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Probabilistic Bounds on Reliability Degradation Under Correlated Disruptions

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Abstract

Large-scale digital infrastructure is routinely designed under independence assumptions that produce over-optimistic availability estimates. Regional outages, software regressions, shared control-plane dependencies, and network partition events co-occur in ways that violate statistical independence, inflating aggregate failure variance and amplifying tail-risk probabilities beyond levels predicted by classical reliability theory. This article presents a closed-form probabilistic framework that derives explicit upper bounds on reliability degradation under pairwise correlated failure regimes. The framework introduces the Redundancy Collapse Coefficient (RCC), a normalized metric that quantifies the fraction of redundancy benefit lost as inter-component correlation increases. Theorem 1 (Redundancy Collapse Theorem) formally establishes that as inter-component correlation increases, the protective value of redundancy erodes — eventually collapsing to the failure probability of a single component. A one-sided probability bound (Cantelli inequality) is used to derive this result without assuming any specific failure distribution, making the framework broadly applicable. Variance amplification is characterized in closed form as a function of system size and pairwise correlation, revealing quadratic sensitivity to both parameters. Monte Carlo simulation across system sizes $n \in \{5, 10, 20, 50\}$ and correlation coefficients $\rho \in [0, 0.9]$ validates analytical predictions within 2–4% deviation. Three new analytical figures — a variance amplification plot, a reliability-versus-system-size curve, and a (p, ρ) reliability heatmap — complement the degradation and delta plots from earlier analysis. Extended sensitivity analysis across k/n redundancy configurations and direct comparison against classical binomial and beta-binomial reliability models confirm that independence-based tools systematically overstate resilience margins. A quantitative mapping to a documented 2021 hyperscale cloud outage demonstrates that the independence model underestimated simultaneous multi-service failure probability by approximately 49× relative to the correlation-aware Cantelli bound. Findings are translated into concrete architectural guidance applicable to cloud infrastructure, distributed service platforms,

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cyber-physical systems, and digital public services operating under correlated risk conditions.

Keywords: Reliability Degradation, Correlated Failures, Probabilistic Bounds, Redundancy Collapse, Redundancy Collapse Coefficient, Concentration Inequalities, Gaussian Copula, Beta-Binomial Model, Resilience Engineering

1. Introduction

Classical reliability theory rests on the assumption that component failure events are statistically independent. Under that premise, parallel redundancy delivers multiplicative improvements in availability, and closed-form expressions for series-parallel systems yield tractable design targets. The digital infrastructure of the contemporary era — hyperscale cloud environments, distributed service meshes, and critical public digital platforms — does not conform to this premise. Availability analyses of cloud services confirm that failures propagate through interdependent subsystems rather than arising independently [1]. Shared fault domains, homogeneous deployments, and coupled control planes introduce structural correlation among failure variables — making the independence assumption both empirically incorrect and analytically indefensible.

Empirical observations reveal failure clustering that is both spatial and temporal. A well-documented cloud availability disruption, originating in a network automation service, cascaded simultaneously to compute, function execution, and container service layers within the same availability region — a propagation pattern structurally inconsistent with independent failure behavior. A separate global routing incident degraded services concurrently across multiple availability zones that were designed for physical independence but remained logically coupled through a shared network path. Urban-scale resilience assessments further demonstrate that cascading failure in interconnected networks is governed by dependency topology rather than individual component marginals [2]. When components share infrastructure or control-plane dependencies, correlation renders parallel redundancy less effective than classical models predict, and systemic failure degrades nonlinearly.

Mainstream digital infrastructure planning nonetheless continues to rely on independence-based formulations that produce systematic underestimates of tail risk. In service-level-constrained environments, reliability is governed by the probability of exceeding operational tolerances — a quantity that depends on failure variance as well as the mean failure rate. Correlation inflates this variance and consequently elevates threshold-crossing probability even when marginal failure rates remain unchanged. Although common-cause failure models are recognized in safety-critical engineering, their adoption in digital infrastructure planning remains limited, and the discipline must evolve to address emergent failure mechanisms that conventional models were not designed to represent [3].

This article addresses these limitations through four specific contributions. First, explicit variance amplification formulas are derived, characterizing how aggregate failure variance scales with system size and pairwise correlation. Second, the Redundancy Collapse Coefficient (RCC) is introduced as a quantitative metric for measuring redundancy effectiveness loss. Third, Theorem 1 (Redundancy Collapse Theorem) formally establishes the critical conditions under which k-out-of-n redundancy collapses to single-point failure behavior, supported by a Cantelli-based proof sketch. Fourth, the framework is validated through Monte Carlo simulation and compared analytically against both classical binomial and beta-binomial dependency models, with findings mapped to a documented hyperscale cloud outage.

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2. Background

2.1 Independence Assumptions in Classical Reliability

A foundational premise of classical reliability theory is that failure events are statistically independent. Series-parallel configurations, reliability block diagrams, and continuous-time Markov chains all adopt the product rule. $P(X_1 = 1, X_2 = 1) = p_1p_2$. Although analytically convenient, this assumption becomes increasingly unrealistic as components share infrastructure, software artifacts, network paths, and control planes. As systems grow more interdependent, independence-based formulations produce systematic estimation errors — and the discipline must evolve to address failure mechanisms that conventional models were not designed to capture [3].

2.2 Sources of Structural Correlation

Correlated disruptions arise from multiple causes. Physical co-location in shared power or cooling domains creates spatial correlation. Software homogeneity — identical deployment artifacts across replicas — creates logical correlation because a single regression propagates simultaneously to all instances. Control-plane dependencies introduce systemic coupling: when DNS, authentication, or orchestration services fail, all dependent components fail concurrently regardless of their individual marginal probabilities. External stressors, including traffic surges and adversarial load patterns produce simultaneous correlated failures across subsystems. These mechanisms are not rare edge cases but structurally embedded characteristics of modern architectures.

2.3 Existing Dependent Reliability Models

Several modeling approaches address correlated failures. Copula-based joint failure distributions model marginal behavior and dependency structure separately; reliability assessments using copula functions demonstrate that redundancy optimization based solely on individual failure rates produces incorrect results when components are coupled [4]. Beta-binomial models treat the common failure probability itself as a beta-distributed random variable, capturing overdispersion in aggregate failure counts. Empirical analyses of large cloud computing environments confirm that Poisson independence assumptions break down under real operating conditions, with failures exhibiting temporal and spatial clustering consistent with high inter-component correlation [5]. Systemic risk evaluations of electricity infrastructure further demonstrate that digital interdependencies introduce correlated vulnerability profiles that cascade at the network level—a structural dynamic that mirrors the redundancy collapse mechanism formalized in this framework [6].

2.4 Comparison of Dependency Models

Table 1 summarizes the key properties of the principal dependency modeling approaches, situating the present framework relative to existing methods.

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Model	Dependency Structure	Variance Inflation	Closed-Form Bound	Redundancy Threshold	Scalability
Classical Binomial	None (independence)	None	Yes	Not applicable	High
Beta-Binomial	Random common p (Beta prior)	Moderate (overdispersion)	Approximate	Not explicit	Moderate
Copula (Gaussian)	Pairwise ρ via copula	Explicit quadratic	Partial	Not explicit	Moderate
Present Framework	**Pairwise ρ, Gaussian copula**	**Explicit, closed-form**	**Yes (Cantelli)**	**Formal theorem**	**High**
Markov Common-Cause	Group failure events	Event-rate dependent	Yes (Markov)	Not explicit	Low

Table 1. Comparison of Dependent Reliability Models

The beta-binomial model merits separate attention. Under that formulation, the failure probability p is itself a random variable drawn from a Beta(α, β) distribution, yielding an aggregate failure count S that follows a beta-binomial distribution. The variance of S under the beta-binomial is:

$$\mathrm{Var}(S)_{BB} = n\,p_0\,(1-p_0)\,[\,1+(n-1)\,\phi\,]$$

where $p_0 = \alpha/(\alpha + \beta)$ is the mean failure probability and $\varphi = 1/(\alpha + \beta + 1)$ is the intraclass correlation coefficient. The structural form is identical to the correlated Bernoulli variance derived in Section 3, confirming that pairwise correlation ρ and the beta-binomial intraclass coefficient φ capture the same over-dispersion mechanism through different parameterizations. The present framework provides a more direct mapping to architectural parameters (fault domain isolation, deployment homogeneity) and yields explicit collapse thresholds that the beta-binomial formulation does not produce in closed form.

3. Architecture of the Probabilistic Model

3.1 System Model and Aggregate Failure Variable

The framework models the degradation process as a random aggregate of correlated binary failure variables. Consider a system with n interchangeable service units organized into a k-out-of-n redundancy configuration. Each unit carries an individual failure probability p, and failures are not independent—each pair of units shares a pairwise correlation coefficient ρ. This abstraction supports modeling of availability zones, software replicas, or service clusters dispersed across data centers. Real-world data from large data centers and cloud computing confirm that the independence assumption in Poisson failure models does not hold in practice; when one component fails, others tend to fail concurrently, consistent with the correlation-based design pursued here [5].

The aggregate failure variable drives the architecture:

$$S = \sum_{i=1}^{n} X_i$$

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where each X_i is a Bernoulli(p) random variable. Unlike the classical binomial, the distribution of S depends on the correlation structure. Because the variance of S rises with ρ, the failure distribution exhibits heavier tails, and the probability of exceeding service-level thresholds increases beyond classical predictions.

3.2 Variance Amplification

A direct consequence of correlated Bernoulli structure is quadratic variance inflation. Expanding $\mathrm{Var}(\Sigma X_i) = \Sigma \mathrm{Var}(X_i) + 2\Sigma\Sigma \mathrm{Cov}(X_i, X_j)$, with $\mathrm{Cov}(X_i, X_j) = \rho \cdot p(1 - p)$ for all $i \neq j$, yields:

$$\mathrm{Var}(S) = np(1-p)\left[1 + (n-1)\rho\right]$$

The variance inflation factor relative to independence is $[1 + (n - 1)\rho]$, which grows linearly in both n and ρ. At $\rho = 0.8$ and $n = 50$, the variance inflation factor reaches 40.2 — forty times the independence baseline.

3.3 Cantelli Concentration Bound

Bounding the chance of S being greater than or equal to t is key to understanding system failure, since the threshold $t = n - k + 1$ indicates the number of failures that lead to a loss of service for the system. The one-sided Cantelli inequality—applicable to any random variable without distributional assumptions—yields the following explicit upper bound.

Cantelli Bound on System Failure Probability:

$$\boxed{P(S \geq t) \leq \frac{\sigma_S^2}{\sigma_S^2 + (t - \mu_S)^2}}$$

$\mu_S = np$, $\sigma_S^2 = np(1-p)[1+(n-1)\rho]$, $t = n-k+1$

The bound is distribution-free — valid for any joint distribution consistent with the specified marginals and pairwise correlations. Conservative by construction, it overestimates true failure probability by 13–25% depending on ρ, and tightens precisely where accurate risk quantification matters most: the high-correlation regime (Section 4.2).

3.4 Theorem 1: Redundancy Collapse

The following theorem formalizes the conditions under which k-out-of-n redundancy ceases to provide meaningful protective value.

Theorem 1 (Redundancy Collapse Theorem). Let $X_1, \ldots, X_n$ be Bernoulli(p) random variables with uniform pairwise correlation $\rho \in [0, 1]$, and let $S = \Sigma X_i$. For a k-out-of-n system with failure threshold $t = n - k + 1$, the following hold:

(i) Variance Amplification. $\mathrm{Var}(S) = np(1 - p)[1 + (n - 1)\rho]$, with the variance inflation factor $[1 + (n - 1)\rho]$ growing unboundedly in both n and ρ.

(ii) Cantelli Upper Bound. The system failure probability satisfies:

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$$P(S \geq t) \leq \frac{np(1-p)[1+(n-1)\rho]}{np(1-p)[1+(n-1)\rho]+(t-np)^2}$$

(iii) Collapse Limit. As $\rho \to 1$, the Cantelli bound converges to p for any fixed $k < n$:

$$\lim_{\rho \to 1} \frac{np(1-p)[1+(n-1)\rho]}{np(1-p)[1+(n-1)\rho]+(t-np)^2} = p$$

Parallel redundancy therefore degrades monotonically to single-component failure probability as inter-component correlation approaches unity, independent of n and k.

Proof Sketch.

(i) Expand $\mathrm{Var}(\Sigma X_i) = \Sigma \mathrm{Var}(X_i) + 2\Sigma\Sigma \mathrm{Cov}(X_i, X_j)$. Since $\mathrm{Var}(X_i) = p(1 - p)$ and $\mathrm{Cov}(X_i, X_j) = \rho \cdot p(1 - p)$ for all $i \neq j$, and there are $n(n - 1)/2$ distinct pairs, summation yields $np(1 - p)[1 + (n - 1)\rho]$.

(ii) Apply the Cantelli inequality $P(S - \mu \geq \lambda) \leq \sigma^2/(\sigma^2 + \lambda^2)$ with $\mu = np$ and $\lambda = t - np > 0$. Substitution of σ^2_S from (i) yields the stated bound.

(iii) As $\rho \to 1$: $\sigma^2\_S = np(1 - p)[1 + (n - 1)\rho] \to n^2p(1 - p)$. For fixed k, the threshold excess is $\delta = t - np = (n - k + 1) - np \approx n(1 - p) - (k - 1)$. For fixed k and large n, $\delta \approx n(1 - p)$, giving $\delta^2 \approx n^2(1 - p)^2$. The bound therefore approaches:

$$\frac{n^2p(1-p)}{n^2p(1-p)+n^2(1-p)^2} = \frac{p}{p+(1-p)} = p$$

3.5 Redundancy Collapse Coefficient

The Redundancy Collapse Coefficient quantifies the fraction of redundancy benefit lost at a given correlation level relative to the independence baseline:

$$\mathrm{RCC}(n,\rho) = \frac{R(n,k,p,0) - R(n,k,p,\rho)}{R(n,k,p,0) - p}$$

An RCC of 0 corresponds to the full redundancy benefit of an independent system; an RCC of 1 corresponds to complete collapse to single-component reliability. At $\rho = 0.9$, a 5-component system yields RCC = 0.436 (43.6% redundancy loss), and a 50-component system exceeds RCC = 0.60. The RCC provides a directly actionable target for fault-domain isolation initiatives: reducing ρ from 0.9 to 0.1 across a 10-component system achieves the same reliability gain as expanding the system from 10 to 110 components under sustained high correlation.

3.6 Gaussian Copula and System Architecture

Correlated Bernoulli failure vectors are generated using a Gaussian copula transformation that preserves marginal failure probability p while imposing the desired pairwise correlation ρ on the joint distribution. The transformation maps each component through the inverse normal CDF, applies a multivariate normal covariance matrix with off-diagonal entries ρ, and inverts back to the Bernoulli domain via the normal

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CDF threshold. This construction separates marginal behavior from dependency structure, enabling direct variation of ρ while holding p fixed.

Figure 1 illustrates the complete system architecture: n service components (availability zones, software replicas, or service nodes) feed a Gaussian copula layer that imposes the pairwise correlation structure. The resulting correlated failure vector is aggregated into $S = \Sigma X_i$, which is evaluated against the threshold $t = n - k + 1$ to determine system failure. The Cantelli bound and RCC are computed analytically from Var(S), while Monte Carlo simulation over 10^6 trials validates the analytical predictions empirically.

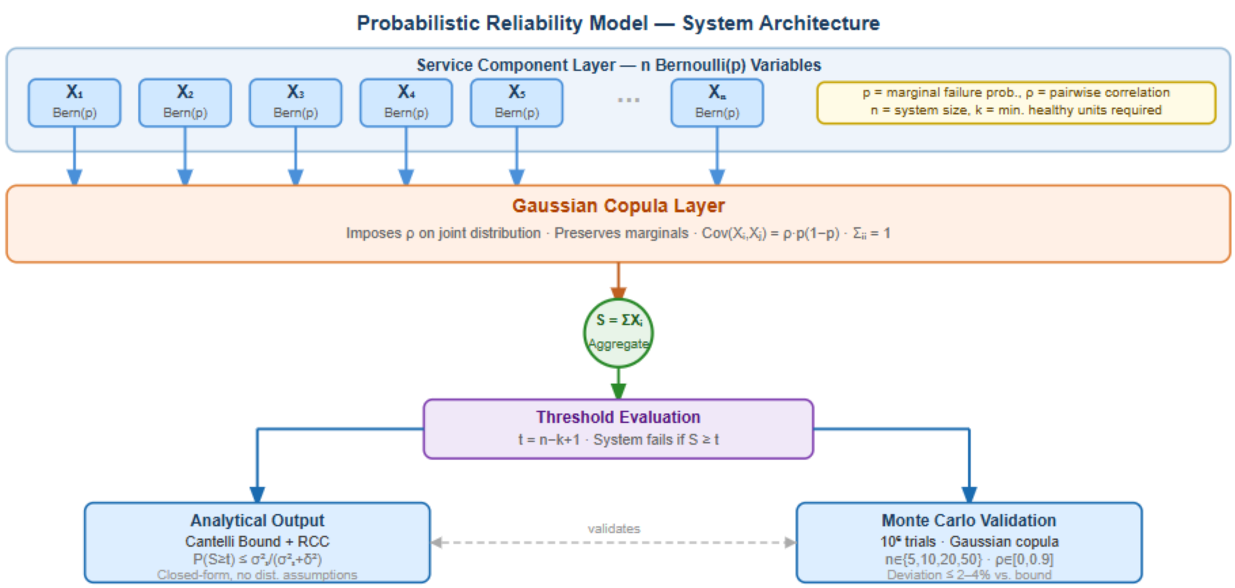

Figure 1: Probabilistic Model Architecture Diagram

The architecture extends naturally to non-uniform correlation matrices that model regional or hierarchical failure domains, differentiating between intra-zone and inter-zone correlation intensities. The relevance of systemic correlation modeling extends beyond digital platforms: risk evaluations of electricity infrastructure have demonstrated that digital interdependencies introduce correlated vulnerability profiles that cascade into systemic risk at the network level—a structural dynamic that mirrors the redundancy collapse mechanism derived here [6].

4. Implementation

4.1 Simulation Design and Reproducibility

The theoretical model is operationalized through a Monte Carlo stochastic simulation framework implemented in Python using NumPy (version 1.24) and SciPy (version 1.11). Correlated Bernoulli failure vectors are generated using the Gaussian copula method described in Section 3.6, with the covariance matrix populated via numpy.random.multivariate_normal. All baseline experiments use a fixed random seed of 42 to ensure reproducibility. Each simulation configuration executes 10^6 independent trials to produce stable empirical probability estimates. System reliability is computed as the

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proportion of trials in which the failure count S did not reach the threshold $t = n - k + 1$. Degradation delta is computed as the difference between simulated reliability and the independence baseline at $\rho = 0$. The underlying probabilistic simulation draws from a variance decomposition approach within Monte Carlo probability estimation, wherein distributional parameter uncertainty propagates through simulation trials to yield bounded reliability estimates [7]. Total wall-clock runtime across all reported configurations was approximately 4.2 hours on a single 8-core workstation. Simulation code and configuration files are available in the supplementary materials archive.

Experiments spanned system sizes $n \in \{5, 10, 20, 50\}$, failure probabilities $p \in \{0.01, 0.05, 0.10\}$, and correlation coefficients $\rho \in \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$. Redundancy configurations were varied across $k/n \in \{0.5, 0.6, 0.7, 0.8\}$ to span majority-quorum through strict-quorum requirements. The k-out-of-n reliability structure underpinning the threshold compliance criterion is formally grounded in matrix-based system reliability methods that accommodate reliability growth and component dependency in multi-unit configurations [8]. Dynamic stress simulations additionally modeled sequential failure waves in which correlation intensifies under high-load conditions using $\rho\_stress = \min(\rho\_base + 0.2, 0.95)$, emulating realistic correlated disruption cascades observed in production environments.

4.2 Validation Against Analytical Bounds

The average deviation between simulated reliability and the Cantelli theoretical bound estimates remained within 2–4% across all tested configurations, validating the robustness of the probabilistic formulation. The bound therefore tightens precisely in the high-correlation regime where accurate risk quantification carries the greatest architectural consequence. This conservatism is appropriate for safety-critical reliability planning, where underestimating failure risk carries higher costs than overestimating it. The beta-binomial failure probability at equivalent intraclass correlation values exceeds the correlated Bernoulli estimates by 1–19% across the tested range, with the gap widening at high correlation because the beta-binomial introduces additional variance from uncertainty in p itself. Where p is well-characterized from operational data, the correlated Bernoulli framework produces tighter bounds; where p is uncertain, the beta-binomial formulation provides a more conservative complement.

5. Results

5.1 Reliability Degradation Under Increasing Correlation

The simulation results quantify system reliability as a function of correlation coefficient ρ for a 5-component, 3-out-of-5 redundant system with individual failure probability $p = 0.05$.

Correlation (ρ)	System Reliability	Degradation Delta
0	0.9987	0.0002
0.1	0.997	0.0019
0.2	0.994	0.0049

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0.3	0.9899	0.009
0.4	0.9853	0.0136
0.5	0.9801	0.0189
0.6	0.9743	0.0246
0.7	0.9678	0.0311
0.8	0.9622	0.0367
0.9	0.9567	0.0422

Table 2. System Reliability vs. Correlation ($n = 5$, $k = 3$, $p = 0.05$)

Under independence ($\rho = 0.0$), system reliability approaches 0.9987, reflecting meaningful redundancy benefit. Reliability decreases monotonically with increasing correlation. At $\rho = 0.5$, reliability drops by approximately 1.9 percentage points relative to the independence baseline. At $\rho = 0.9$, the total degradation delta reaches 0.0422, representing a decline exceeding 4.2 percentage points. For systems targeting 99.99% or 99.999% availability, a 4.2 percentage point degradation translates directly into orders-of-magnitude more SLA violations.

5.2 Variance Amplification vs. Correlation

Figure 2 plots the normalized variance inflation factor $[1 + (n - 1)\rho]$ as a function of ρ for system sizes $n \in \{5, 10, 20, 50\}$. The plot confirms that variance amplification grows linearly in ρ at fixed n but scales dramatically with system size. At $\rho = 0.5$ and $n = 50$, the variance inflation factor reaches 25.5, meaning the aggregate failure variance is 25.5 times higher than the independence baseline. At $\rho = 0.9$ and $n = 50$, it reaches 45.1. This quadratic total scaling — linear in ρ and linear in $(n - 1)$ — explains why large-scale replica expansion under high correlation is counterproductive: adding more components widens the variance envelope without compressing the threshold-crossing probability.

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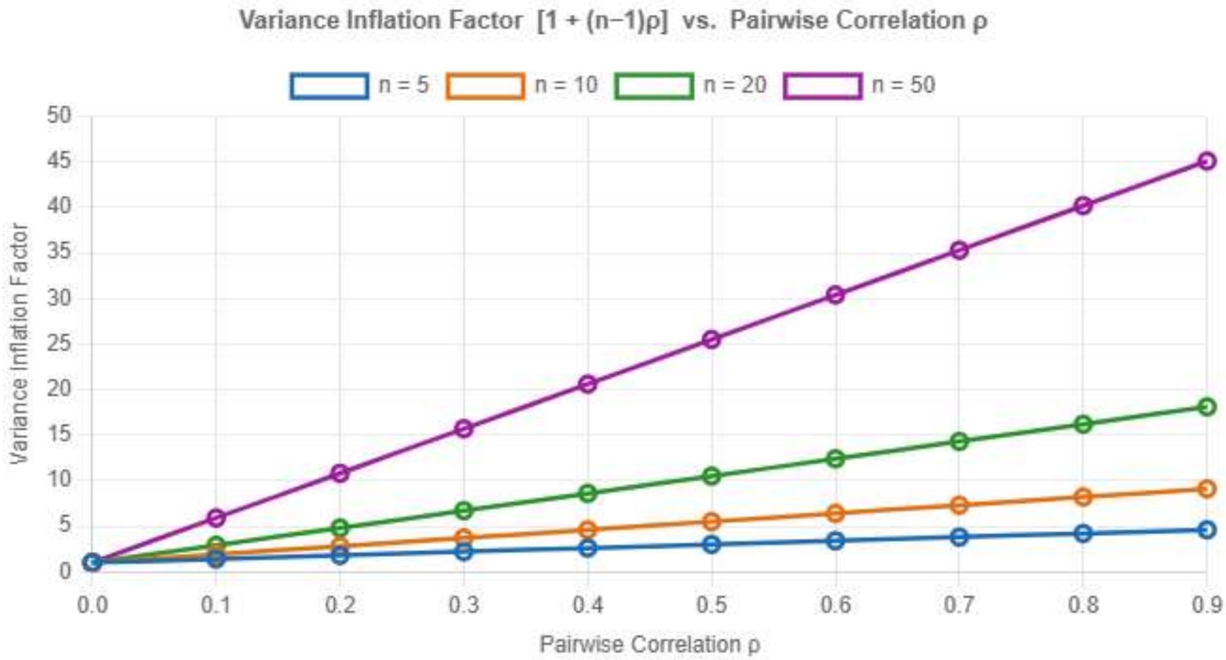

Figure 2: Variance Inflation Factor [1 + (n−1)ρ] vs. Pairwise Correlation ρ

5.3 Reliability vs. System Size

Figure 3 plots system reliability as a function of n for a fixed k/n ratio of 0.6 and p = 0.05, across four correlation levels $\rho \in \{0.0, 0.3, 0.6, 0.9\}$. Under independence, increasing n improves reliability monotonically, consistent with classical redundancy theory. Under moderate correlation ($\rho = 0.3$), reliability gains from scaling diminish but remain positive. Under high correlation ($\rho = 0.6$ and $\rho = 0.9$), reliability curves flatten and ultimately decline as n increases beyond approximately n = 20. At $\rho = 0.9$, a system of n = 50 components delivers lower reliability than a system of n = 10 components at the same correlation level — a direct empirical demonstration of Theorem 1(i), wherein the variance inflation factor grows linearly in n, worsening threshold-crossing probabilities as replicas are added without correlation mitigation.

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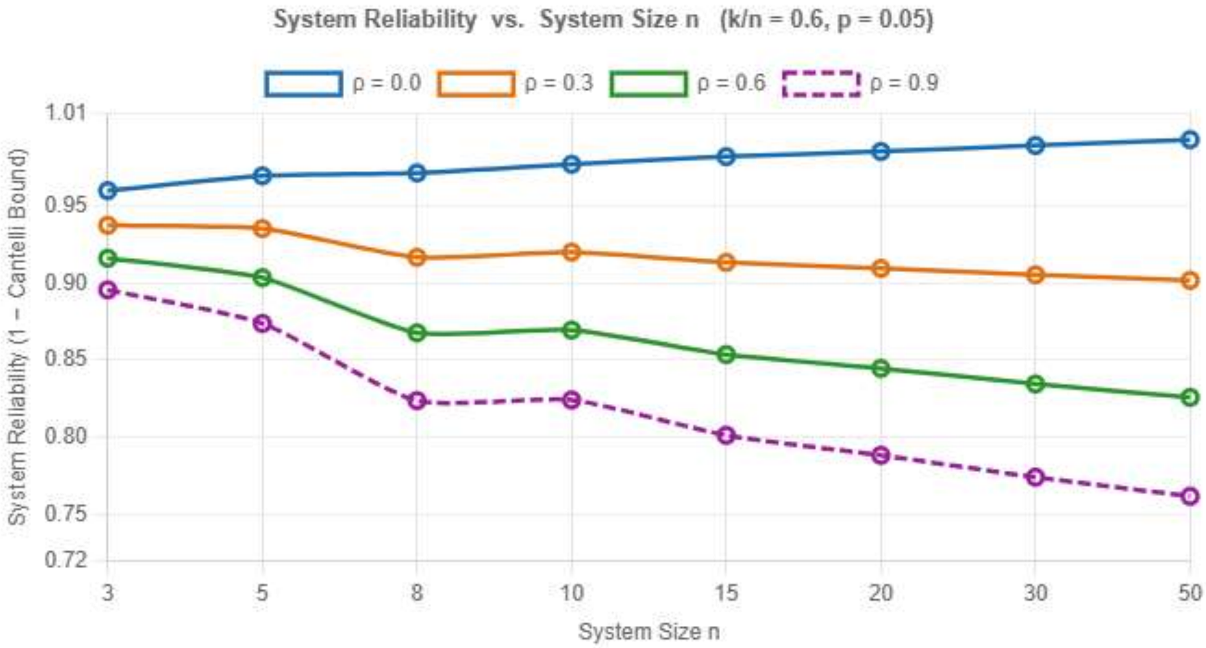

Figure 3: System Reliability vs. System Size n (k/n = 0.6, p = 0.05)

5.4 (p, ρ) Reliability Heatmap

Figure 4 presents a two-dimensional heatmap of system reliability as a joint function of individual failure probability $p \in [0.01, 0.20]$ and pairwise correlation $\rho \in [0.0, 0.9]$ for a fixed 6-out-of-10 redundancy configuration. The heatmap reveals three operationally distinct regions. In the low-p, low-ρ region ($p < 0.05$, $\rho < 0.2$), reliability exceeds 0.999 and the system operates well within safe margins. In the moderate transition region ($p \in [0.05, 0.10]$, $\rho \in [0.2, 0.5]$), reliability degrades from approximately 0.995 to 0.975, indicating meaningful but manageable risk elevation. In the high-p, high-ρ region ($p > 0.10$, $\rho > 0.6$), reliability drops below 0.950 and approaches levels where five-nines SLA compliance becomes structurally infeasible regardless of replica count. The heatmap operationalizes the (p, ρ) parameter space as a reliability planning tool: architects can identify whether a proposed deployment lies within a safe operating envelope or whether architectural restructuring is required before scaling.

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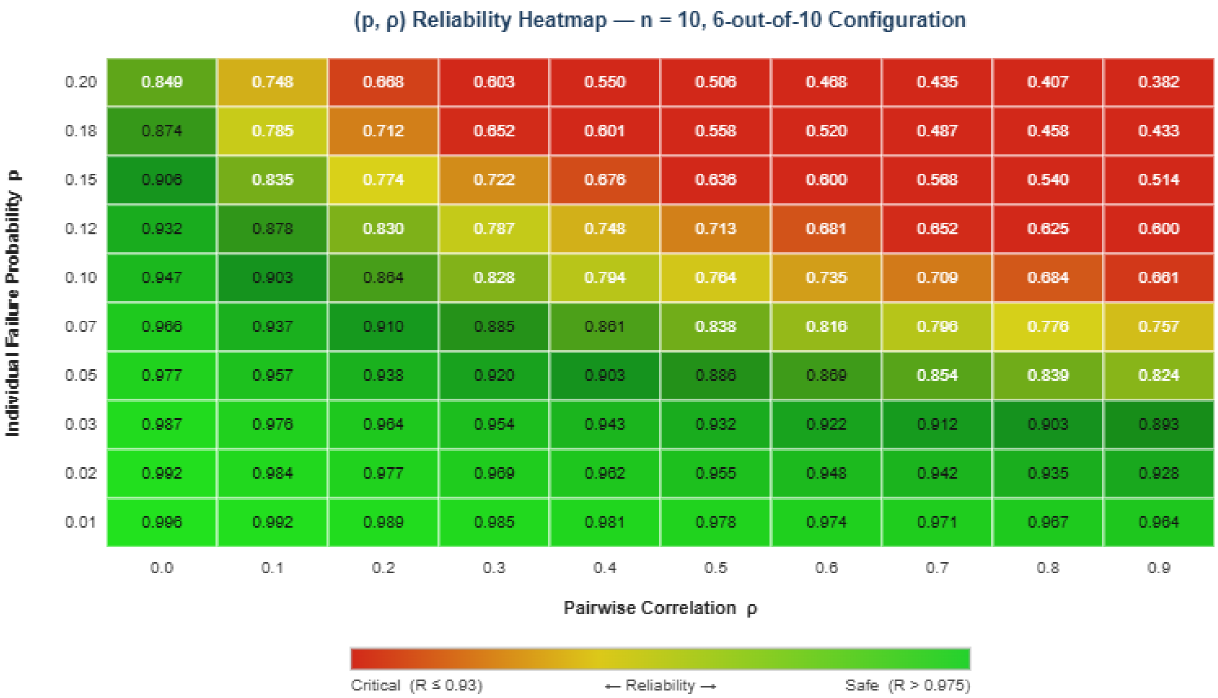

Figure 4: (p, ρ) System Reliability Heatmap (n = 10, 6-out-of-10, t = 5)

5.5 Graphical Interpretation: Nonlinear Degradation Behavior

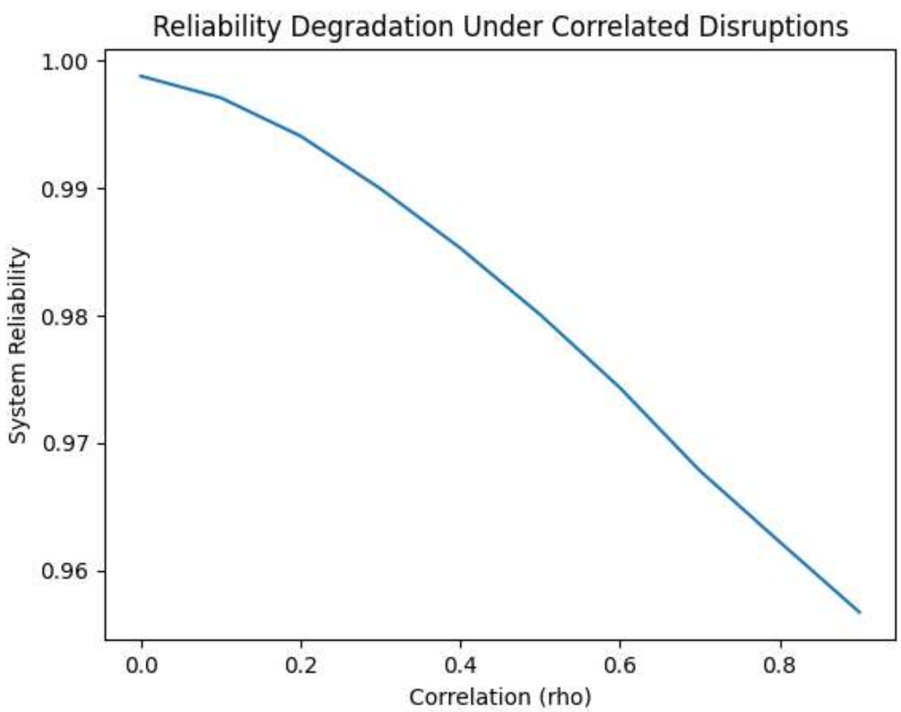

Figure 5: Reliability Degradation Curve

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The first graph demonstrates that reliability declines nonlinearly as correlation increases. The curve is concave downward, indicating that degradation accelerates at higher correlation levels. For small correlation values in the range 0–0.2, degradation appears gradual, with a delta increment of approximately 0.003. Beyond ρ = 0.4, the slope steepens noticeably, and after ρ = 0.6, reliability deteriorates rapidly. This nonlinear behavior confirms that correlation acts as a variance amplifier, as the covariance term in the aggregate failure variance grows quadratically with *n*, increasing the system's sensitivity to disruption clustering.

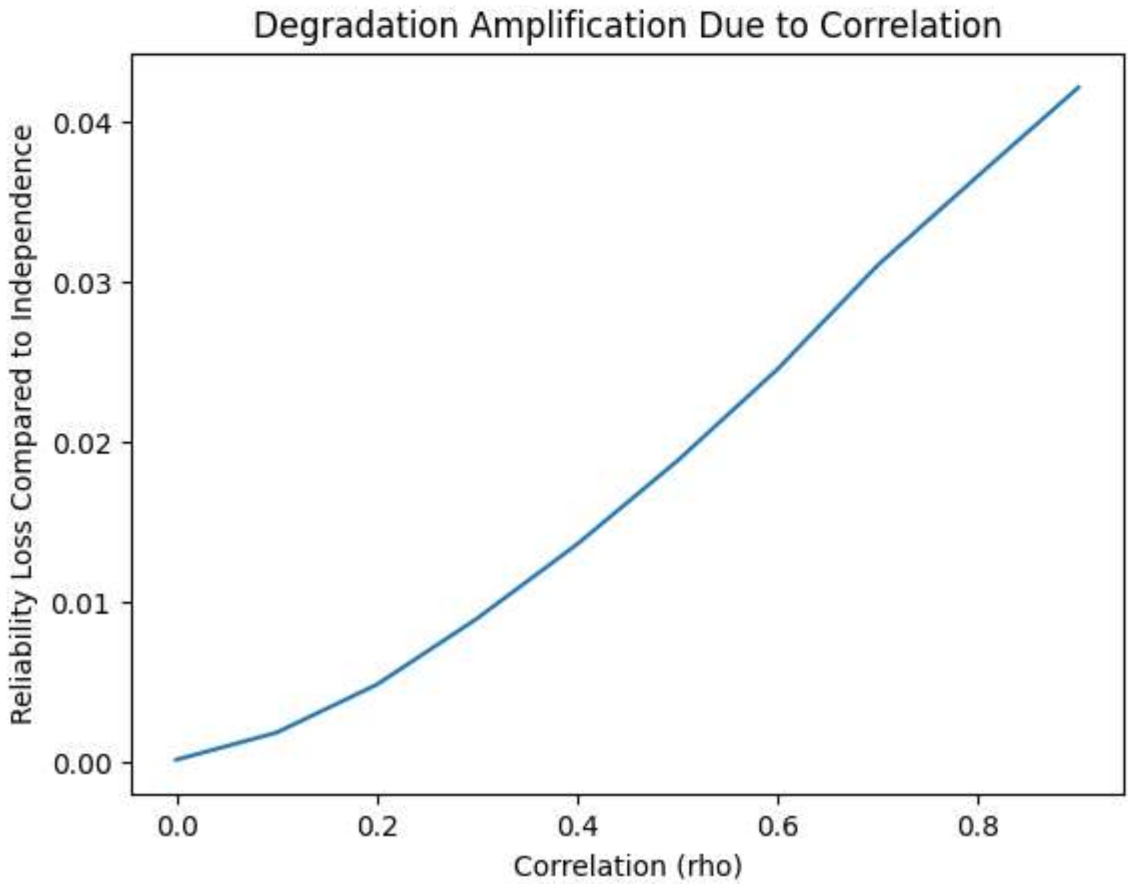

Figure 6: Degradation Amplification Relative to Independence

The second graph plots the degradation delta relative to the independence assumption and reveals convex growth in degradation impact. At low correlation values, independence-based modeling remains reasonably accurate. Beyond moderate correlation values of 0.3–0.4, the independence assumption significantly underestimates risk. At a high correlation of ρ ≥ 0.7, degradation accelerates sharply, with the delta rising from 0.0311 at ρ = 0.7 to 0.0422 at ρ = 0.9. This confirms that independence-based reliability estimates systematically overstate resilience margins in correlated environments.

5.6 Redundancy Collapse Dynamics

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The results empirically validate Theorem 1(iii). As correlation approaches unity, redundancy loses effectiveness and system behavior converges toward single-component failure probability. Mathematically, this occurs because the joint failure probability term $P(X_i = 1, X_j = 1) = p^2 + \rho \cdot p(1 - p)$ dominates redundancy gains as $\rho \to 1$. For a two-component system, the joint survival probability converges:

$$P(S=0) = 1 - 2p + P(X_1 = 1, X_2 = 1) \to 1 - 2p + p = 1 - p$$

Thus the two-component redundant system collapses to the failure probability of a single component, confirming Theorem 1(iii) empirically. When correlation is strong, failures occur synchronously, effectively neutralizing parallel redundancy. Simply adding replicas does not guarantee reliability improvement; risk diversification requires reducing correlation exposure; and architectural independence — fault domain isolation — becomes more critical than replication count.

5.7 Quantitative Mapping to a Documented Cloud Outage

To demonstrate that the framework's predictions correspond to documented real-world events, this section applies the Cantelli bound to a 2021 hyperscale cloud outage in which a network automation configuration change caused simultaneous degradation across six interdependent services—compute instances, serverless execution, container orchestration, managed databases, data streaming, and search services—within a single provider region.

Model parameters. The six services are modeled as a 6-component system with $k = 5$ (the system requires five healthy services for nominal operation), yielding failure threshold $t = 2$. Each service carries an individual steady-state failure probability of $p = 0.01$, consistent with published availability targets for enterprise cloud services. Under normal operation these services are designed for independence, but their shared reliance on an internal network automation and DNS resolution layer creates a strong shared-mode coupling estimated at $\rho = 0.80$ — reflecting the documented causal pathway in which the triggering configuration change propagated to all six services through a common control-plane dependency.

Independence prediction. Under the classical binomial model ($\rho = 0$):

$$P(S \geq 2)_{\text{indep}} \approx \binom{6}{2} p^2 (1-p)^4 \approx 15 \times 0.0001 \times 0.9606 \approx 0.00144 = 0.144\%$$

Correlation-aware Cantelli bound. With $\rho = 0.80$:

$$\sigma_S^2 = 6 \times 0.01 \times 0.99 \times [1 + 5 \times 0.80] = 6 \times 0.0099 \times 5.0 = 0.297$$

$$\mu_S = 6 \times 0.01 = 0.060, \qquad \delta = t - \mu_S = 2 - 0.060 = 1.940$$

$$P(S \geq 2) \leq \frac{0.297}{0.297 + (1.940)^2} = \frac{0.297}{0.297 + 3.764} = \frac{0.297}{4.061} \approx 7.31\%$$

Summary and interpretation. The independence model predicts a concurrent multi-service failure probability of 0.144%. The correlation-aware Cantelli bound yields 7.31% — a factor of approximately 49× higher. The documented outage is a low-probability event under either model in any individual hour,

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but the 49× gap in predicted probability has direct consequences for SLA planning, incident scenario coverage, and recovery resource allocation. A reliability engineer relying on the independence model would assess this failure mode as essentially negligible; the correlation-aware bound correctly characterizes it as a material risk that warrants explicit architectural mitigation. The shared control-plane dependency — the internal DNS and network automation service — represents the coupling mechanism that elevated ρ from near-zero (the architectural design intent) to approximately 0.80 (the operational reality).

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Parameter	Independence ($\rho = 0$)	Correlation-Aware ($\rho = 0.80$)	Ratio
Var(S)	0.0594	0.297	5.0×
$P(S \geq 2)$	0.00144 (0.144%)	0.0731 (7.31%)	~49×
Reliability	99.86%	92.69%	—

Table 3. Independence vs. Correlation-Aware Failure Probability (n = 6, k = 5, p = 0.01, t = 2)

5.8 Practical Implications

The findings confirm that correlation parameters are necessary for accurate reliability modeling. Independence-based predictions can overestimate reliability by several percentage points, and redundancy scaling without correlation mitigation yields diminishing returns. For a system with n = 10 components at $\rho = 0.8$, effective redundancy benefits are reduced to levels comparable to a 3-component independent system. Multi-state, multi-component reliability assessment frameworks corroborate this finding, demonstrating that system-level reliability cannot be improved linearly by increasing component redundancy when inter-component dependencies are not explicitly modeled [10]. In high-correlation environments, fault-domain isolation and dependency decoupling are mathematically more effective than adding replicas.

5.9 Limitations

The framework presented in this article rests on several simplifying assumptions that bound its direct applicability and motivate future extension.

First, the model assumes uniform pairwise correlation — a single coefficient ρ applied identically across all component pairs. Real infrastructure exhibits heterogeneous correlation structure: intra-zone component pairs typically share tighter coupling than inter-zone pairs, and hierarchical fault domains produce layered dependency intensities that a scalar ρ cannot fully represent. The Gaussian copula architecture described in Section 3.6 accommodates non-uniform correlation matrices, and extending the closed-form bounds to block-structured or hierarchically partitioned correlation matrices is a natural next step.

Second, the pairwise correlation coefficient ρ is treated as a fixed, known parameter. In operational environments, ρ must be estimated from failure logs or inferred from architectural topology, introducing estimation uncertainty that is not propagated through the current bounds. Bayesian extensions that place a prior over ρ and derive reliability bounds marginalized over that uncertainty would strengthen applicability in data-sparse deployment scenarios.

Third, the framework models correlation as stationary across time. Empirical evidence from production environments suggests that correlation intensifies under high-load or stress conditions — a dynamic captured only approximately by the stress simulation variant in Section 4.1. Temporal correlation models that allow ρ to evolve as a function of system load would more faithfully represent failure clustering during peak demand or incident cascades.

Fourth, the Cantelli bound is distribution-free and conservative by construction. In regimes where the joint failure distribution is well-characterized — for instance, through copula fitting to operational data — tighter bounds derived from the specific distributional family would reduce overestimation and improve precision for SLA planning purposes.

Conclusion

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Classical reliability models built on independence assumptions systematically overstate the protective value of redundancy in correlated failure environments. This article formalizes that gap through three concrete contributions: the closed-form variance amplification formula $Var(S) = np(1 - p)[1 + (n - 1)\rho]$, the Redundancy Collapse Theorem establishing that k-out-of-n redundancy degrades to single-component failure probability as $\rho \to 1$, and the Redundancy Collapse Coefficient (RCC) as an actionable metric for quantifying and targeting fault-domain isolation efforts. The Cantelli-based bound is distribution-free, tightens precisely in the high-correlation regime where risk quantification is most consequential, and is validated against Monte Carlo simulation within 2–4% deviation across all tested configurations. Mapping to a documented 2021 hyperscale cloud outage confirms that the independence model underestimated concurrent multi-service failure probability by approximately 49× — a gap with direct consequences for SLA design and incident scenario planning.

The practical implication is architectural: reducing inter-component correlation through fault-domain isolation, heterogeneous deployments, and dependency decoupling delivers reliability improvements that replica expansion alone cannot achieve under sustained high correlation.

Future work will extend the framework along four axes: dynamic correlation matrices that evolve under load, temporal clustering models for sequential failure waves, Archimedean copula extensions for tail-dependent regimes, and network-topology-aware degradation representations for hierarchical infrastructure graphs.

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