Octagonal Fuzzy DEMATEL Approach to Study the Risk Factors of Stomach Cancer

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Abstract

Fuzzy number (FN) plays a vital role in decision making problems as it is used to represent the uncertain terms. Researchers in the field of decision-making analysis have used triangular and trapezoidal FN to solve the problem in the uncertain environment. FN have also been extended recently such as pentagonal, hexagonal, and heptagonal and so on. This paper aims to generalize the Hexadecagonal fuzzy number (GHFN) which contains set of 16-tuples. Membership functions and alpha cuts of linear and nonlinear GHFN with symmetry and asymmetry have also been derived.

1 Introduction

Fuzzy sets have been presented by (Zadeh, 1965) to handle imprecision informa tion, all things considered, issues [1]. In 2003 (Coxe & Reiter, 2003), utilized fuzzy automata on a hexagonal foundation utilizing straightforward number juggling mixes of neighboring fuzzy qualities [2]. In 2013, (Rajarajeswari & Sudha, 2013) involved stretch math in another activity for expansion, deduction and duplication of Hexagonal Fuzzy number based on alpha cut sets of fuzzy numbers [3]. (Rajarajeswari & Sudha, 2014) summed up hexagonal fuzzy numbers by rank, mode, uniqueness and spreads to improve the independent direction, estimate and chance investigation [4]. In the extended time of 2015, (Dhurai & Karpagam, 2016) utilized span math to present another enrollment capability and fulfilled the activity of expansion, deduction and duplication of hexagonal fuzzy number based on alpha cut sets of fuzzy numbers [5].Hexagonal, heptagonal, nonagon, decagonal fuzzy numbers have additionally been acquainted with tackle the dubiousness [6, 7, 13].Sankar and Manimohan embraced pentagonal fuzzy number, determined direct and non-straight pentagonal fuzzy number and add- ressed fuzzy conditions utilizing pentagonal fuzzy number [14].Karthik et. al., proposed straight and nonlinear enrollment capabilities for the summed up heptagonal fuzzy number and presented Haar positioning technique for hexagonal fuzzy number [9]. Malini and Kennedy tackled fuzzy transportation issue by utilizing octagonal fuzzy numbers [10]. Felix et.al., proposed the nonagonal fuzzy number and its math activities and determined alpha cuts for nonagonal fuzzy number [7]. Venkatesh and Britto presented a positioning technique utilizing decagonal fuzzy number for diet control [15]. Nagadevi and Rosario tackled transportation issue, in which decagonal fuzzy numbers are utilized to address transportation expenses to find least transportation cost [11]. Naveena and Rajkumar presented turn around request pentadecagonal, nonagonal and decagonal fuzzy numbers and their math activities [12]. Necessary preliminaries are cited therein [8–10].

2 Hexadecagonal Fuzzy Number(HFN) and It's Variation

Hexadecagonal Fuzzy Number: A HFN $\tilde{A} =$
$(\Omega_1, \Omega_2, \Omega_3,\ \Omega_4,\ \Omega_5,\ \Omega_6,\ \Omega_7,\ \Omega_8,\ \Omega_9,\ \Omega_{10},\ \Omega_{11},\ \Omega_{12},\ \Omega_{13},\ \Omega_{14},\ \Omega_{15},\ \Omega_{16})$ where
$\Omega_1, \Omega_2, \Omega_3, \Omega_4, \Omega_5, \Omega_6, \Omega_7, \Omega_8, \Omega_9, \Omega_{10}, \Omega_{11}, \Omega_{12}, \Omega_{13}, \Omega_{14}, \Omega_{15}, \Omega_{16} \in R$ must hold the consequent conditions

•$\mu_{\tilde{A}}(\theta)$ is a continuous function(breifly, $cts.fn$) in $[0, 1]$.

•$\mu_{\tilde{A}}(\theta)$ is strictly increasing and $cts.fn$ on $[\Omega_1, \Omega_2], [\Omega_2, \Omega_3], [\Omega_3,\ \Omega_4]$ and $[\Omega_4,\ \Omega_5], [\Omega_5,\ \Omega_6], [\Omega_6,\ \Omega_7]$ and $[\Omega_7,\ \Omega_8]$.

•$\mu_{\tilde{A}}(\theta)$ is strictly decreasing and $cts.fn$ on $[\Omega_9,\ \Omega_{10}], [\Omega_{10},\ \Omega_{11}], [\Omega_{11},\ \Omega_{12}]$, $[\Omega_{12},\ \Omega_{13}], [\Omega_{13},\ \Omega_{14}], [\Omega_{14},\ \Omega_{15}]$ and $[\Omega_{15},\ \Omega_{16}]$.

3.1.1 Equality of two HFN'S: Two HFN'S $\tilde{A} = (\Omega_1, \Omega_2, \Omega_3,\ \Omega_4,\ \Omega_5,\ \Omega_6,\ \Omega_7,\ \Omega_8,\ \Omega_9,\ \Omega_{10},\ \Omega_{11},\ \Omega_{12},\ \Omega_{13},\ \Omega_{14},\ \Omega_{15},\ \Omega_{16})$ and $\tilde{B} = (\varphi_1,\ \varphi_2,\ \varphi_3,\ \varphi_4,\ \varphi_5,\ \varphi_6,\ \varphi_7,\ \varphi_8,\ \varphi_9,\ \varphi_{10},\ \varphi_{11},\ \varphi_{12},\ \varphi_{13},\ \varphi_{14},\ \varphi_{15},\ \varphi_{16})$ are equal iff $\Omega_1 = \varphi_1, \Omega_2 = \varphi_2, \Omega_3 = \varphi_3,\ \Omega_4 = \varphi_4,\ \Omega_5 = \varphi_5,\ \Omega_6 = \varphi_6,\ \Omega_8 = \varphi_8,\ \Omega_9 = \varphi_9,\ \Omega_{10} = \varphi_{10},\ \Omega_{11} =$

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$\varphi_{11},\ \Omega_{12}=\varphi_{12},\ \Omega_{13}=\varphi_{13},\ \Omega_{14}=\varphi_{14},\ \Omega_{15}=\varphi_{15},\ \Omega_{16}=\varphi_{16}$

Linear Hexadecagonal Symmetry:

$$\mu_x=\begin{cases} p\left(\frac{x-\Omega_1}{\Omega_2-\Omega_1}\right),\Omega_1\le x\le\Omega_2\\ p+(q-p)\left(\frac{x-\Omega_2}{\Omega_3-\Omega_2}\right),\Omega_2\le x\le\Omega_3\\ q+(r-q)\left(\frac{x-\Omega_3}{\Omega_4-\Omega_3}\right),\Omega_3\le x\le\Omega_4\\ r+(s-r)\left(\frac{x-\Omega_4}{\Omega_5-\Omega_4}\right),\Omega_4\le x\le\Omega_5\\ s+(t-s)\left(\frac{x-\Omega_5}{\Omega_6-\Omega_5}\right),\Omega_5\le x\le\Omega_6\\ t+(u-t)\left(\frac{x-\Omega_6}{\Omega_7-\Omega_6}\right),\Omega_6\le x\le\Omega_7\\ u+(1-u)\left(\frac{x-\Omega_7}{\Omega_8-\Omega_7}\right),\Omega_7\le x\le\Omega_8\\ 1,\Omega_8\le x\le\Omega_9\\ u-(u-1)\left(\frac{\Omega_{10}-x}{\Omega_{10}-\Omega_9}\right),\Omega_9\le x\le\Omega_{10}\\ t-(t-u)\left(\frac{\Omega_{11}-x}{\Omega_{11}-\Omega_{10}}\right),\Omega_{10}\le x\le\Omega_{11}\\ s-(s-t)\left(\frac{\Omega_{12}-x}{\Omega_{12}-\Omega_{11}}\right),\Omega_{11}\le x\le\Omega_{12}\\ r-(r-s)\left(\frac{\Omega_{13}-x}{\Omega_{13}-\Omega_{12}}\right),\Omega_{12}\le x\le\Omega_{13}\\ q-(q-r)\left(\frac{\Omega_{14}-x}{\Omega_{14}-\Omega_{13}}\right),\Omega_{13}\le x\le\Omega_{14}\\ p-(p-q)\left(\frac{\Omega_{15}-x}{\Omega_{15}-\Omega_{14}}\right),\Omega_{14}\le x\le\Omega_{15}\\ p\left(\frac{\Omega_{16}-x}{\Omega_{16}-\Omega_{15}}\right),\Omega_{15}\le x\le\Omega_{16}\\ 0,x\le\Omega_1\quad\text{and}\quad x\ge\Omega_{16}\end{cases}$$

$$A_\alpha=\begin{cases} A_{1L}(\alpha)=\Omega_1+(\frac{\alpha}{p})(\Omega_2-\Omega_1)for\alpha\in[0,p]\\ A_{2L}(\alpha)=\Omega_2+(\frac{\alpha-p}{q-p})(\Omega_3-\Omega_2)for\alpha\in[p,q]\\ A_{3L}(\alpha)=\Omega_3+(\frac{\alpha-q}{r-q})(\Omega_4-\Omega_3)for\alpha\in[q,r]\\ A_{4L}(\alpha)=\Omega_4+(\frac{\alpha-r}{s-r})(\Omega_5-\Omega_4)for\alpha\in[r,s]\\ A_{5L}(\alpha)=\Omega_5+(\frac{\alpha-s}{t-s})(\Omega_6-\Omega_5)for\alpha\in[s,t]\\ A_{6L}(\alpha)=\Omega_6+(\frac{\alpha-t}{u-t})(\Omega_7-\Omega_6)for\alpha\in[t,u]\\ A_{7L}(\alpha)=\Omega_7+(\frac{\alpha-u}{1-u})(\Omega_8-\Omega_7)for\alpha\in[u,1]\\ A_{7R}(\alpha)=\Omega_{10}+(\frac{\alpha-u}{u-1})(\Omega_{10}-\Omega_9)for\alpha\in[u,1]\\ A_{6R}(\alpha)=\Omega_{11}+(\frac{\alpha-t}{t-u})(\Omega_{11}-\Omega_{10})for\alpha\in[t,u]\\ A_{5R}(\alpha)=\Omega_{12}+(\frac{\alpha-s}{s-t})(\Omega_{12}-\Omega_{11})for\alpha\in[s,t]\\ A_{4R}(\alpha)=\Omega_{13}+(\frac{\alpha-r}{r-s})(\Omega_{13}-\Omega_{12})for\alpha\in[r,s]\\ A_{3R}(\alpha)=\Omega_{14}+(\frac{\alpha-q}{q-r})(\Omega_{14}-\Omega_{13})for\alpha\in[q,r]\\ A_{2R}(\alpha)=\Omega_{15}+(\frac{\alpha-p}{p-q})(\Omega_{15}-\Omega_{14})for\alpha\in[p,q]\\ A_{1R}(\alpha)=\Omega_{16}-(\frac{\alpha}{p})(\Omega_{16}-\Omega_{15})for\alpha\in[0,p]\end{cases}$$

Linear Haxadecagonal Asymmetry

$$\mu_x = \begin{cases} p\left(\frac{x-\Omega_1}{\Omega_2-\Omega_1}\right), \Omega_1 \le x \le \Omega_2 \\ p+(q-p)\left(\frac{x-\Omega_2}{\Omega_3-\Omega_2}\right), \Omega_2 \le x \le \Omega_3 \\ q+(r-q)\left(\frac{x-\Omega_3}{\Omega_4-\Omega_3}\right), \Omega_3 \le x \le \Omega_4 \\ r+(s-r)\left(\frac{x-\Omega_4}{\Omega_5-\Omega_4}\right), \Omega_4 \le x \le \Omega_5 \\ s+(t-s)\left(\frac{x-\Omega_5}{\Omega_6-\Omega_5}\right), \Omega_5 \le x \le \Omega_6 \\ t+(u-t)\left(\frac{x-\Omega_6}{\Omega_7-\Omega_6}\right), \Omega_6 \le x \le \Omega_7 \\ u+(1-u)\left(\frac{x-\Omega_7}{\Omega_8-\Omega_7}\right), \Omega_7 \le x \le \Omega_8 \\ 1, \Omega_8 \le x \le \Omega_9 \\ e-(e-1)\left(\frac{\Omega_{10}-x}{\Omega_{10}-\Omega_9}\right), \Omega_9 \le x \le \Omega_{10} \\ f-(f-e)\left(\frac{\Omega_{11}-x}{\Omega_{11}-\Omega_{10}}\right), \Omega_{10} \le x \le \Omega_{11} \\ g-(g-f)\left(\frac{\Omega_{12}-x}{\Omega_{12}-\Omega_{11}}\right), \Omega_{11} \le x \le \Omega_{12} \\ h-(h-g)\left(\frac{\Omega_{13}-x}{\Omega_{13}-\Omega_{12}}\right), \Omega_{12} \le x \le \Omega_{13} \\ i-(i-h)\left(\frac{\Omega_{14}-x}{\Omega_{14}-\Omega_{13}}\right), \Omega_{13} \le x \le \Omega_{14} \\ j-(j-i)\left(\frac{\Omega_{15}-x}{\Omega_{15}-\Omega_{14}}\right), \Omega_{14} \le x \le \Omega_{15} \\ j\left(\frac{\Omega_{16}-x}{\Omega_{16}-\Omega_{15}}\right), \Omega_{15} \le x \le \Omega_{16} \\ 0, x \le \Omega_1 and x \ge \Omega_{16} \end{cases}$$

$$A_\alpha = \begin{cases} A_{1L}(\alpha) = \Omega_1 + (\frac{\alpha}{p})(\Omega_2 - \Omega_1) for \alpha \in [0,p] \\ A_{2L}(\alpha) = \Omega_2 + (\frac{\alpha-p}{q-p})(\Omega_3 - \Omega_2) for \alpha \in [p,q] \\ A_{3L}(\alpha) = \Omega_3 + (\frac{\alpha-q}{r-q})(\Omega_4 - \Omega_3) for \alpha \in [q,r] \\ A_{4L}(\alpha) = \Omega_4 + (\frac{\alpha-r}{s-r})(\Omega_5 - \Omega_4) for \alpha \in [r,s] \\ A_{5L}(\alpha) = \Omega_5 + (\frac{\alpha-s}{t-s})(\Omega_6 - \Omega_5) for \alpha \in [s,t] \\ A_{6L}(\alpha) = \Omega_6 + (\frac{\alpha-t}{u-t})(\Omega_7 - \Omega_6) for \alpha \in [t,u] \\ A_{7L}(\alpha) = \Omega_7 + (\frac{\alpha-u}{1-u})(\Omega_8 - \Omega_7) for \alpha \in [u,1] \\ A_{7R}(\alpha) = \Omega_{10} + (\frac{\alpha-e}{e-1})(\Omega_{10} - \Omega_9) for \alpha \in [e,1] \\ A_{6R}(\alpha) = \Omega_{11} + (\frac{\alpha-f}{f-e})(\Omega_{11} - \Omega_{10}) for \alpha \in [f,e] \\ A_{5R}(\alpha) = \Omega_{12} + (\frac{\alpha-g}{g-f})(\Omega_{12} - \Omega_{11}) for \alpha \in [g,f] \\ A_{4R}(\alpha) = \Omega_{13} + (\frac{\alpha-h}{h-g})(\Omega_{13} - \Omega_{12}) for \alpha \in [h,g] \\ A_{3R}(\alpha) = \Omega_{14} + (\frac{\alpha-i}{i-h})(\Omega_{14} - \Omega_{13}) for \alpha \in [i,h] \\ A_{2R}(\alpha) = \Omega_{15} + (\frac{\alpha-j}{j-i})(\Omega_{15} - \Omega_{14}) for \alpha \in [j,i] \\ A_{1R}(\alpha) = \Omega_{16} - (\frac{\alpha}{j})(\Omega_{16} - \Omega_{15}) for \alpha \in [0,j] \end{cases}$$

Nonlinear Haxadecagonal Symmetry:

$$\mu_x = \begin{cases} p\left(\frac{x-\Omega_1}{\Omega_2-\Omega_1}\right)^{S_1}, \Omega_1 \leq x \leq \Omega_2 \\ p+(q-p)\left(\frac{x-\Omega_2}{\Omega_3-\Omega_2}\right)^{S_2}, \Omega_2 \leq x \leq \Omega_3 \\ q+(r-q)\left(\frac{x-\Omega_3}{\Omega_4-\Omega_3}\right)^{S_3}, \Omega_3 \leq x \leq \Omega_4 \\ r+(s-r)\left(\frac{x-\Omega_4}{\Omega_5-\Omega_4}\right)^{S_4}, \Omega_4 \leq x \leq \Omega_5 \\ s+(t-s)\left(\frac{x-\Omega_5}{\Omega_6-\Omega_5}\right)^{S_5}, \Omega_5 \leq x \leq \Omega_6 \\ t+(u-t)\left(\frac{x-\Omega_6}{\Omega_7-\Omega_6}\right)^{S_6}, \Omega_6 \leq x \leq \Omega_7 \\ u+(1-u)\left(\frac{x-\Omega_7}{\Omega_8-\Omega_7}\right)^{S_7}, \Omega_7 \leq x \leq \Omega_8 \\ 1, \Omega_8 \leq x \leq \Omega_9 \\ u-(u-1)\left(\frac{\Omega_{10}-x}{\Omega_{10}-\Omega_9}\right)^{P_1}, \Omega_9 \leq x \leq \Omega_{10} \\ t-(t-u)\left(\frac{\Omega_{11}-x}{\Omega_{11}-\Omega_{10}}\right)^{P_2}, \Omega_{10} \leq x \leq \Omega_{11} \\ s-(s-t)\left(\frac{\Omega_{12}-x}{\Omega_{12}-\Omega_{11}}\right)^{P_3}, \Omega_{11} \leq x \leq \Omega_{12} \\ r-(r-s)\left(\frac{\Omega_{13}-x}{\Omega_{13}-\Omega_{12}}\right)^{P_4}, \Omega_{12} \leq x \leq \Omega_{13} \\ q-(q-r)\left(\frac{\Omega_{14}-x}{\Omega_{14}-\Omega_{13}}\right)^{P_5}, \Omega_{13} \leq x \leq \Omega_{14} \\ p-(p-q)\left(\frac{\Omega_{15}-x}{\Omega_{15}-\Omega_{14}}\right)^{P_6}, \Omega_{14} \leq x \leq \Omega_{15} \\ p\left(\frac{\Omega_{16}-x}{\Omega_{16}-\Omega_{15}}\right)^{P_7}, \Omega_{15} \leq x \leq \Omega_{16} \\ 0, x \leq \Omega_1 and x \geq \Omega_{16} \end{cases}$$

$$A_\alpha = \begin{cases} A_{1L}(\alpha) = \Omega_1 + (\frac{\alpha}{p})(\Omega_2 - \Omega_1) for \alpha \in [0,p] \\ A_{2L}(\alpha) = \Omega_2 + (\frac{\alpha-p}{q-p})(\Omega_3 - \Omega_2) for \alpha \in [p,q] \\ A_{3L}(\alpha) = \Omega_3 + (\frac{\alpha-q}{r-q})(\Omega_4 - \Omega_3) for \alpha \in [q,r] \\ A_{4L}(\alpha) = \Omega_4 + (\frac{\alpha-r}{s-r})(\Omega_5 - \Omega_4) for \alpha \in [r,s] \\ A_{5L}(\alpha) = \Omega_5 + (\frac{\alpha-s}{t-s})(\Omega_6 - \Omega_5) for \alpha \in [s,t] \\ A_{6L}(\alpha) = \Omega_6 + (\frac{\alpha-t}{u-t})(\Omega_7 - \Omega_6) for \alpha \in [t,u] \\ A_{7L}(\alpha) = \Omega_7 + (\frac{\alpha-u}{1-u})(\Omega_8 - \Omega_7) for \alpha \in [u,1] \\ A_{7R}(\alpha) = \Omega_{10} + (\frac{\alpha-u}{u-1})(\Omega_{10} - \Omega_9) for \alpha \in [u,1] \\ A_{6R}(\alpha) = \Omega_{11} + (\frac{\alpha-t}{t-u})(\Omega_{11} - \Omega_{10}) for \alpha \in [t,u] \\ A_{5R}(\alpha) = \Omega_{12} + (\frac{\alpha-s}{s-t})(\Omega_{12} - \Omega_{11}) for \alpha \in [s,t] \\ A_{4R}(\alpha) = \Omega_{13} + (\frac{\alpha-r}{r-s})(\Omega_{13} - \Omega_{12}) for \alpha \in [r,s] \\ A_{3R}(\alpha) = \Omega_{14} + (\frac{\alpha-q}{q-r})(\Omega_{14} - \Omega_{13}) for \alpha \in [q,r] \\ A_{2R}(\alpha) = \Omega_{15} + (\frac{\alpha-p}{p-q})(\Omega_{15} - \Omega_{14}) for \alpha \in [p,q] \\ A_{1R}(\alpha) = \Omega_{16} - (\frac{\alpha}{p})(\Omega_{16} - \Omega_{15}) for \alpha \in [0,p] \end{cases}$$

Nonlinear Haxadecagonal Asymmetry:

$$\mu_x = \begin{cases} p\left(\frac{x-\Omega_1}{\Omega_2-\Omega_1}\right)^{S_1}, \Omega_1 \leq x \leq \Omega_2 \\ p+(q-p)\left(\frac{x-\Omega_2}{\Omega_3-\Omega_2}\right)^{S_2}, \Omega_2 \leq x \leq \Omega_3 \\ q+(r-q)\left(\frac{x-\Omega_3}{\Omega_4-\Omega_3}\right)^{S_3}, \Omega_3 \leq x \leq \Omega_4 \\ r+(s-r)\left(\frac{x-\Omega_4}{\Omega_5-\Omega_4}\right)^{S_4}, \Omega_4 \leq x \leq \Omega_5 \\ s+(t-s)\left(\frac{x-\Omega_5}{\Omega_6-\Omega_5}\right)^{S_5}, \Omega_5 \leq x \leq \Omega_6 \\ t+(u-t)\left(\frac{x-\Omega_6}{\Omega_7-\Omega_6}\right)^{S_6}, \Omega_6 \leq x \leq \Omega_7 \\ u+(1-u)\left(\frac{x-\Omega_7}{\Omega_8-\Omega_7}\right)^{S_7}, \Omega_7 \leq x \leq \Omega_8 \\ 1, \Omega_8 \leq x \leq \Omega_9 \\ e-(e-1)\left(\frac{\Omega_{10}-x}{\Omega_{10}-\Omega_9}\right)^{P_1}, \Omega_9 \leq x \leq \Omega_{10} \\ f-(f-e)\left(\frac{\Omega_{11}-x}{\Omega_{11}-\Omega_{10}}\right)^{P_2}, \Omega_{10} \leq x \leq \Omega_{11} \\ g-(g-f)\left(\frac{\Omega_{12}-x}{\Omega_{12}-\Omega_{11}}\right)^{P_3}, \Omega_{11} \leq x \leq \Omega_{12} \\ h-(h-g)\left(\frac{\Omega_{13}-x}{\Omega_{13}-\Omega_{12}}\right)^{P_4}, \Omega_{12} \leq x \leq \Omega_{13} \\ i-(i-h)\left(\frac{\Omega_{14}-x}{\Omega_{14}-\Omega_{13}}\right)^{P_5}, \Omega_{13} \leq x \leq \Omega_{14} \\ j-(j-i)\left(\frac{\Omega_{15}-x}{\Omega_{15}-\Omega_{14}}\right)^{P_6}, \Omega_{14} \leq x \leq \Omega_{15} \\ j\left(\frac{\Omega_{16}-x}{\Omega_{16}-\Omega_{15}}\right)^{P_7}, \Omega_{15} \leq x \leq \Omega_{16} \\ 0, x \leq \Omega_1 and x \geq \Omega_{16} \end{cases}$$

$$A_\alpha = \begin{cases} A_{1L}(\alpha) = \Omega_1 + (\frac{\alpha}{p})(\Omega_2-\Omega_1) for \alpha \in [0,p] \\ A_{2L}(\alpha) = \Omega_2 + (\frac{\alpha-p}{q-p})(\Omega_3-\Omega_2) for \alpha \in [p,q] \\ A_{3L}(\alpha) = \Omega_3 + (\frac{\alpha-q}{r-q})(\Omega_4-\Omega_3) for \alpha \in [q,r] \\ A_{4L}(\alpha) = \Omega_4 + (\frac{\alpha-r}{s-r})(\Omega_5-\Omega_4) for \alpha \in [r,s] \\ A_{5L}(\alpha) = \Omega_5 + (\frac{\alpha-s}{t-s})(\Omega_6-\Omega_5) for \alpha \in [s,t] \\ A_{6L}(\alpha) = \Omega_6 + (\frac{\alpha-t}{u-t})(\Omega_7-\Omega_6) for \alpha \in [t,u] \\ A_{7L}(\alpha) = \Omega_7 + (\frac{\alpha-u}{1-u})(\Omega_8-\Omega_7) for \alpha \in [u,1] \\ A_{7R}(\alpha) = \Omega_{10} + (\frac{\alpha-e}{e-1})(\Omega_{10}-\Omega_9) for \alpha \in [e,1] \\ A_{6R}(\alpha) = \Omega_{11} + (\frac{\alpha-f}{f-e})(\Omega_{11}-\Omega_{10}) for \alpha \in [f,e] \\ A_{5R}(\alpha) = \Omega_{12} + (\frac{\alpha-g}{g-f})(\Omega_{12}-\Omega_{11}) for \alpha \in [g,f] \\ A_{4R}(\alpha) = \Omega_{13} + (\frac{\alpha-h}{h-g})(\Omega_{13}-\Omega_{12}) for \alpha \in [h,g] \\ A_{3R}(\alpha) = \Omega_{14} + (\frac{\alpha-i}{i-h})(\Omega_{14}-\Omega_{13}) for \alpha \in [i,h] \\ A_{2R}(\alpha) = \Omega_{15} + (\frac{\alpha-j}{j-i})(\Omega_{15}-\Omega_{14}) for \alpha \in [j,i] \\ A_{1R}(\alpha) = \Omega_{16} - (\frac{\alpha}{j})(\Omega_{16}-\Omega_{15}) for \alpha \in [0,j] \end{cases}$$

1. Arithmetic operations on linear HFN with symmetry Let $\tilde{A}_{LS} = (\Omega_1, \Omega_2, \Omega_3, \Omega_4, a_5, a_6, a_7, a_8, a_9, \Omega_{10}, \Omega_{11}, \Omega_{12}, \Omega_{13}, \Omega_{14}, \Omega_{15}, \Omega_{16}; m_1, n_1)$ and

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$\tilde{B}_{LS} = (\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6, \varphi_7, \varphi_8, \varphi_9, \varphi_{10}, \varphi_{11}, \varphi_{12}, \varphi_{13}, \varphi_{14}, \varphi_{15}, \varphi_{16}; m_2, n_2)$ be two linear heptagonal FN's with symmetry, then

(i) The summation of two HFN's is defined as $\tilde{C}_{LS} = \tilde{A}_{LS} + \tilde{B}_{LS} = (\Omega_1 + \varphi_1, \Omega_2 + \varphi_2, \Omega_3 + \varphi_3, \Omega_4 + \varphi_4, \Omega_5 + \varphi_5, \Omega_6 + \varphi_6, \Omega_7 + \varphi_7, \Omega_8 + \varphi_8, \Omega_9 + \varphi_9, \Omega_{10} + \varphi_{10}, \Omega_{11} + \varphi_{11}, \Omega_{12} + \varphi_{12}, \Omega_{13} + \varphi_{13}, \Omega_{14} + \varphi_{14}, \Omega_{15} + \varphi_{15}, \Omega_{16} + \varphi_{16}; m, n)$ Where $m = \min\{m_1, m_2\}$ and $n = \min\{n_1, n_2\}$.

Theorem 2.1.. Let $\tilde{H}_1 = (a^1_{lu}, a^2_{lu}, a^3_{lu}, a^4_{lu}, a^5_{lu}, a^6_{lu}, a^7_{lu}, a^8_{lu} a^9_{lu}, a^{10}_{lu}, a^{11}_{lu}, a^{12}_{lu}, a^{13}_{lu}, a^{14}_{lu}, a^{15}_{lu}, a^{16}_{lu})$ and $\tilde{H}_2 = (a^1_{\alpha\beta}, a^2_{\alpha\beta}, a^3_{\alpha\beta}, a^4_{\alpha\beta}, a^5_{\alpha\beta}, a^6_{\alpha\beta}, a^7_{\alpha\beta}, a^8_{\alpha\beta}, a^9_{\alpha\beta}, a^{10}_{\alpha\beta}, a^{11}_{\alpha\beta}, a^{12}_{\alpha\beta}, a^{13}_{\alpha\beta}, a^{14}_{\alpha\beta}, a^{15}_{\alpha\beta}, a^{16}_{\alpha\beta})$ be two HFN's; then the arithmetic operation of $\tilde{H}_1$and$\tilde{H}_2$, denoted as $\tilde{H}_1 \oplus \tilde{H}_2, \tilde{H}_1 \Theta \tilde{H}_2$ and $\tilde{H}_1 \otimes \tilde{H}_2$ an yield another HFN,

$$1. \tilde{H}_1 \oplus \tilde{H}_2 = \begin{pmatrix} a^1_{lu} + a^1_{\alpha\beta}, a^2_{lu} + a^2_{\alpha\beta}, a^3_{lu} + a^3_{\alpha\beta}, a^4_{lu} + a^4_{\alpha\beta}, a^5_{lu} + a^5_{\alpha\beta}, a^6_{lu} + a^6_{\alpha\beta}, a^7_{lu} + a^7_{\alpha\beta}, a^8_{lu} + a^8_{\alpha\beta} \\ a^9_{lu} + a^9_{\alpha\beta}, a^{10}_{lu} + a^{10}_{\alpha\beta}, a^{11}_{lu} + a^{11}_{\alpha\beta}, a^{12}_{lu} + a^{12}_{\alpha\beta}, a^{13}_{lu} + a^{13}_{\alpha\beta}, a^{14}_{lu} + a^{14}_{\alpha\beta}, a^{15}_{lu} + a^{15}_{\alpha\beta}, a^{16}_{lu} + a^{16}_{\alpha\beta} \end{pmatrix}.$$

$$2. \tilde{H}_1 \Theta \tilde{H}_2 = \begin{pmatrix} a^1_{lu} - a^{16}_{\alpha\beta}, a^2_{lu} - a^{15}_{\alpha\beta}, a^3_{lu} - a^{14}_{\alpha\beta}, a^4_{lu} - a^{13}_{\alpha\beta}, a^5_{lu} - a^{12}_{\alpha\beta}, a^6_{lu} - a^{11}_{\alpha\beta}, a^7_{lu} - a^{10}_{\alpha\beta}, a^8_{lu} - a^9_{\alpha\beta} \\ a^9_{lu} - a^8_{\alpha\beta}, a^{10}_{lu} - a^7_{\alpha\beta}, a^{11}_{lu} - a^6_{\alpha\beta}, a^{12}_{lu} - a^5_{\alpha\beta}, a^{13}_{lu} - a^4_{\alpha\beta}, a^{14}_{lu} - a^3_{\alpha\beta}, a^{15}_{lu} - a^2_{\alpha\beta}, a^{16}_{lu} - a^1_{\alpha\beta} \end{pmatrix}.$$

$$3. \tilde{H}_1 \otimes \tilde{H}_2 = \begin{pmatrix} a^1_{lu} a^1_{\alpha\beta}, a^2_{lu} a^2_{\alpha\beta}, a^3_{lu} a^3_{\alpha\beta}, a^4_{lu} a^4_{\alpha\beta}, a^5_{lu} a^5_{\alpha\beta}, a^6_{lu} a^6_{\alpha\beta}, a^7_{lu} a^7_{\alpha\beta}, a^8_{lu} a^8_{\alpha\beta} \\ a^9_{lu} a^9_{\alpha\beta}, a^{10}_{lu} a^{10}_{\alpha\beta}, a^{11}_{lu} a^{11}_{\alpha\beta}, a^{12}_{lu} a^{12}_{\alpha\beta}, a^{13}_{lu} a^{13}_{\alpha\beta}, a^{14}_{lu} a^{14}_{\alpha\beta}, a^{15}_{lu} a^{15}_{\alpha\beta}, a^{16}_{lu} a^{16}_{\alpha\beta} \end{pmatrix}.$$

$$\lambda \otimes \tilde{H}_1 = (\lambda a^1_{lu}, \lambda a^2_{lu}, \lambda a^3_{lu}, \lambda a^4_{lu}, \lambda a^5_{lu}, \lambda a^6_{lu}, \lambda a^7_{lu}, \lambda a^8_{lu},$$

$$\lambda a^9_{lu}, \lambda a^{10}_{lu}, \lambda a^{11}_{lu}, \lambda a^{12}_{lu}, \lambda a^{13}_{lu}, \lambda a^{14}_{lu}, \lambda a^{15}_{lu}, \lambda a^{16}_{lu}) .$$

Haar Ranking method for Haxadecagonal Fuzzy Number:

Let $\tilde{A} = (\Omega_1, \Omega_2, \Omega_3, \Omega_4, a_5, a_6, a_7, a_8, a_9, \Omega_{10}, \Omega_{11}, \Omega_{12}, \Omega_{13}, \Omega_{14}, \Omega_{15}, \Omega_{16})$ be the HFN. Using HRM(Haar ranking method), the HFN is rewritten as $\tilde{A} = (\Omega_1, \Omega_2, \Omega_3, \Omega_4, a_5, a_6, a_7, a_8, a_9, \Omega_{10}, \Omega_{11}, \Omega_{12}, \Omega_{13}, \Omega_{14}, \Omega_{15}, \Omega_{16})$. The average and elaborate coefficients namely the scaling and wavelet coefficients of HFN can be calculated as follows.

Step-l: Group the HFN in pairs.

$$[\Omega_1, \Omega_2], [\Omega_3, \Omega_4], [a_5, a_6], [a_7, a_8], [a_9, \Omega_{10}], [\Omega_{11}, \Omega_{12}], [\Omega_{13}, \Omega_{14}], [\Omega_{15}, \Omega_{16}]$$

Step-2: Replace the first 4 elements of approximation coefficient with the detailed coefficient.

$\alpha_1 = (\frac{\Omega_1 + \Omega_2}{2}), \alpha_2 = (\frac{\Omega_3 + \Omega_4}{2}), \alpha_3 = (\frac{a_5 + a_6}{2}), \alpha_4 = (\frac{a_7 + a_8}{2})$,

$\alpha_5 = (\frac{a_9 + \Omega_{10}}{2}), \alpha_6 = (\frac{\Omega_{11} + \Omega_{12}}{2}), \alpha_7 = (\frac{\Omega_{13} + \Omega_{14}}{2}), \alpha_8 = (\frac{\Omega_{15} + \Omega_{16}}{2})$

$\beta_1 = (\frac{\Omega_1 - \Omega_2}{2}), \beta_2 = (\frac{\Omega_3 - \Omega_4}{2}), \beta_3 = (\frac{a_5 - a_6}{2}), \beta_4 = (\frac{a_7 - a_8}{2})$

$\beta_5 = (\frac{a_9 - \Omega_{10}}{2}), \beta_6 = (\frac{\Omega_{11} - \Omega_{12}}{2}), \beta_7 = (\frac{\Omega_{13} - \Omega_{14}}{2}), \beta_8 = (\frac{\Omega_{15} - \Omega_{16}}{2})$

The $\tilde{A}_1$ changed into $\tilde{A}_1 = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8)$

Step-3: Group the pair of approximation coefficients of $\tilde{A}_1$. Then, find the new approximation coefficients and the detailed coefficients for the pair of approximation coefficient of $\tilde{A}_1$

$$[\alpha_1, \alpha_2], [\alpha_3, \alpha_4], [\alpha_5, \alpha_6], [\alpha_7, \alpha_8]$$

$$\gamma_1 = (\frac{\alpha_1 + \alpha_2}{2}), \gamma_2 = (\frac{\alpha_3 + \alpha_4}{2}), \gamma_3 = (\frac{\alpha_5 + \alpha_6}{2}), \gamma_4 = (\frac{\alpha_7 + \alpha_8}{2})$$

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$\eta_1 = (\dfrac{\alpha_1 - \alpha_2}{2}), \eta_2 = (\dfrac{\alpha_3 - \alpha_4}{2}), \eta_3 = (\dfrac{\alpha_5 - \alpha_6}{2}), \eta_4 = (\dfrac{\alpha_7 - \alpha_8}{2})$

The $\tilde{A}_1$ changed into $\tilde{A}_2 = (\gamma_1, \gamma_2, \gamma_3, \gamma_4, \eta_1, \eta_2, \eta_3, \eta_4, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8)$

Step-4: Determine the pair of approximation coefficient in $\tilde{A}_2$. Then, find the new approximation and detailed coefficients for the pair of approximation coefficient of $\tilde{A}_2$.

$$[\gamma_1, \gamma_2, \gamma_3, \gamma_4]$$

$\delta_1 = (\dfrac{\gamma_1 + \gamma_2}{2}), \delta_2 = (\dfrac{\gamma_3 + \gamma_4}{2}), \mathcal{E}_1 = (\dfrac{\gamma_1 - \gamma_2}{2}), \mathcal{E}_2 = (\dfrac{\gamma_3 - \gamma_4}{2})$

The $\tilde{A}_2$ changed into $\tilde{A}_3 = (\delta_1, \delta_2, \epsilon_1, \epsilon_2, \eta_1, \eta_2, \eta_3, \eta_4, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8)$

Step-4: Determine the pair of approximation coefficient in $\tilde{A}_3$. Then, find the new approximation and detailed coefficients for the pair of approximation coefficient of $\tilde{A}_3$.

$$[\delta_1, \delta_2]$$

$\mu_8 = (\dfrac{\delta_1 + \delta_2}{2}), \mu_2 = (\dfrac{\delta_1 - \delta_2}{2})$

The $\tilde{A}_3$ changed into $H(\tilde{A}) = (\mu_1, \mu_2, \epsilon_1, \epsilon_2, \eta_1, \eta_2, \eta_3, \eta_4, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8)$

Step-5: Determine the Ranking μ.

• $\tilde{A} \prec \tilde{B}$, if the first element of the ordered tuple of $H(\tilde{A})$ is less than the first element of the ordered tuple of $H(\tilde{B})$.

• $\tilde{A} \succ \tilde{B}$, if the first element of the ordered tuple of $H(\tilde{A})$ is greater than the first element of the ordered tuple of $H(\tilde{B})$.

• $\tilde{A} \approx \tilde{B}$ if and only if all the elements of $H(\tilde{A})$ and $H(\tilde{B})$ are term wise equal.

3 Fuzzy Assignment Problem(FAP)

FAP in general defines as follows

$$\min \text{ (or)}\max\ X = \textstyle\sum_j \sum_{i=1}^{m} \tilde{P_{ij}} y_{ij}$$

Subject to

$$\sum_{i=1}^{m} y_{ij} = 1 for i = 1, 2, \ldots, m.$$

$$\sum_{j=1}^{m} y_{ij} = 1 for j = 1, 2, \ldots, m.$$

$y_{ij} = 1$, if the i^{th} job is assigned to j^{th} person
0, if the i^{th} job is not assigned to j^{th} person

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Example 4.1: A FAP with 4 machines$\mathrm{M}_1, M_2, M_3, M_4$ and 4 jobs J, J_2, J_3, J_4 is premeditated. The cost matrix C_{ij^*} is whose values are depicted by HFN. The problem is to find the minimum assignment cost. Here, the Hungarian method.

After taking the averages of fuzzy cost matrix, the following is obtained

$$A = \begin{bmatrix} 5 & 8.2 & 9.4 & 7.2 \\ 8.3 & 7.1 & 15.1 & 8.3 \\ 10.5 & 9.4 & 10.5 & 10.6 \\ 13.8 & 8.3 & 12.5 & 7.5 \end{bmatrix}$$

Row wise subtraction,

$$A = \begin{bmatrix} 0 & 3.2 & 4.4 & 2.2 \\ 1.2 & 0 & 8 & 1.2 \\ 1.1 & 0 & 1.1 & 1.2 \\ 6.3 & 0.8 & 5 & 0 \end{bmatrix}$$

Column wise subtraction,

$$A = \begin{bmatrix} 0 & 3.2 & 3.3 & 2.2 \\ 1.2 & 0 & 6.9 & 1.2 \\ 1.1 & 0 & 0 & 1.2 \\ 6.3 & 0.8 & 3.9 & 0 \end{bmatrix}$$

Number of rows=Number of squares.

Therefore, the optimal cost is $= 5 + 7.1 + 10.5 + 7.5 = 30.1$.

4 Conclusion

In this present study, the GHFN's have been derived under fuzzy environment which may help to handle uncertainties in the decision-making problems.

These kinds of FN's are helpful when decision maker needs to represent a parameter at 16 different points. The following important outcomes have been attained in this research,

- The membership curve of a generalized linear and nonlinear Haxadecagonal fuzzy number with symmetry and asymmetry has been derived.
- Alpha cuts for all kinds of Haxadecagonal fuzzy number have also been derived.

Generalized Haxadecagonal fuzzy numbers can be used to extend Multi Criteria Decision Making (MCDM) models such as DEMATEL, TOPSIS, VIKOR, and others. These numbers are helpful in transportation problems such fuzzy assignment and transportation.

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