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Inverse domination parameter to the transformation Star and Fan graph

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Abstract

Let G be a non-trivial connected graph with vertex x set V (G). A subset D of V(G) is said to be a dominating set if all the elements of V-D are adjacent with at least one element in D. A minimum dominating set is a dominating set with minimum cardinality and its value is known as its domination number, denoted by γ (G). If D′ is a dominating set in V-D is called the inverse dominating set of G with respect to D. The minimum cardinality taken over all inverse dominating sets of G is called the inverse domination number denoted by γ′. In this paper we find the inverse domination number to the transformation of Star and Fan graphs.

Keywords: Domination, inverse domination, Transformation.

Introduction:

The theory of domination began around 1960. In 1958, Berge formalized this concept. In 1962 O. Ore has used the term dominating set and domination number for the same concept in Graph Theory. This theory has been used in document summarization and designing secure for electrical grids. In 1991, the research paper of Kulli and Sigarkanti help to study the concept of inverse domination. In the chapter we have analysed the inverse domination number and sets to the transformation of Star and Fan graphs. Also our interest is to find some relation between the inverse domination number of G and its transformations.

Definition: 1.1

A graph G is an ordered triple $(V(G),\ E(G),\ \boldsymbol{\Psi}_G)$ consisting of a non empty set $V(G)$ of vertices, a set $E(G)$, disjoint from $V(G)$of edges, and an incidence function $\boldsymbol{\Psi}_G$ that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G. If e is an edge, u and v are vertices such that $\boldsymbol{\Psi}(e) = uv$, then e is said to join u and v, the vertices u and v are called the ends of e.

Definition: 1.2

A bipartite graph is one whose vertex set of G can be partitioned in to two subsets X and Y so that each edge in G has one and in X and other end in Y, Such a partition (X,Y) is called a bipartition of the graph.

A complete bipartite graph is a simple bipartite graph with bipartition (X,Y) in which each vertex of X joined to every vertex of Y, if |X| =m and |Y| = n such a graph is denoted by $K_{m,n}$. A complete bipartite graph $K_{1,n}$ is called a star graph.

Definition: 1.

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In a graph G, a cycle of length k is called k-cycle, a k-cycle is odd or even according as k is odd or even. A 3-cycle is often called a triangle.

Definition: 1.4

The open neighbourhood of $v \in (G)$ is the set of all vertices which are adjacent to v. That is $(v) = \{u/uv \in E(G) \; and \; u \in V(G)\}$. The closed neighbourhood of $v \in (G)$ is defined as $N[v] = N(v) \cup \{v\}$, Let S be a subset of $V(G)$ the open neighbourhood of S is defined as $N(S) = \{u/uv \in E(G) \; for \; all \; v \in s\}$ and the closed neighbourhood of S is

$N[S] = N(S) \cup S$.

Definition: 1.5

A subset$D$ of (G) is said to be a dominating set if the elements of $V - D$ is adjacent with at least one element in D. A minimum dominating set in a graph G is a dominating set of minimum cardinality. The number of elements in the minimum dominating set is called the domination number. It is denoted by γ.

If $V - D$ contains a dominating setD', than D' is called the inverse dominating set of G with respect to D. The minimum cardinality taken over all inverse dominating sets of G is called the inverse domination number denoted by γ' .

Definition: 1.6

Let $G = ((G), E(G))$ be a graph and x, y, z be three variables taking values$+ \; or \; -$. The transformation graph G^{xyz} is the graph having $(G) \cup E(G)$ as the vertex set and for $\alpha, \beta \in V(G) \cup E(G)$,

$\alpha \; and \; \beta$ are adjacent in G^{xyz} if and only if one of the following holds.

(i) $\alpha, \beta \in V(G), \alpha$ and β are adjacent in G if $x = +$; α and β are not adjacent in G if $x = -$.

(ii) $\alpha, \beta \in E(G), \alpha$ and β are adjacent in G if $y = +$; α and β are not adjacent in G if $y = -$.

(iii) $\alpha \in (G), \beta \in E(G), \alpha$ and β are incident in G if $z = +$; α and β are not incident in G if $z = -$.

Definition: 1.7

The Upper and lower ceiling of x is defined as

$$\lceil x \rceil = \begin{Bmatrix} x & if \quad x \in N \\ [x+1] & otherwise \end{Bmatrix}$$

$$\lfloor x \rfloor = \begin{Bmatrix} x & if \quad x \in N \\ [x] & otherwise \end{Bmatrix}$$

Definition: 1.8

The Friendship/Fan graph is a simple planar graph obtained by set of n triangles

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having a common vertex denoted byF_n.That is every pair of distinct adjacent vertices has exactly one common neighbour. F_n containing $2n + 1$ vertices and $3n$ edges.

Theorem: 1.1

Let G_1be any star graph of order n + 1, $[i.\ e.\ G_1 = K_{1,}]$, G is all positive transformation of $G_1[i.\ e.\ G = G_1{}^{+++}]$then$\gamma(G_1) = \gamma(G) = 1$; $\gamma'(G_1) = \gamma'(G) = n$. Also G_1 has a unique dominating and inverse dominating sets but G has 2^n inverse dominating sets.

Proof:

Let $V(G_1) = \{v_i/0 \le i \le n\}, E(G_1) = \{x_j/1 \le j \le n\}$

with $d(u_0) = n$ and $d(v_j) = 1$ for all$1 \le i \le n$; $v_0 v_j = x_j$for allj $= 1,2, \dots$, n

$V(G) = \{v_{i;}\ x_j/0 \le i \le n;\ 1 \le j \le n\}$ is the vertex set of $G = G_1{}^{+++}$ with

$(v_0) = 2n, d(x_j) = n + 1, d(u_i) = 2$ for all $1 \le i, j \le n$

$|(G)| = 2n + 1 \quad with \quad d(v_0) = 2n$.

$\Longrightarrow N[v_0] = V(G)$.

Hence, $D = \{v_0\}$ is the minimum dominating set of G

$\Rightarrow \gamma(G_1) = \gamma(G) = \dots\dots.$ (i)

Also, $(x_i) = \{v_{0;}\ v_{i;}\ x_j/1 \le j \le n \ \&\ j \ne i\}$ for all$1 \le i \le n$ and

$(v_i) = \{u_{0;} x_i\}$ for all$1 \le i \le n$

Hence,

$D' = \{X; \{X - \{u_i\} \cup \{x_i\}\}: 1 \le i \le n\}; \{X - \{u_i, u_j\} \cup \{x_i, x_j\}: 1 \le i, j \le n\}; \dots; \{Y - \{x_i\} \cup \{u_i\}: 1 \le i \le n\}; Y\} \dots\dots.$ (a)

Clearly every set$d' \in D'$

$N[d'] = V(G)$ is the required minimum inverse dominating set of D with $|d'| = n$

By clearly $\gamma'(G_1) = $ n

$\Rightarrow \gamma'(G_1) = \gamma'(G) = n$......... (ii)

By equation (a) , $|D'| = nC_0 + nC_1 + nC_2 + \cdots + nC_n$

$|D'| = 2^n$.

Hence, Equation (i), (ii), and (iii) completes the proof.

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Theorem: 1.2

Let $G_1 = K_{1,}$and $G = G_1^{++-}$ then $\gamma(G) = \gamma'(G) = 2$. G has $2n$ minimum dominating sets with cardinality two also Ghasat most$2(n - 1)$ inverse dominating sets for every dominating set d ∈ D(G).

Proof:

Let $G_1 = K_{1,}$and $G = G_1^{++-}$

Let V$(G_1) = \{v_i/0 \leq i \leq n\}$with $d(v_0) = n, E(G_1) = \{x_i/1 \leq i \leq n\}$ and $x_i = v_0 v_i$ for all $1 \leq i \leq n.$

V$(G) = \{v_{i;}\, x_j/0 \leq i \leq n; 1 \leq j \leq n\}$

Clearly $(u_0) = \{u_i/1 \leq i \leq n\}; N(u_i) = \{u_{0;}\, x_j/1 \leq j \leq n \,\&\, j \neq i\}$ for all $1 \leq i \leq n$ ... (i)

$(x_i) = V(G) - \{u_{0,}u_i\}$: $1 \leq i \leq n$

Choose D_1, D_2 and D_1', D_2' such that

$$\left.\begin{aligned} &D = \{(u_0, x_i); (u_i, x_i)/1 \leq i \leq n\} \\ &D' = \{(u_j, x_j)/(u_j, x_j) \neq (u_i, x_i) \in D: 1 \leq j \leq n\} \end{aligned}\right\} \quad \ldots\ldots\text{(ii)}$$

Clearly N$[d] = N[d'] = V(G)$for all $d \in D \& d' \in D'[\because by\ (i)\}$

from equation (ii) $|D| = 2n$

Choose

$D_1 = \{(u_0, x_i)/1 \leq i \leq n\}; D_2 = \{(u_i, x_i)/1 \leq i \leq n\}$

$D_1' = \{(u_j, x_j)/1 \leq j \leq n\ \&\ x_j \neq x_i \in D_1\}$............(iii)

$D_2' = \{(u_0, x_j); (u_j, x_j)/(u_j, x_j) \neq (u_i, x_i): x_j \neq x_i \in D_2\}$

Every element of D_1' and $D_2'$are the inverse dominating sets of$d_1 \in D_1$ and

$d_2 \in D_2$ respectively. From equation (iv) $|D_1| = n;\ |D_2| = n$

$\Rightarrow |D| = |D_1| + |D_2| = 2n \ \&\ |D_1'| = (n-1); |D_2'| = 2(n-1)$

Hence, $(n-1)$and $2(n-1)$inverse dominating sets for each dominating set $d \in D_1$ and D_2 respectively.

Hence, almost $2(n-1)$ number of inverse dominating sets for each dominating set d belongs to D of G.

Hence the proof.

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Theorem: 1.3

Let $G_1 = K_{1,}$and $, G = G_1{}^{+-+}$ then G has a unique dominating set with cardinality one and 2^n inverse dominating sets with cardinality n.

Proof:

Let $\mathrm{V}(G_1) = \{v_i/0 \le i \le n\}$ be the vertex set of G_1 with $\mathrm{d}(v_0) = n$ and

$\mathrm{E}(G_1) = \{x_i/1 \le i \le n\}$ where $x_i = v_0 v_i$ for all $1 \le i \le n$.

$\mathrm{V}(G) = \{v_i, x_j/0 \le i \le n, 1 \le j \le n\}$ is the vertex set of G.

Clearly, $(v_0) = \{v_i, x_i/1 \le i \le n\}$

$(v_i) = \{v_0, x_j\}$ and $N(x_i) = \{v_{0;} v_i\}$ for all $1 \le i \le n$

Choose,

$X = \{v_i/1 \le i \le n\}; Y = \{x_i/1 \le i \le n\}$

From equation (i), $\mathrm{N}[u_0] = V(G)$

Choose

$$\mathrm{D}' = \left\{ \begin{matrix} X; \{X - \{v_i\} \cup \{x_i\}: 1 \le i \le n\}; \{X - \{v_i, v_j\} \cup \{x_i, x_j\}; 1 \le i, \quad \le n\}; \\ \ldots; \{Y - \{x_i\} \cup \{v_i\}; 1 \le i \le n\}; Y \end{matrix} \right\}$$

Now every element $d' \in D'$, $\mathrm{N}[d'] = V(G)$............(ii)

Hence, every element of D' is an inverse dominating set of G with cardinality $n$and

$|D'| = 1 + nC_1 + nC_2 + \cdots + nC_{n-1} + 1 = 2^n$.

From equation (i), (ii), and (iii)

G has a unique dominating set with cardinality one and 2^n inverse sets with cardinality n.

Theorem: 1.4

Let $G_1 = K_{1,}$and $, G = G_1{}^{-++}$ then G has,$n^2 + n$ dominating set and at most $n^2 + n - 1$ inverse dominating sets with cardinality 2 respectively.

Proof:

Let $\mathrm{V}(G_1) = \{v_i/0 \le i \le n\}$ with $\mathrm{d}(v_0) = n$ and

$\mathrm{E}(G_1) = \{x_j/1 \le j \le n\}$ where $x_i = v_0 v_i/1 \le j \le n$are the vertex and edge sets of G_1 respectively. where $G_1 = K_{1,n}$.

$\mathrm{V}(G) = \{v_i, x_j/0 \le i \le n, 1 \le j \le n\}$ is the vertex set of G where $G = G_1{}^{-++}$

with $d(v_0) = n$, $d(x_i) = n + 1$ and $d(v_i) = n$ for all $i = 1,2, \ldots, n$

$(v_0) = \{x_i/1 \le i \le n\}; N(x_i) = \{v_{0;} v_{i;} x_j/1 \le j \le n \;\&\; i \ne j\}: 1 \le i \le n$

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Let $X = \{v_i/1 \leq i \leq n\}; = \{x_i/1 \leq i \leq n\}$.

choose $= \{(v_0, v_i); (v_i, x_j)/1 \leq i, j \leq n\}$then every element $d \in D$

$\mathrm{N}[d] = V(G)$. Hence every element of $D$is the required minimum dominating set with $|D| = n + n^2$.

If $d = (v_0, v_i) \in D$ then, n^2 inverse dominating sets………(i)

in the from (v_i, v_j): $1 \leq i, J \leq n$. Otherwise if $d = (v_i, v_j) \in D$ then $n + n^2 - 1$ inverse dominating sets for every dominating set $d \in D$ with cardinality 2.

Hence the proof.

Theorem: 1.5

Let $G_1$is a star graph of order $n + 1, G = G_1{}^{+--}$ then $\gamma(G) = \gamma'(G) = 2$. Also G has $n - 1$ inverse dominating sets for every dominating set d belongs to D of .

Proof:

Let $\mathrm{V}(G_1) = \{v_i/0 \leq i \leq n\}$ be the vertex set of $G_1 = K_{1,n}$ with $\mathrm{d}(v_0) = n$ and

$\mathrm{E}(G_1) = \{x_i/1 \leq i \leq n\}$ where $x_i = v_0 v_i$ for all $i = 1,2, \ldots, n$.

$\mathrm{V}(G) = \{v_i, x_j/0 \leq i \leq n, 1 \leq j \leq n\}$ with $x_i = v_0 v_i$; $1 \leq i \leq n$

Choose $X = \{v_i/1 \leq i \leq n\}; Y = \{x_i/1 \leq i \leq n\}$Clearly $N(v_0) = X$

$(v_i) = Y - \{x_i\} \cup \{v_0\}$ for all $1 \leq i \leq n$;$N(x_i) = X - \{v_i, v_0\}$ for all $1 \leq i \leq n$

Choose $D = \{(v_i, x_i)/1 \leq i \leq n\}$……… (i)

Now every element $d_1, d_2 \in D$

$\mathrm{N}[d_1] = \mathrm{N}[d_2] = V(G)$.

Hence, every element of D is a dominating set of G.

Therefore, for every dominating set

$d_i = (v_i, x_i)$; $1 \leq i \leq n$the remaining $n - 1$ sets

$d_j = (v_j, x_j); j = 1,2, \ldots, n$ & $j \neq i$ belongs to D are the inverse dominating set of G with respect to $d_i \in D$.

$\Rightarrow \gamma(G) = \gamma'(G) = 2$.

Hence, G has n dominating sets with cardinality two and $(n - 1)$ inverse dominating sets for each dominating set of G.

Since every pair of (v_i, x_i): $1 \leq i \leq n$ are independent in $G$and the induced sub graph $< (G) - d_i >$ is connected.

Result: 1.5.1

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In any graph G having a isolated vertex then G has no inverse domination.

Theorem: 1.6

Let $G_1$is a star graph of order $n + 1$, $G = G_1^{-+-}$ then G has n^2 dominating sets with cardinality three and no inverse domination.

Proof:

Let $\mathrm{V}(G_1) = \{v_i/0 \leq i \leq n\}$,

$\mathrm{E}(G_1) = \{x_i/1 \leq i \leq n\}$ are the vertex and edge set of $G_1 = K_{1,n}$ respectively.

$\mathrm{V}(G) = \{v_i, x_j/0 \leq i \leq n, 1 \leq j \leq n\}$ with $\mathrm{d}(v_0) = 0$

Let $X = \{v_i/1 \leq i \leq n\}$; $Y = \{x_i/1 \leq i \leq n\}$

Clearly $[v_i] = X \cup Y - \{x_i\}$for all $i = 1,2, \dots, n$

$(x_i) = X \cup Y - \{v_i\}$ for all $i = 1,2, \dots, n$

Choose $D = \{(v_0, v_i, x_i); (v_0, v_i, v_j); (v_0, x_i, x_j)/1 \leq i, j \leq n \ \& \ i \neq j\}$

Clearly every element $\in D$, $\mathrm{N}[d] = V(G)$.

Hence, every element of $d \in D$ is a dominating set of G with

$|D| = n + nC_2 + nC_2 = n^2$.

Since, $v_0 \in (G)$ is an isolated vertex,

By previous result 1:5.1 (i) G has no inverse dominating set.

Theorem: 1.7

$G_1$is a star graph of order $n + 1[i.\ e.\ G_1 = K_{1,n}]$, $G = G_1^{--+}$is a transformation graph then G has n dominating sets with cardinality two and a unique inverse dominating set with cardinality n .

Proof:

Given $G_1 = K_{1,}$, $\mathrm{V}(G_1) = \{v_i/0 \leq i \leq n\}$ with $\mathrm{d}(v_0) = n$ and

$\mathrm{E}(G_1) = \{x_i/1 \leq i \leq n\}$ where $x_i = v_0 v_i$ for all $1 \leq i \leq n$

Let $\mathrm{V}(G) = \{v_i, x_j/0 \leq i \leq n, 1 \leq j \leq n\}$where $G = G_1^{--+}$

Let $X = \{v_i/1 \leq i \leq n\}$, $Y = \{x_i/1 \leq i \leq n\}$

Clearly $(x_i) = \{v_0, v_i\}$: $1 \leq i \leq n$, $N(v_0) = Y$ and $N(v_i) = X \cup \{x_i\}$ for all i

Choose $D = \{(v_0, v_i)/1 \leq i \leq n\}$ then for every element $\in$, $\mathrm{N}[d] = V(G)$

Therefore, every element$d_i = (v_0, v_i)$: $1 \leq i \leq n$

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is the required minimum dominating set of G with $|d| = 2$

Now Y is the only set disjoint from $d \in D$satisfies $[Y] = V(G)$

Hence, $D' = Y$ is the required inverse dominating set with $|D'| =$.

Hence the proof.

Theorem: 1.8

Let $G_1 = K_{1,;}$ $G = G_1^{---}$ then G has $\frac{(n+1)}{2}$dominating sets with cardinality three and G has no inverse dominating set.

Proof:

Given $G_1 = K_{1,}$.

Let V$(G_1) = \{v_i/0 \le i \le n\}$ with d$(v_0) = 0$and

E$(G_1) = \{x_i/1 \le i \le n\}$ such that $x_i = v_0v_i$: $1 \le i \le n$

V$(G) = \{v_i, x_j/0 \le i \le n, 1 \le j \le n\}$ be the vertex set of G where $G = K_{1,n}^{---}$

Clearly $v_0 \in (G)$ is an isolated vertex.

Let $X = \{v_i/1 \le i \le n\}$; $= \{x_i/1 \le i \le n\}$ then

$(v_i) = X \cup Y - \{x_i\} \Longrightarrow N(v_i) = V(G) - \{v_0, x_i\}$:$1 \le i \le n$

Also$(v_i, v_j) = V(G) - \{v_0\}$:$1 \le i, j \le n$

Choose $D = \{(v_0, v_i, v_j); (v_0, v_i, x_i);/1 \le i, j \le n\}$

then every element $d \in D$, N$[d] = V(G)$ is the required minimum dominating set with cardinality 3 with

$|D| = nC_2 + n = \frac{(n+1)}{2}$.

by result 1.5.1 and equation (i)

$v_0 \in (G)$is an isolated vertex.

Hence, G has no inverse dominating set.

Hence the proof.

Theorem: 1.9

Let F_n be any fan graph of order $2n + 1$ and G = F_n^{+++} the $(G) = \gamma'(G) = n + 1$. Also G contains4n inverse dominating sets, for every dominating set $D\ of\ G$.

Proof:

Let $(F_n) = \{v_i/0 \le i \le 2n\}$; $E(F_n) = \{x_i, y_j/1 \le i \le 2n, 1 \le j \le n\}$

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be the vertex and edge set of F_n with $d(v_0) = 2n, d(v_i) = 2$;

$x_i = v_0 v_i$;$_j = v_{2j-1}\, v_{2j,}$ for all $1 \le i \le 2n, 1 \le j \le n$

$(G) = \{v_{i,} x_{j,} y_k / 0 \le i \le 2n: 1 \le j \le 2n: 1 \le k \le n\}$ Where $G = F^{+++}_n$

with $d(v_0)$ = 4n; d$(v_i) = 4$; $d(x_i) = n + 2$; $d(y_j) = 4$ for all$1 \le i \le 2n, 1 \le j \le n$

Choose the subset as

$X = \{x_i/1 \le i \le 2n\}$; $Y = \{y_j/1 \le j \le n\}$; $Z = \{v_i/1 \le i \le 2n\}$

$X_1 = \{x_{2i-1}/1 \le i \le n\}$; $X_2 = \{x_{2i}/1 \le i \le n\}$

$Z_1 = \{v_{2i-1}/1 \le i \le n\}$; $Z_2 = \{u_{2i}/1 \le i \le n\}$; W= $X \cup \{v_0\}$

Clearly N$(v_0) =$ X ∪ Z; $N(v_{2i-1}) = \{v_{0,}\, v_{2i,}\; x_{2i-1,}\, y_i\}$ for all $1 \le i \le n$

N$(v_{2i}) = \{v_{0,}\; v_{2i-1,}\; x_{2i,}\; y_i\}$: $1 \le i \le n$

N$[x_{2i-1}] = X \cup \{v_{0,}\, v_{2i-1,}\, y_i\}$: $1 \le i \le n$

N$[x_{2i}] = X \cup \{v_{0,}\; v_{2i,}\; y_i\}$: $1 \le i \le n$

N$(y_i) = \{v_{2i-1,}\, v_{2i,}\, x_{2i-1,}\, x_{2i}\}$ $for\ all\ i = 1,2, \ldots, n.$

Clearly $X \cup \{v_0\}$ is a complete sub graph of G

Case (i)

Suppose $D_1 = Z_1 \cup \{w/w \in W\}$ Then $[D_1] = V(G)$ for any $w \in W$ is the required minimum dominating set of G with $|D_1| = n + 1$

In this case ,Choose $D'_1 = Z_2 \cup \{w'/w' \ne w \in W\}$ or $D'_2 = Y \cup \{w'/w' \ne w \in W\}$, Clearly N $[D'_1]$ = N $[D'_2] = (G)$ with $|D'_1| = |D'_2| = n + 1$ and $D_1 \cap D'_1 = D_1 \cap D'_2 = \varphi$

Hence,$'_1$and D'_2 are the inverse dominating sets for the dominating Set D_1.

Since |W| = 2n + 1; D'_1 and D'_2 together forms $4n$ inverse dominating sets for a dominating set $D_1 of\ G.$

Case (ii)

Suppose $D_2 = Y \cup \{v_0\}$
In this case $D'_2 = Z_1 \cup \{x/x \in X\}$ or $D'_2 = Z_2 \cup \{x/x \in X\}$
clearly D_2 and D'_2 are the disjoints subsets of V(G) such then N$[D_2] = N[D'_2] = V(G)$are the minimum dominating and inverse dominating sets of V(G) with$|D_2| = |D'_2| = n + 1$ and $4n$ inverse dominating sets for every set $D_2 \subseteq V(G)$

In all cases G contains $4n$ inverse dominating sets for every dominating set $D\ of\ G$

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Theorem: 1.10

Let G be a transformation of the Fan graph F_n of order $2n+1[i.e.\ G = F_n^{++-}]$ then the domination and inverse domination numbers of G are 2 and n respectively.

Proof:

By theorem: 1.9

$(G) = \{v_i,\ x_j,\ y_k/1 \leq i \leq 2n; 1 \leq j \leq 2n; 1 \leq k \leq n\}$ be the vertex set of G where $G = F_n^{++-}$.

Choose $X = \{x_i/1 \leq i \leq 2n\}; Z = \{v_i/1 \leq i \leq 2n\}\ and\ Y = \{y_j/1 \leq j \leq n\}$

clearly $Y \cup Z \subseteq [v_0]$ and the vertices of X forms a complete subgraph of G.

Choose $D = \{(v_0, x)/x \in X\}$, then $N[d] = V(G)$ for an $d \in D$.

Hence, every ordered pair of vertices in D is the required minimum dominating set of G

Therefore, $(G) = 2$.

Now we choose another minimum set Y which is disjoint from the element of D such that

$N[Y] = (G)$.

Hence,Y is the inverse dominating set of G with respect to every dominating set $d \in D$ with$|Y| = n$. Hence, (G) =2 and $\gamma'(G) = n$.

Theorem: 1.11

Let F_n be any Fan graph of order $2n + 1$and $G = F_n^{+-+}$ be a transformation of F_n then G has $2n^2 - 2$ dominating sets with cardinality three and $\gamma'(G) = n + 1$. Also G has $4n - 2$ inverse dominating sets for every dominating set $d \in D$.

Proof:

Given F_n be a fan graph of order $2n + 1$ with

$(F_n) = \{v_i/0 \leq i \leq 2n\}; E(F_n) = \{x_{2i-1},\ x_{2i,}y_i/1 \leq i \leq n\}$ with

$(v_0) = 2n;\ d(v_i) = 2;\quad x_i = v_0v_i$ and $y_i = v_{2i-1}v_{2i}$ for all $1 \leq i \leq n$

Let $(G) = \{v_{i,}\ x_{j,}\ y_k/0 \leq i \leq 2n;\ 1 \leq j \leq 2n;\ 1 \leq k \leq n\}$ with

$(v_0) = 4n; d(x_i) = n + 1; d(y_j) = n + 1; d(v_i) = 4$ for all $1 \leq i \leq 2n, 1 \leq j \leq n$

Let $X = \{x_i/1 \leq i \leq 2n\};\ Y = \{y_j/1 \leq j \leq n\}$

$Z = \{v_i/1 \leq i \leq 2n\}$ and$T = \{(x_i, x_j)/x_i, x_j \in X\}$.

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Clearly $(v_0) = X \cup Z$ and every pair of vertices $t \in TY \subseteq N(t)$ for all $t \in T$

Choose $D = \{(v_0, t)/t \in T\}$.

Clearly every element $d \in D, (d) = V(G)$. Hence, every element $d \in D$is the dominating set with $|d| = 3 \Rightarrow (G) = 3$.

Also $|D| = (2n)_2 = 2n^2 - n$.

Clearly $(Y) = V(G) - \{v_0\}$ and $\{v_0\} \subseteq N(v_i)$ for all $1 \leq i \leq n$

Hence G has $2n^2 - n$ dominating sets with cardinality 3.

$\Rightarrow (G) = 3$.

$Let\ D_{1i}' = \{Y \cup \{x_i\}/x_i \in X\}_i$ is not an element in the pair $t \in T$

$D_{2i}' = \{Y \cup \{v_i\}/v_i \in Z\}$ for all $1 \leq i \leq 2n$

then every element $d_1' \in D_{1i}'$ and $d_2' \in D_{2i}'$,

$(d_1') = N(d_2') = V(G)$ with $|d_1'| = |d_2'| = n + 1$.

$\Rightarrow \gamma'(G) = n + 1$ and

for every element $d = (v_0,\ t)$ where $(x_i, x_j)/x_i, x_j \in X$,…….(i)

$D_1' = \{Y \cup \{x_k\}/1 \leq k \leq 2nand\ x_k \neq x_i\ and\ x_j\}$&................(ii)

$D_2' = \{Y \cup \{v_i\}/v_i \in Z\}$ are the inverse dominating sets to the dominating set(i)

$|D_1'| = 2n - 2; |D_2'| = 2n$.

$\Rightarrow |D_1'| + |D_2'| = 4n - 2$.

Hence G has $4n - 2$ inverse dominating sets for every dominating set of G.

Hence the proof.

Theorem: 1.12

Let G_1 be any Fan graph of order $2n + 1$, G in one of the transformation as $G = G_1^{+--}$then the domination and inverse domination number of G is two and G has $nC_2 - 1$ number of inverse dominating sets for every domination set of G.

Proof:

Let $(G_1) = \{v_i/0 \leq i \leq 2n\}$ and $E(G_1) = \{x_i, y_j/1 \leq i \leq 2n, 1 \leq j \leq n\}$

be the vertex and edge set of the Fan graph of order $2n + 1$ with

$(v_0) = 2n,\ x_i = v_0 v_i$ and $y_j = v_{2j-1} v_{2j}$ for all $1 \leq i \leq 2n, 1 \leq j \leq n$

$(G) = \{v_i,\ x_j,\ y_k/0 \leq i \leq 2n, 1 \leq j \leq 2n, 1 \leq k \leq n\}$is the vertex set of G where $G = G_1^{+--}$ .where $(v_0) = \{v_i/1 \leq i \leq 2n\}$.

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Since, the edge y_k is incident with the vertices $v_{2k-1,2k}$ and adjacent with the edges $x_{2k-1,}$ and x_{2k} in G_1.

Therefore, the vertex y_k is adjacent with all the vertices other than $v_{2k-1,2k,}x_{2k-1}$ and x_{2k} in G. Hence, $(y_k) = V(G) - \{v_{2k-1,}v_{2k,}x_{2k-1\,,}x_{2k}\}$ for all$1 \leq k \leq n$.

Let $Y = \{y_k/1 \leq k \leq n\}$

Choose $D = \{(y_{i,}\, y_j)/y_{i,}\, y_j \in Y\}$ then

every element $d \in D, N[d] = V(G)$ is the required dominating set of G with cardinality 2.

Since D consists of nC_2 element; for every element

$d = (y_{i,}\ y_j) \in D$ the remaining $nC_2 - 1$ elements $d' = (y_{\,,l}) \in D$ and $y_k \neq y_i$and $y_e \neq y_j$are the required inverse dominating set for the dominating set $d \in D$ of G.

Hence, $(G) = \gamma'(G) = 2$ and G has $nC_2 - 1$ number of inverse dominating sets for every dominating set of G.

Hence the proof.

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