

Semi Alpha Generalization of Generalized Star closed sets in Topological Spaces

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Abstract

In this paper, we introduce the concept of semi alpha generalization of generalized star closed sets in topological spaces. Here we study the relation among other closed sets and we derived some of its characteristics. Also, we compared it with other generalized closed sets and we find that the union of two $s\alpha(gg)^*$ -closed sets is not $s\alpha(gg)^*$ -closed set and the intersection of two $s\alpha(gg)^*$ -closed sets is again $s\alpha(gg)^*$ -closed set.

Keywords - $s\alpha(gg)^*$ -closed sets, $\alpha(gg)^*$ -open, semi-closure.

I. INTRODUCTION

In 1970, Levine introduced the concept of generalized closed sets in topology. In 2017, Basavaraj, M. Ittangi and H.G. Govardhana Reddy introduced gg -closed sets in topological spaces. In 2018, I. Christal Bai and T. Shyla Isac Mary introduced $(gg)^*$ -closed sets in topological spaces. In 2022, G.Abhirami and T. Shyla Isac Mary introduced $\alpha(gg)^*$ -closed set in topological spaces. In this paper, a new class of closed set called semi alpha generalization generalized star-closed set is introduced. The concept of semi alpha generalization generalized star-closed set and its characteristics are examined in this paper.

II. PRELIMINARIES

Throughout this paper X and Y Represents the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological spaces, $cl(A)$ and $int(A)$ denote the closure of A and the interior A respectively. $X \setminus A$ denotes the complement of A in X .

Definition 2.1 A subset A of a topological Space (X, τ) is called

1. **Semi-open** [50] if $A \subseteq cl\ int(A)$.
Semi-closed if $int\ cl(A) \subseteq A$.
2. **Pre-open** [28] if $A \subseteq int\ cl(A)$.
Pre-closed if $cl\ int(A) \subseteq A$.
3. **Semi-pre-open** [3] if $A \subseteq cl\ int\ cl(A)$.
Semi-pre-closed if $int\ cl\ int(A) \subseteq A$.
4. **α -open** [33] if $A \subseteq int\ cl\ int(A)$.
 α -closed if $cl\ int\ cl(A) \subseteq A$.
5. **regular open** [48] if $A = int(cl(A))$.
regular closed if $cl\ int(A) = A$.
6. **π -open** [33] if A is the union of regular open sets.
 π -closed if A is the intersection of regular closed sets.
7. **b -open** [4] if $A \subseteq cl\ int(A) \cup int\ cl(A)$.
 b -closed if $cl\ int(A) \cap int\ cl(A) \subseteq A$.

The closure of a subset A of topological space (X, τ) is the smallest closed set containing A , is denoted by $cl(A)$.

The semi-closure of a subset $A \subseteq X$ is the intersection of all semi-closed sets containing A , is denoted by $scl(A)$.

The pre-closure of a subset A of topological space (X, τ) is the smallest preclosed set containing A , is denoted by $pcl(A)$.

The semi-pre-closure of a subset $A \subseteq X$ is the intersection of all semi-pre-closed sets containing A , is denoted by $spcl(A)$.

The α -closure of a subset $A \subseteq X$ is the intersection of all α -closed sets containing A , is denoted by $\alpha cl(A)$.

The b -closure of a subset $A \subseteq X$ is the intersection of all b -closed sets containing A , is denoted by $bcl(A)$.

Definition 2.2 A subset of a topological space (X, τ) is called a

1. Regular semi-open [13] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.
2. generalized-closed [21] (briefly g -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
3. generalization of generalized closed [8] (briefly gg -closed) if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open.
4. generalization of generalized star closed [14] (briefly $(gg)^*$ -closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is gg -open.
5. generalized semi-closed [6] (briefly gs -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
6. α -generalized-closed [22] (briefly αg -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
7. alpha generalization of generalized star-closed [1] (briefly $\alpha(gg)^*$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(gg)^*$ -open.
8. $g^\#$ -closed [49] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg -open.
9. generalization semi-pre-closed [15] (briefly gsp -closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
10. Generalized b -closed [2] (briefly gb -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
11. Semi generalized-closed [12] (briefly sg -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.
12. Strongly generalized semi-pre-closed [36] (briefly $(gsp)^*$ -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gsp -closed.
13. Weakly-closed [43] (briefly w -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.
14. $\hat{g}^*s$ -closed [38] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$ -open.
15. Strongly generalized closed [48] (briefly g^* -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and g -open.

16. $*g$ -closed [48] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.
17. Weakly semi-closed [10] (briefly ws -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is w -open.
18. generalized alpha regular-closed [38] (briefly gar -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open.
19. Generalized pre-regular closed [19] (briefly gpr -closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open.
20. Generalized semi-pre-regular closed [31] (briefly $gspr$ -closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open.
21. Regular weakly-closed [11] (briefly rw -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open.
22. Generalized star semi-closed [39] (briefly g^*s -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs -open.
23. Generalized star b -closed [34] (briefly g^*b -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open.
24. gs^* -closed or $(gs)^*$ -closed [17] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs -open.
25. R^* -closed [20] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open. 26. Generalized star b -omega closed [35] (briefly $g^*b\omega$ -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs -open.
27. α -generalized regular closed [50](briefly αgr -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
28. g^*s^* -closed if closed [18] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* -open.
29. $\hat{g}$ -closed or w -closed [32] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.
30. $R^\#$ -closed set [9] if $gcl \subseteq U$ whenever $A \subseteq U$ and U is R^* -open.

31. generalized pre-closed set [24](briefly *gp*-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
32. $r^{\wedge}g$ -closed set [41] if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open.
33. weakly generalized closed [29] (briefly *wg*-closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open.
34. semi-weakly generalized closed [30] (briefly *swg*-closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.
35. *sg**-closed [19] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* -open.
36. π -generalized-semi-pre-closed [40] (briefly πgsp -closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π -open.
37. $s\alpha g^*$ -closed set [25] if $\alpha cl(A) \subseteq A$ whenever $A \subseteq U$ and U is g^* -open.
38. regular generalized *b*-closed [26] (briefly *rgb*-closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.
39. πgs -closed set [7] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is π -open.
40. generalized *b* star-closed [42] (briefly gb^* -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* -open.
41. generalized αb -closed [46] (briefly gab -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open.
42. πgb -closed set [44] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π -open.
43. g^{**} -closed set [37] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open.
44. gs^{**} -closed set [30] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs^* -open.
45. generalized regular closed set [13] (briefly *gr*-closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

46. regular generalized closed set [33] (briefly rg -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open.

47. πg -closed set [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π -open.

48. $\pi g\alpha$ -closed set [33] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π -open.

49. αg^* -closed set [28] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open.

50. $g\alpha$ -closed set [23] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

The complements of the above closed sets are their open sets and vice versa.

Lemma 2.3 [47]

In an extremally disconnected space X ,

- i) $\alpha cl(A) = scl(A)$ ii)
 $scl(A \cup B) = scl(A) \cup scl(B)$

III. SEMI ALPHA GENERALIZATION GENERALIZED STAR – CLOSED SETS

Definition 3.1 A subset A of a topological space (X, τ) is called semi alpha generalization of generalized star closed (briefly $s\alpha(gg)^*$ -closed) if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is $\alpha(gg)^*$ -open in (X, τ) . The set of all $s\alpha(gg)^*$ -closed sets in (X, τ) is denoted by $s\alpha(GG)^*C$.

Example 3.2 Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$ and $\tau_c =$

$\{\emptyset, \{a, b, d\}, \{a, b, c\}, \{a, b\}, \{b\}, X\}$.

$\alpha(gg)^*$ -open = $\{\emptyset, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{a, d\}, \{a, c\}, \{d\}, \{a\}, \{c\}, \{b\}, X\}$.

$s\alpha(gg)^*$ -closed = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, X\}$.

Theorem 3.3

- i) Every closed set is $s\alpha(gg)^*$ -closed. ii) Every α - closed set is $s\alpha(gg)^*$ -closed. iii) Every π -closed set is $s\alpha(gg)^*$ -closed. iv) Every regular closed is $s\alpha(gg)^*$ -closed.
- v) Every $(gsp)^*$ -closed is $s\alpha(gg)^*$ -closed.
- vi) Every $(gs)^*$ -closed is $s\alpha(gg)^*$ -closed. vii) Every g^*sr - closed is $s\alpha(gg)^*$ -closed.

Proof:

- i) Let A be any closed subset of a space X . Let $A \subseteq U$ and U is $\alpha(gg)^*$ - open in X . Since A is closed, $cl(A) = A \subseteq U$. But $scl(A) \subseteq cl(A) = A \subseteq U$ [5]. Therefore $scl(A) \subseteq U$. Hence A is $s\alpha(gg)^*$ -closed.
- ii) Let A be any α - closed subset of a space X . Let $A \subseteq U$ and U is $\alpha(gg)^*$ - open in X . Since A is α - closed, $\alpha cl(A) = A \subseteq U$. Therefore $\alpha cl(A) \subseteq U$. Since every α - closed set is semi-closed [5] and since A is semi- closed, but $scl(A) \subseteq \alpha cl(A) \subseteq U$ [5]. Thus $cl(A) \subseteq U$. Hence A is $s\alpha(gg)^*$ -closed.
- iii) Let A be any π -closed subset of space X . Let $A \subseteq U$ and U is $\alpha(gg)^*$ -open in X . Since A is π -closed, $\pi cl(A) = A \subseteq U$. But $scl(A) \subseteq \pi cl(A) \subseteq U$ [17]. Therefore $cl(A) \subseteq U$. Hence A is $s\alpha(gg)^*$ -closed.
- iv) Let A be any regular closed subset of space X . Let $A \subseteq U$ and U is $\alpha(gg)^*$ -open in X . Since A is regular closed, $rcl(A) = A \subseteq U$. But $scl(A) \subseteq rcl(A) \subseteq U$ [46]. Therefore $scl(A) \subseteq U$. Hence A is $s\alpha(gg)^*$ - closed.
- v) Let A be a $(gsp)^*$ -closed. Let $A \subseteq U$ and U is $\alpha(gg)^*$ -open. Since every $\alpha(gg)^*$ - open is gsp -open [1]. Therefore U is gsp -open. Since A is $(gsp)^*$ -closed, $cl(A) \subseteq U$. But $scl(A) \subseteq cl(A) \subseteq U$ [5]. Therefore $scl(A) \subseteq U$. Hence A is $s\alpha(gg)^*$ -closed.
- vi) Let A be a $(gs)^*$ -closed. Let $A \subseteq U$ and U is $\alpha(gg)^*$ -open. Since every $\alpha(gg)^*$ - open set is gs -open [1]. Therefore U is gs -open. Since A is $(gs)^*$ -closed, $cl(A) \subseteq U$. But we have $scl(A) \subseteq cl(A) \subseteq U$ [5]. Hence A is $s\alpha(gg)^*$ -closed.

- vii) Let A be g^*sr -closed set in topological space X and U be any $\alpha(gg)^*$ -open set in X such that $A \subseteq U$. Since every $\alpha(gg)^*$ -open set is gs -open [1]. U is gs -open in X . Since A is g^*sr -closed set, $rcl(A) \subseteq U$. But we have $scl(A) \subseteq rcl(A) \subseteq U$ [46]. Therefore $scl(A) \subseteq U$. Hence A is $s\alpha(gg)^*$ -closed.

Remark : 3.4

The converses of the above theorems need not be true in general as seen from the following example.

Example : 3.5

Let $X = \{a, b, c, d\}$, with topology $\tau = \{\emptyset, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$. Then

1. $\{c\}$ is $s\alpha(gg)^*$ -closed but not closed.
2. $\{c\}$ is $s\alpha(gg)^*$ -closed but not α -closed.
3. $\{c\}$ is $s\alpha(gg)^*$ -closed but not π -closed.
4. $\{c\}$ is $s\alpha(gg)^*$ -closed but not regular closed.
5. $\{c\}$ is $s\alpha(gg)^*$ -closed but not $(gsp)^*$ -closed.
6. $\{c\}$ is $s\alpha(gg)^*$ -closed but not $(gg)^*$ -closed.
7. $\{c\}$ is $s\alpha(gg)^*$ -closed but not g^*sr -closed.

Theorem : 3.6

- i) Every $s\alpha(gg)^*$ -closed set is gs -closed.
- ii) Every $s\alpha(gg)^*$ -closed set is gb -closed.
- iii) Every $s\alpha(gg)^*$ -closed set is gsp -closed.
- iv) Every $s\alpha(gg)^*$ -closed set is $gspr$ -closed.
- v) Every $s\alpha(gg)^*$ -closed set is g^*s^* -closed.
- vi) Every $s\alpha(gg)^*$ -closed set is sg^* -closed.
- vii) Every $s\alpha(gg)^*$ -closed set is πgsp -closed.

- viii) Every $s\alpha(gg)^*$ -closed set is rgb -closed.
- ix) Every $s\alpha(gg)^*$ -closed set is πgs -closed.
- x) Every $s\alpha(gg)^*$ -closed set is πgb -closed.
- xi) Every $s\alpha(gg)^*$ -closed set is gb^* -closed.

proof:

- i) Let A be a $s\alpha(gg)^*$ -closed set in X . Let U be an open set in X such that $A \subseteq U$. Since every open set is $\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$. Hence A is gs -closed.
- ii) Let A be a $s\alpha(gg)^*$ -closed set in X . Let U be an open set in X such that $A \subseteq U$. Since every open set is $\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$. But we have $bcl(A) \subseteq scl(A) \subseteq U$ [42]. Hence A is gb -closed.
- iii) Let A be a $s\alpha(gg)^*$ -closed set in X . Let U be an open set in X such that $A \subseteq U$. Since every open set is $\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$. But we have $spcl(A) \subseteq scl(A) \subseteq U$ [46]. Hence A is gsp -closed.
- iv) Let A be a $s\alpha(gg)^*$ -closed set in X . Let U be an open set in X such that $A \subseteq U$. Since every regular set is $\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$. But we have $spcl(A) \subseteq scl(A) \subseteq U$ [46]. Hence A is $gspr$ -closed.
- v) Let A be a $s\alpha(gg)^*$ -closed set in X . Let U be a g^* -open set in X such that $A \subseteq U$. Since every g^* -open set is $\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed $scl(A) \subseteq U$. Hence A is g^*s -closed.
- vi) Let A be a $s\alpha(gg)^*$ -closed set in X . Let U be a g^* -open set in X such that $A \subseteq U$. Since every g^* -open set is $\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$. Hence A is sg^* -closed.
- vii) Let A be a $s\alpha(gg)^*$ -closed set in X . Let U be π -open set in X such that $A \subseteq U$. Since

every π -open set is $\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$.

But we have $spcl(A) \subseteq scl(A) \subseteq U$ [46]. Hence A is πgsp -closed.

Let A be a $s\alpha(gg)^*$ -closed set in X . Let U be regular open set in X such that

$A \subseteq U$. Since every regular open set is $\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$. But we have $bcl(A) \subseteq scl(A) \subseteq U$ [52]. Hence A is rgb -closed.

A be a $s\alpha(gg)^*$ -closed set in X . Let U be π -open set in X such that $A \subseteq U$. Since every π -open set is $s\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$. Hence A is πgs -closed.

- x) Let A be a $s\alpha(gg)^*$ -closed set in X . Let U be π -open set in X such that $A \subseteq U$. Since every π -open set is $s\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$. But we have $bcl(A) \subseteq scl(A) \subseteq U$ [52]. Hence A is πgb -closed.
- xi) Let A be a $s\alpha(gg)^*$ -closed set in X . Let U be a g -open set in X such that $A \subseteq U$. Since every g -open set is $\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$. But we have $bcl(A) \subseteq scl(A) \subseteq U$ [52]. Hence A is gb_* -closed.

Remark : 3.7

The converses of the above theorems need not be true in general as seen from the following example.

Example : 3.8

Let $X = \{a, b, c, d\}$ with topology $\tau = \{\Phi, \{c\}, \{d\}, \{a, c, d\}, X\}$. Then

- i) $\{b, c, d\}$ is gs -closed but not $s\alpha(gg)^*$ -closed. ii) $\{b, c, d\}$ is gb -closed but not $s\alpha(gg)^*$ -closed. iii) $\{b, c, d\}$ is gsp -closed but not $s\alpha(gg)^*$ -closed. iv) $\{b, c, d\}$ is $gspr$ -closed but not $s\alpha(gg)^*$ -closed. v) $\{b, c, d\}$ is g^*s^* -closed but not $s\alpha(gg)^*$ -closed. vi) $\{b, c, d\}$ is sg -closed but not $s\alpha(gg)^*$ -closed. vii) $\{b, c, d\}$ is πgsp -closed but not $s\alpha(gg)^*$ -closed. viii) $\{c, d\}$ is rgb -closed but not $s\alpha(gg)^*$ -closed. ix) $\{b, c, d\}$ is πgs -closed but not $s\alpha(gg)^*$ -closed. x) $\{b, c, d\}$ is πgb -closed but not $s\alpha(gg)^*$ -closed. xi) $\{b, c, d\}$ is gb -closed but not $s\alpha(gg)^*$ -closed.

IV. INDEPENDENCY OF $s\alpha(gg)^*$ -CLOSED SETS WITH OTHER CLOSED SETS

The concept $s\alpha(gg)^*$ -closed is independent from the concepts " rw -closed", " g -

closed", " gs -closed", " R -closed", " αg -closed", " g -closed", " gbr -closed", " gar -closed", " gp -closed", " wg -closed", " $s\alpha g$ -closed", " rg -closed", " gr -closed", " g -closed", " g -closed", " gg -closed", " $(gg)^*$ -closed", " $\alpha(gg)^*$ -closed", " $\hat{g}^*s$ -closed", " R -closed".

Example: 4.1

Let $X = \{a, b, c, d\}$ with topology $\tau = \{\varphi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$.

- i) $\{a\}$ is $sa(gg)^*$ -closed but not rw -closed and $\{c, d\}$ is rw -closed but not $sa(gg)^*$ -closed.
- ii) $\{a\}$ is $sa(gg)^*$ -closed but not $g - ^*$ closed and $\{b, c, d\}$ is $g^\wedge$ -closed but not $sa(gg)^*$ -closed.
- iii) $\{a\}$ is $sa(gg)^*$ -closed but not gs_{τ} -closed and $\{b, c, d\}$ is gs_{τ} -closed but not $sa(gg)^*$ -closed.
- iv) $\{a\}$ is $sa(gg)^*$ -closed but not R_* -closed and $\{c, d\}$ is R^* -closed but not $sa(gg)^*$ -closed.
- v) $\{c\}$ is $sa(gg)^*$ -closed but not αg -closed and $\{b, c, d\}$ is αg -closed but not $sa(gg)^*$ -closed.
- vi) $\{c\}$ is $sa(gg)^*$ -closed but not $g - ^*$ closed and $\{b, c, d\}$ is g -closed but not $sa(gg)^*$ -closed.
- vii) $\{c\}$ is $sa(gg)^*$ -closed but not gpr -closed and $\{a, c, d\}$ is gpr -closed but not $sa(gg)^*$ -closed.
- viii) $\{c\}$ is $sa(gg)^*$ -closed but not gar -closed and $\{b, c, d\}$ is gar -closed but not $sa(gg)^*$ -closed.
- ix) $\{c\}$ is $sa(gg)^*$ -closed but not gp -closed and $\{b, c, d\}$ is gp -closed but not $sa(gg)^*$ -closed.
- x) $\{a, d\}$ is $sa(gg)^*$ -closed but not wg -closed and $\{b, c, d\}$ is wg -closed but not $sa(gg)^*$ -closed.

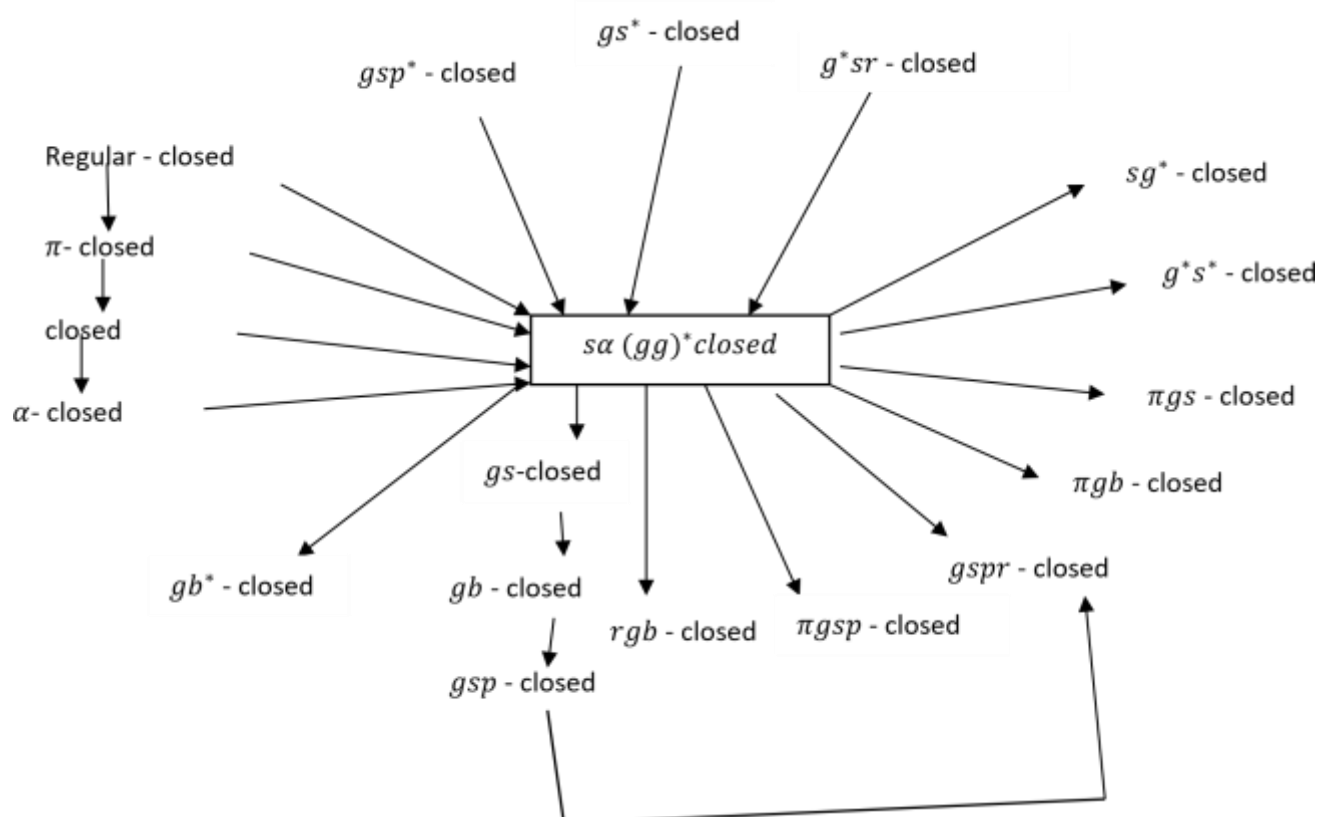
- xi) $\{c\}$ is $s\alpha(gg)^*$ -closed but not $s\alpha g_r$ -closed and $\{b, c, d\}$ is $s\alpha g_r$ -closed but not $s\alpha(gg)^*$ -closed.
- xii) $\{c\}$ is $s\alpha(gg)^*$ -closed but not rg -closed and $\{a, c, d\}$ is rg -closed but not $s\alpha(gg)^*$ -closed.
- xiii) $\{a\}$ is $s\alpha(gg)^*$ -closed but not gr -closed and $\{b, c, d\}$ is gr -closed but not $s\alpha(gg)^*$ -closed.
- xiv) $\{a\}$ is $s\alpha(gg)^*$ -closed but not g -closed and $\{b, c, d\}$ is g -closed but not $s\alpha(gg)^*$ -closed.
- xv) $\{c\}$ is $s\alpha(gg)^*$ -closed but not g -closed and $\{b, c, d\}$ is g -closed but not $s\alpha(gg)^*$ -closed.
- xvi) $\{a\}$ is $s\alpha(gg)^*$ -closed but not gg -closed and $\{c, d\}$ is gg -closed but not $s\alpha(gg)^*$ -closed.
- xvii) $\{a, c\}$ is $s\alpha(gg)^*$ -closed but not $(gg)^*$ -closed and $\{b, c, d\}$ is $(gg)^*$ -closed but not $s\alpha(gg)^*$ -closed.
- xviii) $\{c\}$ is $s\alpha(gg)^*$ -closed but not $\alpha(gg)^*$ -closed and $\{b, c, d\}$ is $\alpha(gg)^*$ -closed but not $s\alpha(gg)^*$ -closed.
- xix) $\{a\}$ is $s\alpha(gg)^*$ -closed but not $\hat{g}^*s$ -closed and $\{b, c, d\}$ is $\hat{g}^*s$ -closed but not $s\alpha(gg)^*$ -closed.
- xx) $\{c\}$ is $s\alpha(gg)^*$ -closed but not $R^\#$ -closed and $\{a, c, d\}$ is $R^\#$ -closed but not $s\alpha(gg)^*$ -closed.
- xxi) $\{c\}$ is $s\alpha(gg)^*$ -closed but not πg -closed and $\{b, c, d\}$ is πg -closed but not

$s\alpha(gg)^*$ -closed.

xxii) $\{c\}$ is $s\alpha(gg)^*$ -closed but not $\pi\alpha g$ -closed and $\{a, c, d\}$ is $\pi\alpha g$ -closed but not $s\alpha(gg)^*$ -closed.

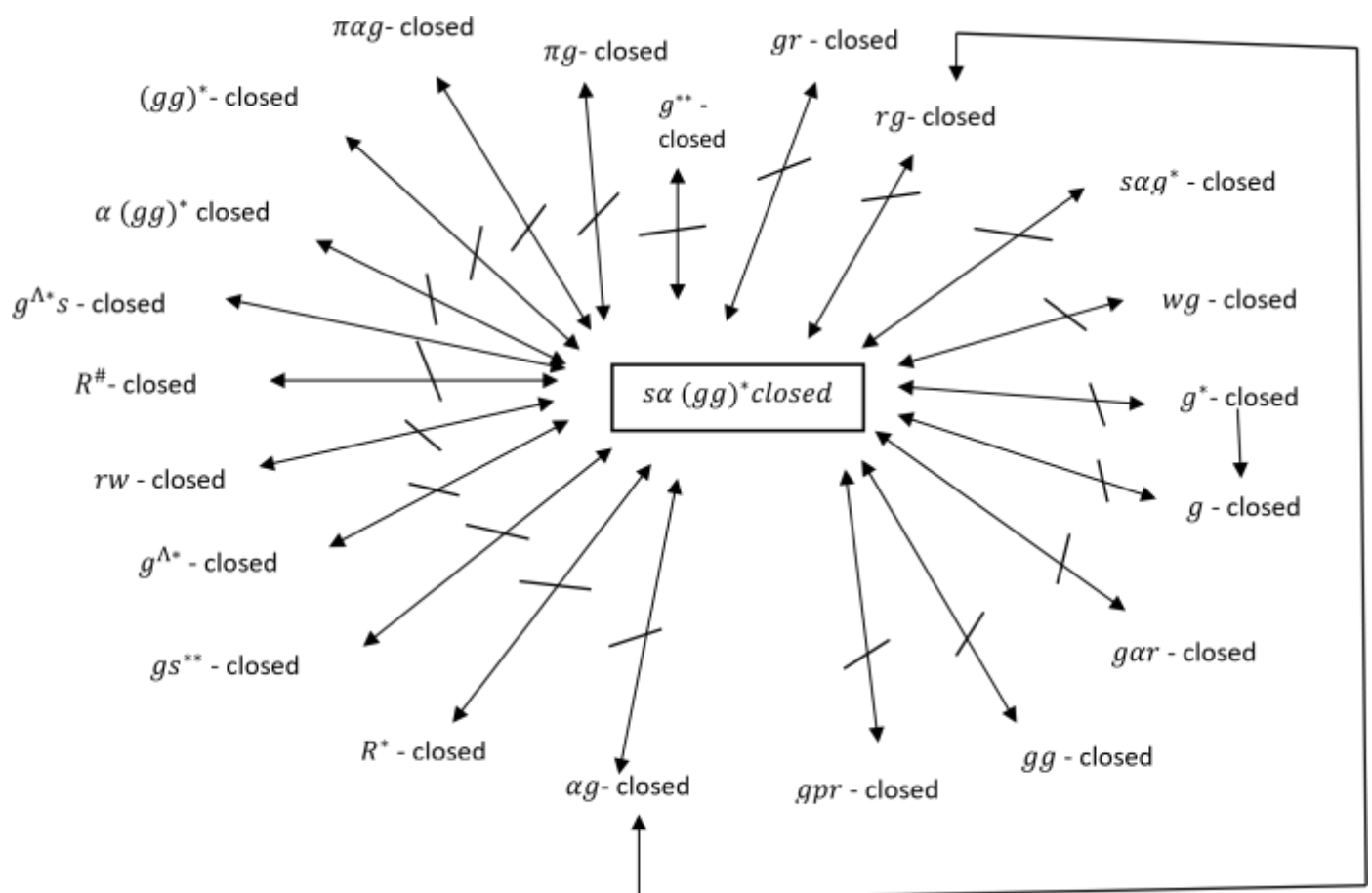
Remark : 4.2

From the above discussions and known results we have the following implications.



Notations:

$A \rightarrow B$ means, A implies B but not conversely.



Notations:

$A \leftrightarrow B$ means, the set A and the set B are independent of each other.

V. CHARACTERISTICS OF $s\alpha(gg)^*$ -CLOSED SETS

Remark: 5.1

The union of any two $s\alpha(gg)^*$ -closed sets need not be a $s\alpha(gg)^*$ -closed.

Example : 5.2

Let $X = \{a, b, c, d\}$ and $\tau = \{\Phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, X\}$. Let $A = \{c\}$ and $B = \{d\}$ be two $s\alpha(gg)^*$ -closed sets in X . But $A \cup B = \{c, d\}$ is not a $s\alpha(gg)^*$ -closed set in X .

Theorem : 5.3

Let A and B be any two $s\alpha(gg)^*$ -closed sets in a topological space X . Then $A \cap B$ is also a $s\alpha(gg)^*$ -closed set in X .

Proof :

Let A and B be any two $s\alpha(gg)^*$ -closed sets. Then $A \subseteq U$ and $B \subseteq U$, U is $\alpha(gg)^*$ -open set. Since A and B are $s\alpha(gg)^*$ -closed sets, $scl(A) \subseteq U$ and $scl(B) \subseteq U$. We know that, $scl(A \cap B) \subseteq scl(A) \cap scl(B) \subseteq U$ [17]. Therefore, $scl(A \cap B) \subseteq U$. Hence $A \cap B$ is also a $s\alpha(gg)^*$ -closed set in X .

Theorem : 5.4

Let X and Y be topological spaces and $A \subseteq Y \subseteq X$. If A is $s\alpha(gg)^*$ -closed set in X then A is $s\alpha(gg)^*$ -closed set relative to Y .

Proof :

Let $A \subseteq Y \subseteq X$ and A is $s\alpha(gg)^*$ -closed in X .

To prove : A is $s\alpha(gg)^*$ -closed relative to Y

Let $A \subseteq Y \cap U$, where U is $\alpha(gg)^*$ -open in X . since A is $s\alpha(gg)^*$ -closed, then by Definition 3.1, $scl(A) \subseteq U$. This implies $Y \cap scl(A) \subseteq Y \cap U$, where $Y \cap scl(A)$ is the semi-closure of A in Y and $Y \cap U$ is $\alpha(gg)^*$ -open in Y . Hence A is $s\alpha(gg)^*$ -closed set relative to Y .

Theorem : 5.5

If A is a subset of X which is $s\alpha(gg)^*$ -closed in X , if and only if $scl(A) \setminus A$ does not contain any non-empty $\alpha(gg)^*$ -closed set in X .

Proof:

Let A be a $s\alpha(gg)^*$ -closed set in X . Assume that, M is a non-empty $\alpha(gg)^*$ -closed subset of

$scl(A) \setminus A$. Then $M \subseteq scl(A) \cap (X \setminus A) \Rightarrow M \subseteq scl(A)$ and $M \subseteq X \setminus A$. Now $M \subseteq X \setminus A \Rightarrow A \subseteq X \setminus M$. Since A is $s\alpha(gg)^*$ -closed and $X \setminus M$ is $\alpha(gg)^*$ -open, $scl(A) \subseteq X \setminus M$. Then $M \subseteq X \setminus scl(A)$. But we have $M \subseteq scl(A)$. Therefore, $M \subseteq (X \setminus scl(A)) \cap scl(A) = \Phi$. This implies $M = \Phi$ which is contradiction to $M \neq \Phi$. Hence $scl(A) \setminus A$ does not contain any non-empty $\alpha(gg)^*$ -closed set in X . Conversely, suppose that $scl(A) \setminus A$ does not contain any non-empty $\alpha(gg)^*$ -closed set in X . Let $A \subseteq N$ and N be $\alpha(gg)^*$ -open. If $scl(A)$ is not a subset of N then $scl(A) \cap N^c$ is a non-empty $\alpha(gg)^*$ -closed subset of $scl(A) \setminus A$, which is a contradiction. Therefore, $scl(A) \subseteq N$ and hence A is $s\alpha(gg)^*$ -closed.

Theorem : 5.6

If A is a $s\alpha(gg)^*$ -closed subset and $A \subseteq B \subseteq scl(A)$, then B is also $s\alpha(gg)^*$ -closed set in X .

Proof :

Let A be a $s\alpha(gg)^*$ -closed set in X such that $A \subseteq B \subseteq scl(A)$.

To Prove : B is also a $s\alpha(gg)^*$ -closed set in X . It is enough to prove that $scl(B) \subseteq U$.

Let U be a $\alpha(gg)^*$ -open set in X such that $B \subseteq U$. Since $A \subseteq B$ and $B \subseteq U$, $A \subseteq U$. Also, since A is $s\alpha(gg)^*$ -closed, then by Definition 3.1, $scl(A) \subseteq U$. By assumption

$B \subseteq scl(A)$ this implies $scl(B) \subseteq scl(scl(A)) = scl(A) \subseteq U$. That is $scl(B) \subseteq U$. Therefore, B is $s\alpha(gg)^*$ -closed set in X .

Theorem : 5.7

If F is regular closed then $A \cap F$ is $s\alpha(gg)^*$ -closed.

Proof :

Let F be a regular closed, by Theorem 3.3(iv), F is $s\alpha(gg)^*$ -closed. Since A is $s\alpha(gg)^*$ -closed by Theorem 5.3, $A \cap F$ is $s\alpha(gg)^*$ -closed.

Theorem : 5.8

Let B be $s\alpha(gg)^*$ -closed and $s\alpha(gg)^*$ -open. Then B is semi-closed. **Proof :**

Let B be $s\alpha(gg)^*$ -closed and $\alpha(gg)^*$ -open. This implies $scl(B) \subseteq B$. Also $B \subseteq scl(B)$.

Therefore $B = scl(B)$. Hence B is semi-closed.

Theorem : 5.9

For every point x in a space X , $X - \{x\}$ is $s\alpha(gg)^*$ -closed or $\alpha(gg)^*$ -open.

Proof :

Suppose that $X - \{x\}$ is not $\alpha(gg)^*$ -open. Then X is the only $\alpha(gg)^*$ -open set containing $X - \{x\}$. Then by using Definition 3.1, $scl(X - \{x\}) \subseteq X$. Hence $X - \{x\}$ is $s\alpha(gg)^*$ -closed.

Theorem : 5.10

In an extremally disconnected space X , the union of two $s\alpha(gg)^*$ -closed sets is $s\alpha(gg)^*$ -closed.

Proof :

Let A and B be two $s\alpha(gg)^*$ -closed subsets of an extremally disconnected space X . Let $A \cup B \subseteq U$ and U be $\alpha(gg)^*$ -open. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $s\alpha(gg)^*$ -closed sets, $scl(A) \subseteq U$ and $scl(B) \subseteq U$. Therefore, $scl(A) \cup scl(B) \subseteq U \dots \dots (1)$.

Since X is an extremally disconnected space, by Lemma 2.3 (ii) $scl(A \cup B) = scl(A) \cup scl(B)$.

From (1) implies $scl(A \cup B) \subseteq U$. Hence $A \cup B$ is also $s\alpha(gg)^*$ -closed.

Theorem : 5.11

In an extremally disconnected space X , every $s\alpha(gg)^*$ -closed set is $g\alpha$ -closed.

Proof :

Let A be a $s\alpha(gg)^*$ -closed subset of X . Let $A \subseteq U$ and U be α -open. Since every α -open set is $\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$. Since X is extremally disconnected, by Lemma 2.2(i), $\alpha cl(A) \subseteq U$. Hence by Definition 2.2[50], A is $g\alpha$ -closed.

Theorem : 5.12

In an extremally disconnected space X , every $s\alpha(gg)^*$ -closed set is αg -closed.

Proof :

Let A be a $s\alpha(gg)^*$ -closed subset of X . Let $A \subseteq U$ and U is open. Since every open set is $\alpha(gg)^*$ -open [1] and since A is $s\alpha(gg)^*$ -closed, $scl(A) \subseteq U$. Since X is extremally disconnected, by Lemma 2.3(i), $\alpha cl(A) \subseteq U$. Hence by Definition 2.2[6], A is αg -closed. **Theorem : 5.13**

If A is gg -open and $(gg)^*$ -closed set in X then A is $s\alpha(gg)^*$ -closed.

Proof :

Let A be a gg -open and $(gg)^*$ -closed set in X . Let U be any $s\alpha(gg)^*$ -open set in X , such that $A \subseteq U$. Since A is gg -open and $(gg)^*$ -closed, then by definition 3.2[4], $rcl(A) \subseteq A \subseteq U$. But we have $scl(A) \subseteq rcl(A) \subseteq A$, by Theorem 3.3(iv). Therefore $scl(A) \subseteq A \subseteq U$. Thus $scl(A) \subseteq U$. Hence A is $s\alpha(gg)^*$ -closed.

Theorem : 5.14

If A is α -open and $g\alpha$ -closed set in X . Then A is $s\alpha(gg)^*$ -closed.

Proof :

Let A be an α -open and $g\alpha$ -closed set in X . Let U be any $\alpha(gg)^*$ -open set in X such that

$A \subseteq U$. Since A is α -open and $g\alpha$ -closed, then by Definition 3.2[50], $\alpha cl(A) \subseteq U$. But we have $scl(A) \subseteq \alpha cl(A) \subseteq A$, by Theorem 3.3(ii). Therefore $scl(A) \subseteq A \subseteq U$. Thus $scl(A) \subseteq U$. Hence A is $s\alpha(gg)^*$ - closed.

VI. CONCLUSION

In this paper, we intent on semi alpha generalization of generalized star closed sets in topological spaces obtained from $\alpha(gg)^*$ -open sets and semi-closure. Also, we impose a relationships among other closed sets and evaluated some of its properties. In future work , this concept can be extended and applied to more specialized structures such as nano topological spaces, fuzzy topological spaces and Bitopological spaces.

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