A Novel Forms of MSL –Closed and MSW –Open Sets in Minuscule Topological Spaces

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ABSTRACT
The main goal of this paper is to explore and characterize new categories of open sets within Minuscule topological spaces through the Minuscule operations, namely, $\gimel R^{MSO}(K)$, $\gimel R^{MRO}(K)$, Mβ O(Q, K), Mγ O(Q, K), $\gimel R^{MPO}(K)$, Mα O (Q, K), MSL – Closed, MSLA – Closed, and MSLB – Closed. Additionally, certain relationshipsand characteristics among these sets are examined.

Keywords: Minuscule topological space, $\gimel R^{MSO}(K)$, Minuscule α- open, Minuscule β- open, Minuscule locally closed.

1. INTRODUCTION

The differences between the two closed subsets dimensional of an n- Euclidean space were investigated in 1921 by Kuratowski and Sierpinski [5,11]. Implicit within their findings lies the concept of a locally closed subset within a topological space (Q, K). Drawing from Bourbaki's work, we define a subset of (Q, K) as locally closed in Q if itrepresents the intersection of an open subset of Q and a closed subset of Q. Nearly seven decades later, in 1989, M. Ganster and I.L.Reilly [3]explored a number of hypotheses regarding the family of locally closed subsets in any given space and examined the relationships between them. The main objective of this research is to use the Minuscule operations to investigate and characterise new types of opensets in Minuscule topological spaces. In this paper, we define some weak forms of opensets and locally closedsets like $\gimel R^{MSO}(K)$, Minuscule regular-open, Minuscule β -open, Minuscule γ - open, Minuscule pre- open, Minuscule α -open, Minuscule locally closed, Minuscule locally closed Z , Minuscule locally closed B sets in Minuscule topological space (Q, $\gimel R(K)$ is de-noted by $\gimel R^{MSO}(K)$, $\gimel R^{MRO}(K)$, Mβ O(Q, K), Mγ O(Q, K), $\gimel R^{MPO}(K)$, Mα O(Q, K), MSL – Closed, MsLCA and MsLCB sets and we investigate severalrelations and characteristics among these subsets.

2. Preliminaries

Definition 2.1 (18)**.** Consider a non-empty finite set U that consists of objects referred to as the universe. Let R be an equivalence relation on U, which is formallyreferred to as the indiscernibility relation. Subsequently, the set U is partitioned into disjoint equivalence classes. Elements that are part of the same equivalence class are considered to be indiscernible from each other. The pair (U, R) is widely referred to as the approximation space in research papers. Let K be a subset of U.

1. The lower approximation of the set K with respect to the relation R refersto the collection of objects that can be unambiguously categorized as belonging to K with respect to the conditions defined by R. This lower approximation can be expressed as $H_L(K)$. That is,

$$H_L(K) = \bigcup_{K\in u}\{R(K) : R(K) \subseteq K\}$$

where R(K) denotes the equivalence class determined by K.

2. The lower minimal approximation:

$$H *_L (K) = \bigcup_{K\in u}\{R(K) : R(K) \subseteq K\} - K = H_L(K) - K$$

3. The upper approximation of K with respect to R is the set of all objects, which can be possibly classified as K with respect to R and it is denoted by $H_U(K)$. That is,

$$H_U(K) = \bigcup_{K\in u}\{R(K) : R(K) \bigcap K \neq \phi\}$$

4. The upper minimal approximation:

$$H*_U(K) = \bigcup_{K\in u}\{R(K) : R(K)\bigcap K \neq \phi\} - K = H_U(K) - K.$$

5. Let $H_L(K)$ and $H_{u^*}(K)$ be two sets. The symmetric difference of the sets $H_L(K)$ and $H_{u^*}(K)$ is $H_L(K)\Delta H_{u^*}(K)$ and it is denoted by,

$$H_L(K)\Delta H*_U(K) = (H_L(K) - H*_U(K))\bigcup(H*_U(K) - H_L(K))$$

Definition 2.2 (18)**.** Let U be the universe, R be an equivalence relation U and $\tau_R(K)$ = {U, ϕ, $H_L(K)$, $H_U(K)$, $H_L{}^*(K)$, $H_U{}^*(K)$, $H_L{}^*(K)\cap H_U(K)$, $H_L(K)\nabla H_{u^*}(K)$}.

Here $H_L{}^*(K)$ and $H_L{}^*(K)\nabla H_U(K)$ is always be ϕ. ϕ is always belongs to the topology.So, $H_L{}^*(K)$ and $H_L{}^*(K)\nabla H_U(K)$ is neglected.

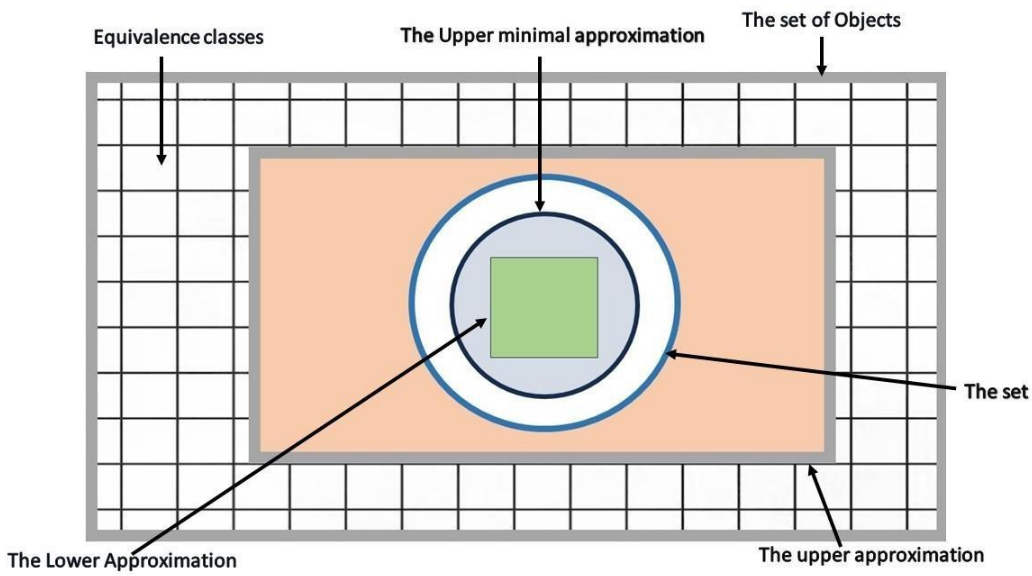
Then the topology $\tau_R(K)$ = {U, ϕ, $H_L(K)$, $H_U(K)$, $H_U{}^*(K)$, $H_L(K)\nabla H_{u^*}(K)$} where $K \subseteq R$. $\tau_R(K)$ satisfies the subsequent axioms.

(i) U and ϕ ∈ $\tau_R(K)$

(ii) The union of the elements of any sub collection of $\tau_R(K)$ is in $\tau_R(K)$.

(iii) The intersection of all elements of any finite subcollection of $\tau_R(K)$ is in $\tau_R(K)$.

That is, $\tau_R(K)$ is a topology on U called the Minuscule topology on U with respect to K. We call (U, $\tau_R(K)$) is a Minuscule topological space. The elements of $\tau_R(K)$ are called Minuscule opensets.



Trimming of Annular Region

Remark 2.3 (18)**.** The basis for the minuscule topological space $H_L(K)$ with respect to K is given by $Y_R(K)$ = {U, $H_L(K)$, $H_L(K)\nabla H_{u^*}(K)$}.

Definition 2.4. If (Q, $\tau_R(K)$ Minuscule topological space with respect to K, where $K \subseteq Q$ and $A \subseteq Q$, then

1. The intersection of all Minuscule closedsets containing in Z defines the Minuscule closure of the set Z. It is represented by the symbol Mscl(A). furthermore, the smallest closed set of Z is Mscl(A).

2. The union of all Minuscule open sets contained in Z defines the Minuscule interior of the set Z. It is represented by the symbol Msint(A). furthermore,the largest open set of Z is Msint(A).

Properties: [18] If (U,R) is an approximation space and $K, L \subseteq U$, then

1. $H_L(K) \subseteq K \subseteq H_U(K)$.
2. $H_L(\phi) = H_U(\phi) = \phi$ and $H_L(K) = H_U(K) = U$
3. $H_U(K \cup L) = H_U(K) \cup H_U(L)$
4. $H_U(K \cap L) \subseteq H_U(K) \cap H_U(L)$
5. $H_L(K \cup L) \supseteq H_L(K) \cup H_L(L)$
6. $H_L(K \cap L) = H_L(K) \cap H_L(L)$

7. $H_L(K) \subseteq H_L(L)$ and $H_U(K) \subseteq H_U(L)$ whenever $K \subseteq L$
8. $H^*_L(K) = \phi$
9. $H^*_L(K) \cap H_U(K) = \phi$
10. $Hu^*(K \cup L) \subseteq Hu^*(K) \cup Hu^*(L)$
11. $Hu^*(K \cap L) = Hu^*(K) \cap Hu^*(L)$
12. $H_U(H^*(K)) = H_L(Hu^*(K)) = Hu^*(K)$.
13. $H^*_L(K) \nabla H_U(K) = H_U(K)$.

3. Weaker Forms of M-Open sets in Minuscule topological spaces

Definition 3.1. A Minuscule topological space (Q, ƛR(K) and A ⊆ Q. Then Z is said to be Minuscule semiopen if A ⊂ Mscl(Msint(A)).

Definition 3.2. A Minuscule topological space (Q, ƛR(K) and A ⊆ Q. Then Z is said to be Minuscule Regular open if Z = Msint(Mscl(A)).

Definition 3.3. A Minuscule topological space (Q, ƛR(K) and A ⊆ Q. Then Z is said to be Minuscule β - open if A ⊆ Mscl(Msint(Mscl(A))).

Definition 3.4. A Minuscule topological space (Q, ƛR(K) and A ⊆ Q. Then Z is said to be Minuscule γ - open if A ⊆ Mscl(Msint(A)) UMscl(Msint(Mscl(A)) .

Definition 3.5. A Minuscule topological space (Q, ƛR(K) and A ⊆ Q. Then Z is said to be $ƛR^{MPO}(K)$ if A ⊂ Msint(Mscl(A)).

Definition 3.6. A Minuscule topological space (Q, ƛR(K) and A ⊆ Q. Then Z is said to be Minuscule α- open if A ⊂ Msint(Mscl(Msint(A))).

Here $ƛR^{MSO}(K)$, $ƛR^{MRO}(K)$, Mβ O(Q, K), Mγ O(Q, K), $ƛR^{MPO}(K)$, Mα O(Q,

denotes the $ƛR^{MSO}(K)$, Minuscule regular-open, Minuscule β -open, Minuscule pre-open, Minuscule α - open subsets of Q respectively.

Definition 3.7. Let (Q, ƛR(K) be a Minuscule topological space and A ⊆ Q. Then Z is said to be Minuscule preclosed (Minuscule semiclosed, Minuscule β - closed, Minuscule α- closed, Minuscule regularclosed) if its complement is $ƛR^{MPO}(K)$ (Minuscule semiopen, Minuscule β - open, Minuscule α- open, Minuscule Regular open) respectively.

Remark 3.8. It is easy to understand the notion $ƛR^{MSO}(K)$, $ƛR^{MRO}(K)$, Mβ O(Q,K), Mγ O(Q,K), $ƛR^{MPO}(K)$, MαO(Q,K) from the subsequent examples.

Example 3.9. Let Q = {ϖxi, ϖxj, ϖxk, ϖxl} with Q/R = {{ϖxi}, {ϖxk}, {ϖxj, ϖxl}}.

Let K = {ϖxi, ϖxj} ⊆ Q, Then ƛR(K) = {Q, Φ, {ϖxi}, {ϖxi, ϖxj, ϖxl}, {ϖxl}, {ϖxi, ϖxl}}and hence the Minuscule closed sets in Q are Q, Φ, {ϖxj, ϖxk, ϖxl}, {ϖxk}, {ϖxi, ϖxj, ϖxk},{ϖxj,ϖxk }. Then,

(i) $ƛR^{MSO}(K)$= {Q, Φ, {ϖxl}, {ϖxj, ϖxk, ϖxl}, {ϖxi, ϖxj}, {ϖxi}, {ϖxk, ϖxl}, {ϖxi, ϖxk}, {ϖxj, ϖxl}, {ϖxi, ϖxj, ϖxk}, {ϖxj, ϖxk}, {ϖxi, ϖxk, ϖxl}, {ϖxi, ϖxj, ϖxl},

(ii) $ƛR^{MRO}(K)$= {Q, Φ, {ϖxi}, {ϖxl}},

(iii) MαO (Q, K) = {Q, Φ, {ϖxi}, {ϖxl}, {ϖxi, ϖxj, ϖxl}, {ϖxi, ϖxl}},

(iv) $ƛR^{MPO}(K)$= {Q, Φ, {ϖxi}, {ϖxl}, {ϖxi, ϖxk, ϖxl}, {ϖxi, ϖxl}, {ϖxi, ϖxj, ϖxl}},

(v) Mβ O (Q, K) = {Q, Φ, {ϖxl}, {ϖxj, ϖxk, ϖxl}, {ϖxi, ϖxj}, {ϖxi}, {ϖxk, ϖxl}, {ϖxi, ϖxk}, {ϖxj, ϖxl}, {ϖxi, ϖxj, ϖxk}, {ϖxj, ϖxk}, {ϖxi, ϖxk, ϖxl}, {ϖxi, ϖxj, ϖxl},

(vi) Mγ O (Q, K) = {Q, Φ, {ϖxl}, {ϖxj, ϖxk, ϖxl}, {ϖxi, ϖxj}, {ϖxi}, {ϖxk, ϖxl}, {ϖxi, ϖxk}, {ϖxj, ϖxl}, {ϖxi, ϖxj, ϖxk}, {ϖxj, ϖxk}, {ϖxi, ϖxk, ϖxl}, {ϖxi, ϖxj, ϖxl}.

Here we note that $ƛR^{MSO}(K)$does not form a topology on Q, Since {ϖxi, ϖxj, ϖxk} and {ϖxj, ϖxk, ϖxl} ∈ $ƛR^{MSO}(K)$ but {ϖxi, ϖxj, ϖxk} ∩ {ϖxj, ϖxk, ϖxl} = {ϖxj, ϖxk} ∉ $ƛR^{MSO}(K)$. Similarly, Mβ O (Q, K) and Mγ O (Q, K) does not form a topology on Q, but $ƛR^{MPO}(K)$ andMα O (Q, K) forms a topology in Q.

Example 3.10. Let Q = {νx, νy, νz, νw} with Q/R = {{νx}, {νz}, {νy, νw}}.

Let K = {νx, νy} ⊆ Q, Then ƛR(K) = {Q, Φ, {νx}, {νx, νy, νw}, {νw}, {νx, νw}}.Then If P = {νx, νy, νw} then P is Minuscule open but not $ƛR^{MRO}(K)$. If S = {νx, νy} then S is $ƛR^{MSO}(K)$ but not MαO (Q, K).

If T = {νx, νz, νw} then T is ʎR^{MPO}(K) but not MαO (Q, K). If V = {νx, νz} then V is ʎR^{MSO}(K) but not ʎR^{MRO}(K).

If W = {νx, νy, νz} then W is MαO (Q, K) but not ʎR^{MRO}(K). If Y = {νy, νz, νw} then Y is MβO (Q, K) but not ʎR^{MPO}(K).

Theorem 3.11. If Z is Minuscule open in (Q, ʎR(K), then it is MαO (Q, K).
Proof. Since Z is Minuscule open in Q, Msint(A) = Z. Then Mscl(Msint(A)) = Mscl(A) ⊇ A. That is, A ⊂ Mscl(Msint(A)). Therefore, Msint(A) ⊆ Msint(Mscl(A) ⊆ A)). That is, A ⊆ Msint(Mscl(Msint(A))). Thus Z is MαO (Q, K).

Theorem 3.12. In a Minuscule topological space, ʎR^{MSO}(K) ⊇ MαO (Q, K). Proof. Let A ∈ MαO (Q, K), A ⊆ Msint(Mscl(Msint(A))) ⊆ Mscl(Msint(A)) and hence A ∈ ʎR^{MSO}(K). □

□

Theorem 3.13. In a Minuscule topological space, ʎR^{MSO}(K) ⊆ ʎR^{MPO}(K). Proof. Obvious.

Theorem 3.14. In a Minuscule topological space, ʎR^{MPO}(K) ⊇ MαO (Q, K). Proof. Similar to the theorem's proof 3.12. □

Remark 3.15. It is not necessary for the aforementioned theorems to be true inconverse. □

Example 3.16. The aforementioned Theorems 3.11, 3.12, 3.13, and 3.14, It canbe observed to be valid generally based on this example.
Let Q = {ϖxi, ϖxj, ϖxk, ϖxl} with Q/R = {{ϖxi}, {ϖxk}, {ϖxj, ϖxl}}.
Let K = {ϖxi, ϖxj} ⊆ Q, Then ʎR(K) = {Q, Φ, {ϖxi}, {ϖxi, ϖxj, ϖxl}, {ϖxl}, {ϖxi, ϖxl}} and hence the Minuscule closed sets in Q are Q, Φ, {ϖxj, ϖxk, ϖxl}, {ϖxk}, {ϖxi, ϖxj, ϖxk}, {ϖxj, ϖxk}}. From the example 3.8 we say that,

1. ʎR^{MSO}(K) ⊇ MαO (Q, K), That is, {Q, Φ, {ϖxl}, {ϖxj, ϖxk, ϖxl}, {ϖxi, ϖxj}, {ϖxi}, {ϖxk, ϖxl}, {ϖxi, ϖxk}, {ϖxj, ϖxl}, {ϖxi, ϖxj, ϖxk}, {ϖxj, ϖxk}, {ϖxi, ϖxk, ϖxl}, {ϖxi, ϖxj, ϖxl} ⊇ {Q, Φ, {ϖxi}, {ϖxl}, {ϖxi, ϖxj, ϖxl}, {ϖxi, ϖxl}}.
2. ʎR^{MPO}(K) ⊆ ʎR^{MSO}(K) That is, {Q, Φ, {ϖxi}, {ϖxl}, {ϖxi, ϖxk, ϖxl}, {ϖxi, ϖxl}, {ϖxi, ϖxj, ϖxl}} ⊆ {Q, Φ, {ϖxl}, {ϖxj, ϖxk, ϖxl}, {ϖxi, ϖxj}, {ϖxi}, {ϖxk, ϖxl}, {ϖxi, ϖxk}, {ϖxj, ϖxl}, {ϖxi, ϖxj, ϖxk}, {ϖxj, ϖxk}, {ϖxi, ϖxk, ϖxl}, {ϖxi, ϖxj, ϖxl}.
3. ʎR^{MPO}(K) ⊇ MαO (Q, K) That is, {Q, Φ, {ϖxi}, {ϖxl}, {ϖxi, ϖxk, ϖxl}, {ϖxi, ϖxl}, {ϖxi, ϖxj, ϖxl}} ⊇ {Q, Φ, {ϖxi}, {ϖxl}, {ϖxi, ϖxj, ϖxl}, {ϖxi, ϖxl}}.

Proposition 3.17. The intersection of ʎR^{MPO}(K) and Mα O (Q,K) is Mα O (Q,K)open.

Proof. Let A ∈ ʎR^{MPO}(K) and Y ∈ MαO (Q, K), then A ⊂ Msint(Mscl(A), Y ⊂ Msint(Mscl(Msint(Y))). So, A ∩ Y ⊂ Msint(Mscl(A) ∩ Msint(Mscl(Msint(Y))) ⊂ Mscl(Msint(Y). Hence A ∩ Y is Minuscule α-open. □

Proposition 3.18. The intersection of a MαO (Q, K)-open set and MβO (Q,K)open set is MαO(Q, K)-open.
Proof. Similar to the proposition 3.17 proof. □

Remark 3.19. It is evident from the following example, that Minuscule α-open is the intersection of Minuscule β-open and Minuscule α-open.

Example 3.20. Let Q = {ϖxi, ϖxj, ϖxk, ϖxl} with Q/R = {{ϖxi, ϖxj}, {ϖxk}, {ϖxl}}. Let K = {ϖxi, ϖxk} ⊆ Q, then ʎR(K) = {Q, Φ, {ϖxk}, {ϖxi, ϖxj, ϖxk}, {ϖxj}, {ϖxj, ϖxk}} and hence the Minuscule closed sets in Q are Q, Φ, {ϖxi, ϖxj, ϖxl}, {ϖxl}, {ϖxi, ϖxk, ϖxl}, {ϖxi, ϖxl}}.
let the intersection of MαO (Q, K) ∩ MβO (Q, K) = {Q, Φ, {ϖxj} {ϖxj, ϖxk, ϖxl}, {ϖxj, ϖxk}, {ϖxi, ϖxj, ϖxk}, {ϖxk}} ∩ {Q, Φ, {ϖxj}, {ϖxk}, {ϖxk, ϖxl}, {ϖxi, ϖxj}, {ϖxj, ϖxl}, {ϖxi, ϖxk}, {ϖxj, ϖxk, ϖxl}, {ϖxi, ϖxj, ϖxk}, {ϖxj, ϖxk}, {ϖxi, ϖxk, ϖxl}, {ϖxi, ϖxj, ϖxl}} = {Q, Φ, {ϖxj}, {ϖxj, ϖxk, ϖxl}, {ϖxj, ϖxk}, {ϖxi, ϖxj, ϖxk}, {ϖxk}} = MαO (Q, K).

Proposition 3.21. The intersection of ʎR^{MSO}(K) and MαO (Q, K)-open set is MαO (Q, K)-open.
Proof. similar to the proposition 3.17 proof.

□

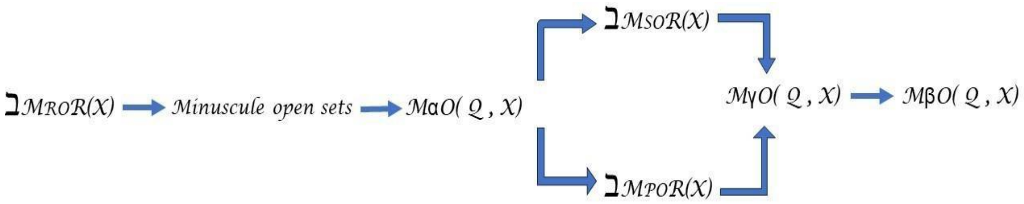


Figure 2

Corollary 3.22. MαO (Q, K) ∪ MβO (Q, K) = MβO (Q, K).
Remark 3.23. The arbitrary intersection of any MβC (Q, K) 's is MβC (Q, K) but the union of two MβC (Q, K) 's may not be MβC (Q, K). This can be seen in the example below.

Example 3.24. Let Q = {ℏp, ℏq, ℏr, ℏs} with Q/R = {{ℏp, ℏq}, {ℏr}, {ℏs}}.
Let K = {ℏp, ℏr} ⊆ Q, Then, ƛR(K) = {Q, Φ, {ℏr}, {ℏp, ℏq, ℏr}, {ℏq}, {ℏq, ℏr}}, thesubsets Z ={ℏq} and W = {ℏp, ℏr}are Minuscule β-closedsets but Z∪W =
{ℏp, ℏq, ℏr} is not Minuscule β-closed.

Proposition 3.25. Each MβO (Q, K) and MαC (Q, K) is ƛR^{MRC} (K). Proof. Let A ⊆ Q be a MβO (Q, K) set and MαC (Q, K) set. Then A ⊆
Mscl(Msint(Mscl(A))) and Mscl(Msint(Mscl(A))) ⊂ A, which implies that Mscl(Msint(Mscl(A))) ⊆ A ⊆ Mscl(Msint(Mscl(A))) So,Z =Mscl(Msint(Mscl(A))).

This proves that Z is Minuscule closed, and so it is Minuscule regularclosed (denoted as ƛR^{MRC}(K)).

Corollary 3.26. Each MβC (Q, K) and MαO (Q, K) is ƛR^{MRO}(K).

4. Novel forms of LC sets in Minuscule topological spaces

Definition 4.1. A Minuscule topological space (Q, ƛR(K) and A ⊆ Q. Then Z is said to be MsLC if Z =Q∩P, where Q is Minuscule open and P is Minuscule closed in Q.
Example 4.2. let Q = {ℏα, ℏβ, ℏγ, ℏδ} with Q/R = {{ℏαℏβ}, {ℏγ}, {ℏδ}}.
Let K = {ℏα, ℏγ} ⊆ Q, Then ƛR(K) = {Q, Φ, {ℏγ}, {ℏα, ℏβ, ℏγ}, {ℏβ}, {ℏβ, ℏγ}}. The Minuscule closed sets in Q are Q, Φ, {ℏα, ℏβ, ℏδ}, {ℏδ}, {ℏα, ℏγ, ℏδ}, {ℏα, ℏδ}}. let us take Q = {ℏα, ℏβ, ℏγ}, P = {ℏα, ℏβ, ℏδ}, Z = {ℏα, ℏβ,} ⊆ Q. This implies Z is MsLC.

Definition 4.3. A Minuscule topological space (Q, ƛR(K) and A ⊆ Q. Then Zis said to be
MsLCt-set if Msint(A) = Msint(Mscl(A)).
MsLCA-set if Z=Q ∩ P, where Q is Minusculeopen and P is ƛR^{MRC}(K) in Q
MsLCB- set if Z=Q ∩ P, where Q is Minusculeopen and P isMsLCt-set in Q
MαZ-set if Z=Q∩P, where Q is Minuscule α-open and P is ƛR^{MRC}(K) in Q.
MS-set if Z=Q∩P, where Q is Minuscule semiopen and P is Minusculeclosed in Q.

Proposition 4.4. Let Minuscule topological space (Q, ƛR(K) and A ⊆ Q. Then
1. Every Minuscule closed set isMsLCt-set.
2. Every MsLCA set is MS-set.
3. EveryMsLCt set is MsLCB-set.
4. Every ƛR^{MSO}(K) is MS-set.
5. Every MαZ set is MS-set.

Remark 4.5. The contrary of what is above an argument need not be correct. Theexample provides a clear understanding of the abovementioned proposition.
Example 4.6. Let Q = {ℏp, ℏβ, ℏγ, ℏδ} with Q/R = {{ℏp}, {ℏγ}, {ℏβ, ℏδ}}.
Let K = {ℏp, ℏβ} ⊆ Q, Then ƛR(K) = {Q, Φ, {ℏp}, {ℏp, ℏβ, ℏδ}, {ℏδ}, {ℏp, ℏδ}} and hence the Minuscule closed sets in Q are Q, Φ, {ℏβ, ℏγ, ℏδ}, {ℏγ}, {ℏp, ℏβ, ℏγ},

{ℏβ,ℏγ}}.

(i) A ={ℏp} is MsLCt set but not Minuscule closed set.

(ii) A = {ℏp,ℏβ,ℏδ} is MsLCB set but not MsLCt set.

(iii) A ={ℏγ} is MS set but not MsLCA set and MαZ set.

Theorem 4.7. If Z is MsLCt set if and only if Minuscule semiclosed. Proof. Let Z be a MsLCt set. Then Msint(A) = Msint(Mscl(A)) and so
Msint(Mscl(A)) = Msint(A) ⊂ A . Thus, Z is Minuscule semiclosed. Conversely,let Z be Minuscule semi closed. Then Msint(Mscl(A)) ⊂ Z and Msint(Mscl(A)) ⊂ Msint(A). Since Msint(A) ⊂ Msint(Mscl(A)), Msint(A) = Msint(Mscl(A)).
Thus, is MsLCt set.

□

Theorem 4.8. If Z is ƛR^{MRO}(K) iff Z is ƛR$^{MP O}$(K) and MsLCt-set.
Proof. Let Z be Minuscule regular open, then it is Minuscule pre-open and by the definition of Minuscule regular open, Msint(A) = Msint(Mscl(A)). Conversely, Z be Minuscule pre-open and MsLCt -set. Then Z is Minuscule open and so, Msint(Mscl(A))
= Z. Thus, Z is Minuscule regular open.

Theorem 4.9. If Z and Y are two MsLCt-set, then Z ∩ Y is a MsLCt-set. □
Proof. Let Z and Y are two MsLCt-set. Then Msint(A ∩ Y) = Msint(A) ∩
Msint(Y) = Msint(Mscl(A) ∩ Msint(Mscl(Y)). Hence A ∩ Y is MsLCt-set.

□

Theorem 4.10. Let Minuscule topological space (Q, ƛR(K) and A ⊆ Q. Then the following are equivalent:

1. Z is Minuscule open.
2. Z is ƛR^{MPO}(K) and MsLCB -set.

Proof. 1 ⇒ 2 Let Z be a Minuscule open. Every Minuscule open is Minuscule pre-open and Z = ZU U. Thus, Z is MsLCB-set.
2 ⇒ 1 Let Z be a MsLCB-set. Then we have Z = D ∩ E, where D is Minusculeopen and E is MsLCt-set. □
Since Z is Minuscule pre-open, Z ⊂ Msint(Mscl(A).
Hence Z = D∩E = (D∩E)∩D ⊂ [Msint(Mscl(C)∩Msint(E)]∩D = D ∩Msint(E) .Therefore, Z is Minuscule open.

Theorem 4.11. Every MαZ set is ƛR^{MSO}(K) set.
Proof. Let Z be MαZ set. Then Z = D ∩E, where D is MαZ open set and
E is Minuscule- regularclosed, E= Mscl(Msint(E)). So, E is ƛR^{MSO}(K). By
Proposition 3.19, Z is ƛR^{MSO}(K).

□

Theorem 4.12. Every MsLCA set is MαZ-set.
Proof. It is Obvious that every Minuscule open Mα-open.

□

Theorem 4.13. Every MαZ set is Mβ -set.
Proof. It is Obvious from Theorem 4.11 and it follows from Proposition 3.17.

□

Theorem 4.14. Every MsLCA set is MsLCB -set.Proof. It is Obvious from Proposition 4.4.

Theorem 4.15. Let Minuscule topological space (Q, ƛR(K) and A ⊆ Q. Then,the subsequent statements are equivalent.: □

1. Z is MsLCA set.
2. Z is MαZ set and a MsLC.
3. Z is ƛR^{MSO}(K) and MsLC.

Proof. 1 ⇒ 2 Let Z be a M set, Z = D∩E where D is M open set and E is
M- regularclosed. Since every ƛR^{MRO}(K) is M-closed and M- open is Mα-open,
Z is MsLC and MαA-set.
2 ⇒ 3 It is Obvious from the theorems 4.11,4.12 and 4.13.

3 ⇒ 1 Let Z be M semi-open and MsLC. Then $Z \subset$ Mscl(Msint(A)) and Z = D $\cap$ Mscl(A), where D is Minuscule open, Mscl(Msint(A)) = Z. Thus, Z is MsLCAset. □

CONCLUSION

This paper explains the concepts of open sets in Minuscule topological spaces through the Minuscule operations, namely $\mathcal{R}^{MSO}(K)$, $\mathcal{R}^{MRO}(K)$, Mβ O(Q, K), Mγ O(Q , K), $\mathcal{R}^{MP\ O}$ (K), Mα O(Q , K), MSL − Closed , MSLA − Closed , and MSLB − Closed . Furthermore, various relationships and characteristics between these sets are investigated.

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