Degree in the Cartesian Product of Two constant Intutionistic Fuzzy Graphs

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ABSTRACT
Intuitionistic fuzzy is an extension of Fuzzy sets. Intuitionistic fuzzy graphs is an extension of Fuzzy graphs whereas Fuzzy graphs are derived from crisp graphs. The constant graph is a special IF graph with the same degrees of membership and non – membership values. Cartesian product of two constant IF graphs extend the application similar to IF graphs in various fields like operations research, medical and visual communication. In this paper let us derive the degree of the cartesian product of two constant IF graphs in various conditions.

Key words: Constant IF graphs, Cartesian Product of constant IF graphs, Degrees of the vertices and edges of the Cartesian product of Constant IF graphs.

1. INTRODUCTION

The origin of graph theory started with the problem of Konigsberg bridge, in 1735.This problem leads to the concept of Eulerian Graph. The concept of tree was implemented by Kirchhoff, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. Then, Kirkman and Hamilton studied cycles on polyhedral and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. Even the four colour problem was invented it was solved only after a century by Appel and Haken. This time is considered as the birth of Graph Theory. This leads us towards Fuzzy graph. It is quite well known that graphs are simply models of relations. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a 'Fuzzy Graph Model.'

Most of our traditional tools for formal modelling, reasoning, and computing are crisp, deterministic, and precise in character. In set theory, an element can either belong to a set or not; and in optimization, a solution is either feasible or not. So, for a tie situation the cases are not perfectly placed.

Fuzzy set theory gives an idea for solving this type of problem. The theory of fuzzy set was proposed by Zadeh[9] to handle the various uncertainties in many real applications. The theory of fuzzy sets is, basically, a theory of graded concepts-a theory in which everything is a matter of degree or, to put it figuratively, everything has elasticity. In the two decades since its inception, the theory has matured into a collection of concepts and techniques for dealing with complex phenomena that do not lend themselves to analysis by classical methods based on probability theory and bivalent logic. Since complete information in science and technology is not always available, we need some other idea to solve those types of problems.

After that Rosenfeld introduced fuzzy graphs, though Yeh and Bang also introduced this independently. Fuzzy graph is useful to represent some special relationship which is related with uncertainty.

Atanassov introduced the concept of intuitionistic fuzzy(IF) relations and intuitionistic fuzzy graphs (IFG). Theory of intuitionistic fuzzy sets (IFSs) creates an exponential growth in Mathematics and its applications. This ranges from traditional Mathematics to Information Sciences. This influences us to consider IFGs and their applications. Parvathy and Karunambigai's idea introduced the concept of IFG and its components.

In this paper, some properties of Intuitionistic fuzzy graphs are studied. Also, some results on maximal product of constant intuitionistic fuzzy graphs are shown.

2. Preliminaries

In this part of the article, few descriptions of an IF graphs, Constant IF graphs and maximal product of an IF graphs are presented.

Definition: 2.1[1] Let the set E be fixed. An IF set A in E takes the form

$A = \{\alpha, \mu_A(\alpha), \gamma_A(\alpha)/\alpha \epsilon E\}$ where the degrees of membership and non – membership of the element $\alpha \in E$ are indicated by the functions $\mu_A: E \to [0,1]$ and $\gamma_A: E \to [0,1]$ where $0 \le \mu_A(\alpha) + \gamma_A(\alpha) \le 1$

Definition: 2.2 [2] The set$G = \{< \alpha, \beta >, \mu_G(\alpha, \beta), \gamma_G(\alpha, \beta)/< \alpha, \beta > \epsilon VxV\}$ is said to be an IF graph if these functions $\mu_G: VxV \to [0,1]$ and $\gamma_G: VxV \to [0,1]$ define the corresponding degrees of membership and non – membership of the elements $(\alpha, \beta) \in VxV$ over IFSs for all $(\alpha, \beta) \in VxV$ such that $0 \le \mu_G(\alpha, \beta) + \gamma_G(\alpha, \beta) \le 1$.

Using one of this Cartesian Product the following definition is obtained.

Definition: 2.3 [4] Maximum IF graph takes the form $G = (V, E)$ where $V = \{v_1, v_2, \ldots . v_n\}$ such that $\mu: V \to [0,1]$ and $\gamma: V \to [0,1]$ represent the degrees of membership and non – membership of the element $v_1 \in V$ respectively with $0 \le \mu(v_i) + \gamma(v_i) \le 1$ for $i = 1,2, \ldots \ldots . n$.

If $E < VxV$ where $\mu: VxV \to [0,1]$ and $\gamma: VxV \to [0,1]$

such that $\mu(v_i, v_j) \le \max[\mu(v_i), \mu(v_j)]$ and $\gamma(v_i, v_j) \le \min[\gamma(v_i), \gamma(v_j)]$

with $0 \le \mu(v_i, v_j) + \gamma(v_i, v_j) \le 1$ for every $v_i, v_j \in E$ for $i, j = 1,2, \ldots . . n$.

Definition: 2.4 [7] Let $G(\mu, \gamma)$ be an IF graph, the $\mu -$ degree of a vertex $v_i$is

$$d_\mu(v_i) = \sum_{(v_i,v_j)\in E} \mu(v_i, v_j)$$

and the $\gamma -$ degree of the vertex v_i is

$$d_\gamma(v_i) = \sum_{(v_i,v_j)\in E} \gamma(v_i, v_j)$$

the degree of the vertex is

$$d(v_i) = \{\sum_{(v_i,v_j)\in E} \mu(v_i, v_j), \sum_{(v_i,v_j)\in E} \gamma(v_i, v_j)\}$$

and $\mu(v_i, v_j) = \gamma(v_i, v_j) = 0$ if $(v_i, v_j) \notin E$.

Definition: 2.5 [5] Let $G(\mu, \gamma)$ be an IF graph with $d_\mu(v_i) = k_i$ and $d_\gamma(v_j) = K_j$ for all $v_i, v_j \in V$ of the IF graph $G(V, E)$, the graph is denoted as (k_i, k_j) - IFG (or) Constant IFG of degree (k_i, k_j)

Definition: 2.6 [7] Let $G(V, E)$ be an IF graph with $G(\mu, \gamma)$, the total degree of a vertex $v \in V$ is defined as

$$td(u) = \sum_{(v_i,v_j)\in E} d_\mu(v_i, v_j) + \mu(v_i), \sum_{(v_i,v_j)\in E} d_\gamma(v_i, v_j) + \gamma(v_i)$$

If the total degree of each vertex in G is the same and it is denoted as (r_1, r_2) , then G is called an IF graph of total degree (r_1, r_2) or a (r_1, r_2) totally Constant IF graph.

3. Cartesian product of two Constant IF graphs

Definition: 3.1

Let $G^*(V^*, E^*, \mu^*, \gamma^*)$ and $G^{**}(V^{**}, E^{**}, \mu^{**}, \gamma^{**})$ be two constant IF graphs then

$G^*. G^{**} = (V, E, \mu, \gamma)$ is called the cartesian product of these constant IF graphs with the set of vertices $V = V^* \times V^{**}$exist with

$\mu(u, v) = \mu(u). \mu(v)$and $\gamma(u, v) = \gamma(u). \gamma(v)$ for all $(u, v) \in V$the set of edges

$E = \left((u_i, v_i)(u_j, v_j)\right)$ such that $u_i = u_j$and $v_iv_j \in E^{**}, v_i = v_jandu_iu_j \in E^*$

exist with membership value of the edges as,

$$\mu\left((u_i, v_i)(u_j, v_j)\right) = \begin{cases} \mu^*(u_i). \mu^{**}(v_iv_j) \text{ if } u_i = u_j \text{ and } v_iv_j \in E^{**} \\ \mu^*(u_iu_j). \mu^{**}(v_i) \text{ if } v_i = v_j \text{ and } u_iu_j \in E^* \end{cases}$$

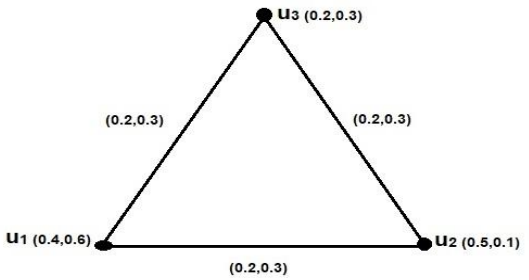
and non - membership value of the edge as

$$\gamma\left((u_i, v_i)(u_j, v_j)\right) = \begin{cases} \gamma^*(u_i). \gamma^{**}(v_iv_j) \text{ if } u_i = u_j \text{and } v_iv_j \in E^{**} \\ \gamma^*(u_iu_j). \gamma^{**}(v_i) \text{ if } v_i = v_j \text{and} u_iu_j \in E^* \end{cases}$$

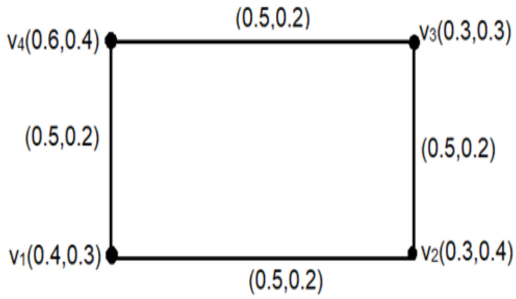
thus, the edge exists in the cartesian product if there exist edges already in the anyone of the constant IF graph.

Example: 3.2

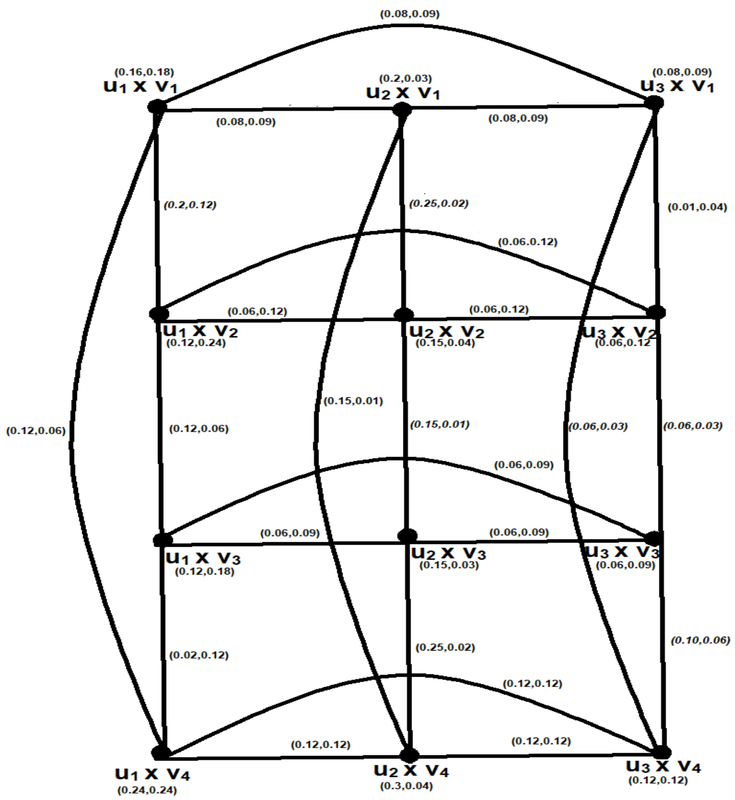
Consider the following two constant IF graphs with 3 and 4 edges as $G^*(V^*, E^*, \mu^*, \gamma^*)$ and $G^{**}(V^{**}, E^{**}, \mu^{**}, \gamma^{**})$ and their cartesian product as $G^*.G^{**} = (V, E, \mu, \gamma)$ in graph 1, graph 2 and graph 3 as follows:



Graph 1



Graph 2



Graph 3

Definition: 3.3

The degree of the vertex of the cartesian product of two constant IF graphs $G^*(V^*, E^*, \mu^*, \gamma^*)$ and $G^{**}(V^{**}, E^{**}, \mu^{**}, \gamma^{**})$ denoted as $G^*.G^{**}$ and it is defined as follows for its membership and non - membership values as

$\deg_\mu(G^*.G^{**})(u_i, v_i) = \sum \mu^*(u_i u_j).\mu^{**}(v_i) + \sum \mu^*(u_i)\mu^{**}(v_i v_j)$ and

$\deg_\gamma(G^*.G^{**})(u_i, v_j) = \sum \gamma^*(u_i u_j).\gamma^{**}(v_j) + \sum \gamma^*(u_i)\gamma^{**}(v_i v_j)$and its degree is denoted as

$\deg\left((G^*.G^{**})(u_i, v_j)\right) = \deg_\mu(G^*.G^{**})(u_i, v_i), \deg_\gamma(G^*.G^{**})(u_i, v_j)$

Consider the graphs 1,2 and 3 given in example 3.2. The degree of each vertex can be calculated using the above definition as follows:

$\deg_\mu(u_1, v_1) = \mu^*(u_1 u_2).\mu^{**}(v_1) + \mu^*(u_1 u_3).\mu^{**}(v_1) + \mu^*(u_1).\mu^{**}(v_1 v_2) + \mu^*(u_1).\mu^{**}(v_1 v_4)$

= (0.2) (0.4) + (0.2) (0.4) + (0.4) (0.5) + (0.4) (0.3)

= 0.08 + 0.08 + 0.20 + 0.12

$= 0.48$

And, $\deg_\gamma(u_1, v_1) = \gamma^*(u_1u_2).\gamma^{**}(v_1) + \gamma^*(u_1u_3).\gamma^{**}(v_1) + \gamma^*(u_1).\gamma^{**}(v_1v_2) + \gamma^*(u_1).\gamma^{**}(v_1v_4)$

$= (0.3)(0.3) + (0.3)(0.3) + (0.6)(0.2) + (0.6)(0.1)$

$= 0.09 + 0.09 + 0.12 + 0.06$

$= 0.36$

thus, $\deg(u_1, v_1) = (0.48, 0.36)$

$\deg_\mu(u_2, v_1) = \mu^*(u_2u_1).\mu^{**}(v_1) + \mu^*(u_2u_3).\mu^{**}(v_1) + \mu^*(u_2).\mu^{**}(v_1v_2) + \mu^*(u_2).\mu^{**}(v_1v_4)$

$= (0.2)(0.4) + (0.2)(0.4) + (0.5)(0.5) + (0.5)(0.3)$

$= 0.08 + 0.08 + 0.25 + 0.15$

$= 0.56$

and, $\deg_\gamma(u_2, v_1) = \gamma^*(u_2u_1).\gamma^{**}(v_1) + \gamma^*(u_2u_3).\gamma^{**}(v_1) + \gamma^*(u_2).\gamma^{**}(v_1v_2) + \gamma^*(u_2).\gamma^{**}(v_1v_4)$

$= (0.3)(0.3) + (0.3)(0.3) + (0.1)(0.2) + (0.1)(0.1)$

$= 0.09 + 0.09 + 0.02 + 0.01$

$= 0.21$

thus, $deg(u_2, v_1) = (0.56, 0.21)$

similarly, all other degrees of vertices in the Cartesian product can be calculated using the definition as follows,
$deg(u_3, v_1) = (0.32, 0.27)$, $deg(u_1, v_2) = (0.44, 0.42)$, $deg(u_2, v_2) = (0.52, 0.27)$,
$deg(u_3, v_2) = (0.28, 0.33)$, $deg(u_1, v_3) = (0.44, 0.36)$, $deg(u_2, v_3) = (0.52, 0.21)$,
$deg(u_3, v_3) = (0.28, 0.27)$, $deg(u_1, v_4) = (0.56, 0.42)$, $deg(u_2, v_4) = (0.64, 0.27)$,
and, $deg(u_3, v_4) = (0.40, 0.31)$

Theorem: 3.4
Cartesian product of two constant IF graphs is not a constant IF graph.
Proof
Let $G^*(V^*, E^*, \mu^*, \gamma^*)$ and $G^{**}(V^{**}, E^{**}, \mu^{**}, \gamma^{**})$ are two constant IF graphs with degrees $({c_1}^*, {c_2}^*)$ and $({c_1}^{**}, {c_2}^{**})$ respectively. Let $V^* = \{u_1, u_2, \dots u_m\}$ and $V^{**} = \{v_1, v_2, \dots v_n\}$ then by the definition of cartesian product $G^*.G^{**}$ has mnvertices in the form of (u_i, v_j) where $i = 1,2, \dots m$ and $j = 1,2, \dots n$.
Then by the definition of cartesian product (3.1) the membership and non - membership values of the vertices of the cartesian product of the above two constant IF graphs are,
$\mu(u_i, v_j) = \mu(u_i).\mu(v_j)$ and
$\gamma(u_i, v_j) = \gamma(u_i).\gamma(v_j)$ where $i = 1,2, \dots m$ and $j = 1,2, \dots n$.
The edges exist in their cartesian product between the vertices if there exist an edge in their constant IF graphs.
Case(i)
If $u_i = u_j$ in G^* and v_iv_j exist in G^{**},
$\mu\left((u_i, v_j)(u_j, v_i)\right) = \mu^*(u_i).\mu^{**}(v_iv_j)$
$< {c_1}^*{c_1}^{**}$
Case(ii)
If $v_i = v_j$ in G^{**} and u_iu_j exist in G^*,
$\mu\left((u_i, v_j)(u_j, v_i)\right) = \mu^*(u_iu_j).\mu^{**}(v_i)$
$< {c_1}^*{c_1}^{**}$
these values are not equal in both the cases as membership values of the vertices in both the constant IF graphs are not equal.
For non - membership values of the edges in their cartesian product are:
Case(i)
If $u_i = u_j$ in G^* and v_iv_j exist in G^{**},
$\gamma\left((u_i, v_j)(u_j, v_i)\right) = \gamma^*(u_i).\gamma^{**}(v_iv_j)$
$< {c_2}^*{c_2}^{**}$
Case(ii)
If $v_i = v_j$ in G^{**} and u_iu_j exist in G^*,
$\gamma\left((u_i, v_j)(u_j, v_i)\right) = \gamma^*(u_iu_j).\gamma^{**}(v_i)$
$< {c_2}^*{c_2}^{**}$
Here also these membership values are not same as they differ in both the constant IF graphs. The edge has different membership and non - membership values in their cartesian product even though they have same values in their constant IF graphs lead to the different degrees of the vertices by the definition 3.3.

This concludes the result that the cartesian product of two constant IF graphs is not a constant IF graph.

Example: 3.5

Consider the graph 1,2 and their cartesian product graph 3 given in the example 3.2.
Either by the definition 3.3 or by the direct calculation of the degrees of the verities we can clearly understand that the theorem 3.4 is proved.
By direct calculation of degrees in graph 3,
$deg_\mu(u_1, v_1) = 0.08 + 0.08 + 0.20 + 0.12 = 0.48$
$deg_\gamma(u_1, v_1) = 0.09 + 0.09 + 0.12 + 0.06 = 0.36$
thus, $deg(u_1, v_1) = (0.48{,}0.36)$
and $deg_\mu(u_1, v_2) = 0.06 + 0.06 + 0.12 + 0.20 = 0.44$
$deg_\gamma(u_1, v_2) = 0.12 + 0.12 + 0.06 + 0.12 = 0.42$
thus, $deg(u_1, v_2) = (0.44{,}0.42)$
and $deg_\mu(u_1, v_3) = 0.06 + 0.06 + 0.12 + 0.20 = 0.44$
$deg_\gamma(u_1, v_3) = 0.09 + 0.09 + 0.12 + 0.06 = 0.36$
thus, $deg(u_1, v_3) = (0.44{,}0.36)$
similarly, we can derive the degrees of other vertices in graph 3 and assert our result in theorem 3.4

Theorem: 3.6

Cartesian product of two connected constant IF graphs is a connected IF graph.

Proof

Let $G^*(V^*, E^*, \mu^*, \gamma^*)$ and $G^{**}(V^{**}, E^{**}, \mu^{**}, \gamma^{**})$ are two constant IF graphs with vertices $V^* = \{u_1, u_2, \dots u_m\}$ and $V^{**} = \{v_1, v_2, \dots v_n\}$ of G^* and G^{**} respectively.
Also, $\mu^*(u_i u_j) > 0, \gamma^*(u_i u_j) > 0, for\ all\ u_i, u_j \in V^*, i, j = 1,2, \dots m$
and $\mu^{**}(v_i v_j) > 0, \gamma^{**}(v_i v_j) > 0, for\ all\ v_i, v_j \in V^{**}, i, j = 1,2, \dots n$
The cartesian product of G^* and G^{**} is $G = G^*.G^{**}$ with $G = (V, E, \mu, \gamma)$. These are m-subgroups with vertices $\{u_i v_1, u_i v_2 \dots u_i v_n\}$, where $i = 1,2, \dots m$. Since each v_i is adjacent to v_j for $i, j = 1,2, \dots n$ in G^{**} and is connected with G^* connected as u_i is adjacent to u_j for $i, j = 1,2, \dots m$, by the definition of cartesian product of two constant IF graphs, as G has m - subgroups there exist at least one edge between the vertices of the subgroups. As the membership and non - membership values in G^* and G^{**} are > 0 in $G = G^*.G^{**}$ also they are greater than zero.
That is, $\mu\left((u_i, v_j)(u_j, v_i)\right) > 0$ and
$\gamma\left((u_i, v_j)(u_j, v_i)\right) > 0$ for $i, j = 1,2, \dots m$ or $i, j = 1,2, \dots n$
Thus, G is a connected IF graph.

Remark

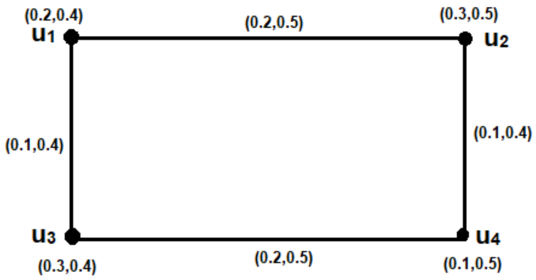
As the cartesian product of two constant IF graph is not a constant IF graph, the above connected IF graph need not be constant IF graph.

Definition :3.7

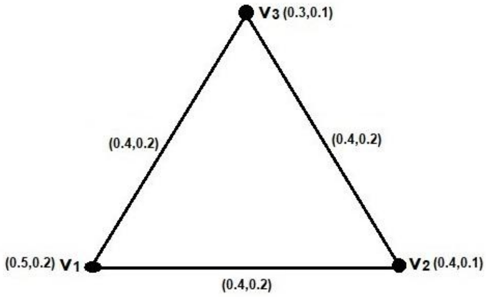
The total degree of the vertex of the cartesian product of two constant IF graphs $G^*(V^*, E^*, \mu^*, \gamma^*)$ and $G^{**}(V^{**}, E^{**}, \mu^{**}, \gamma^{**})$ for the membership and non - membership value of $G = G^*.G^{**}$ denoted as $tdeg_\mu$ and $tdeg_\gamma$ is defined as,
$tdeg_\mu G(u_i, v_j) = \sum \mu^*(u_i u_j).\mu^{**}(v_j) + \sum \mu^*(u_i).\mu^{**}(v_i\ v_j) + \mu(u_i, v_j)$ and
$tdeg_\gamma G(u_i, v_j) = \sum \gamma^*(u_i u_j).\gamma^{**}(v_j) + \sum \gamma^*(u_i).\gamma^{**}(v_i\ v_j) + \gamma(u_i, v_j)$
The total degree in $G = G^*.G^{**}$ is denoted by
$tdegG(u_i, v_j) = \left(tdeg_\mu G(u_i, v_j), tdeg_\gamma G(u_i, v_j)\right)$.

Example: 3.8

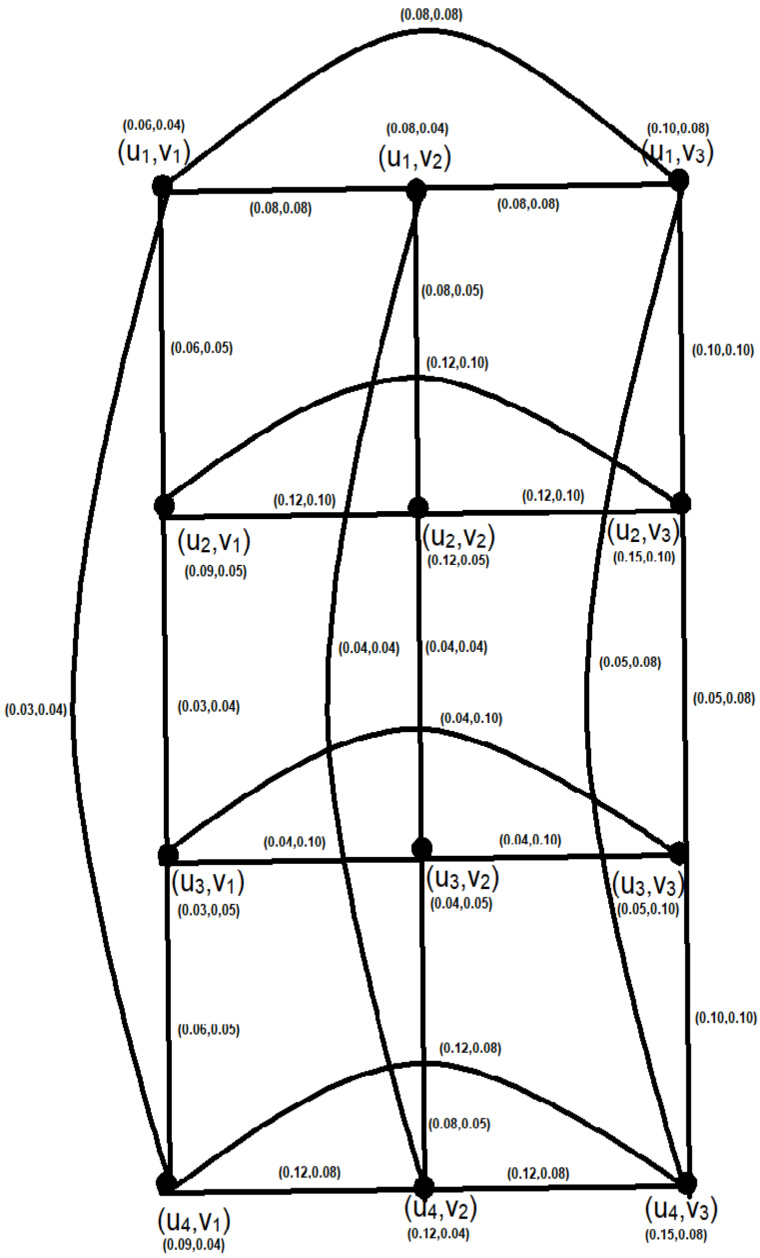
Consider the following two constant IF graphs and their cartesian product in graph 4, graph 5 and graph 6. The total degree of each vertex in their cartesian product is calculated explicitly.



Graph 4



Graph 5



Graph 6

$tdeg_\mu G(u_1,v_1) = 0.08 + 0.08 + 0.03 + 0.06 + 0.06 = 0.31$
$tdeg_\gamma G(u_1,v_1) = 0.08 + 0.08 + 0.04 + 0.05 + 0.04 = 0.29$
$\Rightarrow tdegG(u_1,v_1) = (0.31,0.29)$
$tdeg_\mu G(u_1,v_2) = 0.08 + 0.08 + 0.08 + 0.04 + 0.08 = 0.36$
$tdeg_\gamma G(u_1,v_2) = 0.08 + 0.08 + 0.05 + 0.04 + 0.04 = 0.29$
$\Rightarrow tdegG(u_1,v_2) = (0.36,0.29)$
$tdeg_\mu G(u_1,v_3) = 0.10 + 0.08 + 0.08 + 0.05 + 0.10 = 0.41$
$tdeg_\gamma G(u_1,v_3) = 0.10 + 0.08 + 0.08 + 0.08 + 0.08 = 0.42$
$\Rightarrow tdegG(u_1,v_3) = (0.41,0.42)$
Similarly total degree of all other vertices of G, the cartesian product of two constant IF graphs are as follows

$tdegG(u_2,v_1) = (0.42,0.34)$
$tdegG(u_2,v_2) = (0.48,0.34)$
$tdegG(u_2,v_3) = (0.54,0.48)$
$tdegG(u_3,v_1) = (0.20,0.34)$
$tdegG(u_3,v_2) = (0.24,0.34)$
$tdegG(u_3,v_3) = (0.28,0.48)$
$tdegG(u_4,v_1) = (0.42,0.29)$
$tdegG(u_4,v_2) = (0.48,0.29)$
$tdegG(u_4,v_3) = (0.54,0.42)$

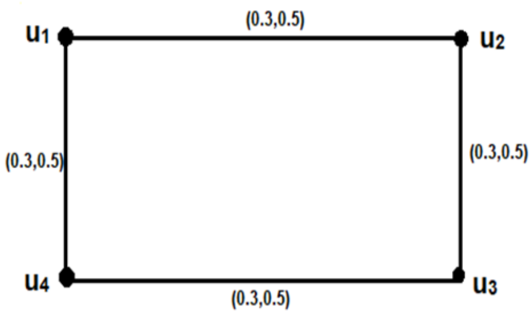
Theorem: 3.9
Cartesian product of both constant and totally constant IF graphs is a constant and totally constant graph.
Proof
Let $G^*(V^*,E^*,\mu^*,\gamma^*)$ and $G^{**}(V^{**},E^{**},\mu^{**},\gamma^{**})$be two constant and totally constant IF graphs. Then $V^* = \{u_1,u_2,\dots u_m\}$ and $V^{**} = \{v_1,v_2,\dots v_n\}$ in G^* and G^{**} gives mn vertices in its cartesian product $G(V,E,\mu,\gamma) = G^*.G^{**}$ with their vertex degree as
$\mu(u_i,v_j) = \mu^*(u_i).\mu^{**}(v_j)$ and
$\gamma(u_i,v_j) = \gamma^*(u_i).\gamma^{**}(v_j)$for $i = 1,2\dots,m$ and $j = 1,2,\dots,n$
These values are same for all i and j as they are same in both the graphs (by the definition of totally constant IF graphs). Hence all the vertices of the cartesian product has the membership and non – membership values.
Also, the edges in their cartesian product,

$$\mu\left((u_i,v_j)(u_j,v_i)\right) = \begin{cases}\mu^*(u_iu_j).\mu^{**}(v_i) \text{ if } v_i = v_j\\ \mu^*(u_i).\mu^{**}(v_iv_j) \text{ if } u_i = u_j\end{cases} \text{for all } i = 1,2\dots,m \text{ and } j = 1,2,\dots,n$$

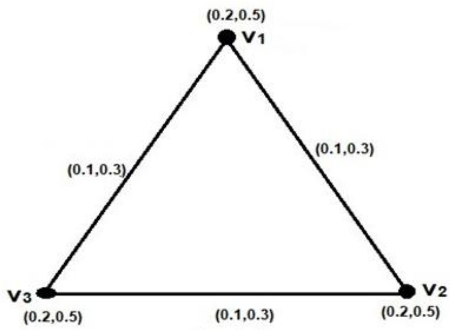
$$\gamma\left((u_i,v_j)(u_j,v_i)\right) = \begin{cases}\gamma^*(u_iu_j).\gamma^{**}(v_i) \text{ if } v_i = v_j\\ \gamma^*(u_i).\gamma^{**}(v_iv_j) \text{ if } u_i = u_j\end{cases} \text{for all } i = 1,2\dots,m \text{ and } j = 1,2,\dots,n$$

As, $\mu^*(u_i).\mu^{**}(v_j)$ and $\gamma^*(u_i).\gamma^{**}(v_j)$ are same in G^* and G^{**} by totally constant IF graphs, all the edges in its cartesian product are same leads to the constant and totally constant IF graphs.

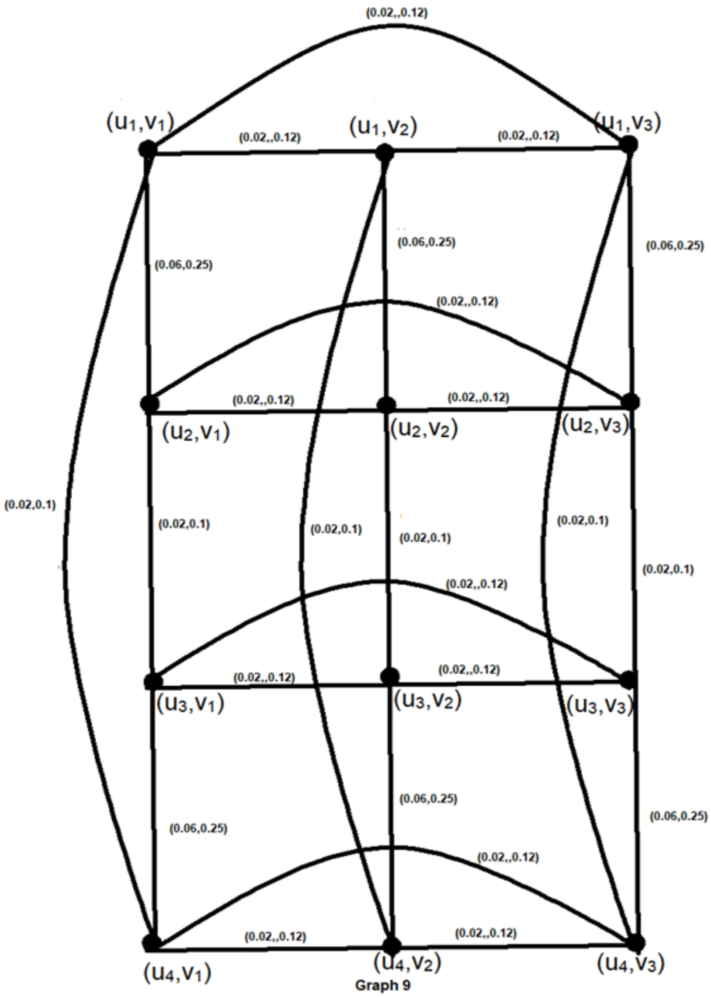
Example: 3.10
Consider the following two constant and totally constant IF graphs graph 7 and graph 8. Also, its cartesian product in Graph 9 as follows.



Graph 7



Graph 8



Graph 9

By the definition of 3.3 from Graph 9
$deg_{\mu}G(u_i, v_j) = 0.12$ and $deg_{\gamma}G(u_i, v_j) = 0.59$ for $i = 1,2,3,4$ and $j = 1,2,3$
Hence $degG(u_i, v_j) = (0.12\ , 0.59)$ for all the vertices in the cartesian product of the constant as well as totally constant IF graphs.
By the definition of 3.7 from Graph 9
$tdeg_{\mu}G(u_i, v_j) = 0.16$ and $tdeg_{\gamma}G(u_i, v_j) = 0.68$ for $i = 1,2,3,4$ and $j = 1,2,3$
Hence $tdegG(u_i, v_j) = (0.16\ , 0.68)$

CONCLUSION

As graph represents the relation, the cartesian product of two constant IF graphs represent the relation between the vertices and edges of their product.
In this paper it is explained with examples by defining the cartesian product degree and total degree of the vertices of the cartesian product of two constantIF graphs. Also, some theorems on connectivity and total degree of vertices are proved with examples.

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