10.48047/jocaaa.2024.33.08.390

Behavioural Analysis of an m-out-of-n Units Cold Standby System with Perfect ARM

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Received: 20.09.2024	Revised: 22.10.2024	Accepted: 25.11.2024

ABSTRACT

2-out-of-3 unit redundant system of identical units is considered in this paper. Initially two units work in parallel and third unit kept as cold standby. An automatic repair machine is considered which repairs a failed unit. The standby unit will automatically starts working whenever an operative unit fails. In this model we assume that the automatic repair machine repairs a failed unit without failure. It is assumed that a repaired unit is as good as new and switch over time is negligible.

Keywords: Regenerative Point, MTSF, Availability, Busy Period.

1. INTRODUCTION

A large amount of work has been done by many researchers in field of reliability taking two-unit models under different conditions. Kumar, Gupta and Taneja [1996] and Giri, Goel and Singh [2009] and many more have given various results for two unit systems. People are trying to manufacture most automatic system in their industries to meet public demands. The provision of spare unit is felt necessary to increase the reliability of a system. Such systems may need more than one of its parallel components. For example two of the four generators in a generating station may be necessary to supply the required power to the customers. The other two are added to increase the supply reliability. Another example is a four-engine aircraft where only two engines are required for successful operation. Many more such systems exist in industries and other areas.

Here, three units' system is considered in which two units are sufficient to perform its work. Such a three unit redundant system is applied in real world to attain high reliability and performance. Considering this view a model is studied in this chapter consisting of three-units in which operation of two is necessary. Here in this model three units are considered out of which two works at a time and one is kept as a cold standby. Keeping in view more automation of the system an Automatic Repair Machine (ARM) is considered which detects a problem automatically and repair it whenever a unit is found faulty or failed. The standby unit will automatically starts working whenever an operative unit fails. In this model it is assumed that ARM will not fail while repairing the failed unit. Model is accessed by taking particular cases on the reliability parameters like as Availability, MTSF, Busy Period of Repair machine etc., and using mathematical tools like Markov Process, Markov chain and Regenerative Point Technique.

Assumptions and System Description:

- System is made-up of three identical units. Initially two units are operative and third are kept as cold standby.
- System is considered in up-state if two units are working, in partial up-state if only one unit is working and in down-state if no unit is working.
- A unit of the system has two forms failed or normal operative.
- Failure of a unit is self-detected.
- A failed unit is repaired by an Automatic Repair Machine (ARM) which is always with the system.
- In this model it is assumed that ARM will never fail.

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- Failure rate of unit have an exponential distribution of time and repair rates are taken general distribution.
- The switching is perfect and instantaneous.
- Expert repairman is called whenever required.
- The random variables are considered as independent in nature.

NOTATIONS:

E :	Set of regenerative states
$\overline{E}$:	Set of non-regenerative states
λ :	Constant failure rate of a unit
g (t), G (t):	probability density function and cumulative density function of repair time of a failed unit
O:	Unit is in operative state
C_S:	Unit is in cold standby state
F_r:	Failed unit under repair
F_R:	Repair of failed unit continuous from previous state
F_{wr}, F_{WR}:	A failed unit waiting for repair, a failed unit waiting for repair from previous state
ARM_O:	Automatic Repair Machine in operative state
Θ:	Symbol for Stieltjes Convolution
©:	Symbol for Convolution

2. MODEL

In this model ARM is considered in such a way that it will not fail. There are total four states (St_0, St_1, St_2, and St_3) in which St_0 and St_1 are in up-state, St_2 is in partial up-state and St_3 is in down-state. Here

E :	(St_0, St_1, St_2)	--	regenerative states
$\overline{E}$:	(St_3)	--	non- regenerative state

Transition Probabilities:

The transition probabilities from the state St_i to St_j are given by dQ_{ij} and in steady state p_{ij} denote the transition probability from state St_i to St_j are given as under

$dQ_{01} = \int_0^t \lambda e^{-\lambda t} dt$, $dQ_{10} = \int_0^t e^{-\lambda t} g(t) dt$, $dQ_{12} = \int_0^t \lambda e^{-\lambda t} \overline{G}(t) dt$

$dQ_{21} = \int_0^t e^{-\lambda t} g(t) dt$, $dQ_{23} = \int_0^t \lambda e^{-\lambda t} \overline{G}(t) dt$, $dQ_{2}{}^{(3)}{}_{2} = \int_0^t (\lambda e^{-\lambda t} © 1) g(t) dt$

The non-zero p_{ij} are as follows

$$p_{ij} = dQ_{ij}(\infty) \quad \text{or} \quad p_{ij} = \lim_{t\to\infty} dQ_{ij}(t)$$

$p_{01}=1$, $p_{10}=g^*(\lambda)$, $p_{12}=1- g^*(\lambda)$, $p_{21}=g^*(\lambda)$, $p_{23}=1- g^*(\lambda)$, $p_{2}{}^{(3)}{}_{2}= 1- g^*(\lambda)$

We may easily verify the following relations

$p_{01}=1$, $p_{10} + p_{12}=1$, $p_{10} + p_{23}=1$, $p_{10} + p_{2}{}^{(3)}{}_{2}=1$, $p_{21} + p_{12}=1$, $p_{21} + p_{23}=1$, $p_{21} + p_{2}{}^{(3)}{}_{2}=1$

Mean Sojourn Times:

To calculate mean time of stay μ_i in state St_i, let T_i be this time. Then we have following relations

$\mu_0=\frac{1}{\lambda}$, $\mu_1=\frac{1}{\lambda}[1- g^*(\lambda)]$, $\mu_2=\frac{1}{\lambda}[1- g^*(\lambda)]$,

$m_{01}=\frac{1}{\lambda}$, $m_{10}= - [g^*(\lambda)]'$, $m_{12}=\frac{1}{\lambda}[1- g^*(\lambda)] + [g^*(\lambda)]'$, $m_{21}=- [g^*(\lambda)]'$,

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$m_{23}=\frac{1}{\lambda}[1-g^*(\lambda)]+[g^*(\lambda)]'$ $\quad\quad m_2{}^{(3)}{}_2 = -[g^*(0)]'+[g^*(\lambda)]'$

MTSF:

The mean time for system failure is given by

$$\pi_0(t) = dQ_{01}(t)\ \Theta\ \pi_1(t)$$
$$\pi_1(t) = dQ_{10}(t)\ \Theta\ \pi_0(t) + dQ_{12}(t)\ \Theta\ \pi_2(t)$$
$$\pi_2(t) = dQ_{23}(t) + dQ_{21}(t)\ \Theta\ \pi_1(t)$$

Taking Laplacian Stieltjes Transforms for $\pi_0^{**}(s)$, we get

$$\pi_0^{**}(s) = \frac{N(s)}{D(s)}$$

Here

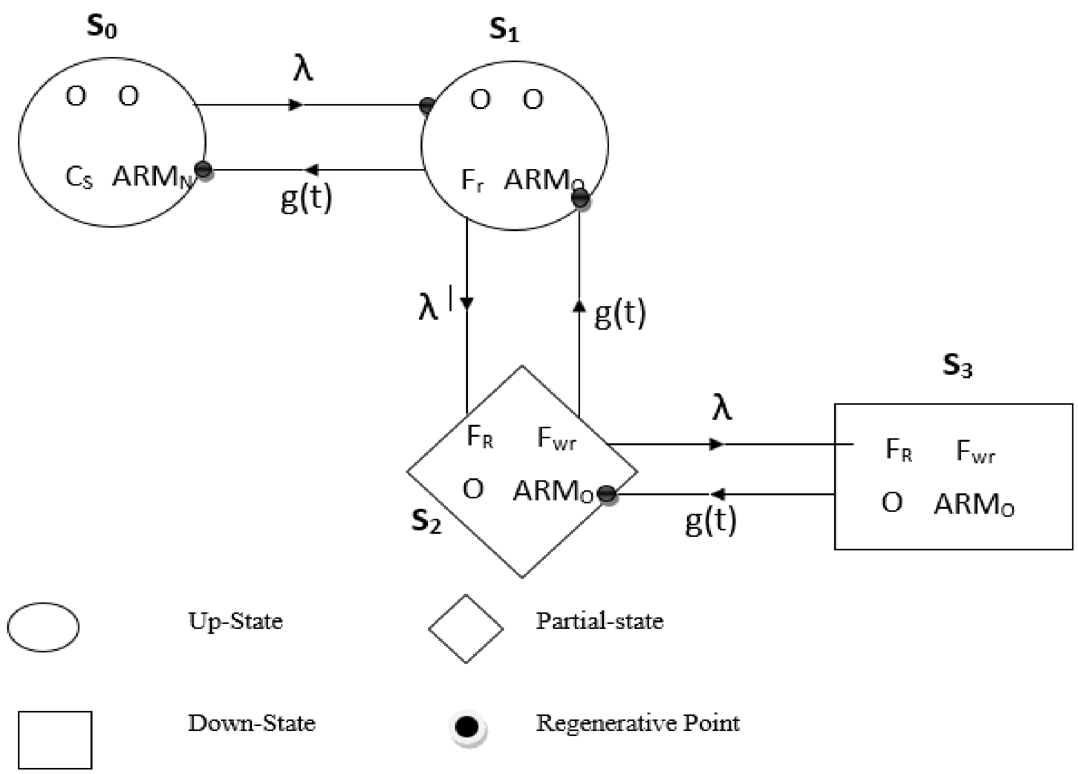
$$N(s) = -q_{01}q_{12}q_{23}$$
$$D(s) = -1 + q_{12}q_{21} + q_{01}q_{10}$$

$$\text{MTSF} = \pi_0 = \lim_{s\to 0}\frac{1-\pi_0^{**}(s)}{s}$$
$$= \frac{D'(0)-N'(0)}{D(0)} = \frac{N}{D}$$
$$= -\frac{\mu_0(p_{10}-1)(p_{10}-3)}{[1-g^*(\lambda)]^2}$$

Here

$N(0) = -[1-g^*(\lambda)]^2$, $\quad D(0) = -[1-g^*(\lambda)]^2$

State Transition Diagram:



Availability of the System:

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Let $Av_i(t)$ be the probability that a system is available at a point of time t after the unit entered the regenerative position at t = 0. The relations for $Av_i(t)$ are as follows:

$$Av_0(t) = M_0(t) + q_{01}(t) © Av_1(t)$$
$$Av_1(t) = M_1(t) + q_{10}(t) © Av_0(t) + q_{12}(t) © Av_2(t)$$
$$Av_2(t) = q_{21}(t) © Av_1(t) + q_{2\,2}^{(3)}(t) © Av_2(t)$$

Here $M_0(t) = \int_0^t e^{-\lambda t} d(t)$ $\quad M_1(t) = \int_0^t e^{-\lambda t}\bar{G}(t)d(t)$

Now taking Laplacian Transforms for $Av_0^*(s)$, we get

$$Av_0^*(s) = \frac{N_1(s)}{D_1(s)}$$

Availability in the long run is as follows;

$$Av_0 = \lim_{s\to 0}(sAv_0^*(s)) = \frac{N_1(0)}{D_1'(0)}$$

Here

$$N_1(0) = -\mu_0[1-p_{2\,2}^{(3)}-p_{12}p_{21}] + p_{01}\mu_1[-1 + p_{2\,2}^{(3)}]$$
$$D_1'(0) = -[m_{2\,2}^{(3)} + m_{12}p_{10} + p_{12}m_{21} + \mu_0 p_{10}(1-p_{12}) + m_{10} - \mu_0 p_{23} - p_{1o}\, m_{2\,2}^{(3)}]$$

Busy Period Analysis of ARM:

Let $Bp_i(t)$ is the busy period of an Automatic Repair Machine starting at a point at t=0 is as follows;

$$Bp_0(t) = q_{01}(t) © Bp_1(t)$$
$$Bp_1(t) = W_1(t) + q_{10}(t) © Bp_0(t) + q_{12}(t) © Bp_2(t)$$
$$Bp_2(t) = W_2(t) + q_{21}(t) © Bp_1(t) + q_{2\,2}^{(3)}(t) © Bp_2(t)$$

Here $W_1(t) = \int_0^t e^{-\lambda t}\bar{G}(t)d(t)$

$$W_2(t) = \int_0^t e^{-\lambda t}\bar{G}(t)d(t)$$

Taking Laplacian Transforms for $Bp_0^*(s)$, we have

$$Bp_0^*(s) = \frac{N_2(s)}{D_1(s)}$$

In the steady state

$$Bp_0 = \lim_{s\to 0} sBp^*(s) = \lim_{s\to 0}\frac{sN_2(s)}{D_1(s)} = \frac{N_2(0)}{D_1'(0)}$$

Here $N_2(0) = -\mu_1 = -\frac{1}{\lambda}[1- g^*(\lambda)]$

$D_1'(0)$ is already defined

Profit Function:

If P is profit function, then in long run it is given by

$$P = r_1 * Av_0 - C_1 * Bp_0$$

Where

C_1 = operating cost per unit time of ARM

Particular cases:

By taking particular cases for repair rate, $g(t) = \omega e^{-\omega t}$ and drawing graphs we can find the impact of different parameters. Then we get,

$$p_{01} = 1,\ p_{10} = \frac{\omega}{\lambda+\omega} \quad p_{12} = \frac{\lambda}{\lambda+\omega} \quad p_{21} = \frac{\omega}{\lambda+\omega}, \quad p_{23} = \frac{\lambda}{\lambda+\omega} \quad p_{2\,2}^{(3)} = \frac{\lambda}{\lambda+\omega}$$

Mean Sojourn Time:

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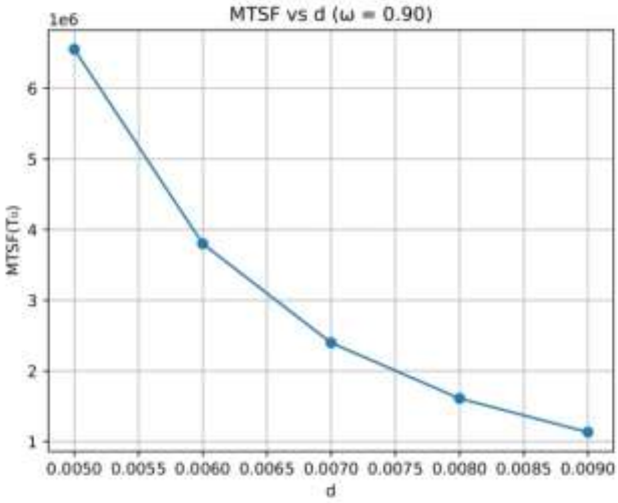
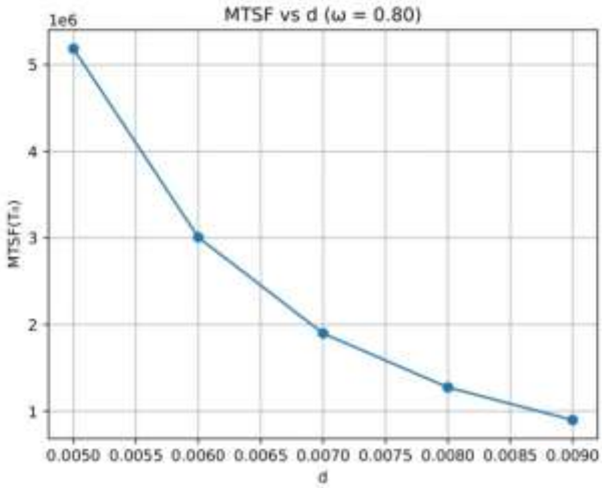
$\mu_0 = \dfrac{1}{\lambda}$, $\mu_1 = \dfrac{1}{\lambda+\omega}$, $\mu_2 = \dfrac{1}{\lambda+\omega}$

Unconditional Mean Time:

$m_{01} = \dfrac{1}{\lambda}$, $m_{10} = \dfrac{\omega}{(\omega+\lambda)^2}$, $m_{12} = \dfrac{\lambda}{(\omega+\lambda)^2}$ $m_{21} = \dfrac{\omega}{(\omega+\lambda)^2}$ $m_{23} = \dfrac{\lambda}{(\omega+\lambda)^2}$ $m_{2^{(3)}2} = \omega\left\{\dfrac{1}{\omega^2} - \dfrac{1}{(\lambda+\omega)^2}\right\}$

$$\textbf{MTSF} = \frac{\omega\lambda(1+2\lambda)+\omega(\omega+\lambda)+\lambda^2(\omega+2\lambda)}{\lambda^3}$$





$$\textbf{Availability} = \frac{\omega^2(\lambda+\omega)^2}{\lambda\omega(\lambda^2+\omega^2)+\lambda^4+\omega^4+\omega^2\lambda^2}$$

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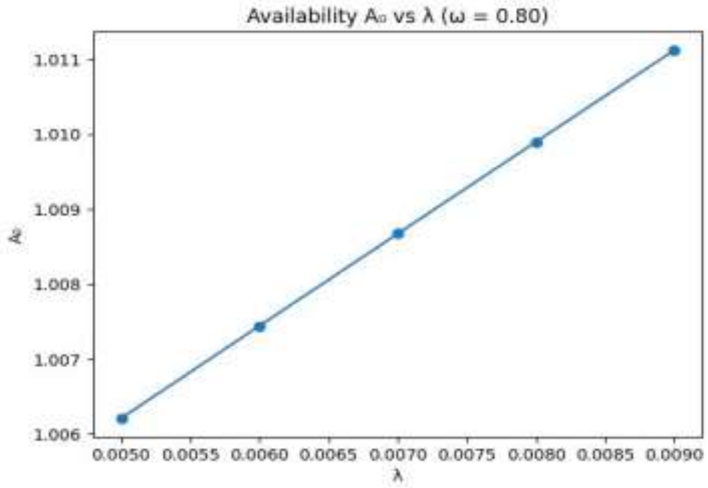

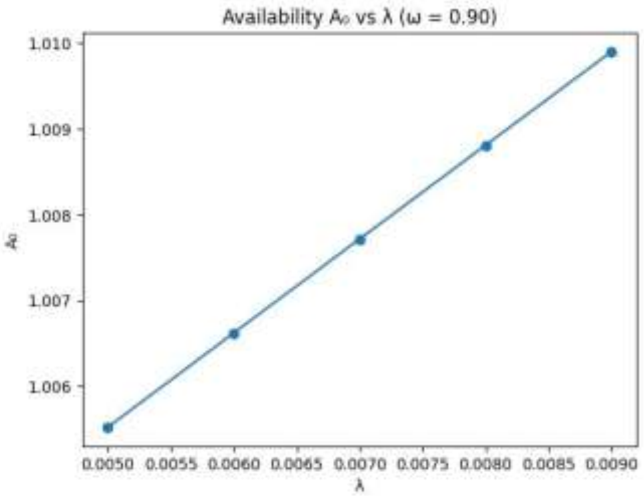

3. CONCLUSION

In this paper, it is observed with the help of Graphs and Tables, the Failure Rate(λ) increases, MTSF and Availability of the System is decrease, which should be. In this study, m-out-of-n Units Cold Standby System with Perfect ARM , the perfect ARM i.e. Automatic Repair Machine means it should not fail and we used RPT to solve the various parameters of System. In future, Researchers can evaluated the parameters, when repair rate and failure rate are time-dependant and also discuss the cost and profit benefit analysis. Further results can also be apply to find the Waiting Time of Units and Number of Server's visits. Generally, the Management have fixed resources and by fixing the target Availability, we can provide the repair rate for optimum solution.

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