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The Upper Total Steiner Global Domination Number of a Graph

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Abstract
The total Steiner global dominating set W in a connected graph G is called a minimal total Steiner global dominating set of G if no proper subset of W is a total Steiner global dominating set of G. The upper Steiner hop dominating number $\overline{\gamma}_{st}(G)$ is the maximum cardinality of a minimal total Steiner global dominating set of G. Some general properties satisfied by this concept are studied. The upper total Steiner global domination number for certain standard graphs are determined. In addition, it is shown that for every pair of integers a and b with $4 \leq a \leq b$, there exists a connected graph G such that $\overline{\gamma}_{st}(G) = a$ and $\overline{\gamma}_{st}(G) = b$.

Keywords: Steiner number, Total Steiner number, Steiner domination number, Steiner global domination number, total Steiner global domination number, upper total Steiner global domination number.
AMS Subject Classification: 05C69, 05C12.

1. INTRODUCTION

By a graph $G = (V, E)$, we mean a finite, undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by n and m respectively. For basic graph theoretic terminology, we refer [1]. For every vertex $v \in V$, the *open neighborhood* $N(v)$ is the set $\{u \in G / uv \in E(G)\}$. The degree, $deg_G(v)$, of a vertex v of G is the number of edges incident with it. The degree of a vertex $v \in V$ is $deg(v) = |N(v)|$. If $e = uv$ is an edge of a graph G with $deg(u) = 1$ and $deg(v) > 1$, then we call e a *pendant edge* or *end edge* , u a *leaf* or *end vertex* and v a *support vertex* of u. The minimum degree of G, $\delta(G)$, is the smallest of the degrees of the vertices in G, and the maximum degree, $\Delta(G)$, of G is the largest of the degrees of the vertices in G. A vertex that is adjacent to every other vertex in the graph G is called *universal vertex.* The *distance* $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u - v$ path in G. The maximum distance between a vertex to all other vertices is considered as the *eccentricity* of vertex. The minimum eccentricity among the vertices of G is the *radius* $radG$ and the maximum eccentricity is its *diameter*, $diamG$ of G. Two vertices x and y are antipodal if $d(x, y) = diamG$. For any set S of vertices of G, the *induced subgraph* $\langle S \rangle$ is the maximal subgraph

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of G with vertex set S and $E(\langle S\rangle) = \{uv \in E(G): u, v \in S\}$. A vertex is said to be an *extreme vertex* if the subgraph induced by its neighbor is complete. An end vertex is an extrme vertex. A vertex v in a graph G is called a *cut-vertex* if deleting v from G increases the number of components of G. An $u - v$ path of length $d(u, v)$ is called an $u - v$ geodesic.

For a nonempty set W of vertices in a connected graph G, the *Steiner distance* $d(W)$ of W is the minimum size of a connected subgraph of G containing W. Necessarily, each such subgraph is a tree and is called a *Steiner tree* with respect to W or a *Steiner W-tree*. For a given set $W \subseteq V(G)$, there may be more than one Steiner $W -$ tree in G. Let $S(W)$ denotes the set of vertices that lies in Steiner $W -$ trees. Let G be a connected graph with at least 2 vertices. A set $W \subseteq V(G)$ is called a Steiner set of G if $S(W) = V(G)$. The *Steiner number* $s(G)$ is the minimum cardinality of its Steiner sets and any Steiner set of cardinality $s(G)$ is a *minimum Steiner* set of G. The Steiner number of a graph was studied in [2,4-7,10].

A set $S \subseteq V(G)$ is a *dominating set* of G if for every $v \in V(G)\backslash S$ is adjacent to some vertex in S. A dominating set S is said to be minimal if no subset of S is a dominating set of G. The minimum cardinality of a minimal dominating set of G is called the *domination number* of G and is denoted by $\gamma(G)$. The domination number of a graph was studied in [3].

A dominating set is called *global dominating* set if it is also a dominating set for the graph G and its complement $\bar{G}$[8-10].

Let G be a connected graph with atleast two vertices. A *total Steiner set* of G is a Steiner set W such that subgraph $\langle W\rangle$ induced by W has no isolated vertex. The minimum caarrdinality of a toal Steiner set of G is the *total Steiner number* of G and is donated by $s_t(G)$. A total Steiner set of cardinality $s_t(G)$ is called s_t-set of G or a minimum total Steiner set of G[5].

Let G be a graph. A set of vertices W in G is called *Steiner global dominating set* or simply a (S, GD)-set if W is both a Steiner and global dominating set of G. The minimum cardinality of a Steiner global dominating set of G is the *Steiner global domination number* of G is denoted by $\bar{\gamma}_s(G)$. A (S, GD)-set of size $\bar{\gamma}_s(G)$ is said to be $\bar{\gamma}_s$-set [10].

In this article we determine the upper total Steiner global domination number for some certain standard graphs. This concept helps in network designing like telecommunication networks, internet networks and also in transportation plannings, location allocation, shortest route for emergency services and helps for solving more real world problems.

The following Theorems are used in the sequel.

Theorem 1.1. [10] Each extreme vertex of a connected graph G belongs to every (S, GD)-set of G, if G is connected.

Theorem 1.2 [5] Let G is a connected graph and v an extreme vertex of G. Then every total Steiner set of G contains at least one element of $N(v)$.

2. The upper total Steiner global dominating number of a graph

Definition 2.1. Let W is a $\overline{\gamma}_{st}$ -set of G. A subset W of vertices in a graph G is called *total Steiner global dominating set* of G if W is both total Steiner set and global dominating set of G. The

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minimum cardinality of a total Steiner global dominating set of G is its *total Steiner global domination number* $\overline{\gamma}_{st}\,(G)$. A total Steiner global domination set of size $\overline{\gamma}_{st}\,(G)$ is said to be a $\overline{\gamma}_{st}$ -set of G.

Definition 2.2. A total Steiner global dominating set W in a connected graph G is called *a minimal total Steiner global dominating set* of G if no proper subset of W is a total Steiner global dominating set of G. The *upper total Steiner global dominating number* $\overline{\gamma}_{st}^{+}\,(G)$ is the maximum cardinality of a minimal total Steiner global dominating set of G.

Example 2.3. For the graph G given in Figure 1, $W_1 = \{v_2, v_3, v_4, v_7, v_8, v_9, v_{12}, v_{13}, v_{14}\}, W_2 = \{v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{14}, v_{15}\}$ are the minimum total Steiner global dominating sets of G so that $\overline{\gamma}_{st}\,(G) = 9$ and $\overline{\gamma}_{st}^{+}\,(G) = 12$.

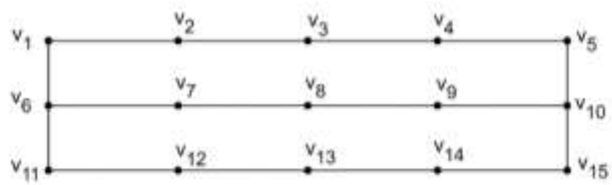

Figure 1

Observation 2.4

Let G be a connected graph of order $n \geq 2$. Then

(a) Each extreme vertex of a graph G belongs to every minimal total Steiner global dominating set of G. In particular, each end-vertex of G belongs to every minimal total Steiner global dominating set of G.
(b) Each universal vertex of G belongs to every minimal total Steiner global dominating set of G.
(c) Let G is a connected graph and v an extreme vertex of G. Then every minimal total Steiner global dominating set of G contains at least one element of $N(v)$.
(d) Let G be a connected graph and v be a cut vertex of G. If W is a minimal total Steiner global dominating set of G, then every component of $G - v$ contains as element of W.
(e) For a connected graph G of order $n \geq 2$, $2 \leq \overline{\gamma}_{st}\,(G) \leq \overline{\gamma}_{st}^{+}\,(G) \leq n$.
(f) For a connected graph G of order $n \geq 2$, $\overline{\gamma}_{st}\,(G)$ if and only if $\overline{\gamma}_{st}^{+}\,(G) = n$.
(g) For the complete bipartite graph $K_{r+s}\,(1 \leq r \leq s), \overline{\gamma}_{st}^{+}\,(G) = r + s$.

Theorem 2.5. For the complete graph $K_n\,(n \geq 2), \overline{\gamma}_{st}^{+}\,(G) = n$.

Proof: Let $G = K_n$ be the complete graph of order $n \geq 2$. Since $W = V(G)$, W is a Steiner set of G. Since every vertex of $\bar{G}$ is isolated vertex, W dominates G and $\bar{G}$. Therefore, W is a total Steiner global dominating set of G and W is a minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}\,(G) = n$.

Theorem 2.6. For the path $P_n\,(n \geq 2)$,

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$$\overline{\gamma}_{st}^{+}(G)=\begin{cases} n \; if \; n=2 \; or \; n=3 \\ 4 \; if \; n=5 \; or \; n=6 \\ 5 \; if \; n=7 \\ 2k+2 \; if \; n=4k \\ 2k+5 \; if \; n=4k+5 \; or \; 4k+6 \\ 2k+6 \; if \; n=4k+7, where \; k\geq 1. \end{cases}$$

Proof: Let $G = P_n$ be $v_1, v_2, \dots, v_n$.

For $n = 2$ and $n = 3$. Then $W = V(G)$ is the only minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G) = n$.

Case 1. Let $n = 5$ and $n = 6$. Then $W_1 = \{v_1, v_2, v_{n-1}, v_n\}$ is the only minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G) = 4$.

Case 2. Let $n = 7$. Then $W_2 = \{v_1, v_2, v_3, v_6, v_7\}$ and $W_3 = \{v_1, v_2, v_5, v_6, v_7\}$ are the minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G) = 5$.

Case 3. Let $n = 4k$ and $k \geq 1$. For $k = 1$, then $W_4 = \{v_1, v_2, v_3, v_4\}$ is the only minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G) = 4$. So let $k \geq 2$, then $W_5 = \{v_1, v_2, v_5, v_6 \dots, v_{4k-3}, v_{4k-2}\} \cup \{v_{4k-1}, v_{4k}\}$ is a total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G) \geq 2k + 2$. We demonstrate that W_5 is a minimal total Steiner global dominating set of G. On the contrary, suppose that W_5 is not a minimal total Steiner global dominating set of G. Then there exists a total Steiner global dominating set W_5' of G such that $|W_5'| \subset W_5$. Then there exists a vertex $x \in W_5$ such that $x \notin W_5'$. If $x = v_{4k-1}$, then $x \notin I(W_5')$, which is a contradiction. If $x \in \{v_1, v_2, v_5, v_6 \dots, v_{4k-3}, v_{4k-2}\} \cup \{v_{4k}\}$ then $\langle W_5' \rangle$ has isolated vertices, which is a contradiction. Therefore W_5' is not a total Steiner global dominating set of G. Hence it follows that W_5 is a minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G) \geq 2k + 2$. Similarly we can demonstrate that another minimal total Steiner global dominating set $W_6 = \{v_1, v_2\} \cup \{v_3, v_4, v_7, v_8, \dots, v_{4k-1}, v_{4k}\}$ is $\overline{\gamma}_{st}^{+} - set$ of G. We demonstrate that W_6 is a minimal total Steiner global dominating set of G. On the contrary, suppose that $\overline{\gamma}_{st}^{+}(G) \geq 2k + 3$. Then there exists a minimal total Steiner global dominating set W_6' of G such that $\overline{\gamma}_{st}^{+}(G) \geq |W_6'| \geq 2k + 3$. Since $|W_6'| \geq 2k + 3$, it follows that W_5, W_6 and $W_5'$are proper subset of W_6', which is a contradiction. Hence $\overline{\gamma}_{st}^{+}(G) = 2k + 2$.

Case 4(a). Let $n = 4k + 5$ ($k \geq 1$). Then $W_7 = \{v_1, v_2\} \cup \{v_3, v_4, v_7, v_8 \dots, v_{4k-2}, v_{4k-1}, v_{4k}\}$. Then as in the Case 3, we can demonstrate that W_7 is a $\overline{\gamma}_{st}^{+} - set$ of G resulting in $\overline{\gamma}_{st}^{+}(G) = 2k + 5$.

Case 4(b). Let $n = 4k + 6$ ($k \geq 1$). Then $W_8 = \{v_1, v_2\} \cup \{v_4, v_5, v_8, v_9 \dots, v_{4k-2}, v_{4k-1}, v_{4k}\}$. Then as in the Case 3, we can demonstrate that W_8 is a $\overline{\gamma}_{st}^{+} - set$ of G resulting in $\overline{\gamma}_{st}^{+}(G) = 2k + 5$.

Case 5. Let $n = 4k + 7$ ($k \geq 1$). Then $W_9 = \{v_1, v_2, v_4, v_5, \dots, v_{4k-1}, v_{4k}\}$. Then as in the Case 3, we can demonstrate that W_9 is a $\overline{\gamma}_{st}^{+} - set$ of G resulting in $\overline{\gamma}_{st}^{+}(G) = 2k + 6$.

Theorem 2.7. For the cycle C_n ($n \geq 3$),

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$$\overline{\gamma}_{st}^{+}(G)=\begin{cases}3 \; if \; n=3\\ 4 \; if \; n=4 \; or \; n=6 \; or \; n=8\\ 5 \; if \; n=5 \; or \; n=7 \; or \; n=9\\ 2k+5 \; if \; n=4k+6 \; or \; 4k+7 \; or \; n=4k+8\\ 2k+6 \; if \; n=4k+9, where \; k\geq 1.\end{cases}$$

Proof: Let $G=C_n$ be $v_1,v_2,\ldots,v_n,v_1$.

For $n=3$, by the result of the complete graph K_n $(n\geq 2)$, $\overline{\gamma}_{st}^{+}(G)=n$. Therefore, $\overline{\gamma}_{st}^{+}(G)=3$.

Case 1(a). For $n=4$, then $W=V(G)$ is the only minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G)=4$.

Case 1(b). Let $n=6$. Then every pair of antipodal vertices of G is a Steiner set of G. Since every pair of antipodal vertices contains at least one adjacent vertex, resulting in $W_2=\{v_1,v_2,v_4,v_5\}$ is a minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G)=4$.

Case 1(c). Let $n=8$. Then, as in the Case 1(b), $W_3=\{v_1,v_2,v_5,v_6\}$ is a minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G)=4$.

Case 2(a). For $n=5$, then $W=V(G)$ is the only minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G)=5$.

Case 2(b). Let $n=7$. Then, as in the Case 1(b), $W_4=\{v_1,v_2,v_4,v_5,v_7\}$ is a minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G)=5$.

Case 2(c). Let $n=9$. Then, as in the Case 1(b), $W_5=\{v_1,v_2,v_5,v_6,v_9\}$ is a minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G)=5$.

Case 3(a). Let $n=4k+6$ and $k\geq 1$. Then $W_6=\{v_1,v_2\}\cup\{v_4,v_5,v_8,v_9\ldots,v_{4k-6},v_{4k-5},v_{4k-4},v_{4k-3},v_{4k-2}\}$ is a minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G)\geq 2k+5$. We demonstrate that W_6 is a minimal total Steiner global dominating set of G. On the contrary, suppose that W_6 is not a minimal total Steiner global dominating set of G. Then there exists a total Steiner global dominating set $W_6{}'$ of G such that $|W_6{}'|\subset W_6$. Then there exists a vertex $x\in W_6$ such that $x\notin W_6{}'$. If $x=v_4$, then $x\notin I(W_6{}')$, which is a contradiction. If $x\in\{v_1,v_2\}\cup\{v_5,v_8,v_9\ldots,v_{4k-6},v_{4k-5},v_{4k-4},v_{4k-3},v_{4k-2}\}$ then $\langle W_6{}'\rangle$ has isolated vertices, which is a contradiction. Therefore $W_6{}'$ is not a total Steiner global dominating set of G. Hence it follows that W_5 is a minimal total Steiner global dominating set of G resulting in $\overline{\gamma}_{st}^{+}(G)\geq 2k+5$. We demonstrate that W_6 is a minimal total Steiner global dominating set of G. On the contrary, suppose that $\overline{\gamma}_{st}^{+}(G)\geq 2k+6$. Then there exists a minimal total Steiner global dominating set W_7 of G such that $\overline{\gamma}_{st}^{+}(G)\geq|W_7|\geq 2k+6$. Since $|W_7|\geq 2k+6$, it follows that W_6 and $W_6{}'$are proper subset of W_7, which is a contradiction. Hence, $\overline{\gamma}_{st}^{+}(G)=2k+5$.

Case 3(b): Let $n=4k+7$ ($k\geq 1$). Then $W_8=\left\{v_1,v_2,v_5,v_6,v_7,v_{8,}\ldots,v_{4k-4},v_{4k-3},v_{4k}\right\}$. Then as in the Case 3(a), we can demonstrate that W_8 is a $\overline{\gamma}_{st}^{+}-set$ of G resulting in $\overline{\gamma}_{st}^{+}(G)=2k+5$.

Case 3(c): Let $n=4k+8$ ($k\geq 1$). Then $W_9=\{v_1,v_2\}\cup\{v_3,v_4,v_8,v_9,\ldots,v_{4k-4},v_{4k-3},v_{4k-2}\}$. Then as in the Case 3(a), we can demonstrate that W_9 is a $\overline{\gamma}_{st}^{+}-set$ of G resulting in $\overline{\gamma}_{st}^{+}(G)=2k+5$.

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Case 4: Let $n = 4k + 9$ ($k \geq 1$). Then $W_{10} = \{v_1, v_2\} \cup \{v_5, v_6, v_7, v_8, v_{11}, v_{12}, \dots, v_{4k-2}, v_{4k-1}\}$. Then as in the Case 3(a), we can demonstrate that W_{10} is a $\overline{\gamma}_{st}^{+} - set$ of G resulting in $\overline{\gamma}_{st}^{+}(G) = 2k + 6$.

Theorem 2.8. Let G be a connected graph of order $n \geq 2$, then $\overline{\gamma}_{st}^{+}(G \odot K_1) = 2n$.
Proof: Let $Z = \{z_1, z_2, \dots, z_n\}$ be the set of all end vertices of G. Then by Theorem1.1 and 1.2, Z is the subset of every total Steiner global dominating set of G and the adjacent vertices z_i $(1 \leq i \leq n)$ belongs to each total Steiner global dominating set of G and so $\overline{\gamma}_{st}^{+}(G \odot K_1) \geq 2n$. Hence it follows that $W = V(G \odot K_1)$ is the unique total Steiner global dominating set of G. Therefore, $\overline{\gamma}_{st}^{+}(G \odot K_1) = 2n$.

Theorem 2.9. For any positive integers a and b with $4 \leq a \leq b$, there exists a connected graph G such that $\overline{\gamma}_{st}(G) = a$ and $\overline{\gamma}_{st}^{+}(G) = b$.
Proof: Let $H = K_{a-3}$ and $F = K_{b-a+1}$ with $V(K_{a-3}) = \{z_1, z_2, \dots, z_{a-3}\}$ and $V(K_{b-a+1}) = \{h_1, h_2, \dots, h_{b-a+1}\}$. Let $P: v_1, v_2, v_3, v_4$ be a path of order four. Let G be a graph obtained from H and F, by joining each vertex z_i $(1 \leq i \leq a-3)$ with v_1 and v_2. Also, by joining each vertex h_i $(1 \leq i \leq b-a+1)$ with v_1 and v_4. The graph G is shown in Figure 2.

First, we prove that $\overline{\gamma}_{st}(G) = a$. Let $Z = V(K_{a-3})$ is the set of all extreme vertices of G. By Theorem 1.1, Z is a subset of every total Steiner global dominating set of G and so $\overline{\gamma}_{st}(G) \geq a-3$. Since $S(W) \neq V(G)$, Z is not a total Steiner global dominating set of G and so $\overline{\gamma}_{st}(G) \geq a-2$. It is easily observed that $Z \cup \{x\}, x \notin Z$ or $Z \cup \{x, y\}$ $x, y \notin Z$ is not a total Steiner global dominating set of G and so $\overline{\gamma}_{st}(G) \geq a$. Let $W = Z \cup \{v_1, v_3, v_4\}$. Then $S(W) = V(G)$ and W is a dominating set in G and $\overline{G}$. Hence it follows that W is a Steiner global dominating set of G. Since $\langle W \rangle$ has no isolated vertices. Therefore, W is a total Steiner global dominating set of G so that $\overline{\gamma}_{st}(G) = a$.

Next, we prove that $\overline{\gamma}_{st}^{+}(G) = b$. Let $W_1 = \{v_2, v_3\} \cup Z \cup V(K_{b-a+1})$. Then W_1 is a total Steiner global dominating set of G. We prove that W_1 is a minimal total Steiner global dominating set of G. On the contrary suppose that W_1 is not a minimal total Steiner global dominating set of G. Then there exists a $W_2 \subset V(G)$ such that W_2 is a total Steiner global dominating set of G and $W_2 \subset W_1$. Let $x \in W_1$ and $x \notin W_2$. By Observation 2.4 (a), $x \neq z_i$ $(1 \leq i \leq a-3)$. If $x = h_i$ for some i $(1 \leq i \leq b-a+1)$, then $x \notin S(W_2)$. If x is either v_3 or v_4, then $x \notin S(W_2)$. Hence it follows that W_2 is not a total Steiner global dominating set of G, which is a contradiction. Therefore, W_1 is a minimal total Steiner global dominating set of G and so $\overline{\gamma}_{st}^{+}(G) \geq b$. We prove that $\overline{\gamma}_{st}^{+}(G) = b$. On the contrary suppose that $\overline{\gamma}_{st}^{+}(G) \geq b+1$. Then there exists a minimal total Steiner global dominating set of G W_3 such that $|W_3| \geq b+1$. Since $|V(G)| = b+2$, $|W_3| = b+2$. Hence $\langle W_3 \rangle$ is connected which implies $S(W_3) = W_3 \neq V(G)$, which is a contradiction. Therefore, W_3 is not a minimal total Steiner global dominating set of G. Hence W_1 is a total Steiner global dominating set of G so that $\overline{\gamma}_{st}^{+}(G) = b$.

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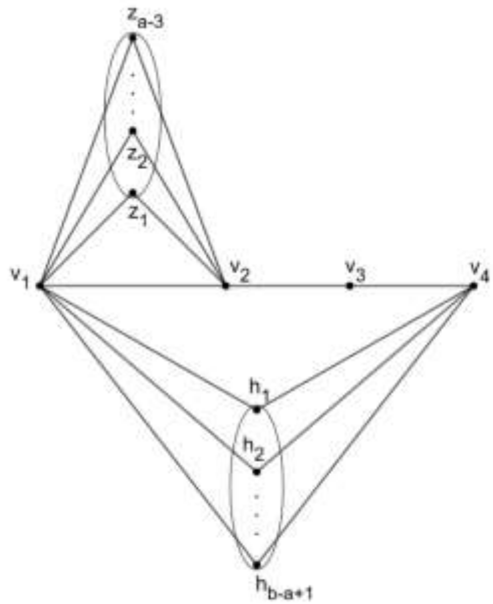

Figure 2

3. CONCLUSION

In this article, the upper total Steiner global domination number for some standard graphs were observed and determined and a realization result is given. This can be extended with other parameters of graphs in further studies.

REFERENCES

[1] F. Buckley and F. Harary, Distance in Graphs, *Addition - Wesley Publishing Company*, Redwood City, CA, 1990.

[2] G. Chartrand and P. Zhang, The Steiner number of a graph, *Discrete Mathematics*, vol. 242, pp. 41 – 54, 2002.

[3] Haynes. T. W, Hedetniemi. S.T and Slater. P. J, Fundamentals of Domination in Graphs, *Marcel Dekker*, NewYork, 1998.

[4] J. John, The vertex Steiner number of a graph, *Transactions on Combionatorics*, vol. 9, no. 02, pp. 115-124, 2020.

[5] J. John, The total Steiner number of a graph, *Discrete Mathematics, Algorithms and Applications*, vol.12, no.03, 2050038, 2020.

[6] J. John and M.S. Malchijah Raj, The upper restrained Steiner number of a graph, *Discrete Mathematics Algorithms and Applications*, vol.12, no.01, 2050004, 2020.

[7] J. John and M.S. Malchijah Raj, Graphs with Large Steiner Number, *Ukrainian Mathematical Journal*, vol. 76, pp.805-815, 2024.

[8] S. Kalaiselvi and R. Jeyamani, A Review on Global Domination Number of a Graph, *International Jounrnal of Mathematics Trends and Technology (IJMTT)*, vol. 44, no.2, pp.58-60, 2017.

[9] E. Sampathkumar, The global domination number of a graph, J. Math. Phys.Sci., vol. 23, pp.377-385, 1989.

10.48047/jocaaa.2024.33.07.100

[10] J. Suja and V. Sujin Flower, The Steiner Global Domination Number of a Graph, *Library Progress International*, vol.44, no.3, pp.27998-28003, 2024.