

Hydrodynamic Forces and Sloshing Analysis of Steel Ground Tanks Supported on Rigid Inclusions or Stone Columns

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ABSTRACT

Background: Given the critical seismic performance requirements of steel ground storage tanks in earthquake-prone areas, ground improvement solutions such as stone columns proved to be cost-effective alternatives to conventional deep foundation systems. This paper numerically investigates the seismic performance of 16 m diameter $\times$ 10 m high open-top steel above-ground storage tanks founded on soft clay improved using either rigid inclusion foundations (RIF) or stone column foundations (SCF). Three-dimensional nonlinear finite-element models were developed in ADINA (2025) to simulate soil–structure–fluid interaction under static loading and time-history seismic excitation using the 1989 Loma Prieta earthquake record. The soil profile comprised a 6 m soft clay layer underlain by dense sand, with foundations consisting of 13 rigid concrete inclusions or 13–19 stone columns having elastic moduli ranging from 50 to 200 MPa. Hydrodynamic pressures, hoop stresses, axial shell stresses, settlements, and fluid sloshing were quantitatively evaluated. The results indicate that RIF provided the lowest static settlement (1.76 cm) compared to SCF, where settlements increased up to 7.30 cm for high-modulus stone columns; however, all values remained within permissible limits. Under seismic loading, peak dynamic settlement increased from 1.35 cm (RIF) to 2.75–3.50 cm (SCF), with a residual settlement of approximately 1.0 cm observed for stone column cases. Tanks supported on SCF exhibited up to 5% reduction in axial compressive stresses and approximately 10% reduction in hoop stresses relative to RIF, significantly mitigating the risk of elephant-foot buckling. Furthermore, replacing rigid inclusions with stone columns of 100–150 MPa modulus reduced maximum fluid sloshing height from 20 cm to 17 cm (15%), while increasing the number of stone columns from 13 to 19 achieved up to 40% reduction in sloshing amplitude. Hydrodynamic pressure variations were marginal, with reductions limited to approximately 1%.

Keywords: Steel storage tanks; Seismic performance; Stone column foundations; Rigid inclusions; Soft clay soils; Soil–structure interaction; Hydrodynamic forces; Fluid sloshing

1. INTRODUCTION

Soft clay soils are commonly encountered in various parts of the world, particularly in coastal regions. In Egypt, several areas are covered by relatively thick soft clay layers. Coastal zones of Port Said, Damietta, parts of Alexandria, and Tina Plain in Sinai are characterized by surface layers of soft clay. These soils present two major geotechnical challenges: being highly compressible and having lower shear strength values, which can lead to significant structural issues when supporting tanks and other structures. The design and stability of steel tanks constructed on soft clay deposits pose considerable engineering challenge due to the tanks' substantial loads, susceptibility to both differential and total settlements, and the hazardous nature of the stored fluids.

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To address these challenges, various soil improvement and stabilization methods can be utilized. Among these, piled-raft is the most widely used solution, as they effectively distribute heavy loads to deeper, more stable soil layers. End-bearing rigid inclusions are particularly advantageous since they directly transfer the tank's load to a strong underlying stratum, without the need for steel reinforcements. On the other hand, friction rigid inclusions are suitable for lighter structures in situations where no competent soil layer is present. These rigid inclusions primarily depend on side friction to resist loads, with minimal support from the pile tip capacity.

Piled-raft is the most commonly used foundations in supporting steel tanks for both the cases static and dynamic loading. On the other hand, an alternative cost-effective solution is the use of granular columns. Stone columns serve as an effective ground improvement method for enhancing the properties of saturated soft clay due to their relatively high load-bearing capacity. Additionally, their high permeability allows them to function as vertical drains, allowing for the accelerated consolidation of the soft clay soil, Dash & Bora (2013).

A stone column is a ground improvement element formed by placing and compacting crushed stone aggregates within a vertically excavated cylindrical hole in soft clay. Due to their superior stiffness and strength as compared to the surrounding soft clay soil, stone columns bear most of the applied structure loads, significantly enhancing the underlying soils as presented in; Juran and Guermazi, 1988, Hughes and Withers, 1974, Wood et al., 2000, McKelvey et al., 2004, and Dash and Bora, 2013.

Above-ground liquid storage tanks experience both horizontal and vertical ground accelerations during earthquakes, which can result in structural damages. Such seismic forces may compromise the integrity of these tanks, posing significant risks. The leakage of hazardous liquids can have severe consequences for the health of humans and the environment. Furthermore, failure of tanks storing flammable substances, such as liquid chemicals or oil, has historically caused uncontrolled fires, as observed in Niigata and Alaska 1964 earthquakes, Roopkumdee (2017).

Earthquakes caused many damages of storage tanks as those took place in Alaska (1964), San Fernando (1971), Imperial Valley (1979), Coalinga (1983), along with Loma Prieta (1989), Landers (1992), Northridge (1994), and Kobe (1995). Damages due to earthquake in cylindrical storage tanks took several shapes and forms. As presented in American Lifelines Alliance (ALA) (2001), has documented various failure mechanisms affecting above-ground steel tanks. The failure modes include; shell buckling, damage of the roof and other structural components, failures in anchorage and support system, foundation instability, hydrodynamic pressure, connecting pipelines, in addition to manhole units structural failure, Roopkumdee (2017).

Analysis and design of steel tanks must consider not only structural strength of the materials but also buckling resistance and serviceability criteria. Since steel tanks are often used to store hazardous or contaminant materials, any leakage could lead to severe environmental damage, with associated costs potentially exceeding those caused by the earthquake itself. Additionally, maintaining the functionality of storage tanks after an earthquake is crucial for emergency response. Therefore, these tanks must be designed to comply with serviceability limits, such as preventing buckling and sloshing, while also meeting strength requirements to withstand seismic events of a specified magnitude based on the seismic zone. Properly engineered tanks should be capable of resisting applied loads to mitigate the risks of sloshing, uplift, and most critically sloshing, Hafez (2012).

Razzaghi (2023) presented a review on the seismic behavior assessment of above ground steel storage cylindrical tanks. The paper presented a brief review of many studies on the seismic performance of above ground steel cylindrical tanks, focusing on analytical methods only.

Zulfiqar et al. (2023) presented an analysis of steel cylindrical tank under various earthquake Peak Ground Accelerations (PGA) employing Fluid-Structure-Interaction (FSI) approach. The upper part of the moving fluid is presented as convective mass, whereas the lower portion of the moving fluid is presented as impulsive mass, as presented in Celik et al. (2018). The convective and impulsive forces of the liquid inside the steel tank are all accounted for in the seismic analysis. Karamanos et al. (2006) presented a methodology for estimating of the total horizontal forces due to seismic sloshing in horizontal spherical and cylindrical vessels.

Konar and Das (2025) presented review of numerical modeling of sloshing of liquids in containers. Sloshing took place in liquids within partially filled containers. Sloshing significantly affect the dynamic stability of the fluid containers and their supporting structures. The problem is a multi-disciplinary

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engineering problem in which numerical methods are usually employed to model especially for dealing with large amplitude sloshing.

Salem et al. (2021) studied the behavior of steel storage oil tanks using ADINA software. The tank is supported on reinforced concrete piles in one case and stone columns in other cases. Research results showed that stone columns are viable alternative to end bearing piles in both static and dynamic analyses. However, in some cases using stone columns can effectively reduce hoop stresses and sloshing height that took place in oil surface due to earthquakes.

This current analysis investigates the dynamic response of steel cylindrical above-ground storage tanks supported by either Rigid Inclusions Foundations (RIF) or a Stone Columns Foundation (SCF). A comparative analysis of RIF and SCF is presented under both the static in addition to the dynamic conditions considering the feasibility of SCF as a viable alternative to RIF. ADINA software is used in performing nonlinear time-history analysis to achieve the research objectives. The key parameters include; distribution of hydrodynamic forces, settlement behavior, settlement to seismic forces, effect of sloshing, and variations in hoop stress.

Furthermore, the effect of foundation type on the seismic performance of storage tanks is analyzed to determine the effectiveness of SCF in mitigating earthquake-induced deformations. Results provide insights into optimizing foundation design for enhanced structural stability and operational safety.

2. Numerical Modeling

In the numerical analysis part, 3-dimensional finite element models of the storage tank and the underlying soils are developed using the nonlinear finite element analysis software ADINA (2025). The analysis is performed to investigate the static and the dynamic behavior of above ground steel storage tanks supported on a blanket soil layer of specified thickness, underlain by 13 stone columns or rigid inclusions. The subsoil beneath the tank consists of a 6-meter-thick layer of soft clay followed by a deep extended sand deposit. The properties of soils utilized in the analysis are detailed in Table 1.

The steel tank shell and tank base plates are modeled as an isotropic elastic material, adhering to the ASTM-A-516 Grade 70 steel properties. The required thickness of the steel tank shell is determined using the Variable-Design-Point Method in addition to satisfying the 1-Foot Method, as outlined in API-650 (2014). Additionally, the raft and rigid inclusions are also represented as elastic isotropic materials, incorporating the mechanical properties of both reinforced and plain concrete.

Table 1. Characteristics of the soft clay layer and the underlying sand layers.

Soil Properties	Soft clay	Sand	Blanket soil
Poisson's ratio (ν)	0.48	0.35	0.35
Elastic modulus of soil (E), (MPa)	2.00	50.0	100.0
Bulk unit weight, (kN/m ³)	16.00	19.0	20.0
Lambda (λ)	-	-	-
Kappa (κ)	-	-	-
Untrained cohesion (C), (kPa)	15.00	0.0	0.0
Angle of friction (ϕ), (°)	0.0	36.0	40.0
Layer thickness, (m)	6.00 m	Extended	2.00 m

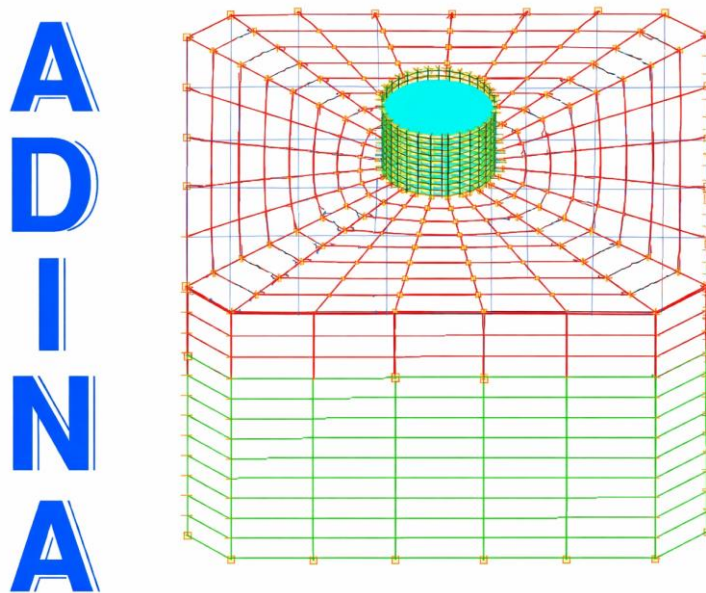
The tank's liquid content (oil) is simulated using a three-dimensional fluid linear potential-based element with a free surface, allowing for interaction of the fluid-structure and the effect of the applied ground acceleration. The stored oil specific gravity is assumed equal to 0.9. The steel and concrete material properties utilized in this study are provided in Table 2.

Table 2. Concrete and steel material properties utilized in the current study.

Material Properties	Plain Concrete	Reinforced Concrete	Steel
Poisson's ratio (ν)	0.20	0.20	0.30
Elastic modulus (E), (MPa)	20,000	20,000	200,000
Unit weight, (kN/m ³)	22.00	25.00	78.00

The tanks analyzed in this study are having a standard diameter of 16.0 m and a 10.0 m height, resulting in a tank aspect ratio (D/H) of 0.625. These dimensions are selected due to their widespread use in petroleum storage applications. The foundation system includes a 1.20 m thick reinforced concrete raft, which is supported by either circular concrete rigid inclusions or stone columns. The rigid inclusion or stone column is having 1.20 m diameter and extends to a depth of 16.0 m. The tanks used are open-top structures without any roof. The tank shell thickness is equal to 6.0 mm, conforming to API standards. The analyzed finite element model is presented in Figure 1.

These foundation configurations are designed to enhance the structural performance of the tanks, ensuring stability under the effect of static and dynamic loading cases. The presence of stone columns or rigid inclusions contributes to improved load distribution, reducing differential settlements and enhancing overall soil-structure interaction. Additionally, the study evaluates the effect of these foundation systems on the seismic behavior of storage tanks, particularly their resistance to hydrodynamic forces and sloshing effects. The numerical models used in the research incorporate nonlinear behavior to provide realistic representation of the actual site conditions. This comprehensive analysis aims to optimize the design and assure safe operation of the storage tanks in soft soil environments.

**Figure 1.** 3-Dimensional FEM model for open-top above-ground liquid storage tank used in the analysis.

2.1. Finite Element Analysis Procedure

Nonlinear static and dynamic analyses are conducted to investigate the behaviour of the storage tanks. The static analysis phase involves defining the material properties, tank structure, soil profile, foundation system, liquid contents, and boundary conditions. During this stage, hydrostatic pressure distribution and static hoop stresses within the tank shell are determined. Time is allowed for the soil to settle under own weight and the generated pore water pressure to dissipate. The seismic response

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of the tanks is evaluated using the recorded ground motion from the 1989 Loma Prieta earthquake record. The horizontal component of the earthquake excitation is applied at the lower boundary of the soil domain to simulate realistic seismic loading. The acceleration-time history of the Loma Prieta earthquake, as shown in Figure 2, is utilized for the dynamic analysis. This approach enables a detailed assessment of the tank's response, including sloshing effects, hydrodynamic pressures, dynamic settlements, and structural stresses under seismic conditions.

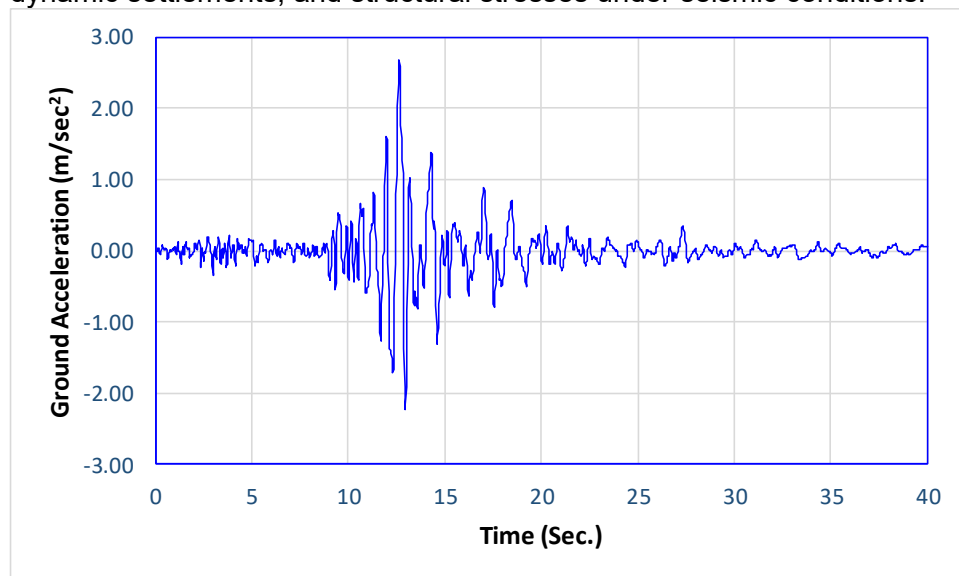


Figure 2. Loma Prieta earthquake (1989) acceleration time-history.

3. Results and Discussion

The structural behavior of above-ground steel liquid storage tanks supported by sand blanket underlain by soft soil reinforced with either stone columns or concrete rigid inclusions. A detailed analysis is conducted to compare the effectiveness of the two foundation types in improving the load-bearing capacity of the two foundation types. The study evaluates settlement patterns, stress distribution, and structural performance for both the static and dynamic conditions.

Table 3 shows the different cases analysed in this paper, along with the engineering properties of the stone columns and rigid inclusions used in the study. Additionally, this research explores the feasibility of stone columns as a cost-effective and practical alternative to traditional rigid inclusions. The influence of varying material properties, geometric configurations, and installation techniques on the overall tank stability is also examined. The results provide valuable insights for engineers designing foundations for tanks constructed on soft and compressible soil deposits.

One case of plain concrete rigid inclusion is studied herein as a reference case. Different cases of stone columns are also analyzed using the same tank configuration and soil properties. The chosen elastic modulus values of the stone column material represent different relative densities with angles of internal friction (ϕ) increasing from 36° to 42° with moduli ranging from 50 to 200 MPa reflecting different stiffness values.

Table 3. Engineering characteristics of stone columns and rigid inclusions implemented in the current study.

Case	Foundation Type	Elastic modulus, E, (MPa)	Angle of internal friction, (ϕ)
1	Concrete Rigid Inclusions	20,000	-
2	Stone column	200	42
3	Stone column	150	40
4	Stone column	100	38
5	Stone column	50	36

3-1- Axial Compressive Stress

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Analyzing axial compression stresses in tank shells is crucial, as excessive stress can result in structural failure, particularly buckling. To evaluate the stability and performance of the tanks, the maximum longitudinal shell membrane compression stress is determined through numerical modeling. Figure 3 presents color contour shading of the axial compressive stress (static and dynamic) for case 1. From figure, it can be seen that the maximum axial compression stress takes place at the lower boundary of the tank steel shell in all the analyzed cases.

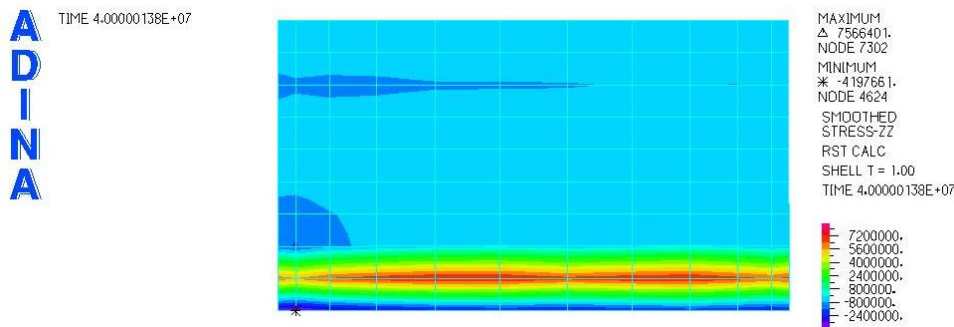


Figure 3. Contour shading of the axial compressive stress (static and dynamic) for case 1.

Figure 5 through **Figure 10** present variations and changes of the axial compressive stresses with time during the seismic excitation. From figures, it is noticed that the maximum axial compression stress for all cases occurred at a time of about 13.00 sec, which the time of peak ground acceleration. Results show that the best case which gives lower values of axial compression stresses is case 2 (when using stone columns with 200 MPa elastic modulus). The reduction of axial compressive stress reaches 5% when using stone columns with 200 MPa (case 2) instead of rigid inclusions.

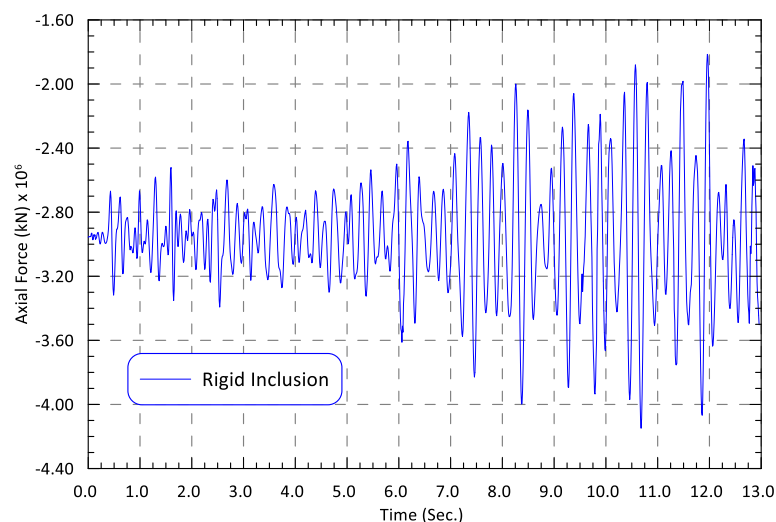


Figure 4. Variation of the axial compressive stresses with time at the tank base for case 1, rigid inclusions.

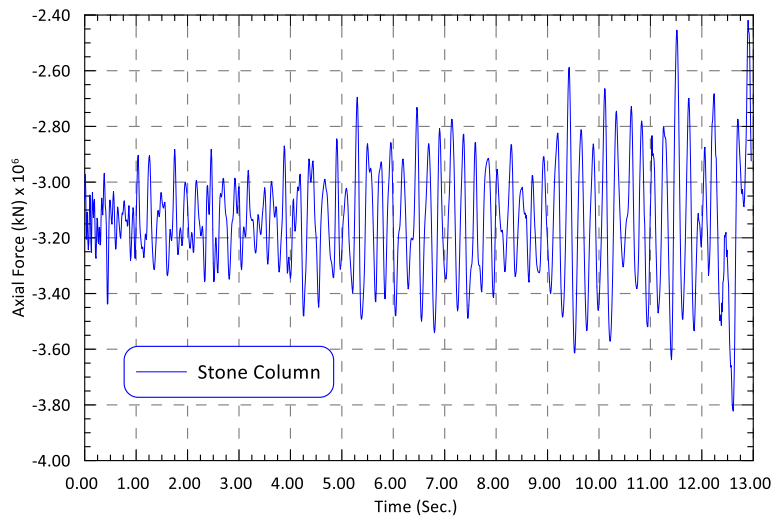


Figure 5. Variation of the axial compressive stresses with time at the tank base for stone columns, case 2.

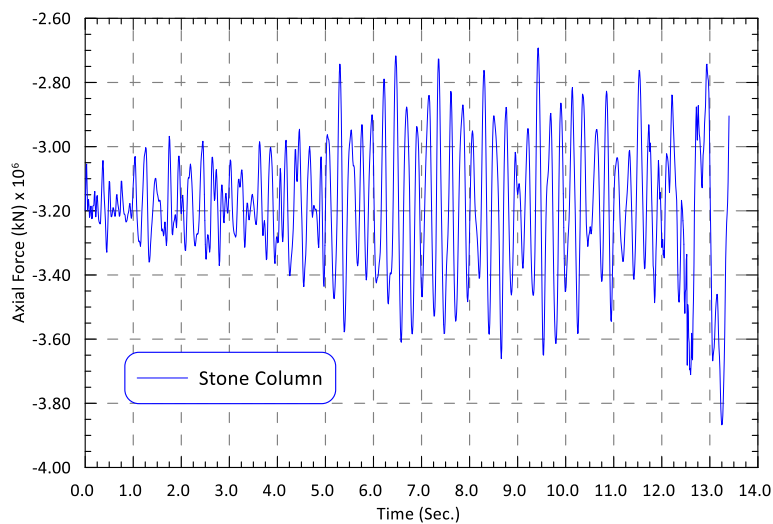


Figure 6. Variation of the axial compressive stresses with time at the tank base for stone columns, case 3.

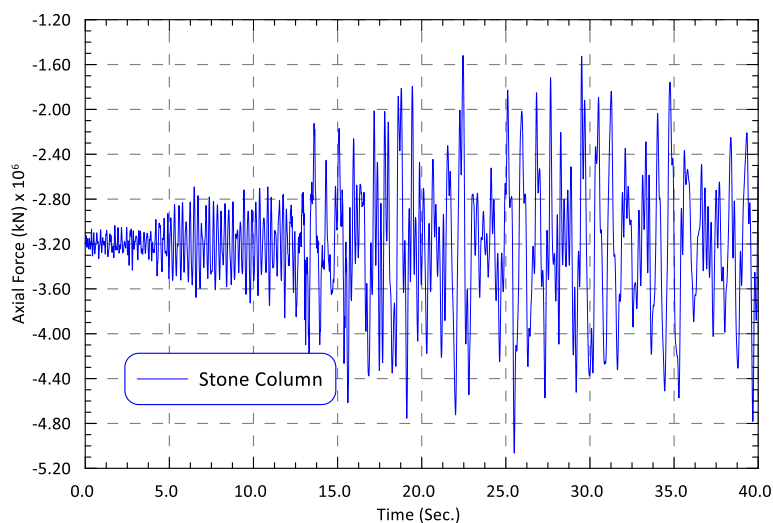


Figure 7. Variation of the axial compressive stresses with time at the tank base for stone columns, case 4.

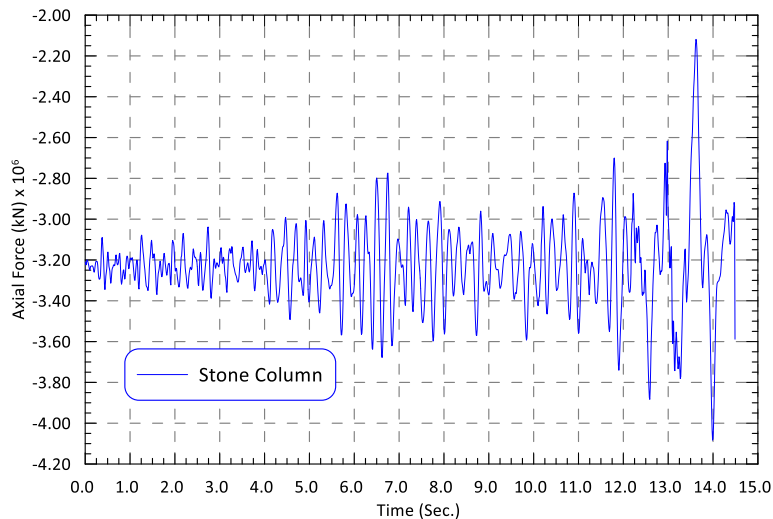


Figure 8. Variation of the axial compressive stresses with time at the tank base for stone columns, case 5.

3-2- Hydrodynamic Pressure

Stresses due to variable fluid pressures play an important role in generating stresses in tank shell which may lead to failure of the tank. Figure 11 shows contour shading of the water pressures (hydrostatic + hydrodynamic) for Case (3). Dynamic analysis results showed that the maximum stresses took place at the liquid base. Results of the hydrodynamic pressure versus time for all studied cases are presented in Figure 10 through Figure 14. Cases 4 and 5 exhibit the lowest hydrodynamic pressure values compared to other cases.

This indicates that utilizing stone columns with a low elastic modulus plays a major role in reducing hydrodynamic pressure. The flexibility of using stone columns helps dissipate seismic energy more effectively, leading to lower pressures on the tank walls. Additionally, the improved drainage properties of stone columns may contribute to enhanced soil behavior under dynamic loading conditions, further stabilizing the tank structure.

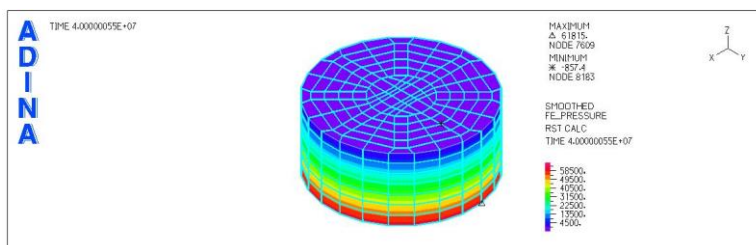


Figure 9. Contour shading of the fluid (oil) pressures (hydrostatic + hydrodynamic) for Case 3.

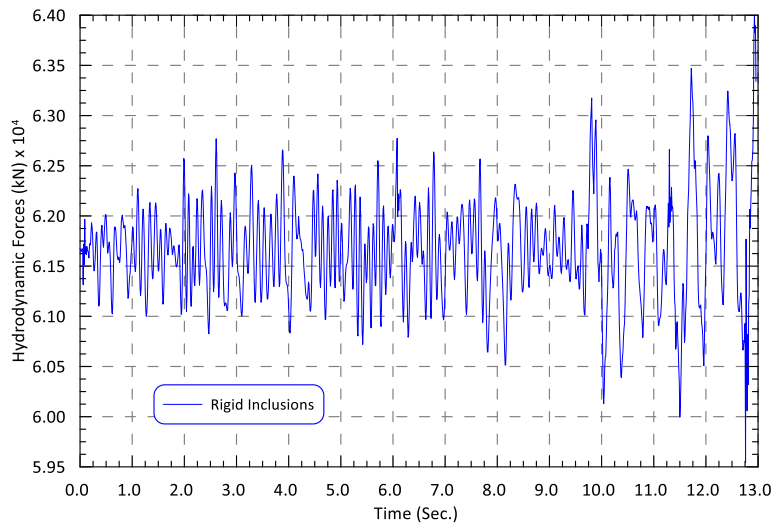


Figure 10. Hydrodynamic stresses at the liquid base for RI, Case 1.

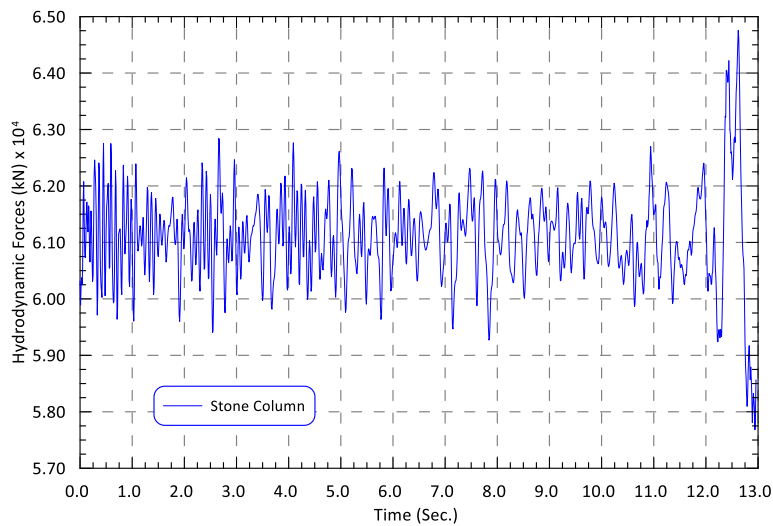


Figure 11. Hydrodynamic stresses at the liquid base for SC, Case 2.

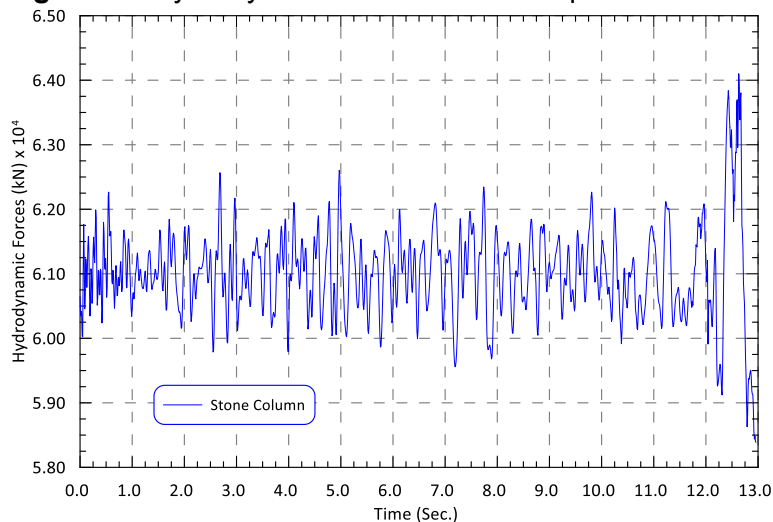


Figure 12. Hydrodynamic stresses at the liquid base for SC, Case 3.

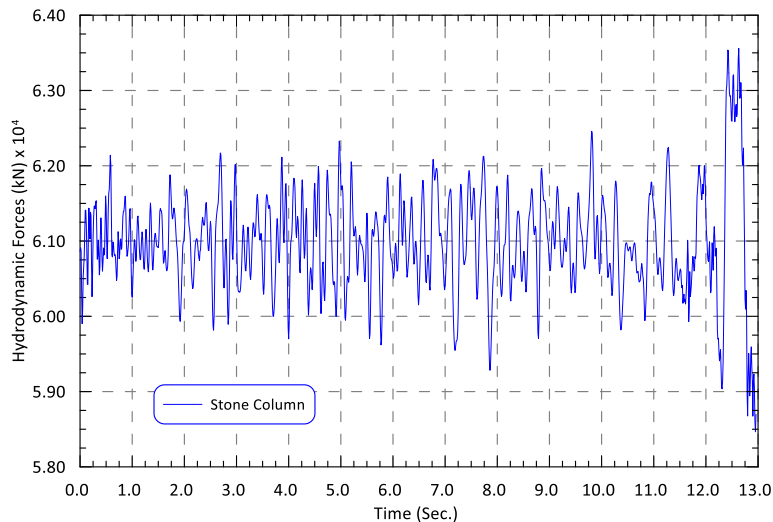


Figure 13. Hydrodynamic stresses at the liquid base for SC, Case 4.

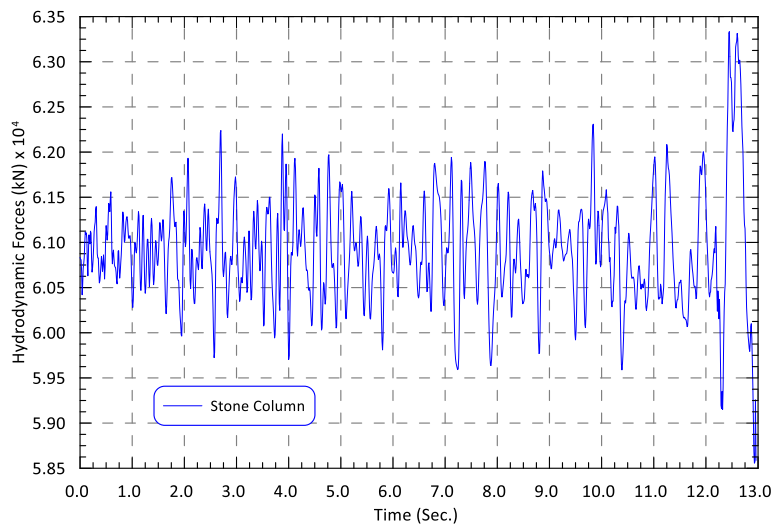


Figure 14. Hydrodynamic stresses at the liquid base for SC, Case 5.

3-3- Hoop Stresses within the Tank Shell

Elephant-foot buckling refers to an outward deformation that occurs just above the base of the tank, typically within the lower third portion of the shell where hoop stresses are at their highest. As a result, analyzing the hoop stresses developed in the tank shell is crucial to ensuring that buckling remains within acceptable limits. Observations indicate that SCF are more effective than RIF in minimizing the maximum hoop stresses within the tank shell, thereby enhancing its structural stability under loading conditions.

As shown in Figure 15 through Figure 19, the results indicate that cases 4 and 5 exhibit the lowest hoop stresses, with a reduction of approximately 10% compared to case 1. This suggests that SCF are more effective than RIF in mitigating hoop stresses due to their lower elastic modulus, which allows for better load redistribution. However, the calculated settlements under the tank center are slightly higher inducing stress reduction by accommodating deformations more efficiently. Additionally, the maximum values of hoop stresses in the studied cases are consistently located around 1.0 m from the tank lower base, likely due to the combined effects of hydrostatic pressure and differential settlement-induced bending stresses in this region. Contour shading of the hoop stresses for Case 2 is shown in Figure 20.

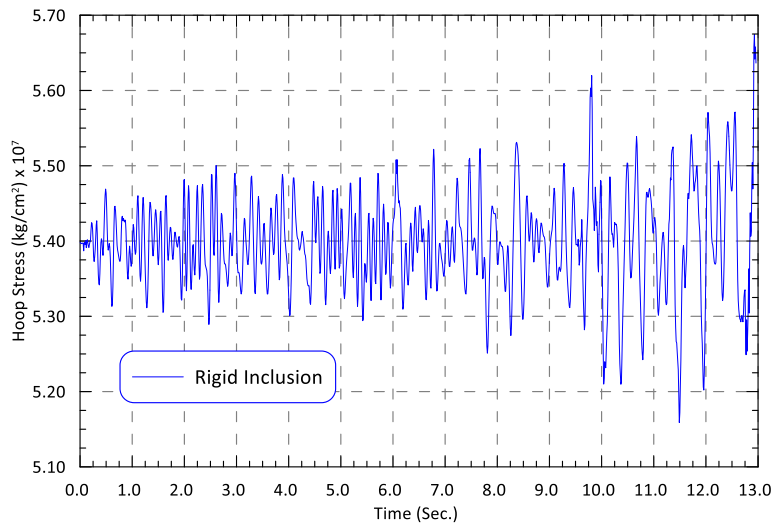


Figure 15. Hoop stresses at 1m above the tank base for RIF, Case 1.

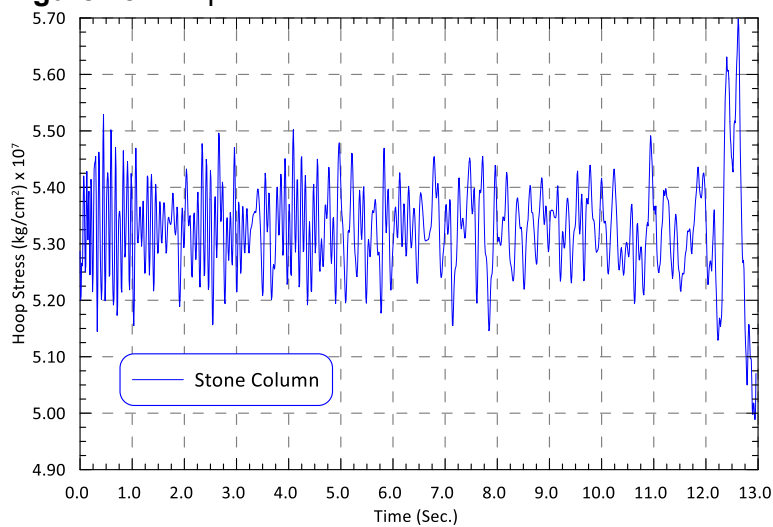


Figure 16. Hoop stresses at 1m above the tank base for SCF, Case 2.

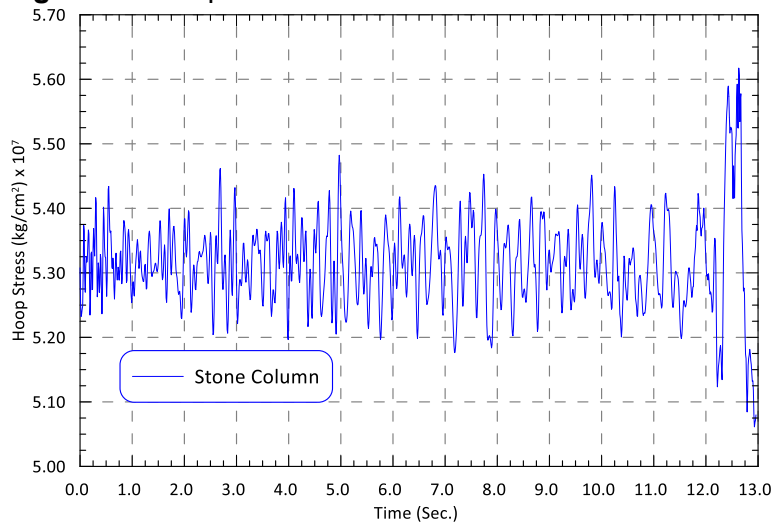


Figure 17. Hoop stresses at 1m above the tank base for SCF, Case 3.

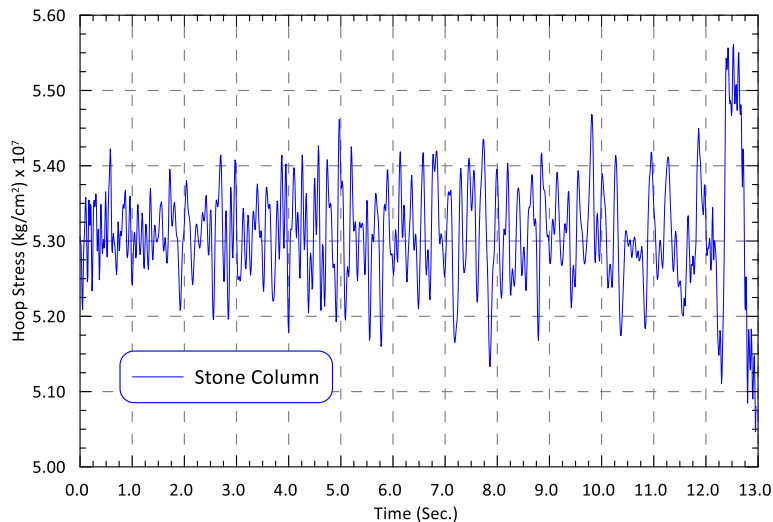


Figure 18. Hoop stresses at 1m above the tank base for SCF, Case 4.

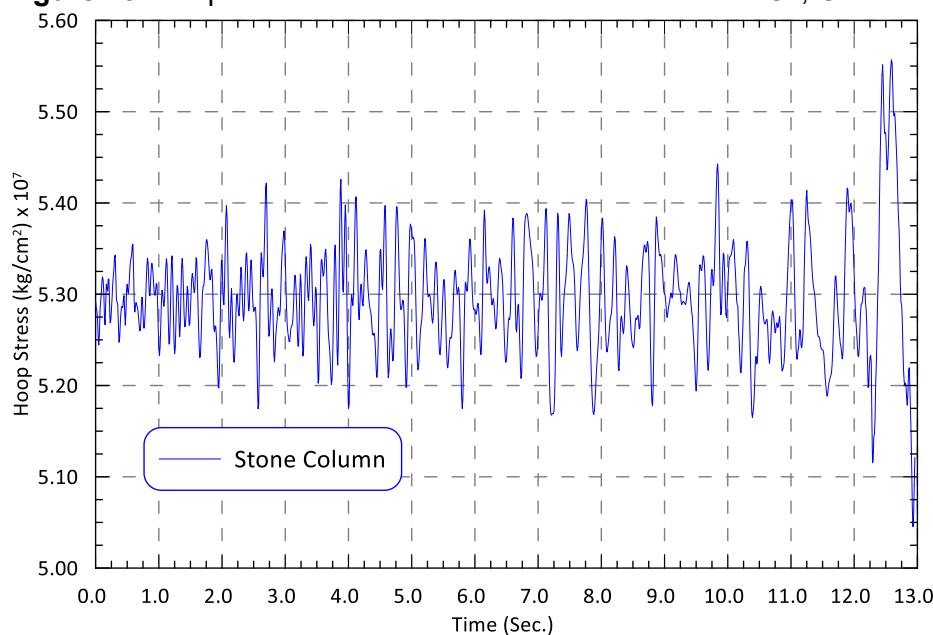


Figure 19. Hoop stresses at 1m above the tank base for SCF, Case 5.

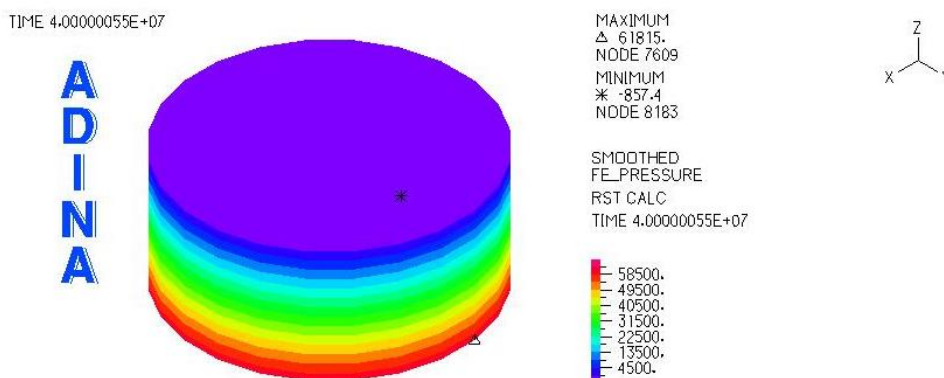


Figure 20. Contour shading of the hoop stresses for Case 2.

3-5- Sloshing

The sloshing of liquid at the tank's surface can lead to severe consequences, including fires and explosions, especially since most ground storage tanks are used for containing chemicals or hazardous substances. As a result, this study places significant emphasis on analyzing the sloshing behavior of fluids within storage tanks to assess its impact and potential risks. Excessive sloshing can also compromise the structural integrity of the tank by exerting additional dynamic forces on the shell

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and roof. Furthermore, prolonged sloshing may lead to the weakening of weld joints, increasing the likelihood of leaks or structural failure. Therefore, understanding the influence of different foundation types on sloshing behavior is essential for enhancing the seismic performance and safety of steel storage tanks.

It is noted that the calculated sloshing value reduced from 20 to 17 cm, or by about 15%, when RIF (Case 1) is changed to be SCF with modulus $E = 100$ MPa (Case 4), as presented in Figure 21 and Figure 22. Therefore, SCF are especially better than RIF in inhibiting sloshing fluid height during seismic events, and tend to introduce higher damping and soothing of the earthquake excitation. Figure 23 shows variation of the sloshing of the fluid during earthquake for Case 1.

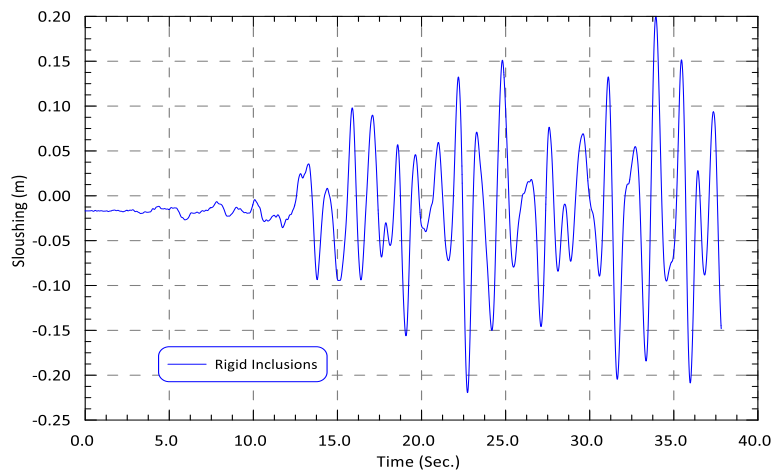


Figure 21. Sloshing at liquid surface for RIF, Case 1.

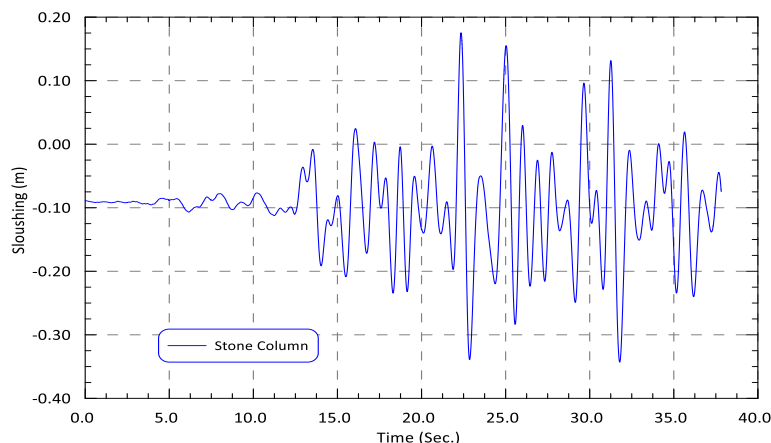


Figure 22. Sloshing at liquid surface for SCF, Case 4.

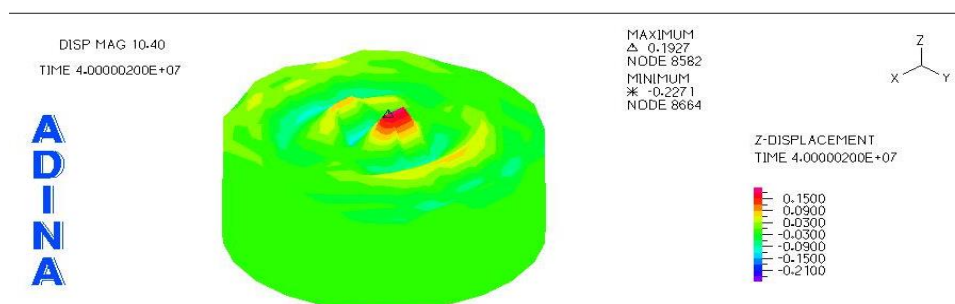


Figure 23. Fluid sloshing during earthquake for Case 1.

4. Conclusions

This study numerically evaluated the seismic behavior of steel ground storage tanks supported on RIF or SCF using three-dimensional nonlinear finite element modeling in ADINA. Based on the results of this research, it can be concluded that: Utilizing stone column foundations as a substitute for R.C.

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piles beneath above-ground steel storage tanks has been shown to be a financially viable alternative. Using rigid inclusions as tank foundations is noticeably efficient in reducing the settlements under both static and dynamic cases in the liquid storage tanks. On the contrary, stone column foundations are presenting better performance than RIF in hoop stresses that took place in tank steel shell during static analysis. Moreover, SCF are also better in reducing sloshing during seismic events due to their ability to inhibit the seismic excitation that reaches the tank body. The computed static settlement for 13 rigid inclusions supporting the tank is 1.76 cm. However, when 19 stone columns having high modulus are used, the calculated settlement increases to 7.30 cm. Nonetheless, in all the studied cases, the computed settlement values in the static cases remain within the permissible limits specified by relevant codes and are considered structurally safe. The peak settlement experienced during the earthquake rises from 1.35 cm into 3.50 cm and 2.75 cm when RIF are replaced with SCF in Cases (2) and Case (6), respectively. Additionally, for the stone column cases, a permanent settlement of approximately 1.0 cm is observed. Replacing concrete piles with stone columns of higher elastic modulus significantly reduces the calculated fluid sloshing height at the liquid surface during the seismic analysis. When the number of support elements increases from 13 reinforced concrete piles to be 19 stone columns with modulus of elasticity equal to 150 MPa, the sloshing height decreases by 40%, which provides higher safety under earthquakes. Replacing 13 concrete piles with either 13 or 19 stone columns results in slight reduction in the hoop stresses, approximately 5%. However, the impact on hydrostatic pressure in the tank shell is minimal, with a decrease of only about 1%.

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Data Availability Statement: The data presented in this study are available on request from the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

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