Analysis Of Nonlocal Thermo-Mechanical Vibration Of Functionally Graded Carbon Nanotube Reinforced Nanorod Under The Influence Of External Magnetic Field

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ABSTRACT
Thermo-mechanical vibration of carbon nanotube (CNT) reinforced functionally graded nanorods are explored in this work under the external influence of magnetic field. The nanorod is modelled based on Love-Bishop rod theory and the small-scale effects are considered based on nonlocal elasticity theory. Axial gradation of CNT reinforcement is modelled in the formulation. The complete governing differential equation of motion of axially graded CNT reinforced composite nanorod is derived and solved for the clamped-clamped and clamped-free boundary conditions. Closed form expressions are derived for the nonlocal frequencies of nanorods. The effects of material properties, temperature, magnetic field, aspect ratio, external stiffness and inhomogeneity parameters on nonlocal frequencies of CNT reinforced nanorods were analyzed thoroughly. The results presented in this work are very useful for the design of futuristic nanodevices where the CNT reinforced nanorod is the primary element.

Keywords: Love-Bishop theory; Magnetic field; CNT reinforced composite; Nonlocal elasticity and Inhomogeneity

1. INTRODUCTION

One-dimensional materials such as carbon nanotubes, boron nitride nanotubes, silicon carbide nanotubes, nanowires, and nanorods are largely responsible for the revolutionary uses of nanotechnology in contemporary technology. Due in large part to the contributions of nanotechnology, these materials have set the foundation for innovative technological developments. Miniaturization is the major trend in the world today, especially when it comes to materials and devices like micro/nano electro-mechanical systems (MEMS/NEMS). Understanding the characteristics of nano and microscale structures is becoming more and more important due to the development of imaging technologies that are specifically designed for very small sizes. Investigating the material properties of nanoscale structures is the current emphasis of nanotechnology. The main goal now is to incorporate these qualities into traditional disciplines.

Going on to the study of Danilo et al.[1], they evaluated the impact of shear stiffness and lateral motion inertia on the longitudinal vibrations of a novel resonator model. Using a Love-Bishop theory technique, Civalek et al.[2] developed four different nanorod models with accuracy comparable to numerical methods across various nonlocal parameters. Lastly, by taking into account constrained boundary conditions at one end and attachment to the nonlinear spring at the other, Bahrami et al.[3] investigated the relationship between the stiffness of a nonlinear spring and the natural frequency of a functionally graded nanorod. This appears to be a varied collection of research that offers insightful information to the area.

Long et al. [4] discovered that the distribution of carbon nanotubes (CNT) in face sheets has little effect on the thermal post-buckling reaction and buckling temperatures of sandwich plates. Touloukian [5] illustrated the effect of temperature and longitudinal magnetic fields on the natural frequency and thermal buckling of composite plates, both porous and non-porous. Vodenitcharova and Zhang [6] carefully examined the effects of length scale, magnetic field, and mode shape on the flexural vibration response of nanobeams, offering critical insights for the construction of magnetically precise nanomotors. Karličić et al. [7] found that nonlocal axial vibration in Bishop nanorod systems is critical for designing and analyzing nano resonator devices and nanoelectromechanical systems (NEMS). Li X-F et al. [8] used nonlocal elasticity theory, which accounts for small-scale effects, to provide a more accurate depiction of the dynamic behavior of nanorods than classical models. Behar et al. [9] and Atashafrooz et al. [10] used Galerkin-based closed-form methods to solve the equations of motion and forecast the vibration response of a spinning and surface effect on a smart nanotube under electrical loads. Recognize that external voltage affects critical speed, structural integrity, vibration, instability, and mechanical behavior of nanotubes that transport fluid. Barretta et al. [11] investigated the stress-driven nonlocal integral theory to describe nano-sensors and nano-actuators. Wan and Ma [12] investigated the thermodynamic behavior of functionally graded rotating piezoelectric rods in the presence of moving heat sources. Nonlocal nanorods are an important topic of research in nanotechnology due to their distinct mechanical properties and applications in nanoelectromechanical systems (NEMS). Hani et al. [13], Moustafa et al. [14], and Gennadi and Mikhasev [15] used nonlocal elasticity theory, specifically Eringen's nonlocal theory, to examine these features, accounting for the small-scale effects that classical theories typically neglect. Busra et al. [16] tested this theory to a variety of issues, including longitudinal vibrations of nanorods, and shown that raising the nonlocal parameter lowers natural frequencies, particularly at higher modes. Mohammad Ali et al. [17] obtained counterintuitive findings in differential models by ensuring well-posed and consistent elastic issues over confined regions. Furthermore, Ashraf M and Zenkour [18] and Babak et al. [19] demonstrated that the spectral properties and dynamic behavior of nanorods can be effectively analyzed using port-Hamiltonian formulations and nonlocal differential models, revealing the hyperbolic nature of these systems as well as their inherent control and stabilization challenges. Chinnawut et al. [20] and Yuan et al. [21] investigated the free vibration analysis of nanorods, including those with functionally graded materials, and discovered that material qualities and boundary conditions have a substantial influence on their vibrational characteristics. Furthermore, Hanif et al. [22] employed the nonlocal strain gradient theory to investigate torsional vibrations in nanorods with noncircular cross-sections, emphasizing the significance of cross-sectional geometry in their dynamic response. The buckling behavior of nanorods, taking into account of shear and normal deformations, is also important, with different end conditions influencing the critical buckling loads. Finally, advanced approaches such as the Wentzel Kramers Brillouin approximation method and the Galerkin method are used to solve complex boundary-value problems and account for nonlocal effects on nanorod eigen frequencies. Overall, including nonlocal theories into the study of nanorods provides a thorough understanding of their mechanical behavior, which is critical for their application in advanced nanotechnologies. The investigation of nonlocal effects in nanorods exposed to magnetic fields is a diverse topic of study that combines theoretical and experimental approaches. Keivan and Kamil [23] and Danilo et al. [24] used the nonlocal elasticity theory to study the free vibration of nanorods and discovered that nonlocal parameters, surface energy, and magnetic field strength all have a significant influence on the natural frequencies and vibrational modes, particularly in the presence of defects and different boundary conditions. Von et al. [25] and Vladimir et al. [26] found that the interaction of optical fields with nanostructures, which cannot be adequately described by the dipole approximation, necessitates the use of non-local response functions that include all multipoles, resulting in a more comprehensive understanding of optical processes at the nanoscale. Experimental studies on magnetoresistance in nanostructures, such as Pt strips on yttrium iron garnet and multi-terminal Ni nanostructures, have been conducted by Prasoon and Saurabh [27] and Rüffer et al. [28]. Their findings show that non-local magnetoresistance can differ significantly from local magnetoresistance, with the former being influenced by elements such as magnon spin accumulation and current spreading in non-isotropic conductors. Analytical findings and three-dimensional particle-in-cell simulations corroborate Zsolt and Alexander's [29] identification of the effects of magnetic fields in nano-structured targets, such as nanorods arranged in forests, which demonstrates that self-generated quasi-static magnetic fields can reach amplitudes up to 1 MT, which is supported by both analytical results and three-dimensional particle-in-cell simulations. In order to precisely explain the behavior of nanorods in magnetic fields, Xiong et al. [30] discovered the significance of nonlocal effects and sophisticated modeling tools. This is important for the improvement of laser ion acceleration technologies as well as the development of novel spintronic devices. All things considered, the combination of sophisticated optical modeling, experimental magnetoresistance research, and nonlocal elasticity theory offers a thorough foundation for comprehending and adjusting the magnetic characteristics of nanorods.

The vibration analysis of functionally graded carbon nanotube-reinforced nanorods (FG-CNTR) is a complicated topic that has been studied using a variety of theoretical and numerical methods. These nanorods have improved mechanical properties due to the integration of carbon nanotubes (CNTs) within a polymeric matrix, which are frequently functionally graded (FG) across their thickness. Eddine et al. [31] and Wang et al. [32] utilized refined third-order shear deformation theory (TSDT), while higher-order shear deformation theory (HSDT) is commonly used to capture shear deformation effects without the need for shear correction factors, ensuring accurate modeling of vibration behavior. Ezzraimi et al. [33] and Yu X L et al. [34] assessed the mechanical properties of CNTs using the rule of mixing or modified rule of mixture, which takes into consideration the nonlinear distribution of CNTs across the thickness, resulting in increased stiffness and higher natural frequencies. Huan et al. [35], Manish and Sarangi [36], and Vishal et al. [37] used analytical and numerical methods such as the Differential Quadrature Finite Element Method (DQFEM) and the Rayleigh-Ritz method to solve the governing equations derived from Lagrange's and Hamilton's principles, ensuring high convergence speed and numerical stability. Surya and Sahoo [38], Ravindra and Ashok [39], and Tham et al. [40] investigated how CNT volume percentage, distribution type, and boundary conditions effect dynamic performance, including natural frequencies and loss factors. Furthermore, the combination of piezoelectric materials and FG-CNTR nanorods has showed promise in active vibration control, allowing feedback algorithms to efficiently govern dynamic reactions. Studies have also investigated the impacts of fluid-structure interaction, in which the nanorods are submerged in a fluid media, complicating the vibration analysis due to mass effects. Overall, the analysis of FG-CNTR nanorod vibration behavior takes a multifaceted approach, combining advanced theoretical models, numerical methodologies, and parametric investigations to maximize their dynamic performance for actual engineering applications.

Inhomogeneity factors have a major impact on the mechanical and vibrational properties of functionally graded nanorods. These factors, which are frequently represented by a power-law index, control the fluctuation of material properties over the length or thickness of the nanorod. Reza and Hassan [41] investigated axially functionally graded nanorods. Surface energy factors such as surface stress, surface density, and surface Lame constants are important because they affect the non-homogeneous governing equations of motion and the resulting nanorod frequencies. Similarly, Shishesaz and Hosseini [42] investigated the mechanical behavior of functionally graded nano-cylinders under radial pressure and found that the material inhomogeneity index has a substantial impact on radial and circumferential stress. Shishesaz et al. [43] investigated functionally graded nanodisks, finding that the material inhomogeneity parameter effects the magnitudes and peak values of high-order stresses, particularly under heat loads. Changjian et al. [44] found that the vibration behavior of functionally graded Euler nanobeams decreases with an increase in the gradient index, demonstrating the susceptibility of vibrational features to material gradation. Ebrahimi and Barati [45] Furthermore, the gradient index influences the buckling and free vibration of functionally graded nanobeams sitting on elastic foundations, changing the nanobeam's shear deformation and overall stability. Arefi [46] discovered that the wave propagation characteristics in functionally graded piezoelectric nanorods are similarly influenced by material property gradation, affecting the electromechanical response. Dang-Van et al. [47] discovered that the nonlinear vibration responses of functionally graded nanobeams are also reliant on the power-law index, which influences the nonlinear frequency and overall dynamic behavior. Furthermore, Aydogdu et al. [48] observed that axial grading in nanorods and beams, modeled using stress gradient elasticity theory, exhibits significant changes in frequency when compared to constant nonlocal parameter situations. Pham [49] discovered that variations in nonlocal characteristics, which depend on the material constituents across the thickness, have a considerable impact on the free vibration response of functionally graded nanoplates. Finally, inhomogeneity factors play an important role in determining the mechanical and vibrational properties of functionally graded nanorods, which influence their performance in a variety of applications.

As previously stated, many researchers have conducted thermo-mechanical vibration analyses of functionally graded carbon nanotube reinforced composite rods in the literature. However, none of the researchers have studied the combined impacts of inhomogeneity parameters, stiffness, and magnetic forces on the micro/nano rod. For the first time, an analytical equation for a carbon nanotube reinforced nanorod with distinct physical properties under various boundary circumstances is examined. These findings help to develop engineering applications in magneto-mechanical systems.

Nomenclature

L	length of the nanorod
d	diameter of the nanorod
W_{CNT}	mass fraction of CNT
ρ_{CNT}	density of CNT
ρ_m	mass density
k, p	inhomogeneity parameters
η_1, η_3	efficiencies of CNT reinforced composite rod
V_{CNT}, Vp	volume fractions of CNT and matrix
E_{11}^{CNT}	Young's modulus of CNT in X-direction
G_{12}^{CNT}	shear modulus of CNT in X-direction
E_P	youngs modulus of polymer matrix
G_P	shear modulus of polymer matrix
ρ	mass density of the nanocomposite rod
ν_{12}	equivalent Poisson's ratio of the nanocomposite rod
α_{11}	coefficient of linear thermal expansion of the nanocomposite rod
T_0	ambient temperature
ΔT	temperature difference
P_0, P_{-1}, P_1, P_2 and P_3	temperature coefficients
u	component of motion equation along X axes
v	component of motion equation along Y axes
w	component of motion equation along Z axes
ν	poison's ratio
t	time constant
σ_{ij}	nonlocal stress
C_{ijkl}	fourth order elasticity tensor
$e_0 a$	nonlocal scale parameter
K_m	stiffness of the external elastic medium
F	applied external force per unit length
A	cross-sectional area of the nanocomposite rod
S_{xx}, S_{yy}, S_{zz}	classical axial stresses
S_{xy}, S_{yz}, S_{xz}	classical shear stresses
I_P	polar moment of inertia
F_m	external magnetic force acting on the CNT reinforced composite nanorod
F_E	external elastic force acting on the CNT reinforced composite nanorod
λ	mode number
H_x, H_y	magnetic flux along X and Y direction
ω	natural frequency of the system
C-C	clamped-clamped
C-F	clamped-free

2. Mathematical Formulation

2.1. Configuration of CNT reinforced composite nanorod

A schematic of CNT reinforced composite nanorod considered in the present problem is shown in Fig 1. The longitudinal and lateral directions of the composite rod are represented by the X- Y- and Z-axes, respectively. The length of nanorod is L and its diameter is d.

The total volume fraction of CNTs (V_{CNT}^*) is calculated using the following empirical equation [4]

$$V_{CNT}^* = \frac{W_{CNT}}{W_{CNT} + \left({}^{\rho_{CNT}}/_{\rho_m}\right)(1 - W_{CNT})} \tag{1}$$

where W_{CNT}, ρ_{CNT} and ρ_m are the mass fraction, density, and mass density, respectively, of the polymer matrix.

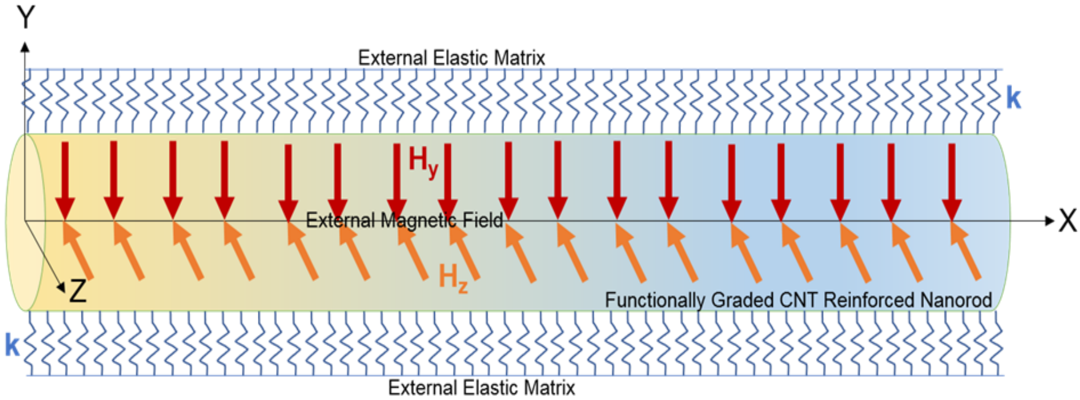


Fig 1. A schematic of CNT-reinforced composite nanorod under the effects of external elastic matrix and magnetic field

2.2 Axial Functional Gradation

The composite nanorod is composed of single-walled carbon nanotubes, whose distribution is smoothly graded in the X-direction from the right surface with 100% CNTs to the left surface with 100% epoxy. Using the power law function with two inhomogeneity parameters k and p, the range is from 0 to 1 (0≤p≤1 and 0≤k≤1). Where k secures the CNT's intensity along the length direction, while the inhomogeneity parameter p ensures the parabolic distribution as shown in Fig. 2.

The volume fraction of axially functionally graded CNTRC can be expressed as

$$V_{CNT} = 2k\left(\frac{x}{L}\right)^{p} V_{CNT}^{*} \tag{2}$$

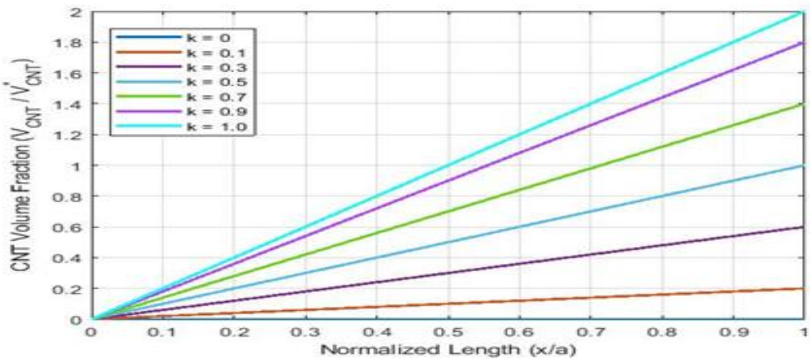


Fig. 2(a)

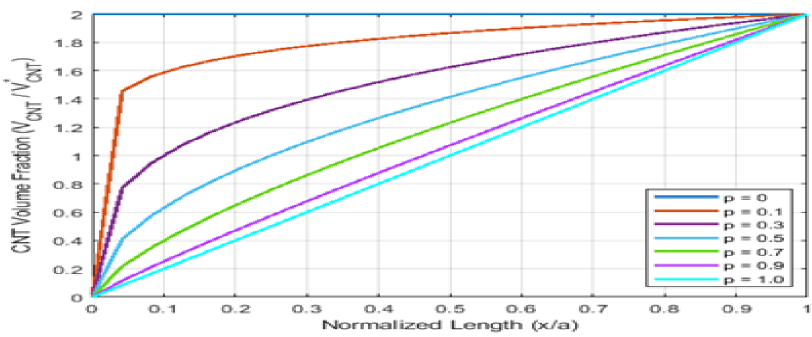


Fig. 2(b)

Fig. 2 Variation of CNTs volume fraction in composite nanorod for inhomogeneity parameters (a) k and (b) p

By using Rule of mixture to homogenize the characteristics of materials involved in composite rod as a single material [4]

$$E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_P E_P$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_P}{G_P} \quad (3)$$

where η_1 and η_3 are the efficiencies of CNT reinforced composite rod, V_{CNT}, Vp are the volume fractions of CNT and matrix, respectively, E_{11}^{CNT} is the Young's modulus of CNT in X-direction, G_{12}^{CNT} is the shear modulus of CNT in X-direction. E_P and G_P are the Youngs modulus and shear modulus of polymer matrix. Further, mass density, equivalent Poisson's ratio and coefficient of linear thermal expansion of the nanocomposite rod are written as

$$\rho = V_{CNT}\rho_{CNT} + V_P\rho_P \quad (4)$$

$$\nu_{12} = V_{CNT}\nu_{12}^{CNT} + V_P\nu_P \quad (5)$$

$$\alpha_{11} = V_{CNT}\alpha_{CNT} + V_P\alpha_P \quad (6)$$

The relationship between volume fractions of CNT and polymer matrix is written as

$$V_{CNT} + V_P = 1 \quad (7)$$

Let us assume that the temperature distribution is uniform over the surface of the CNT reinforced nanocomposite nanorod. It is also possible to analyze the mechanical properties of SWCNT as a function of the temperature. The temperature dependent material properties of CNT by Touloukian principle [5] is given by the following expression

$$P = P_0(P_{-1}T^{-1} + 1 + p_1T + P_2T^2 + P_3T^3) \quad (8)$$

where $T=T_0+\Delta T$ here T_0=300 K at ambient temperature and ΔT is the temperature difference. The temperature coefficients P_0, P_{-1}, P_1, P_2 and P_3 are as shown in the table 1

Table 1. Temperature coefficients

Parameter	P_0	P_{-1}	P_1	P_2	P_3
E_{11}^{CNT} (TPa)	6.3998	0	-6.77898x10-4	1.16097x10-6	-6.96636x10-10
α_{11} ($10^{-6}/^0C$)	-1.12515	0	-2.63678x10-2	2.56588x10-5	-1.00986x10-8
$\boldsymbol{\nu}_{11}^{CNT}$	0.175	0	0	0	0

2.3 Governing Equation of Motion

The stress-strain relations are fundamental for obtaining the governing equation of motion of the Love-Bishop nanorod. In a simple rod analysis, the complexity will be decreased by ignoring the lateral effects; however, in the Love-Bishop problem, it will be necessary to obtain an exact analysis. The components of displacement equations are taken from Civalek et al. [6]

$$u(x,t) = u\,;\;\; v(x,t) = -\nu y\frac{\partial u}{\partial x}\,; w(x,t) = -\nu z\frac{\partial u}{\partial x} \quad (9)$$

where u, v and w are the components of motion equations along X, Y and Z axes respectively. ν is the poison's ratio and t is time constant. The constitutive equation of nonlocal elasticity model is given as

$$\sigma_{ij} - (e_0a)^2(\partial^2\sigma_{ij}/\partial x^2) = C_{ijkl}\varepsilon_{ij} \quad (10)$$

where σ_{ij} is nonlocal stress, C_{ijkl} is the fourth order elasticity tensor and e_0a is nonlocal scale parameter. The dynamic equilibrium equation of the nanocomposite rod is given as [21]

$$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} + \frac{\partial\sigma_{xz}}{\partial z} = \rho\frac{\partial^2 u}{\partial t^2} + \frac{K_m}{A}u - \frac{F}{A} \quad (11)$$

where K_m is the stiffness of the external elastic medium, F is the applied external force per unit length and A is the cross-sectional area of the nanocomposite rod. Using Eq. (9), the classical stresses can be written as

$$S_{xx} = E\varepsilon_{xx} = E\frac{\partial u}{\partial x},\; S_{xy} = 2G\varepsilon_{xy} = -G\nu y\frac{\partial^2 u}{\partial x^2},\; S_{xz} = 2G\varepsilon_{xz} = -G\nu z\frac{\partial^2 u}{\partial x^2}$$

$$S_{yy} = S_{zz} = Syz = o \quad (12)$$

Where S_{xx}, S_{yy}, S_{zz} and S_{xy}, S_{yz}, S_{xz} are classical axial and shear stresses respectively.
The nonlocal stresses can be written as [6]

$$\sigma_{xx} = (e_0a)^2\rho(1+2\nu)\frac{\partial^3 u}{\partial x\,\partial t^2} + (e_0a)^2\frac{K_m}{A}\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial x} - \frac{(e_0a)^2}{A}\frac{\partial f}{\partial x}$$

$$\sigma_{xy} = -(e_0a)^2\rho\nu y\frac{\partial^4 u}{\partial x^2\,\partial t^2} - G\nu y\frac{\partial^2 u}{\partial x^2}$$

$$\sigma_{xz} = -(e_0a)^2\rho\nu z\frac{\partial^4 u}{\partial x^2\,\partial t^2} - G\nu z\frac{\partial^2 u}{\partial x^2} \quad (13)$$

The equation of motion can be obtained by substituting Eq. (13) in Eq. (12) as

$$(EA + (e_0a)^2K_m)\frac{\partial^2 u}{\partial x^2} - \rho A\frac{\partial^2 u}{\partial t^2} - K_m u + F + (\rho A(e_0a)^2(1+2\vartheta) + \rho I_P\vartheta^2)\frac{\partial^4 u}{\partial x^2\,\partial t^2} - GI_P\vartheta^2\frac{\partial^4 u}{\partial x^4} - (e_0a)^2\rho I_P\vartheta^2\frac{\partial^6 u}{\partial x^4\,\partial t^2} - (e_0a)^2\frac{\partial^2 F}{\partial x^2} = 0$$

where I_P is the polar moment of inertia.

(14)

The external magnetic and elastic forces acting on the CNT reinforced composite nanorod are given as

$$F_m = \eta A(H_y^2 + H_z^2)\frac{\partial^2 u}{\partial x^2}$$

$$F_E = K_m A u \qquad (15)$$

The total external force acting on the composite nanorod is F = F_m + F_E Substituting these forces in Eq. (14) leads to the Love-Bishop governing equation of the present CNT reinforce composite nanorod as

$$(EA + (e_0a)^2K_m)\frac{\partial^2 u}{\partial x^2} - \rho A\frac{\partial^2 u}{\partial t^2} - K_m u + (\rho A(e_0a)^2(1+2\vartheta) + \rho I_P\vartheta^2)\frac{\partial^4 u}{\partial x^2\,\partial t^2} - GI_P\vartheta^2\frac{\partial^4 u}{\partial x^4} - (e_0a)^2\rho I_P\vartheta^2\frac{\partial^6 u}{\partial x^4\,\partial t^2} + \eta(H_y^2 + H_z^2)\frac{\partial^2 u}{\partial x^2} - \eta(H_y^2 + H_z^2)(e_0a)^2\frac{\partial^4 u}{\partial x^4} = 0$$

(16)

By equating the nonlocal parameter (e_0a) to zero, the above equation leads to a classical Love-Bishop model.

3. Analytical Solutions

By taking into consideration of the harmonic type of wave solution for the displacement field in dimensionless form to analyze the dispersion properties of the CNT reinforced composite love-bishop nanorod.

$$u = \hat{u}e^{\iota\lambda x}e^{\iota\omega t} \qquad (17)$$

Substituting the above equation into the governing equation of motion and solving for natural frequency for nontrivial solution of (uˆ) as

$$\omega = \sqrt{\frac{\lambda^2[EA+(e_0a)^2K_m + GI_P\vartheta^2\lambda^2 + \eta(H_y^2+H_z^2)(1+(e_0a)^2)\lambda^2] + K_m}{\rho A(1+\lambda^2(e_0a)^2(1+2\vartheta)) + \lambda^2\rho I_P\vartheta^2(1+(e_0a)^2\lambda^2)}} \qquad (18)$$

By using the boundary conditions for Clamped-Clamped (C-C) and Clamped-Free (C-F) nanorod from [5],

For C-C: $\lambda = \frac{n\pi}{L}$, whereas C-F: $\lambda = \frac{(2n-1)\pi}{L}$

Finally, for free vibration in clamped-clamped boundary condition the natural frequency of the system will be

$$\omega = \sqrt{\frac{\left(\frac{n\pi}{L}\right)^2\left[EA+(e_0a)^2K_m + GI_P\vartheta^2\left(\frac{n\pi}{L}\right)^2 + \eta(H_y^2+H_z^2)(1+(e_0a)^2)\left(\frac{n\pi}{L}\right)^2\right] + K_m}{\rho A\left(1+\left(\frac{n\pi}{L}\right)^2(e_0a)^2(1+2\vartheta)\right) + \left(\frac{n\pi}{L}\right)^2\rho I_P\vartheta^2\left(1+(e_0a)^2\left(\frac{n\pi}{L}\right)^2\right)}} \qquad (19)$$

Similarly, for clamped-free boundary condition

$$\omega = \sqrt{\frac{\left(\frac{(2n-1)}{L}\pi\right)^2\left[EA+(e_0a)^2K_m + GI_P\vartheta^2\left(\frac{(2n-1)}{L}\pi\right)^2 + \eta(H_y^2+H_z^2)(1+(e_0a)^2)\left(\frac{(2n-1)}{L}\pi\right)^2\right] + K_m}{\rho A\left(1+\left(\frac{(2n-1)}{L}\pi\right)^2(e_0a)^2(1+2\vartheta)\right) + \left(\frac{(2n-1)}{L}\pi\right)^2\rho I_P\vartheta^2\left(1+(e_0a)^2\left(\frac{(2n-1)}{L}\pi\right)^2\right)}} \qquad (20)$$

At the end, the various numerical experimental data already available in the literature are used to study the effect of the external magnetic field, stiffness, mode number, slenderness ratio, temperature variation, volume fraction and inhomogeneity parameters on the natural frequency of the system.

4. RESULT AND DISCUSSION

Numerical experiments were carried out by varying the volume fraction of CNT by 0.11, 0.14, and 0.28, with efficiencies of CNTRC rods of 0.149, 0.150, and 0.149, respectively. The solid lines in the figure represent the local parameter, whereas the dashed line represents the nonlocal parameter effect. The effects of varying the parameters on the axial vibration of the nanocomposite rod are discussed below.

For the present analysis, a (10,10) SWCNT is considered as the reinforcement with the Vodenitcharova-Zhang Criterion [6] with an effective thickness h = 0.067 nm, radius R = 0.68 nm and length L = 9.26 nm. The Thermo-mechanical properties of (10,10) SWCNT are given by [4] as shown in Table 2.

Table 2. Mechanical properties of polymer matrix PmPV based on temperature variation

T(K)	E_{11}^{CNT} (TPa)	ν_{11}^{CNT}	α_{11}^{CNT} (10^{-6})/K)
300	5.6466	0.175	3.4584
400	5.5679	0.175	4.1496
500	5.5308	0.175	4.5361
700	5.4744	0.175	4.6677
1000	5.0330	0.175	4.2841

The Polymer matrix PmPV properties are given by: Young's modulus E_P=(3.51-0.0047T) GPa, linear thermal expansion coefficientα_P=45 (1+0.0005 ΔT)×10^{-6}/℃, Poisson's ratio ν_P=0.34 and mass density of the matrix ρ_P=1150 kg/m^3.

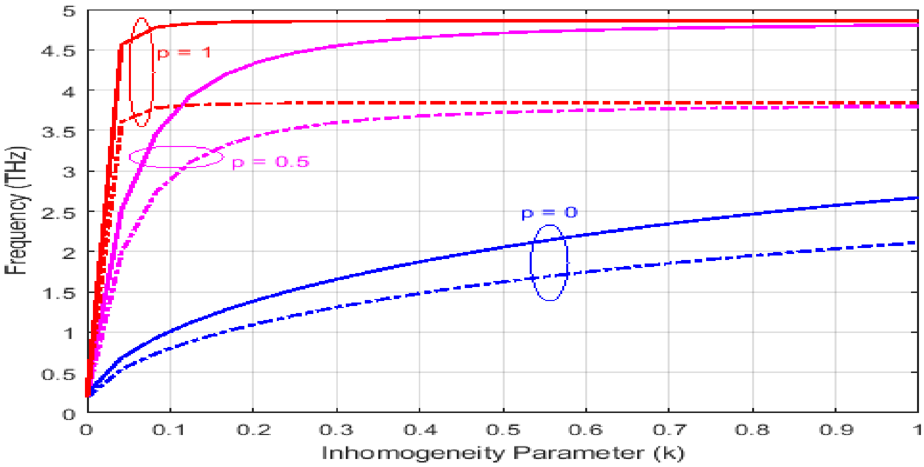


Fig 3. Variation of natural frequency as a function of inhomogeneity parameters k & p in local and nonlocal environment

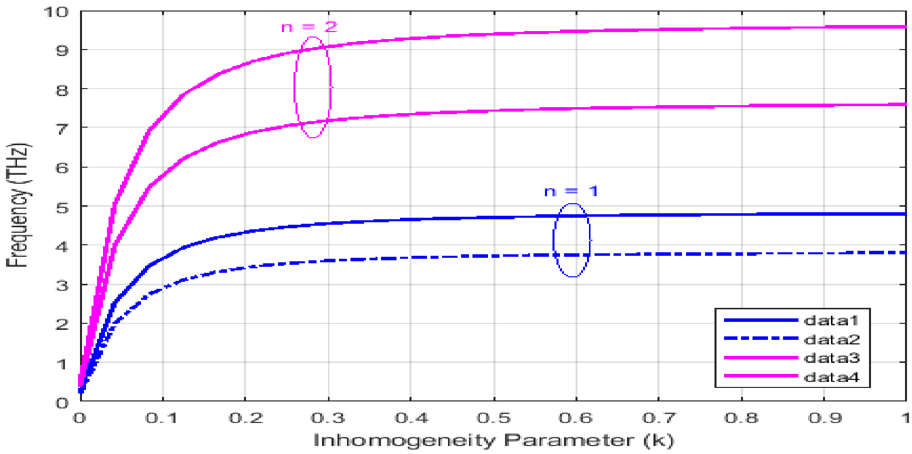


Fig 4. Variation of natural frequency as a function of mode number with inhomogeneity parameter 'k' in local and nonlocal environment

The natural frequency of a nanorod is affected by several factors, including its material characteristics, geometrical dimensions, and environmental circumstances. While studying, the natural frequency of a nanorod fluctuates with inhomogeneity parameters in local and nonlocal settings such as material composition, geometrical variations, boundary conditions, scale effects, and surface effects. Local and nonlocal inhomogeneities can coexist, and the combined influence must be considered accurately forecast the natural frequency. Advanced modelling techniques are used to account for both material and geometrical changes, as well as experimental validation. Fig.3 shows the effect of the inhomogeneity and nonlocal parameters on the axial vibration. Keeping n=1, V^*_{CNT}=0.28 and T=300 k by increasing the p and k parameters, the local natural frequency increases gradually when compared to the nonlocal natural frequency. Similarly, the natural frequency of a nanorod as a function of mode number in relation to an inhomogeneity parameter k in both local and nonlocal environment affects vibrational characteristics throughout the nanorod in various modes. The natural frequency of a nanorod is calculated as a function of mode number and inhomogeneity parameter 'k' by considering both local and nonlocal effects. Fig.4 shows the effect of axial frequency for different mode numbers with data1 as n=1, p=0.5, e_0a=o and data2 as n=1, p=0.5, e_0a =1 and data3 as n=2, p=0.5, e_0a =0 and data4 as n=2,p=0.5 and e_0a =1. The Natural axial frequency was gradually increased by increasing the number of modes.

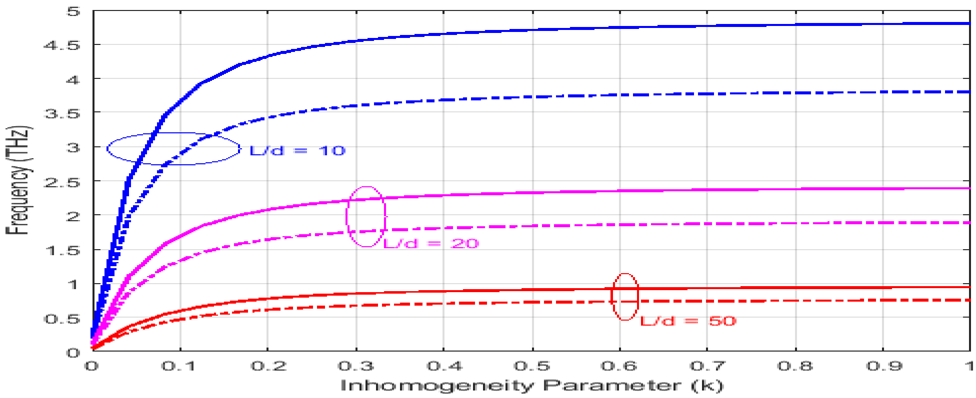


Fig 5. Effect of L/d ratio on natural frequency with inhomogeneity parameter ‘k’ in local and nonlocal environment

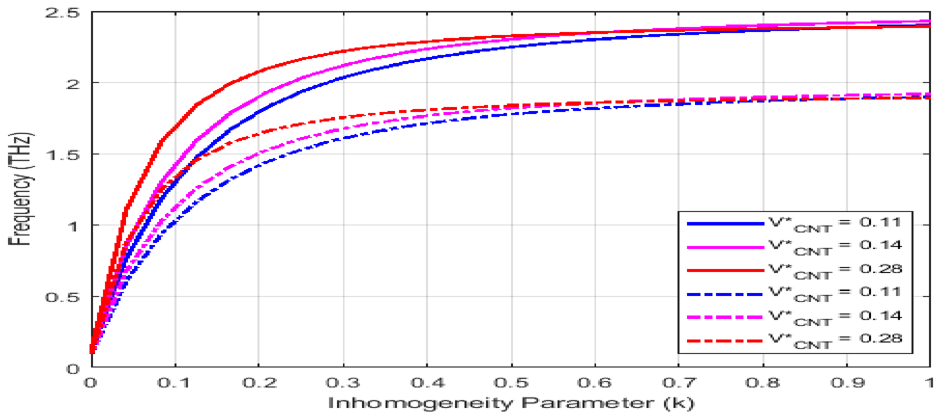


Fig 6. Influences of volume fraction of reinforcement V^*_{CNT} on natural frequency in local and nonlocal environment

The length-to-diameter (L/d) ratio and the inhomogeneity parameter 'k' figure out natural frequency in various contexts. At large L/d ratios, the nanorod acts more like a beam, in such circumstances bending modes dominate, and natural frequencies tend to fall, as the L/d ratio increases due to the structure's increased flexibility. Local inhomogeneities can result in large changes in natural frequencies. The frequencies may differ from those predicted by classical beam theory due to changes in local stiffness. The variation in axial frequency in terms of local and nonlocal parameters is elaborated, keeping its mode number=1, V^*_{CNT} =0.28,T=300k by varying the l/d ratio. From fig 5: It is clear that, the axial natural frequency is more in the local parameter when compared to a nonlocal parameter that is also proportional to the l/d ratio; however, if the k value is up to 0.2, there is a chance of huge variation in the natural frequency beyond that, the variation of frequency is negligible. Similarly, the percentage of the nanorod's volume occupied by inclusions or reinforcing elements is known as the volume fraction of reinforcement. The reinforcement has the potential to enhance mechanical attributes, like strength and rigidity, which could impact the nanorod's inherent frequency. To fully capture the intricate relationships between material properties and structural dynamics, the impact of reinforcement volume percent on natural frequencies necessitates a comprehensive method that combines theoretical models, numerical simulations, and experimental validation. The local frequency is higher than the nonlocal frequency when the parameters n=1, l/d=20, p=0, T=300k with the variation of k and volume fraction of CNTCR are shown in fig.6. If the k value is up to 0.5, there is a variation in the frequency beyond that value, and the frequency is marginally dependent on the V^*_{CNT}.

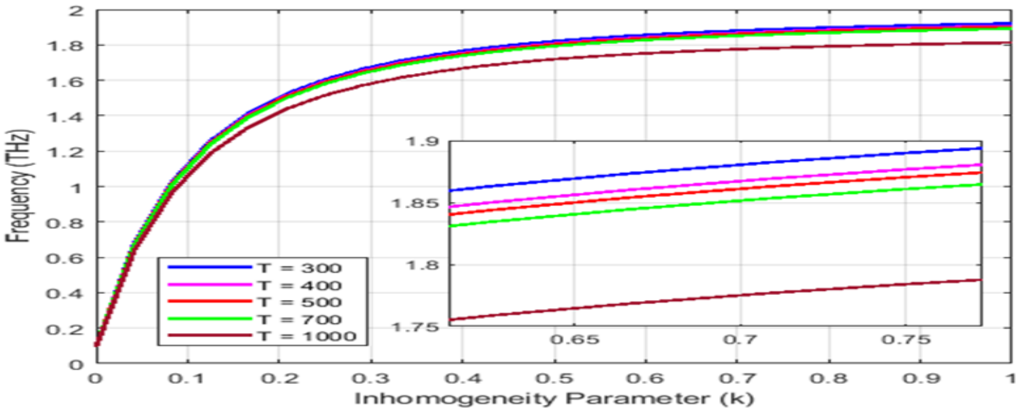


Fig 7. Effect of temperature variation on natural frequency with inhomogeneity parameter 'k' in local and nonlocal environment

Young's modulus, density, and internal damping are among the material qualities that are influenced by temperature changes. Young's modulus typically decreases as temperature rises, although dimensional changes are brought about by thermal expansion. The effects of temperature and inhomogeneity may not be as noticeable for shorter and thicker nanorods, but temperature still influences the dimensions and material properties, which can alter the natural frequency. Fig.7 shows that there is a small variation in frequency with the increase in temperature. Here, frequency variation is inversely proportional to temperature. By changing the local stiffness distribution along the nanorod, the parabolic distribution of reinforcement in the local environment modifies the natural frequencies. Higher local stiffness resulting from higher reinforcement fractions in particular places might raise the natural frequencies, particularly for larger values of p. The effective nonlocal parameter in a nonlocal environment is influenced by the reinforcement distribution, which in turn affects the overall vibrational response. Natural frequencies are altered by variations in both nonlocal interactions and local stiffness. These modifications result in different natural frequencies. Fig 8 shows the effect of the inhomogeneity parameter p and polymer matrix on the frequency of the CNTRC Love-Bishop nanorod. In this case, the PmPV polymer matrix gives the frequency which is a little bit higher when compared to the PMMA polymer matrix.

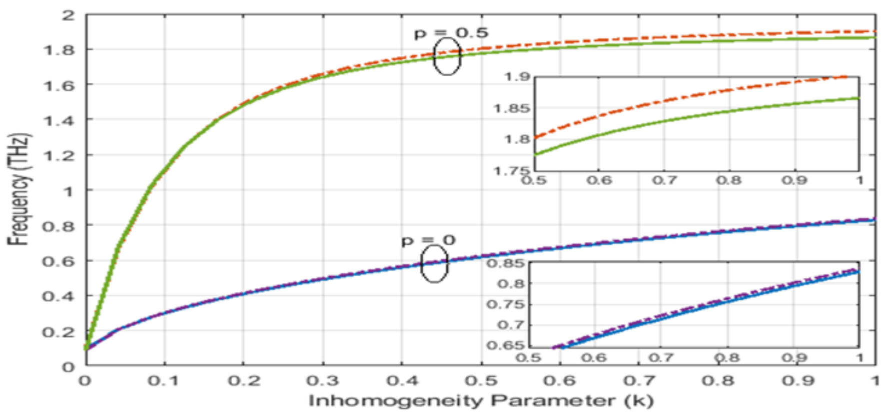


Fig 8. Effect of parabolic distribution of reinforcement 'p' on natural frequency along with inhomogeneity parameter 'k' in local and nonlocal environment

We must consider the combined effects of spatially varying reinforcement, magnetic effects, and nonlocal elasticity to analyze the influences of a parabolic distribution of carbon nanotubes (CNTs) and magnetic flex on the natural frequency of a nanorod, along with the inhomogeneity parameter 'k' in a nonlocal environment. The term 'magnetic flex' describes how an external magnetic field affects a nanorod and can change its mechanical characteristics. The distribution and alignment of CNTs can be influenced by the magnetic field, which in turn affects the mechanical response of the nanorod. Fig 9 shows that magnetic flex plays a crucial role on the natural frequency of the CNTRC nanorod. Frequency is proportional to magnetic flex and the inhomogeneity parameter p.

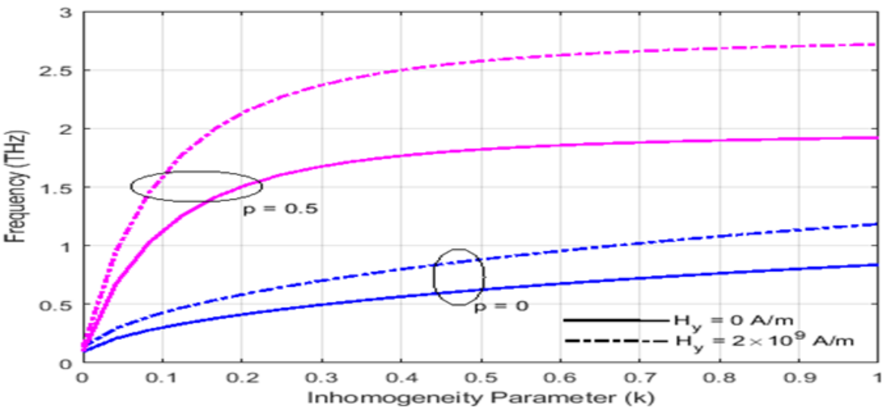


Fig 9. Influences of parabolic distribution of CNT's and magnetic flex on natural frequency along with inhomogeneity parameter 'k' in nonlocal environment

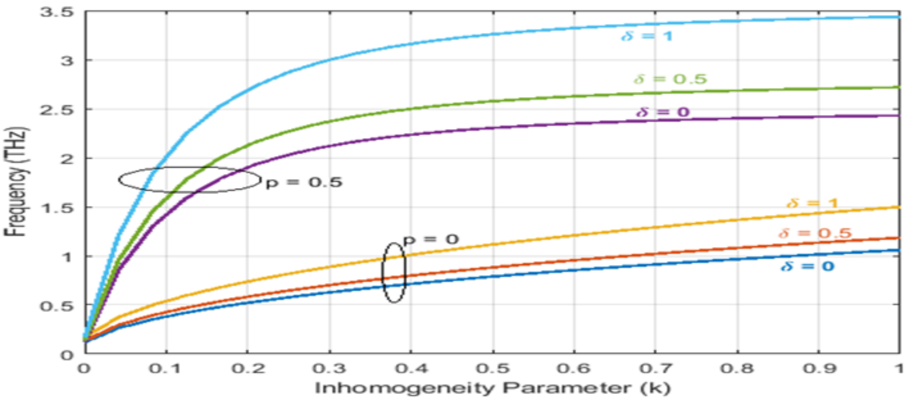
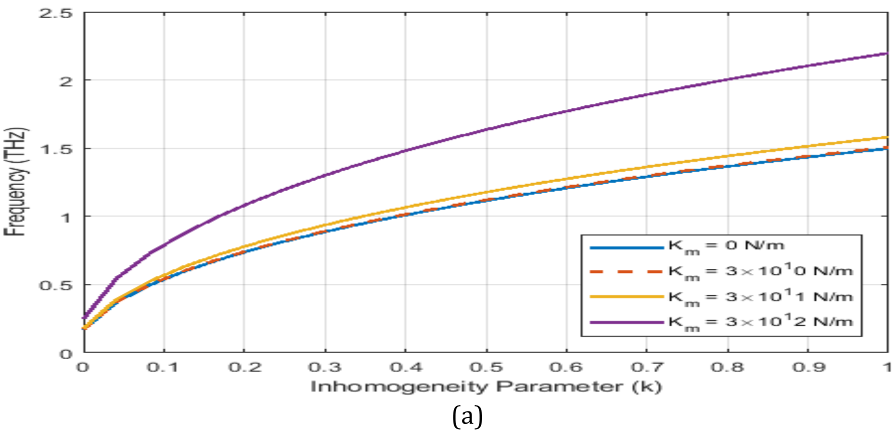
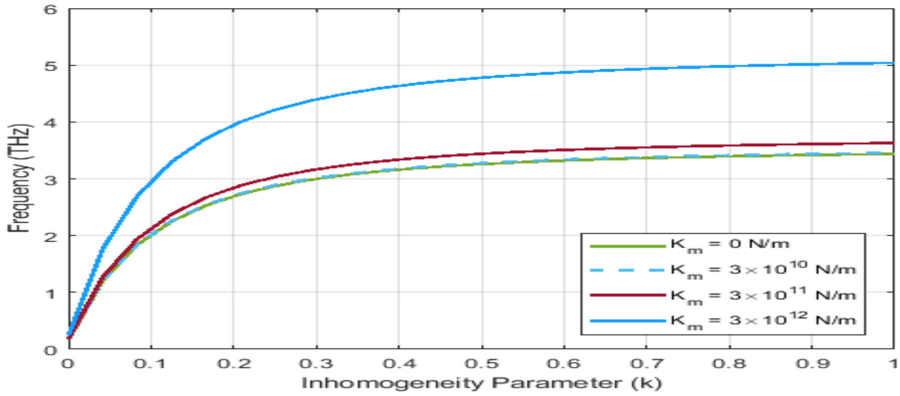


Fig 10. Effect of p and magnetic permeability on frequency

While magnetic permeability influences these qualities and can vary the natural frequencies, the inhomogeneity parameter 'p' in the local environment affects natural frequencies by changing the local material properties. Greater changes in stiffness are caused by higher 'p' values and higher natural frequencies are usually associated with enhanced magnetic permeability. However, in a nonlocal setting, the natural frequency spectrum is altered because of the effects of both magnetic permeability and p on the effective nonlocal parameter 'k'. In general, higher magnetic permeability and inhomogeneity result in higher natural frequencies. Fig 10 shows that magnetic permeability and inhomogeneity parameters are proportional to the natural frequency of the CNTRC nanorod, keeping its remaining parameters as constants. The variation in frequency is not uniform for given ranges of magnetic permeability



(a)



(b)

Fig :11(a) & 11(b) Effect of natural frequency on variation of thermal conductivity and inhomogeneity parameter 'k' without and with parabolic distribution of reinforcement 'p'.

In the absence of Parabolic distribution, the natural frequency is mostly determined by thermal conductivity, and it influences the density and effective modulus of the nanorod. Through thermal stresses and variations in material qualities, changes in thermal conductivity can indirectly change the natural frequency. Heat conductivity has a more complicated effect when there is a parabolic distribution because it interacts with the parabolic reinforcement distribution. Because of the combined effects of thermal stresses and material characteristics, this can result in naturally occurring frequencies that vary spatially. Figs. 11(a) and 11(b) demonstrated that stiffness and inhomogeneity parameters have much greater effects on the frequency of the system keeping δ=1, Hy=2e9 A/m. On increasing the values of k, p and K_m the natural frequency of the CNTRC also increases in C-C boundary condition.

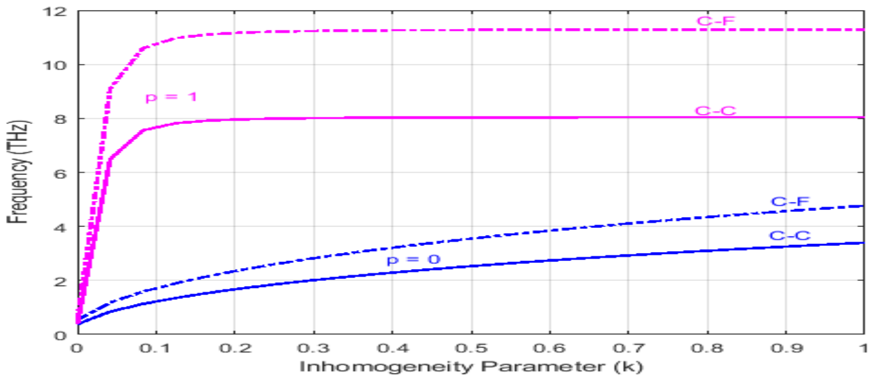


Fig 12. Effect of parabolic distribution of reinforcement 'p' and boundary conditions on natural frequency along with inhomogeneity parameter 'k'

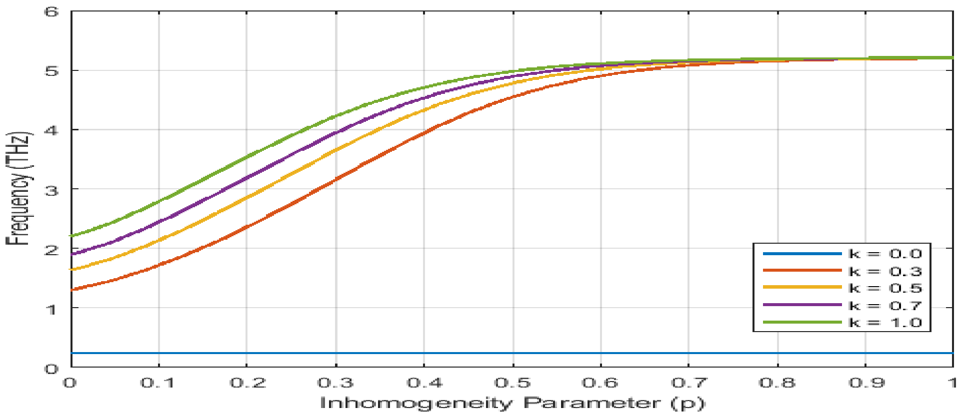


Fig 13. Effect of inhomogeneity parameter 'k' on natural frequency along with parabolic distribution of CNT's.

The natural frequency of a structure is influenced by complicated interactions between the boundary conditions (clamped-free and clamped-clamped), the inhomogeneity parameter and the parabolic distribution of reinforcement. The vibrational performance of engineering structures can be significantly improved by considering and optimizing these parameters. The variation of natural frequency in response to an increase in inhomogeneity parameters while keeping remaining parameters constant is demonstrated in fig.12 for C-C and C-F boundary conditions. The inhomogeneity parameter k effect is considerable when p=0. If the p value is increased to 1, then the natural frequency is more variation for small values of k and it is linear by increasing the k value. Under the influence of inhomogeneity parameters C-F is having more frequency when compared to C-C boundary conditions. Several complex interactions must be examined to determine how the inhomogeneity parameter 'k' affects the natural frequency of structures that use a parabolic distribution of CNTs as reinforcement. Stiffness variation along the structure is assumed to be more significant with a greater inhomogeneity parameter. Natural frequency will naturally rise in parts with higher stiffness and fall in regions with lower stiffness. sFig.13 reveals that the natural frequency is increased by increasing the inhomogeneity parameter k. Also, for small values of the inhomogeneity parameter p, the effect is more on the natural frequency of the CNTRC nanorod.

5. CONCLUSIONS

Based on Eringen's nonlocal elasticity theory, the natural frequency of the nanorod with an external magnetic field was analyzed by adopting the Love-Bishop theory. After completion of the analysis, it was concluded that

1. Nonlocal analysis of the nanorod gives less frequency than the local model.
2. Out of two different matrix, PmPV matrix gives more frequency than the PMMA.
3. The high volume fraction of reinforcement, inhomogeneity parameters (k, p), magnetic flex, magnetic permeability and stiffness are proportional to the natural frequency.
4. The natural frequency of nanorod is inversely proportional to size effect and temperature.
5. The Stiffness effect is more when compared to remaining parameters.
6. Clamped-Free have a higher frequency than the Clamped-Clamped boundary condition.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interest.

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