Connected Domatic Number On Anti Fuzzy Graph

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Received: 05.04.2024 Revised : 12.05.2024 Accepted: 16.05.2024

ABSTRACT
The idea of connected domatic number of an anti fuzzy graph [AFG] is defined in this research work. We determine the bounds on the connected domatic number of an anti fuzzy graph. We provide and prove certain connected domatic number theorems on anti fuzzy graphs.

Keywords Anti fuzzy graph, Domatic Number, Dominating set, Connected dominating set, Connected domatic Number.

1. INTRODUCTION

In this research article all graphs are considers connected simple, finite graphs without loops, numerous edges and undirected graphs. The domatic number of a graph was defined by S. T. Hedetniemi and E. J. Cockayne[1]. Eventually, several associated ideas were presented. The total domatic number was first presented by the same authors along with R. M. Dawes [2].Sampathkumar, E. and Walikar, H.B. established the idea of a connected dominating set[3] and the connected domatic number was first presented by R. Laskar and S. T. Hedetniemi [4].The concept of anti fuzzygraphs[AFG] was first proposed by R. Seethalakshmi and R. B. Gnanajothi [5]. Besides this, R. Muthuraj and A. Sasirekha expanded on the notion of an anti fuzzy graph [AFG] and proposed the idea of an AFG domination [6, 7]. The connected domatic number [CDN] on anAFG is defined in this study.An AFG's connected domatic number limitations are identified. In an AFG, connected domatic number-related theorems are given and proved. Certain standard AFG's are examined using connected domatic numbers.

An anti fuzzy graph $A_G = (\sigma, \mu)$ is a pair of functions σ: N→ [0,1] and μ: N×N→ [0,1], with $\mu(k, l) \geq \sigma(k) \vee \mu(l)$ for all (k, l) ϵ N.The orderρ and size q of an AFGA_G= (N,A, σ, μ) are defined to be $\rho = \sum_{k\in N} \sigma(k)$ and $q = \sum_{kl \in N} \mu(k, l)$. It is denoted by O($A_G$) and S($A_G$).

2. MAIN RESULTS

Definition 2.1

Let A_G= (N, A, σ, μ) be an anti fuzzy graph. A partition CDP = {CDS_1, CDS_2, ……, CDS_k} of N (A_G) is called connected domatic partition [CDP] of A_G if for each CDS_i is a connected dominating set[CDS] of an anti fuzzy graph A_G.

The maximum fuzzy cardinality taken over all maximum number of classes with a minimal connected domatic partition of A_G is called the connected domatic number [CDN] of A_G and it is denoted by $d_c(A_G)$.

The maximum number of classes with maximum fuzzy cardinality of a partition $CDS_i(A_G)$ is called the anti fuzzy connected domatic number [ACDN] of an anti fuzzy A_G and it is denoted by $d_{afc}(A_G)$.

Example 2.2

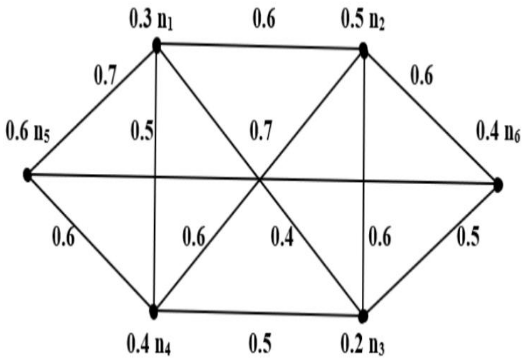

Fig 1. Anti Fuzzy Graph A_G

From figure 1, the CDS's are
$CDS_1 = \{n_1, n_3\} = \{0.3, 0.2\} = 0.5$
$CDS_2 = \{n_2, n_4\} = \{0.5, 0.4\} = 0.9$
$CDS_3 = \{n_5, n_6\} = \{0.6, 0.4\} = 1$
$CDP = \{CDS_1, CDS_2, CDS_3\}$
CDN of an anti fuzzy graph A_G, $d_c(A_G) = 3$
Anti fuzzy CDN of an anti fuzzy graph A_G,
$d_{afc}(A_G) = Max\{0.5, 0.9, \quad 1\} = 1$

Definition 2.3
Let $A_G = (N, A, \sigma, \mu)$ be an anti fuzzy graph. A partition CDP = $\{CDS_1, CDS_2, \ldots\ldots, CDS_k\}$ of N (A_G) is called partial connected domatic partition [PCDP] of A_G if for each CDS_i is a connected dominating set [CDS] of an anti fuzzy graph A_G and atleast one nodedoes not in any one of CDS_i.
The maximum fuzzy cardinality taken over all maximum number of classes with a minimal partial connected domatic partition of A_G is called the partial connected domatic number of [PCDN] A_G and it is denoted by $d_{pc}(A_G)$.
The maximum number of classes with maximum fuzzy cardinality of a partition $CDS_i(A_G)$ is called the anti fuzzy partial connected domatic number [APCDN] of an anti fuzzy graph A_G and it is denoted by $d_{afpc}(A_G)$.

Example 2.4

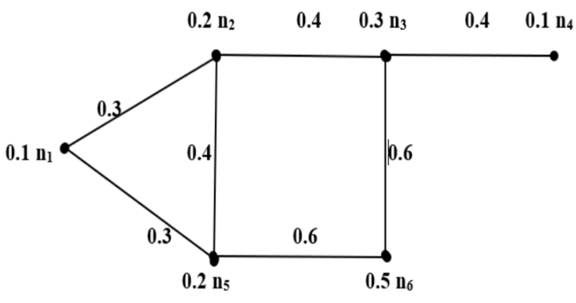

Fig 2. Anti Fuzzy Graph A_G

From figure 2, the CDS is
$CD_1 = \{n_2, n_3\} = \{0.2, 0.3\} = 0.5$
$CDP = \{CD_1\}$
PCDN of an anti fuzzy graph A_G, $d_{pc}(A_G) = 1$
APCDN of an anti fuzzy graph A_G, $d_{afpc}(A_G) = 0.5$

Theorem 2.5
Let A_G=(N, A, σ, μ) be an anti fuzzy path then (i)$d_{pc}\,(A_G) = 1$ (ii) $d_{afpc}\,(A_G) = \rho - \sigma(n_{1)} - \sigma(n_{u)}$.
proof
(i) Let $A_G$be an anti fuzzy path graph with {n_1, n_2, n_3, ..., n_u} nodes. Then node $n_1$is adjacent to node n_2, node n_2 is adjacent to n_3. Similar way node n_{u-1} is adjacent to n_u. It is obvious that n_1 dominates n_2 and n_2 dominates n_3 and so an n_{u-1} dominates n_u. By the definition of connected domatic partition we have to form only one connected dominating set. At least one node in the group of nodes is missing in the connected domatic partition.By the definition of partial connected domatic partition we form one connected domatic partition. Hence $d_{pc}\,(A_G) = 1$.
(ii) Already we know that,the order p of an AFGA$_G$ = (N, A, σ, μ) is known as$p = \sum_{n \in N} \sigma(n)$. In an anti fuzzy path graph A_G, consider the partial connected domatic partition. By the above proof we have only one partial connected domatic partition. In this partition we have all nodes except first and last nodes. Because here we consider first partial connected domatic partition consist minimal partial connected dominating set. Hence$d_{afpc}\,(A_G) = \rho - \sigma(n_{1)} - \sigma(n_{u)}$.

Proposition 2.6
Let A_G= (N, A, σ, μ) be an anti fuzzy uninodal path then $d_{afpc}\,(A_G) = \rho - 2\sigma(n)$.

Example 2.7

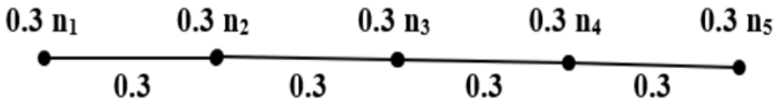

Fig 3. Anti Fuzzy Graph A_G

From figure 3, the CDS is
CDS = {n_2, n_3, n_4} = {0.3, 0.3, 0.3} = 0.9
CDP = {CDS}
PCDN of an anti fuzzy graph A_G, $d_{pc}\,(A_G) = 1$
APCDN of ananti fuzzy graph A_G, $d_{afpc}\,(A_G) = 0.9$ -----(1)
ρ = 0.3+0.3+0.3+0.3+0.3
=1.5
σ (n_i) = 0.3
2σ (n_i) = 2(0.3)= 0.6
$\rho - 2\sigma(n)$= 1.5 – 0.6 = 0.9 ---------------(2)
From (1) & (2), $d_{afpc}\,(A_G) = \rho - 2\sigma(n)$.

Theorem 2.8
Let A_G= (N, A, σ, μ) be an anti fuzzy cycle C_n then $d_{pc}\,(A_G) = 1$.
Proof
LetC_n be the anti fuzzy cycle with n nodes {n_1, n_2, n_3, ..., n_u}.n_i is adjacent with n_{i-1} and n_{i+1} nodes. Also n_i node is dominated by n_{i-1} and n_{i+1} nodes where i = 2, 3, 4, ...u. By the definition of partial connected domatic partition we have to form only one partition in an anti fuzzy cycle. Hence $d_{pc}\,(A_G) = 1$.

Theorem 2.9
Let A_G= (N, A, σ, μ) be an anti fuzzy complete bipartite with u and v nodes then $d_c(A_G) = min\{u, v\}$ for all $n_i \in N_1(A_G)$ and $m_i \in N_2(A_G)$
Proof
Let N_1,N_2 be the complete bipartition of the node set of A_G with $N_1(A_G)$ = {n_1, n_2, n_3, ..., n_u}and $N_1(A_G)$ = {m_1, m_2, m_3, ..., m_v}. Let us assume that u ≤ v. Let CD be a domatic partition of A_G. We claim that CD is connected domatic partition of A_G. Every node in the $N_1(A_G)$ set connects to all the other nodes in the $N_1(A_G)$ set according to the definition of the complete bipartition graph.Let CD = $N_2(A_G)$ and n_1, n_2 ϵ CD. If n_1, n_2 ϵ $N_2(A_G)$ then n_1 and n_2 are not adjacent nodes in $N_2(A_G)$ then there exists n_1-n_2 path as n_1m_1, m_1n_2. Similarly we have to form the Connected Domatic partitions like CD_1, CD_2,…, CD_n. If u < v, the last one node in V to be taken in the last connected domatic partition<CD> is connected domatic partition. Hence $d_c(A_G) = min\{u, v\}$.

Proposition 2.10

IfA_G is connected anti fuzzy graph and n ≥ 3, then $(i)\ d_c(A_G) = n-2\ (or)\ d_{pc}(A_G) = n-2$.

Theorem 2.11

Let A_G= (N, A, σ, μ) be an complete uninodal AFG with n nodes, then

$$(i)\ d_c(A_G) = \begin{cases} \frac{n}{2} & ; if\ n\ is\ even \\ \left\lfloor \frac{n}{2} \right\rfloor & ; if\ n\ is\ odd \end{cases} \quad (ii)\ d_{afc}(A_G) = \begin{cases} \frac{n}{2}\sigma(n_i) & ; if\ n\ is\ even \\ \left\lfloor \frac{n}{2} \right\rfloor \sigma(n_i) & ; if\ n\ is\ odd \end{cases}$$

Proof

(i) Let A_G is a complete uninodal AFG and CD is a connected domatic partition of A_G. Connected domatic partition of A_G has CD_1, CD_2,….. classes. The classes CD_1, CD_2,…..$CD_{\frac{n}{2}}$ are connected dominating sets with same cardinality.

Let $n_1 \in CD_1$ and it has adjacent to n-1 nodes with degree (n-1)$n_1.n_2 \in$ N (A_G) and $n_1, n_2 \in CD_1$. $n_1$and n_2 are also adjacent and dominates all other nodes in A_G. If n is even number we get CD_1, CD_2,…..$CD_{\frac{n}{2}}$ classes in connected domatic partition of A_G.If n is odd number again we get CD_1, CD_2,…..$CD_{\frac{n}{2}}$ classes because a node at the end must be taken into some connected domatic partition.

Hence $d_c(A_G) = \begin{cases} \frac{n}{2} & ; if\ n\ is\ even \\ \left\lfloor \frac{n}{2} \right\rfloor & ; if\ n\ is\ odd \end{cases}$.

(ii) By the above proof and by the definition of an anti fuzzy connected domatic number of an anti fuzzy graph A_G hence $d_{afc}(A_G) = \begin{cases} \frac{n}{2}\sigma(n_i) & ; if\ n\ is\ even \\ \left\lfloor \frac{n}{2} \right\rfloor \sigma(n_i) & ; if\ n\ is\ odd \end{cases}$.

Proposition 2.12

Anti fuzzy connected domatic number of an AFG of A_G= (N, A, σ, μ) is not exist for the complement of a complete AFG A_G.

Theorem 2.13

Let A_G= (N, A, σ, μ) be an anti fuzzy Petersen graph then (i) $d_c(A_G) = 2$ (ii) $d_{afc}(A_G) \geq \frac{\rho}{2}$.

Proof

(i) Let A_G = (N, A) be a Petersen Anti fuzzy graph and N be the node set defined as N(A_G) = {n_1, n_2, n_3, ….., n_{10}} such that $|N(A_G)| = 10$ and A be the Arc set defined as A(A_G) = a_1, a_2, a_3,…., a_{15}} such that $|A(A_G)| = 15$.In the discipline of graph theory mathematics, the Petersen graph is an undirected anti fuzzy graph. In study of graph theory that has 10 nodes and 15 Arcs. It is a tiny graph that may be used as both a counter example and an example for a variety of graph theory issues.

Let us consider the partition of the node set N(A_G) as D_1= {n_1, n_2, n_3, n_4, n_5}and D_2= {n_6, n_7, n_8, n_9, n_{10}}. Then N - D_1= {n_6, n_7, n_8, n_9, n_{10}} and N - D_2 = {n_1, n_2, n_3, n_4, n_5}. Since D_1 is adjacent to all the vertices of N - D_1, we say that D_1 dominatesD_2, Similarly D_2 is adjacent to all the vertices of N – D_2, we say that D_2dominatesD_1.Here, the partition of node set N(A_G) that is D_1 and D_2 are also connected dominating sets.

Because D_1 is a set of nodes of an AFGA_G = (N, A) such that every node in N - D_1 is adjacent to a minimum of one nodein D_1 and sub AFG<D_1>induced by the set D_1 is connected.

Similarly, D_2 is a set of nodes of an AFGA_G = (N, A) such that every node in N – D_2 is adjacent to at least one node in $D_2$and sub AFG<D_2> induced by the set D_2 is connected. Therefore, we conclude that D_1 dominates D_2 and connected dominating set. Similarly, D_2 dominates D_1 and connected dominating set. Since there are only two connected partition D_1 and D_2 of node set N(A_G), The maximum number of classes is 2. Hence CDN of the PetersenAFG is 2. Therefore, $d_c(A_G) = 2$.

(ii) In Petersen AFG, the nodes have minimum number of neighborhoods. That is, the nodes are adjacent to atmost n/2 nodes and not an isolated node in A_G. Hence $d_{afc}(A_G) \geq \frac{\rho}{2}$.

Example: 2.14

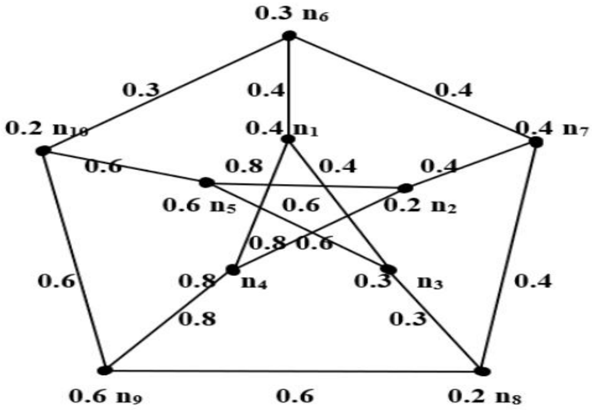

Fig 3. Anti Fuzzy Graph A_G

From the above Fig.3.,

$\rho = \sum \sigma(n_i)$

$= \sigma(n_1) + \sigma(n_2) + \dots + \sigma(n_{12})$

$= 0.4+0.2+0.3+0.8+0.6+0.3+0.4+0.2+0.6+0.2$

$\rho = 4$

$\rho/2 = 2$

$CDS_1 = \{n_1, n_2, n_3, n_4, n_5\} = \{0.4+0.2+0.3+0.8+0.6\} = 2.3$

$CDS_2 = \{n_6, n_7, n_8, n_9, n_{10}\} = \{0.3+0.4+0.2+0.6+0.2\} = 1.7$

$CDP = \{CDS_1, CDS_2\}$

CDN of an AFG A_G, $d_c(A_G) = 2$

ACDN of anAFG A_G,

$d_{afc}(A_G) = Max\{2.3, 1.7\} = 2.3$

Here, $\rho/2 = 2$

Therefore, $d_{afc}(A_G) \geq \frac{\rho}{2}$

Theorem 2.15

Let $A_G = (N, A, \sigma, \mu)$ be an uninodal anti fuzzy Petersen graph then $d_{afc}(A_G) \geq \frac{\rho}{2}$.

Results on Named Graphs

1. If A_G is a Bidiakis cube anti fuzzy graph then $d_c(A_G)=2$ and $d_{afc}(A_G)=\rho/2$.
2. If A_G is a Brinkmann anti fuzzy graph then $d_{pc}(A_G)=1$ and $d_{afpc}(A_G)\geq\left\lceil\frac{2\rho}{3}\right\rceil$.
3. If A_G is a Bull anti fuzzy graph then $d_{pc}(A_G)=1$ and $d_{afpc}(A_G)<q/2$.
4. If A_G is a Butterfly anti fuzzy graph then $d_{pc}(A_G)=1$ and $d_{afpc}(A_G)>\rho/3$.
5. If A_G is a Chratal uninodal anti fuzzy graph then $d_c(A_G)=3$ and $d_{afc}(A_G)=\rho/3$.
6. If A_G is a Diamond anti fuzzy graph then $d_c(A_G)=2$ and $d_{afc}(A_G)>\rho/3$.
7. If A_G is a Durer anti fuzzy graph then $d_{pc}(A_G)=1$ and $d_{afpc}(A_G)>\rho/3$.
8. If A_G is a Franklin anti fuzzy graph then $d_c(A_G)=2$ and $d_{afc}(A_G)\geq\rho/4$.
9. If A_G is a Franklinuninodal anti fuzzy graph then $d_c(A_G)=2$ and $d_{afc}(A_G)=\rho/2$.
10. If A_G is a Frucht anti fuzzy graph then $d_{pc}(A_G)=1$ and $d_{afpc}(A_G)\geq\rho/2$.

CONCLUSION

In this paper, Connected and partial domatic number on an anti fuzzy graph $A_G$is defined and it is determined for some types of anti fuzzy graphs.

REFERENCES

[1] Cockayne, E.J. and Hedetniemi, S.T., 1977. Towards a theory of domination in graphs. Networks, 7(3), pp.247-261.

[2] E. J. Cockayne, R. M. Dawes, S. T. Hedetniemi, Total domination in graphs, Networks, 10(1980), 211–219. https://doi.org/10.1002/net.3230100304

[3] Sampathkumar, E. and Walikar, H.B., 1979. The connected domination number of a graph. J. Math. Phys.
[4] Hedetniemi, S.T. and Laskar, R.C., 1984. Connected domination in graphs, B. Bollabas, ed.: Graph Theory and Combinatorics.
[5] Seethalakshmi, R. and Gnanajothi, R.B., 2016. Operations on anti fuzzy graphs. Mathematical Sciences International Research Journal, 5(2), pp.210-214.
[6] Muthuraj. R., and Sasireka. A., 2017, On Anti Fuzzy Graph, Advances in Fuzzy Mathematics, 12(5), pp: 1123-1135.
[7] Muthuraj, R. and Sasireka, A., 2018. Domination on anti fuzzy graph. International Journal of Mathematical Archive, 9(5), pp.82-92.
[8] Zelinka, B., 1986. Connected domatic number of a graph. Mathematica Slovaca, 36(4), pp.387-392.