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INDEPENDENT ANNIHILATOR DOMINATION NUMBER OF CARTESIAN PRODUCT OF TWO GRAPHS

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Abstract

Let $G = (V, E)$ be a graph. An independent dominating set $D \subseteq V(G)$ is called an independent annihilator dominating set if the induced subgraph $\langle V - D \rangle$ contains only isolated vertices. The minimum cardinality of an independent annihilator dominating set denoted by $\gamma_{ia}(G)$ is called an independent annihilator domination number. In this paper, we determine the independent annihilator domination number of cartesian product of P_n and P_m, $\gamma_{ia}(P_n \times F)$, $\gamma_{ia}(P_n \times K_{1,m-1})$.

Key words: Domination number, Independent Annihilator Domination Number, Paths, Cartesian Product.

1. Introduction

Let $G = (V, E)$ be a graph. As usual, we denote $V(G)$ as set of vertices of G and $E(G)$ as set of edges of G. A set $S \subseteq V(G)$ is a dominating set of graph G if every vertex of $V(G) - S$ is adjacent to atleast one vertex of set S. Domination number is the cardinality of the smallest dominating set of G which is denoted by $\gamma(G)$. A dominating set of a graph G is called Annihilator Dominating set if its induced subgraph is containing only isolated vertices. The annihilator domination number of G, denoted by $\gamma_a(G)$ is the minimum cardinality of Annihilator Dominating set. In graph theory, a product graph is a graph formed by combining two or more graphs according to a specific rule. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two graphs. The product of G_1 and G_2 which is denoted as $G_1 \times G_2$ is given as follows. Consider two points u = (u_1, u_2) and v = (v_1, v_2) in V then u and v are adjacent in the product graph $G_1 \times G_2$ if, i. $u_1 = v_1$ and u_2 is adjacent to v_2. ii. $u_2 = v_2$ and u_1 is adjacent to v_1. This paper is organized as follows: In section 2, contains the Independent Annihilator Domination Number of Cartesian Product of two paths, $\gamma_{ia}(P_n \times F)$, $\gamma_{ia}(P_n \times K_{1,m-1})$. In section 3 discussions and conclusion is given.

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2. MAIN RESULTS

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Lemma 2.1

Let P_n be a path graph with $n \geq 2$ vertices. Then $\gamma_{ia}(P_n) = \left\lfloor \frac{n}{2} \right\rfloor$.

Proof:

By choosing alternate vertices from P_n, the vertices choosen are independent and can dominate P_n.

By removing those vertices, we get a graph containing only isolated vertices.

Hence, $\gamma_{ia}(P_n) = \left\lfloor \frac{n}{2} \right\rfloor$

Theorem 2.2

Let n and m be two integers such that $n \geq 2$ and $m \geq 2$. Then $\gamma_{ia}(P_n \times P_m) = \begin{cases} \frac{nm-1}{2}, & if\ n \equiv 1(mod\ 2)\ and\ m \equiv 1(mod\ 2) \\ \frac{nm}{2}, & otherwise \end{cases}$.

Proof:

Let P_n and P_m be two path graph with n and m vertices labeled as $\{u_1, u_2 \ldots \ldots u_n\}$ and $\{v_1, v_2 \ldots \ldots v_m\}$.

Let $P_n \times P_m$ denote the cartesian product of P_n and P_m.

We have $V(P_n \times P_m) = \{(u_i, v_j) | 1 \leq i \leq n, 1 \leq j \leq m\}$

The vertices $(u_i, v_j)$and (u_{i+1}, v_{j+1}) are adjacent in $P_n \times P_m$ if $u_i = u_{i+1}$ and v_j is adjacent to v_{j+1} or $v_j = v_{j+1}$ and u_i is adjacent to u_{i+1}.

Let D be an independent annihilator dominating set of $P_n \times P_m$.

We choose D in such a way that D is independent and hence the induced subgraph $\langle V - D \rangle$ is isolated.

We shall prove this theorem by two cases.

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Case (i)

$n \equiv 1(mod\ 2)$ and $m \equiv 1(mod\ 2)$.

In this case the set,

$D = \left\{(u_{2i-1}, v_{2j}) \middle| 1 \leq i \leq \frac{n+1}{2}, 1 \leq j \leq \left\lfloor\frac{m}{2}\right\rfloor\right\} \cup \left\{(u_{2i}, v_{2j-1}) \middle| 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor, 1 \leq j \leq \left\lfloor\frac{m+1}{2}\right\rfloor\right\}$

forms an independent annihilator dominating set of $P_n \times P_m$.

Hence $\gamma_{ia}(P_n \times P_m) = \frac{nm-1}{2}$

Case (ii)

$n \equiv 0(mod\ 2)$ and $m \equiv 1(mod\ 2)$, $n \equiv 1(mod\ 2)$ and $m \equiv 0(mod\ 2)$, $n \equiv 0(mod\ 2)$ and $m \equiv 0(mod\ 2)$.

The set,

$D = \left\{(u_{2i-1}, v_{2j}) \middle| 1 \leq i \leq \left\lceil\frac{n}{2}\right\rceil, 1 \leq j \leq \left\lfloor\frac{m}{2}\right\rfloor\right\} \cup \left\{(u_{2i}, v_{2j-1}) \middle| 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor, 1 \leq j \leq \left\lfloor\frac{m+1}{2}\right\rfloor\right\}$

forms an independent annihilator dominating set of $P_n \times P_m$.

Clearly D is independent and the induced subgraph $\langle V - D \rangle$ contains only isolated vertices.

Hence $\gamma_{ia}(P_n \times P_m) = \frac{nm}{2}$

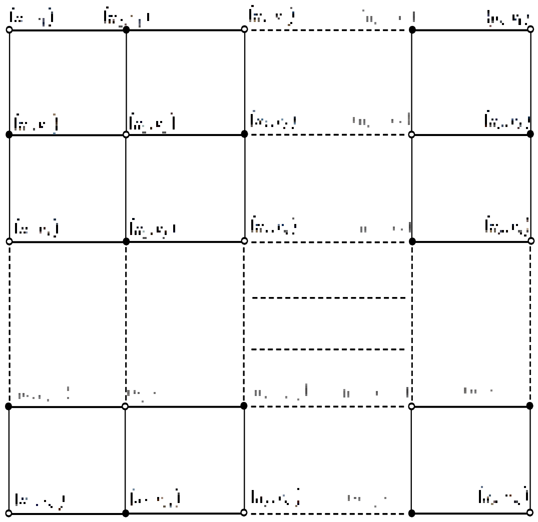

Figure 2.1 An independent annihilator dominating set for $P_n \times P_m$

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Theorem 2.3

Let P_n be a path graph with $n \geq 2$ vertices and F be a fork graph. Then $\gamma_{ia}(P_n \times F) = \begin{cases} \frac{5n-1}{2}, & if\ n \equiv 1(mod\ 2) \\ \frac{5n}{2}, & if\ n \equiv 0(mod\ 2) \end{cases}$.

Proof:

Let P_n be a path graph with vertices labeled as $\{u_1, u_2 \dots \dots u_n\}$

Let $\{v_1, v_2, v_3, v_4, v_5\}$ be the vertices of the Fork graph.

Let $V(P_n \times F) = \{(u_i, v_j) | 1 \leq i \leq n, 1 \leq j \leq 5\}$

Let $(u_i, v_j), j = 1, 2, 3, 4, 5$ represent the vertices in the i^{th} column of $P_n \times F$.

Let D be an independent annihilator dominating set of $P_n \times F$.

Let $D_1 = \left\{(u_{2i-1}, v_2), (u_{2i-1}, v_5) \middle| 1 \leq i \leq \left\lceil\frac{n}{2}\right\rceil\right\}$

Let $D_2 = \left\{(u_{2i}, v_1), (u_{2i}, v_3), (u_{2i}, v_4) \middle| 1 \leq i \leq \left\lfloor\frac{n}{2}\right\rfloor\right\}$

We note that $D = D_1 \cup D_2$ forms an independent annihilator dominating set of $P_n \times F$.

For n is odd, the number of columns containing vertices in D_1 is greater than the number of columns containing vertices in D_2.

Hence we get, $\gamma_{ia}(P_n \times F) = \frac{5n-1}{2}, if\ n \equiv 1(mod\ 2)$.

For n is even, the number of columns containing vertices in D_1 is equal to the number of columns containing vertices in D_2.

Hence we get, $\gamma_{ia}(P_n \times F) = \frac{5n}{2}, if\ n \equiv 0(mod\ 2)$.

Theorem 2.4

Let P_n be a path graph with $n \geq 2$ vertices and $K_{1,m-1}$ be a star graph with $m \geq 5$ vertices. Then we have $\gamma_{ia}(P_n \times K_{1,m-1}) = \begin{cases} \frac{mn}{2}, & if\ n \equiv 0(mod\ 2) \\ \frac{mn-(m-2)}{2}, & if\ n \equiv 1(mod\ 2) \end{cases}$.

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Proof:

Let P_n be a path graph with n vertices labeled as $\{u_1, u_2 \dots \dots u_n\}$.

Let $K_{1,m-1}$ be a star graph with vertices labeled as $\{v_1, v_2 \dots \dots v_m\}$, where $m \geq 5$.

Let $V(P_n \times K_{1,m-1}) = \{(u_i, v_j) | 1 \leq i \leq n, 1 \leq j \leq m\}$

We shall prove this theorem by two cases.

Case (i)

$n \equiv 0 \ (mod\ 2)$

In this case we represent the vertices of $P_n \times K_{1,m-1}$ namely $(u_i, v_1),\ (u_i, v_2), \dots \dots (u_i, v_m)$ as vertices of i^{th} column.

Two vertices (u_i, v_j) and (u_{i+1}, v_{j+1}) are adjacent in $P_n \times K_{1,m-1}$ if u_i and u_{i+1} are adjacent and $v_j = v_{j+1}$ or $u_i = u_{i+1}$ and v_j is adjacent to v_{j+1}.

Since $K_{1,m-1}$ is a star graph we have a center vertex (let us take the center vertex as v_2) adjacent to all other vertices.

Hence in $P_2 \times K_{1,m-1}$, the vertices (u_i, v_2), where $1 \leq i \leq n$ can dominate the entire graph.

But they cannot form an independent dominating set, Since P_n is a path graph with vertices $\{u_1, u_2, u_3, \dots \dots, u_n\}$, we have (u_i, v_2) and (u_{i+1}, v_2) where $1 \leq i \leq n-1$ are adjacent which does not meet our independent annihilator domination condition.

Hence, it is necessary to choose independent annihilator dominating set in a more flexible way.

Thus we can choose an independent annihilator dominating set as,

$$D = \left\{(u_{2i-1}, v_2) \middle| 1 \leq i \leq \tfrac{n}{2}\right\} \cup \left\{(u_{2i}, v_1) \middle| 1 \leq i \leq \tfrac{n}{2}\right\} \cup \left\{(u_{2i}, v_j) \middle| 1 \leq i \leq \tfrac{n}{2}, 3 \leq j \leq m\right\}$$

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It is clear that $|D| = \frac{mn}{2}$

If we remove D from $P_n \times K_{1,m-1}$, our induced graph $\langle V - D \rangle$ contains only isolated vertices.

Case (ii)

$n \equiv 1(mod\ 2)$

Like in case (i), we construct a graph $P_n \times K_{1,m-1}$.

In this case we choose our independent annihilator dominating set as,

$$D = \left\{(u_{2i-1}, v_2) \middle| 1 \le i \le \left\lceil \frac{n}{2} \right\rceil \right\} \cup \left\{(u_{2i}, v_1) \middle| 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \right\} \cup \left\{(u_{2i}, v_j) \middle| 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor, 3 \le j \le m \right\}$$

It is clear that $|D| = \frac{mn-(m-2)}{2}$

Removing D from $P_n \times K_{1,m-1}$, the induced subgraph $\langle V - D \rangle$ contains only isolated vertices.

This completes the proof.

3. CONCLUSION

In this paper we investigated the independent annihilator domination number on the cartesian product of two graphs we examined $\gamma_{ia}(P_n \times P_m)$, $\gamma_{ia}(P_n \times F)$, $\gamma_{ia}(P_n \times K_{1,m-1})$.

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