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EVOLUTION OF MAGNETIZED LRS BIANCHI TYPE-III UNIVERSE WITH A LIQUID-STATE DARK ENERGY

S. A. BHOJNE

Department of Mathematics P. R. Pote (Patil) College of Engineering and Management, Amravati (M.S.),(India)- 444604, Email:
st1731982@gmail.com

ABSTRACT:

This study investigates magnetized locally-rotationally symmetric Bianchi type-III cosmological model filled with Wet dark fluid within the framework of General Relativity. By imposing non shearing condition. A new equation of state for the dark energy component of the universe has been used. It is modeled on the equation of state $p = \gamma(\rho - \rho_*)$ which can describe a liquid, for example water. The exact solutions to the corresponding field equations are obtained in quadrature form. We provide a detailed analysis of the physical and geometrical properties to evaluate the model's cosmological implication.

1.INTRODUCTION:

The Presence of galactic-scale magnetic fields is a well-established fact in modern cosmology. As noted by Zel'dovich[1],these fields play a critical role in various astrophysical phenomena,
a significance that is now widely recognized withen the scientific community. Also, Harrison [2] has suggested that magnetic field could have a cosmological origin, as a natural consequence. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological model more general than Robertson-Walker model [3], Singh et al. [4] has examined spatially homogeneous and anisotropic Bianchi type-III with massive string in presence of viscous fluid and electromagnetic field. The presence of primordial magnetic fields in the early stage of evolution of the universe has been discussed by several authors Kim et al. [5], Barrow [6], Melvin [7]. The authors Soni et al. [8] have examined non shearing locally rotationally symmetric Bianchi type-III string cosmological models in presence of bulk viscosity with the assumption that coefficient of bulk viscosity is proportional to scalar expansion. This model describes expanding, non shearing and non rotating universe. Patil et al. [9] have obtained Non Shearing Locally –Rotationally Symmetric Bianchi type-III string cosmological model with bulk viscous fluid and magnetic flux with some tractable assumption that expansion scalar θ is proportional to eigen value σ_1^1 of shear tensor[10]. The above situations motivate us and to think what happens, when we use wet dark fluid with locally-rotationally symmetric Bianchi type-III

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cosmological model and we have include electromagnetic field as a energy momentum tensor of early universe.

2. FORMATION OF FIELD EQUATIONS

We have considered locally rotationally symmetric Bianchi type-III space-time,

$$ds^2 = -dt^2 + A^2dx^2 + B^2(e^{2x}dy^2 + dz^2) \quad (2.1)$$

in which A, B are cosmic scale functions of t.

$g = \det g_{ij} = -A^2B^4e^{2x}$, therefore $\sqrt{-g} = AB^2e^x$

The Einstein's field equation is,

$$R_{ij} - \frac{1}{2}g_{ij}R = T_{ij} \quad (2.2)$$

The energy momentum tensor for system of wet dark fluid and magnetic field is

$$T_j^i = (\rho_{WDF} + p_{WDF})u_ju^i - p_{WDF}\,\delta_j^i + E_j^i \quad (2.3)$$

in which u^i is the flow vector satisfying the relation

$$g_{ij}u^iu^j = 1$$

The electromagnetic field E_j^i is defined as,

$$E_j^i = \frac{1}{4\pi}\left[-F_{jl}F^{il} + \frac{1}{4}g_j^iF_{lm}F^{lm}\right]$$

with the Maxwell's equation,

$\frac{\partial}{\partial x^j}\left(F^{ij}\sqrt{-g}\right) = 0$, which leads to, $F_{23} = I$

$$\Longrightarrow E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = \frac{I^2}{8\pi B^2C^2} \quad (2.4)$$

Using co-moving system of co-ordinate and equation (2.4) in equation (2.2) we have obtain the energy momentum tensors as

$$T_0^0 = \rho_{WDF} + \frac{I^2}{8\pi B^2C^2} \quad (2.5)$$

$$T_1^1 = -p_{WDF} + \frac{I^2}{8\pi B^2C^2} \quad (2.6)$$

$$T_2^2 = -p_{WDF} - \frac{I^2}{8\pi B^2C^2} \quad (2.7)$$

$$T_3^3 = -p_{WDF} - \frac{I^2}{8\pi B^2C^2} \quad (2.8)$$

After substituting the value of Ricci Scalar R_{ij}, Ricci tensor R and Energy momentum tensor T_{ij} for i, j = 0,1,2,3 in equation (2.2) we have

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = -p_{WDF} + \frac{I^2}{8\pi B^2C^2} \quad (2.9)$$

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$$\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\ddot{A}}{A}-\frac{1}{A^2}= -p_{WDF}-\frac{I^2}{8\pi B^2C^2} \tag{2.10}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{3}{A^2}= \rho_{WDF}+\frac{I^2}{8\pi B^2C^2} \tag{2.11}$$

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}= 0 \tag{2.12}$$

Using equations (2.12), we have

$A = mB$, where m is integration constant. (2.13)

$$\Longrightarrow \frac{\dot{A}}{A}=\frac{\dot{B}}{B} \quad \text{and} \quad \frac{\ddot{B}}{B}=\frac{\ddot{A}}{A} \tag{2.14}$$

The Scalar expansion θ is defined as

$$\theta = u^i_{;i} = \left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)=\frac{3\dot{B}}{B} \tag{2.15}$$

We define the generalized mean Hubble parameter H as,

$$H = \frac{1}{3}\left(H_x+H_y+H_z\right)$$

where H_x, H_y, H_z are the directional Hubble parameter in the direction of x, y and z-axis respectively, such that $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{B}}{B}$.

Therefore, $H = \frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{B}}{B}\right) \Longrightarrow H = \frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)$

Using equation (2.14) in above equation, we have

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) = \frac{1}{3}\left(\frac{3\dot{B}}{B}\right)=\frac{\dot{B}}{B} \tag{2.16}$$

Using equation (2.15) in equation (2.9), we obtain

$$2\frac{\ddot{B}}{B}+(1-9k)\frac{\dot{B}^2}{B^2}+\frac{I^2}{2B^4}=0 \tag{2.17}$$

Let us take, $\dot{B} = f$, $\ddot{B} = ff'$ where $f' = \frac{df}{dB}$

Using these values in equation (2.17), we have

$$2\frac{ff'}{B}+(1-9l)\frac{f^2}{B^2}+\frac{I^2}{2B^4}=0$$

Since l is positive constant, let us assume that, $l = \frac{2}{9}$ for simplification only.

$$\Longrightarrow 2\frac{ff'}{B}-\frac{f^2}{B^2}+\frac{I^2}{2B^4}=0$$

Therefore, $d\left(\frac{f^2}{B}\right)+\frac{I^2}{2B^4}=0$

On solving above equation, we obtain

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$$f^2 = \frac{I^2}{6B^2}$$

For simplification, here we consider integration constant to be zero.

$$\Rightarrow f = \sqrt{\frac{I^2}{6\,B^2}}$$

$$\Rightarrow \frac{dB}{dt} = \dot{B} = \sqrt{\frac{I^2}{6\,B^2}} = \sqrt{\frac{1}{6}}\frac{I}{B}$$

Therefore, $B\;dB = \sqrt{\frac{1}{6}}\;I\;dt$

On integrating above equation, we obtain

$B^2 = I\sqrt{\frac{2}{3}}\;t + 2k_1$ **,** where k_1 is integration constant.

$$B^2 = N\,t + K, \qquad \text{where, } I\sqrt{\frac{2}{3}} = N \text{ and } 2k_1 = K \tag{2.18}$$

Using equation (2.18) in equation (2.13), we obtain

$$A^2 = m^2(N\,t + K) \tag{2.19}$$

Hence, the geometry of the space-time in the presence of electromagnetic field by using equation (2.18) and (2.19) in equation (2.1), we have obtained the cosmological model as,

$$ds^2 = -dt^2 + m^2\,(Nt + K)dx^2 + (Nt + K)(e^{2x}dy^2 + dz^2) \tag{2.20}$$

3. GEOMETRICAL AND PHYSICAL SIGNIFICANCE

Consider equation (2.18),

$$B^2 = (N\,t + K)$$

On differentiating w. r. t.[1] t, we have

$$2B\dot{B} = N \Longrightarrow \frac{\dot{B}}{B} = \frac{N}{2(N\,t+K)}$$

Again differentiating, we obtain

$$2\left(\dot{B}^2 + B\ddot{B}\right) = 0$$

$$\dot{B}^2 = -B\ddot{B}$$

$$\frac{\ddot{B}}{B} = -\frac{\dot{B}^2}{B^2} \Longrightarrow \frac{\ddot{B}}{B} = \frac{-N^2}{4(Nt+K)^2}$$

From above equation, we have

$$\frac{\dot{B}}{B} = \frac{N}{2(N\,t+K)} \quad \text{and} \quad \frac{\ddot{B}}{B} = \frac{-N^2}{4(Nt+K)^2} \tag{3.1}$$

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[1] w. r. t. means "with respect to"

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Using equation (3.1) in equation (2.15), we have $\theta = \frac{3N}{2(N\,t+K)}$ (3.2)

Using equation (3.1) in equation (2.16), we obtain

$$H = \left(\frac{N}{2(N\,t+K)}\right) \tag{3.3}$$

and shear tensor σ_{ij} is defined as,

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;\alpha}P_j^{\alpha} + u_{j;\alpha}\,P_i^{\alpha}\right) - \frac{1}{3}\theta P_{ij}, \quad P_{ij} = g_{ij} + u_i u_j$$

$$u_{i;\alpha} = u_{i,\alpha} - \Gamma_{i\alpha}^{m} u_m \quad \text{and} \quad P_i^{\alpha} = g_i^{\alpha} + u_i u^{\alpha}$$

Therefore,

$$\sigma_{00} = 0\,, \quad \sigma_{11} = \frac{1}{2}\left(2A\dot{A}\right) - \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)A^2,$$

$$\sigma_{22} = e^{2x}B^2\left(\frac{\dot{B}}{B} - \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\right), \quad \sigma_{33} = B^2\left(\frac{\dot{B}}{B} - \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\right)$$

and $\sigma^{00} = 0\,, \quad \sigma^{11} = \frac{1}{A^2}\left(\frac{\dot{A}}{A} - \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\right)$

$$\sigma^{22} = \frac{1}{e^{2x}B^2}\left(\frac{\dot{B}}{B} - \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\right), \quad \sigma^{33} = \frac{1}{B^2}\left(\frac{\dot{B}}{B} - \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\right)$$

And $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$

$$\Rightarrow \sigma^2 = \frac{1}{2}\left[\frac{A^2}{A^2}\left(\frac{\dot{A}}{A} - \frac{1}{3}\left[\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right]\right)^2 + \frac{B^2e^{2x}}{B^2e^{2x}}\left(\frac{\dot{B}}{B} - \frac{1}{3}\left[\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right]\right)^2 + \frac{B^2}{B^2}\left(\frac{\dot{B}}{B} - \frac{1}{3}\left[\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right]\right)^2\right] \tag{3.4}$$

$$\Rightarrow \sigma^2 = 0$$

$$\Rightarrow \sigma = 0$$

4. CONCLUSION

In this chapter, We have discussed the Geometrical and Physical Significance of model, we have observed that, the energy expansion scalar (θ), Hubble parameter (H) are inversely proportional to that of cosmic time t. It implies that, as t increases the above parameters decreases and the model expand very large and when t decreases, the model get shrink. Also, in the presence of electromagnetic field, the model expands and the parameters θ, H increases, since $\sigma = 0$ therefore, $\lim_{t \to \infty} \frac{\sigma}{\theta} = 0$, hence model is isotropic for large value of t .

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