wedge \mu(v_j)$ and $\alpha(v, v_i) \leq \mu(v) \wedge \mu(v_i)$
i.e., $\mu(v) \wedge \mu(v_i) = \left(3n + \frac{i}{3} - 4\right)$ and $\alpha(v, v_i) = n * |\mu(v_i) - \mu(v_j)|$
Therefore, $n * |\mu(v_i) - \mu(v_j)| < 3n + \frac{i}{3} - 4$ which holds true.
This relationship verifies that $\alpha(v_i, v_j) \leq \mu(v_i) \wedge \mu(v_j)$ for all i.
Consequently, in a C_n circular graph, the true membership value μ assigned to vertices and edges must be unique for each element under picture fuzzy graceful labeling. This requirement ensures that every vertex and edge in the graph has a distinct membership value.

Indeterminacy function
Vertex Labels: $\sigma(v)$ = (3n+7) z
$\sigma(v_i) = \left(3n + \frac{i}{3} - 7\right)$zFor all $1 \leq i \leq n$
Edge labels: $\boldsymbol{\beta}(\mathbf{e_i}) = (\mathbf{n} + \mathbf{1}) * (|\sigma(v_i) - \sigma(v_j)|$membership of the edges for all i $(1 \leq i \leq n-1)$ edge labels
A picture Fuzzy graph verified the condition
$\beta(v_i, v_j) \leq \sigma(v_i) \wedge \sigma(v_j)$ and $\beta(v, v_i) \leq \sigma(v) \wedge \sigma(v_i)$
i.e., $\sigma(v) \wedge \sigma(v_i) = 3n + \frac{i}{3} - 7$ and $\boldsymbol{\beta}(\mathbf{e_i}) = (\mathbf{n} + \mathbf{1}) * (|\sigma(v_i) - \sigma(v_j)|$
Therefore, $(\mathbf{n} + \mathbf{1}) * (|\sigma(v_i) - \sigma(v_j)| < 3n + \frac{i}{3} - 7$ which holds true.

This relationship verifies that $\beta(v, v_i) \leq \sigma(v) \wedge \sigma(v_i)$ for all i.
Therefore, under the picture fuzzy graceful labeling of the C_n circular graph, each vertex and each edge must be assigned a unique membership value. This means that no two vertices or edges can share the same membership value, ensuring that every element (vertex or edge) in the graph has a distinct label.

False function
Vertex Labels: $\eta(v) = (3n-7)z$

$$\eta(v_i) = \left(3n - 7 - \frac{i}{3}\right)z \text{ for all } 1 \leq i \leq n$$

Edge labels: $\gamma(e) = |\eta(v_j) - \eta(v_i)|$
$\gamma(e_i) = (n + 2) * (|\eta(v_j) - \eta(v_i)|)$
Membership of the edges for all i (1≤ i ≤ n-1) edge labels
A picture Fuzzy graph verified the condition
$\gamma(v_i, v_j) \leq \eta(v_i) \wedge \eta(v_j)$ and
$\gamma(v, v_i) \leq \eta(v) \wedge \eta(v_i)$
i.e., $\eta(v) \wedge \eta(v_i) = 3n - 7 - \frac{i}{3}$ and $\gamma(v_i, v_j) = (n + 2) * (|\eta(v_j) - \eta(v_i)|)$
Therefore, $(n + 2) * (|\eta(v_j) - \eta(v_i)|) < 3n - 7 - \frac{i}{3}$ which holds true.
This relationship verifies that $\gamma(v, v_i) \leq \eta(v) \wedge \eta(v_i)$ for all i.
Thus, the circular graph C_n satisfies the conditions of a fuzzy graph, exhibiting clear and valid false membership values for both vertices and edges. In essence, these memberships align with the principles of fuzzy graceful labeling, thereby confirming the uniqueness and validity of the graph's arrangement.
Therefore, all the membership values of vertices and edges are distinct. So, each value given to the points and lines is different, proving the picture fuzzy graceful labeling on the circular graph.

Example: 3.4
Let us consider the Circular graph which contains 4 vertices and 4 edges which satisfies the picture fuzzy graceful labeling conditions. Here is an example for such type of a graph as shown in figure 3.5
The membership value of vertices and edges are given below:

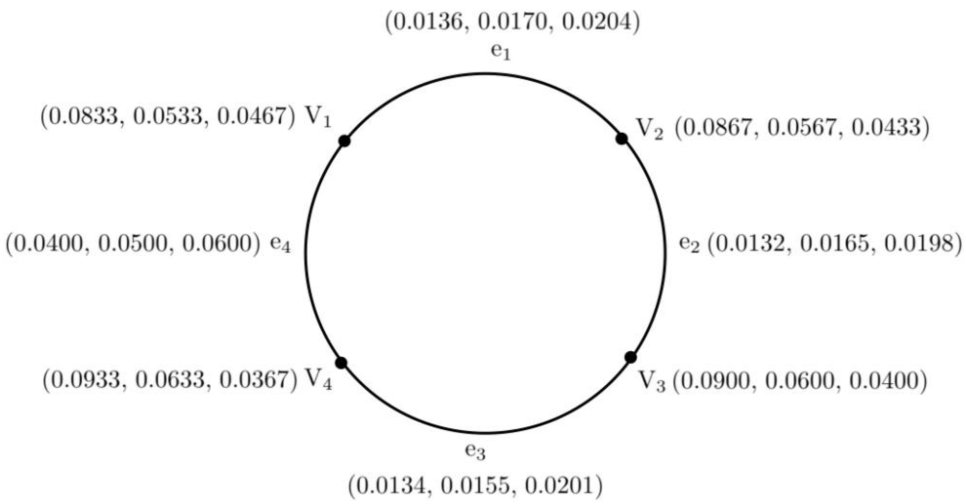

Figure 3.4. Circular Graph C_n

Theorem: 3.5
S_m denote a star graph with n vertices, where m≥1. Then for any star graph S_m, there exists a picture fuzzy graceful labeling such that the differences between adjacent labels of vertices and edges form a set of consecutive integers.
Proof
Let S_m be a star graph with n vertices, where m ≥1. We will construct a picture fuzzy graceful labeling using the true membership, false membership and hesitant membership functions. General star is given in figure 3.5.

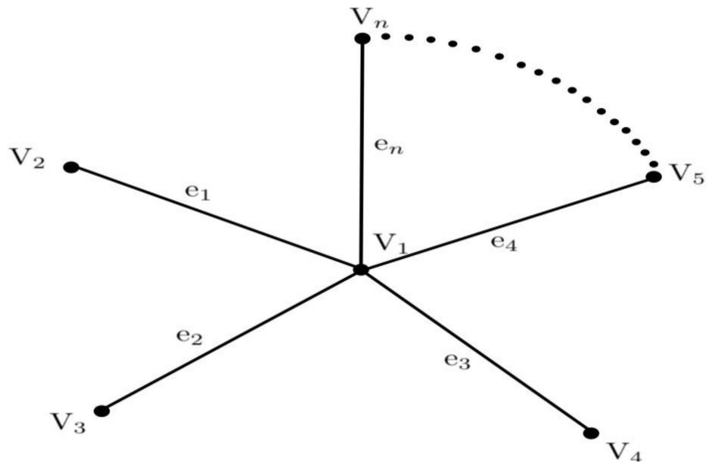

Figure 3.5. Star Graph S_n

True function

Vertex Labels: $\mu(v) = \frac{m-1}{2(2m+1)}$

$\mu(v_i) = \frac{m+i-1}{2(2m+1)}$

Edge labels: The edge membership label is defined based on the difference between the memberships of the two vertices it connects, normalized to be between 0 and 1

$$\alpha(e_i) = \frac{|\mu(v) - \mu(v_i)|}{1 + |\mu(v) - \mu(v_i)|}$$

To satisfy the conditions of picture fuzzy graceful labeling we have the requirement that $\mu(v_i) \geq 0$ since $m \leq 1$ and $i \geq 1$ then the numerator $m + i - 1$ always greater than or equal to 1. and the denominator $2(2m + 1)$ is always positive. Let's set an upper bound value for $\mu(v_i)$, such that $\mu(v_i) \leq 1$ we ensure that if the numerator is less than or equal to the denominator, then we have:

$m + i - 1 \leq 2(2m + 1)$

Simplifying the inequality we get, $m + i - 1 \leq 4m + 2$

$$\Rightarrow i \leq 3m + 3$$

Given $i \leq m$ holds true. Thus, $\alpha(e_i) \leq \mu(v) \wedge \mu(v_i)$

The true membership function satisfies the necessary conditions for picture fuzzy graceful labeling while maintaining non-negativity and an upper bound. Each vertex and edge in the star graph has a unique membership value, ensuring distinctiveness and meeting the requirements of picture fuzzy graceful labeling.

Indeterminacy function

Vertex Labels: $\sigma(v) = \frac{2m}{3(2m+1)}$

$\sigma(v_i) = \frac{2m-i}{3(2m+1)}$

Edge labels: $\beta(v, v_i) = \frac{|\sigma(v) - \sigma(v_i)|}{1 + |\sigma(v) - \sigma(v_i)|}$

This keeps it distinct and within bounds $\sigma(v_i) \geq 0$

$$2m - i \geq 0 \Rightarrow i \leq 2m$$

Since $m \geq 1$ and $i \leq m$ then the numerator $2m - i$ always greater than or equal to 1. and the denominator $3(2m + 1)$ is always positive. Let's set an upper bound value for $\sigma(v_i) \leq 1$ this is ensured if the numerator is less than or equal to the denominator:

$2m - i \leq 3(2m + 1)$

Simplifying the inequality we get, $2m - i \leq 6m + 3$

Given $i \leq m$ holds true. Thus, $\alpha(e_i) \leq \mu(v) \wedge \mu(v_i)$

The indeterminacy membership function satisfies the necessary conditions for picture fuzzy graceful labeling while maintaining non-negativity and an upper bound. Each vertex and edge in the star graph has a unique membership value, ensuring distinctiveness and meeting the requirements of picture fuzzy graceful labeling.

False function

Vertex Labels: $\boldsymbol{\eta}(\mathbf{v}) = \frac{\mathbf{4m-3}}{\mathbf{3(2m+1)}}$

$\boldsymbol{\eta}(\mathbf{v_i}) = \frac{\mathbf{4m-i-3}}{\mathbf{3(2m+1)}} z$ for all $1 \le i \le n$

Edge labels: $\beta(v, v_i) = \frac{|\boldsymbol{\eta}(v) - \boldsymbol{\eta}(v_i)|}{1 + |\boldsymbol{\eta}(v) - \boldsymbol{\eta}(v_i)|}$ for all i ($1 \le i \le n-1$)

A picture Fuzzy graph verified the condition

$\gamma(v_i, v_j) \le \eta(v_i) \wedge \eta(v_j)$ and

$\gamma(v, v_i) \le \eta(v) \wedge \eta(v_i)$

i.e., $\eta(v) \wedge \eta(v_i) = \frac{\mathbf{4m-i-3}}{\mathbf{3(2m+1)}}$ and $\gamma(v_i, v_j) = \frac{|\boldsymbol{\eta}(v) - \boldsymbol{\eta}(v_i)|}{1 + |\boldsymbol{\eta}(v) - \boldsymbol{\eta}(v_i)|}$

Therefore, $\frac{|\boldsymbol{\eta}(v) - \boldsymbol{\eta}(v_i)|}{1 + |\boldsymbol{\eta}(v) - \boldsymbol{\eta}(v_i)|} < \frac{\mathbf{4m-i-3}}{\mathbf{3(2m+1)}}$ which holds true.

This relationship verifies that $\gamma(v, v_i) \le \eta(v) \wedge \eta(v_i)$ for all i.

Thus, the star graph S_m meets the conditions of a fuzzy graph, showcasing clear and valid false membership values for both vertices and edges. When m=6, the star graph (S_6) as shown in figure: 3.6. These memberships adhere to the principles of fuzzy graceful labeling, thereby confirming the uniqueness and validity of the graph's structure.

The distinctiveness of all membership values assigned to vertices and edges ensures that each value attributed to the vertices and edges is unique. Consequently, the star graph exemplifies a valid instance of picture fuzzy graceful labeling, reinforcing its distinct and valid arrangement within the framework of fuzzy graph theory.

Example: 3.6

Let us consider the star graph S_6 with 6 vertices and 5 edges as given below:

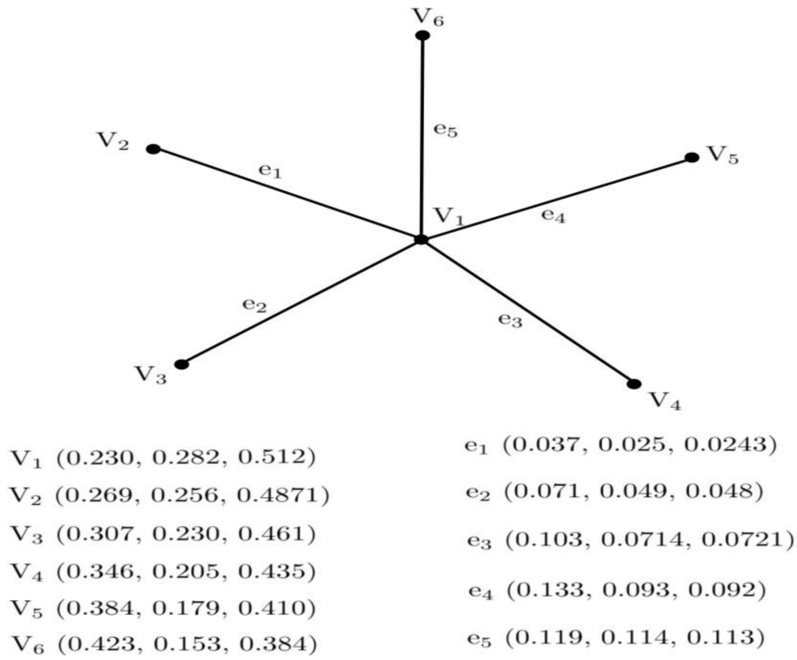

Figure 3.6. Star Graph S_6

CONCLUSION

Compared to simple fuzzy graphs, a picture fuzzy graph is a substantial development in fuzzy graph models, more capable to handle the complexity of real-world uncertainties. This work provides various theorems meeting the pictures fuzzy graceful labeling criteria and discusses examples of it by providing the idea of picture fuzzy graceful labeling.

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