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ON REVERSE SUPER EDGE MAGIC GRAPHS

Dr. Nanda S Warad
Associate professor Department of Mathematics
Government First Grade College kembhavi Dist. Yadgir, Karnataka, India.

ABSTRACT

In [2], Akka et. al. introduced a new concept reverse super edge magic labling. A graph G=(V,E) is said to be Reverse super edge magic labling is there exists a one- to-one correspondence f from VUE on to {1,2,.....,|V|+|E|} such that f(V) =(1,2,...,|V|}and f(xy)-f(x)- f(y) is constant $k^{-}$for every edge xy of G. We find whether or not some families of graphs BT_n, (n,t)- kite, K_2UC_2 and S($K_{1,n}$) are Reverse super edge magic.

Key Words: Edge magic labeling, Reverse edge magic labeling, Reverse super edge magic labeling.

2010 Mathematics Subject Classification: Primary 05C07 ,Secondary 05C90

1. introduction

In this paper, we consider only finite, simple,undirected graphs. Our nations and terminology are as in [6]. In particular a graph G has vertex set V(G) and edge set E(G) and we let |V(G)|=p and |E(G)|=q. Given a graph G, let V= V(G),E= E(G). A one-to-one correspondence f from VUE onto the integers {1,2,...,p+q} is an edge magic labeling if there is a constant k so that for any edge xy, f(x)+f(y)+ f(xy)=k the constant k is called the edge magic number**.** Moreover f is called a super edge magic labeling if f(V(G))= {1,2,.....,p} and f is called a super magic labeling if f(E(G))= {1,2,.....,q}. A one-to-one correspondence f: VUE→{1,2,...,p+q} is a reverse edge magic labling if there is a constantk^{-1} so that any edge xy, f(xy)-f(y)- f(x)= k^{-1}*the constant* k^{-1} is called the reverse edge magic number**.** Moreover f is called reverse super edge magic labeling if f(V(G))= {1,2,.....,P} and f is called reverse super magic labeling f(E(G))= {1,2,.....,q}. A graph G is called edge magic(super edge magic, super magic, reverse super edge magic, reverse super magic) if there exists an edge magic(super edge magic, super magic, reverse super edge magic, reverse super magic) labeling of G. The above first two definitions where first introduced by Kotzig and Rose[7] in 1970 and Enomoto Llado, Nakamigawa and Ringel[5] in 1998 respectivly. It is worth while to mention that Kotzig and Rosa called magic valuations, the current term is due to Ringel[9]. The third definition was first introduced Akka and Warad[3,1]. The last two definitions where first introduced by Akka and Hunagund[2] and Akka and Warad[4] respectively. BT_n is a graph with maximum edges 2p-3, (n,t) is a kite consisting of cycle of length n with a t-edge path attached to one vertex[Fig.1), K_2UC_n is union of edge K_2 *and cycle* C_n and S($K_{1,n}$) is a subdivision graph of a star $K_{1,n}$.

In this paper we investigate whether or not several not worty families of graphs. BT_n, (n,t)- kite, K_2UC_2

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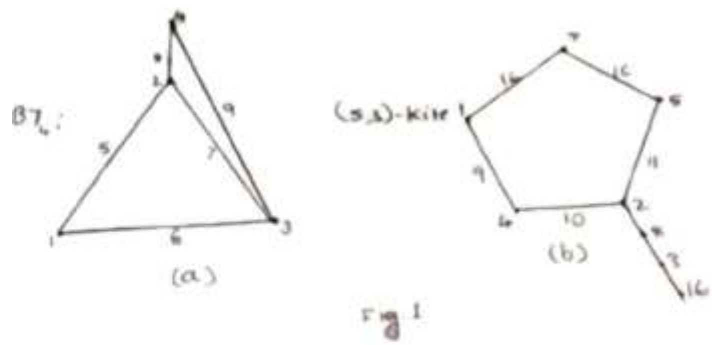
and S($K_{1,n}$) are , reverse super edge magic or not.

Fig.1



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2. Main Results

We find whether BT_n, (n,t)- kite, K_2UC_2 and S($K_{1,n}$) are reverse super edge magic or not.

I. BT_n

Theorem 2.1. The graph BT_n is a reverse super edge magic where n≥ 4

Proof. Let V(BT_n)={v_1, v_1.....,v_n} and

E(BT_n) $= \{v_i v_{i+1}/1 \leq n \leq n-1\} U\{v_{2i-1}v_{2i+1}/1 \leq i \leq$ ₹ n- 2 $i \leq$ 1[]}$U\{v_{2i}v_{2i+2}$ /

₹ n-2 []}.Since 2 the number of edges of BT_n $is\ 2n-3, the\ labeling$ f: VUE→{1,2,...,3n-3 } defined as follows:

F(v_i) $= i$ $for\quad i \leq\quad \leq i\quad n$

$f(v_i v_{i+1})$ = n+2$i-1$ $for\quad i \leq i \leq n-1$

f($v_{2i-1}v_{2i+1}$) $- n + 4i - 2$ $for\quad 1 \leq i \leq$ ₹ [n−1 2

f($v_{2i}v_{2i+2}$) = n +4i $for\quad 1 \leq i \leq$ ₹ [n−2 2

It is easily seen that f is a reverse super edge magic labeling of BT_n with , reverse super edge magic number n-2. Hence BT_n $n \geq 4$,is reverse super edge magic graph.

II. (n,t)- kite

An (n,t)- kite consists of a cycle of length n with a t-edge path (the tail)attached to vertex of the cycle. Wallis[12]posed a problem to investigate the edge magic properties of (n,t)- kite for general t. In [10] Suh- Ryung Kim et al (Akka et al [3]) obtained a necessary condition for an (n,t)- kite being super edge magic(super magic). We give a necessary condition for an (n,t)- kite being super edge magic as followings:

Theorem 2.2. Let v be the vertex of degree 3 and w be the pendent vertex of a reverse super edge magic of (n,t)- kite.Then the following are true: 1. n and t must have the same parity

2. The reverse edge magic number is either k^{-1}= n+t 2 and f(v)-f(w)= − n+t 2 or k^{-1}= n+t 2 − n+t 2 1 2

$and\ \lambda(v) - \lambda(w) =$

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Proof. Let v, v_1, v_2 … ..,v_nv be a vertex set of C_n. *Let* f(v)= α and $f(w) = \beta$. Then

$k^{-1}\{n + t) = \Sigma$ $\underset{xy\in\in E(G)}{[f(xy) - f(y) - f(x)]}$

$=\sum_{xy\in\in E(G)} f(xy) - 2\sum_{x\in V(G)}[f(x) -$ f(v) + f(w)]

$\frac{(n+t)[(n+t+1)+(2n+2t)]}{2} = \frac{}{2}$ $\frac{2(n+t)(n+t+1)}{2} - -\alpha + \beta$

$\frac{(n+t)|n+t-1\}|-(\alpha-\beta)}{2} =$

This implies that k^{-1}= $\frac{n+t-1}{2} - \frac{(\alpha-\beta)}{n+t}$ is an intiger.Sinc a< $|\frac{(\alpha-\beta)}{n+t}|$< 1 it is true that $\frac{(\alpha-\beta)}{n+t} = \frac{1}{2}$

and that n and t must have the same parity.Then

$k_{-1} = \frac{n+t}{2}$ if f(v) - $f(w)$ =$-\frac{n+t}{2}$ and $k^{-1} = \frac{n+t}{2}$-1 f(v)- $f(w)$=$\frac{n+t}{2}$

It has been shown that the Theorem 2.23 in [12] an (n,t)- kite is edge magic.In[10] Suh-Ryung Kim et al proved that an (n,t)- kite is super edge magic if n is odd and that the converse is also true. We prove that it is also reverse super edge magic if and only if n is odd in the following Theorem 2.3

Theorem 2.3. An (n,t)- kite is reverse super edge magic if and only if n is odd

Proof. The necessary part immediately follows from Theorem 2.2. To show sufficient condition, let n=2m+1 for m=0,1,2,…. .We define a function f: VUE→{1,2,…,4m+m} as follows:

$f(v_i) = \frac{i}{2}+1$ if i is even

$=m+2+\frac{i+1}{2}$ if i is odd

f(v)=1, f(w) $= m + 2$, f(vw)=2m+4 , f(vv_{n-1}) = 2m+3, f(vv_1) = 2m+5

f(v_iv_{i+1}) = 2m+5+I where 1≤ $i \leq n - 2$

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It easy to see that f is a reverse super edge magic labeling of an (n,t)- kite with the reverse edge magic number m+1.Hence an (n,t)- kite reverse super edge magic graph.

Park et al [8] proved that (n,2)- kite is super edge magic for every positive even integer n. In[11] showed that (n,3)- kite might not be super edge magic even if n is odd.Analogously in the following theorem we show that (n,3)- kite might not be reverse super edge magic even if n is odd.

Theorem 2.4. An (3,3)- kite is not reverse super edge magic

Proof. Assume that a (3,3)- kite=G is a reverse super edge magic graph. Let k^{-1} be a reverse super edge magic number.Then there exists a labeling f: VUE→{1,2,...,12}.Let v be the vertex of degree 3 and w be the the pendent vertex. Let E(G)={ v_1, v_2 ... ..,v_6}, f(v)= α and f(w)= $\beta. Then$ by Theorem 2.2 either k=3 and $\alpha-\beta=-3$ or $k=2$ and $\alpha-\beta=3$. If k=3 and $\alpha-\beta=-3$ then for each reverse edge label the possible labels of its end vertices are as follows: From Table 1 we show that labels of the end vertices of e_6 are 1 and 3. For
convenience we denote by $\lambda(e_i)$ the set of labels for endvertices of edge e_i.We consider three cases (i) $\alpha=1, \beta=4$ (ii) $\alpha=2, \beta=5$ $(iii) \alpha=3, \beta=6$

***Case* 1**. if $\alpha=1, \beta=4$ $then$ $\lambda(e_5)\neq\{1,4\}$, $since$ v and w are not adjacent. Thus $\lambda(e_5)=\{2,3\}$. $Since$ 3 is $used$ $trice$ for $vertex$ label, $\lambda(e_1)$={4,5} and $\lambda(e_2)$ ={2,6}.Since 2 is used twice $\lambda(e_3)$={1,6} and $\lambda(e_4)$={1,5}. From the fact that $\lambda(e_1)=$ {4,5} and $\lambda(e_4)$={1,5} have a common element 5, we know that edges e_1 and e_4 are adjacent and that the vertex with label 1 and that with label 4 are at distance two, we have contradiction.

***Case* 2**. if $\alpha=2, \beta=5$ then $\lambda(e_3)\neq$ {2,5} since v and w are not adjacent. Since 2 is a label for v of degree 3, 2 must be used 3 times. Thus $\lambda(e_2)$= {2,6}, $\lambda(e_4)$= {2,4}

Edges	e_1	e_2	e_3	e_4	e_5	e_6
Edge labels	12	11	10	9	8	7
Possible sets of labels of end vertices	{3.6} {4,5}	{2,6} {3,5}	{1,6} {2,5} {3,4}	{1,5} {2,4}	{1,4} {2,3}	{1,3}

Table.1. Possible labels for k=3

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and $\lambda(e_5)$= {2,3}. Since 3 is used twice, $\lambda(e_1)$={4,5}.Then edges e_1 *and* e_4 *are adjacent* and so the vrtex with label 2 and that with label 5 are at distance two. Now we reach a contradiction.

***Case* 3**.If $\alpha = 3$, $\beta = 6$ then $\lambda(e_1)$≠{3,6} and so $\lambda(e_1)$={4,5}. Since label 6 must be used once either $\lambda(e_2)$={2,6} or $\lambda(e_3)$={1,6}.

***SubCase* 3. 1**. If $\lambda(e_3)$={1,6}.then e_3 *and* e_6 are adjacent and so the vertex with labe 3 and that with label 6 are at distance two, which is a contradiction

***Case* 3. 2**.If $\lambda(e_2)$= {2,6} then $\lambda(e_3)$={3,4} and $\lambda(e_6)$={2,3}.Since 3 must be used trice. Then edge e_2 and e_5 are adjacent. Thus the vertex with label 3 and that with label 6 are at distance two and we come to the contradiction. Thus k^{-1}=3 and $\alpha - \beta =$
-3 *is wrong*.

Suppose $k^{-1} = 2$ and $\alpha - \beta = 3$. *Then for each edge label, the possible labels of its end vertices are as follows*:

Edges	e_1	e_2	e_3	e_4	e_5	e_6
Edge labels	12	11	10	9	8	7
	{4.6}	{3,6}	{2,6}	{1,6 }	{1,5}	{1,4}
$\lambda(e_i)$		{4,5}	{3,5}	{2,5}	{2,4}	{2,3}
				{3,4}		

Table.2. Possible labels for k^{-1}=2

From table 2, we have that $\lambda(e_1)$={4,6}. There fore exactly 3 cases to consider

(i) $\alpha = 4, \beta = 1$ (*ii*) $\alpha = 5, \beta = 2$ (*iii*)$\alpha = 6, \beta = 3$. ***Case* 3. 3**. If $\alpha = 4, \beta = 1$ *then* $\lambda(e_6) \neq \{1,4\}$ and so $\lambda(e_6) = \{2,3\}$. Since 1 must be used once, either $\lambda(e_4)$={1,6} or $f(e_5)$={1,5}.If $f(e_4)$={1,6} then e_1 *and* e_4 are adjacent and so the vertex with label 1 and that with label 4 are at distance two. This is a contradiction. Thus $f(e_5)$={1,5}.Since 4 is used 3 time, $f(e_2)$={4,5}, $f(e_4)$={3,4}. Then e_2 *and* e_5 are adjacent and so the vertex with 1 and that with label 4 are at distance two. Thus we reach a contradiction.

***Case* 3. 4**. If $\alpha = 5, \beta = 2$ *then* $f(e_4) \neq \{2,5\}$. Since 5 must be used trice $\lambda(e_2)$={4,5}

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$f(e_3)$={3,5} and λ (e_5)={1,5}.Since 4 is used trice, $f(e_6)$={2,3}. Then e_3 *and* e_6 are adjacent and so vertex with label 2 and that with label 5 are at distance two, a contradiction.

***Case* 3. 5.** If $\alpha = 6, \beta = 3$ *then* $f(e_2) \neq \{3,6\}$ and so $f(e_2) =$
{4,5}. Since 6 must be used trice , $f(e_3) \neq \{2,6\}$ $and \lambda(e_4)$={1,6}. Since 4 is used twice, $f(e_6)$={2,3}. Then
e_3 *and* e_6 *are adjaent and so the vertex with label* 3 *and that with* label 6 are at a distance two. There fore we have a contradiction. Thus it is not possible that k^{-1}=2 and $\alpha - \beta = 3$. *This completes* the proof of the Theorem

An (n,3)- kite except that (3,3)- kite is reverse super edge magic as long as n is odd.

Theorem 2.5. An (n,3)- kite is reverse super edge magic if and only if n(≥ 5) is odd.

Proof. The necessary condition follows from Theorem 2.2 and 2.4. To prove sufficient condition, let C_n= {$v_0, v_1 v_2$, …..,$v_{n-1} v_0$} be a vertex sequence with the toil end attached to
v_0. Since n is an odd and ≥5, either n= 4m+1 or 4m+3 where m is positive integer ≥1.

***Case* 1**. *Assume* thatbn=4m+1. In Fig(b), a labeling of (5,3)-kite is given for m=1, For m≥ 2 *we define* f: VUE → {1,2, … ,8m + 8} as follows:

$f(v_i) = \frac{i}{2}+1$ if i= 0,1,….,2m+ 2

$= 2m+6+\frac{i-1}{2}$ if i=1,3,……,2m-3

= m+3 if i=2m-1

=3m+5 if i=2m+1

$= \frac{i-1}{2}+3$ if i=2m+3, 2m+5,…….,4m-1

$= \frac{i}{2}+2m+4$ if i= 2m+4, 2m+6,……,4m.

f(x)= 2m+5 f(y)=2m+4 f(w)=2m+3

$f(v_i v_{i+1})$ = 4m+9+i if $0\leq i \leq 2m-3$

= 2m+7+i if i= 2m-2. 2m-1

= 4m+8+i if i=2m, 2m+1

= 4m+8 if i= 2m+2

= 4m+9+i if $2m+3 \leq i \leq 4m-1$

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f($v_{4m}v_o$) =6m+7, f($v_o y$) = 4m+7, f(xy)= 6m+11, f(xw) = 6m+10

It can be seen easily that f is a reverse super edge magic lalebing of (n,3)-kite with reverse edge magic number 2m+2

***Case* 2** . *Suppose that* n=4m+3

We define a function f: VUE → $\{1,2, \ldots ,8m + 12\}$ labeled as follows:

i

f(v_i) = +m if i= 0, 1,.,2m+ 2

2

i−1

= 3m+6+ ___ if i=1,3,……,2m+1

2

= 2m+2 if i=2m+3

i

= +m+1 _ if i= 2m+4, 2m+6,……,4m+2

2

i−3

=--m if i=2m+5, 2m+7,…….,4m+1

2

f(x)= 2m+5 f(y)=3m+4 f(w)=3m+3 f($v_i v_{i+1}$) = 6m+9+i if 0≤ $i \leq$

$2m$+1

= 6m+6 if i= 2m+2.

=6m+8 if i=2m+3

=2m+3+i if 2m+4≤ $i \leq 4m + 1$

f($v_{4m+2}v_o$) =6m+5, f($v_o y$) = 6m+7, f(xy)= 8m+12, f(xw) = 8m+11

It is easy to see that f is a reverse super edge magic labeling of (n,3)-kite with the reverse edge magic number 2m+3.

III. K_2UC_n

Wallis [12] proved that K_2UC_n is not edge magic but K_2UC_4 is edge magic. Then Wallis[12] proposed the following problem. For which values of n, is K_2UC_n *edge magic. Park et al*[8]*proved that* K_2UC_n *is* super edge magic for even integer n≠10. In [12] they left the case when n=10 open. In[10] Suh- Ryang Kim et al presented a super edge magic labeling of $K_2UC_nK_2UC_n$. *to compute the case where n is even. They found this labeling by an exhsustive sear*

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$In\ the\ same\ manner\ we\ present$ a reverse super edge magic labelimg of K_2UC_n to complete case when n is even.We could find this labeling by an $exhsustive\ search\ based$ on the following observation

Proposition 1. If the graph K_2UC_n has a reverse super edge magic labelimg f for a positive even integer then the reverse super edge magic number for f is 2n+3.

Proof. Let G= K_2UC_n. Let v_o, v_1,, ...,v_{n-1} be a vertex sequence of C_n and let u and w be the vertices of K_2. Let n=2l wherever l is positive integer and k^{-1} $be\ a$ reverse edg magic number for l. Let f(u)= $\alpha\ and$ f(w) $= \beta. Since\ p = 2l + 2$ and q=2l+1.

$$k^{-1}q = \sum_{xy\in\in E(G)} |f(xy) - f(x) - f(y)|$$

$$k^{-1}(2l+1) = \sum_{xy\in\in E(G)} f(xy) - 2\sum_{x\in V(G)} [f(x) + f(u) + f(w)]$$

$$=(2l+1)\frac{(2l+3)[(2l+3)+(4l+3)]\ 2}{} \quad \frac{2(2l+2)(2l+3)}{2} \quad - \quad + (\alpha+\beta)$$

$=8l^2 +8l +(\alpha+\beta)$

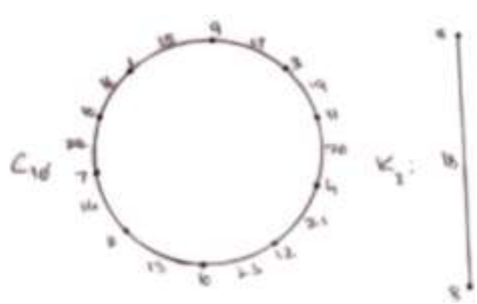
Since $3\leq \alpha+\beta) \leq 4l+3$, $it\ holds\ that$ $8l^2 + 8l + 3 \leq k(2l+1) \leq 8l^2 + 12l + 3$

Since k^{-1} is an integer k^{-1} =4l+3=2n+3

Proposition 2. The graph K_2UC_{10} reverse super edge magic

Proof. A reverse super edge magic labeling is given in Fig. 2 when a reverse edg magic number is 5.

Fig.2. A reverse super edge magic labeling of K_2UC_{10}.



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The following theorem completely characterizes reverse super edge magic of K_2UC_n.

Theorem 2.6. The graph K_2UC_n. is reverse super edge magic if and only if n is even.

Proof. The necessary condition immediately follows from the result of Park et al [8] and proposition 2.

To prove the sufficient condition, let G=K_2UC_n. and let
$v_o, v_1, \ldots, v_{n-1}, v_o$ *be a vertex sequence* of cycle C_n. *Let* u and w be the vertices of K_2.

We prove the theorem by contradiction methed. Assume that n is odd and that G has a reverse super edge magic labeling f with reverse super edge number k^{-1}. *Then* n=2m+1 where m=1,2,3,……. Let f(u)=a and f(w)=b. Since p=2m+3 and q=2m+2

$$k^{-1} q = \sum_{xy\in E(G)} |f(xy) - f(x) - f(y)|$$

$$k^{-1}(2m+1) = \sum_{xy\in E(G)} f(xy) - 2\sum_{x\in V(G)} [f(x) + f(u) + f(w)]$$

$$=(2m+2)\frac{(2m+4)[+(4m+5)]}{2} - \frac{2(2m+3)(2m+4)}{2} + (a+b)$$

=$2m^2$ +m-3 +$(a+b)$ ………………………………………………………………………………………… (I)

Since $3 \leq a+b \leq 4m+5$, *it follows that from the equality* (I)that $2m^2 + m \leq k^{-1}(2m+2) \leq 2m^2 + 5m + 2$

Since k^{-1} is an integer k^{-1} = m or m+1

Case I *. Suppose that* k^{-1} = m

For any distinct vertices x and y. since 2m+4≤ $f(xy) \leq 4m+5$ and $f(xy) + f(x) + f(y)$=m
m+4≤ $f(x) + f(y) \leq 3m+5$ ………………………………………………………………………………… (II)

However it follows from (I) that m(2m+2)= $2m^2$ +m-3 +$(a+b)$ and so a+b=

$f(u) + f(w)$=m+3 This is a contradiction to (II)

Case II *. Suppose that* k^{-1} = m +1

Then for any distinct vertices x and y by a similar methed to the case k^{-1} = m +1 we can prove that

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m+3$\leq \mathrm{f(x)} + \mathrm{f(y)} \leq 3\mathrm{m} + 4$ .... (III)

However it follows from (I) that (m+1)(2m+2)= $2m^2$ +m-3 +$(a+b)$

and so a+b= $\mathrm{f(u)} + \mathrm{f(w)}$=3m+5 This is a contradiction to (III)

IV. S($K_{1,n}$)

A star subdivision graph S($K_{1,n}$) derived from a star by adding a pendent edge to each vertex of degree 1. It is known that the star $K_{1,n}$ is a super edge magic. In[11], it is known that S($K_{1,n}$) is a super edge magic. The following proves that S($K_{1,n}$) is reverse super edge magic.

Theorem 2 7. The graph S($K_{1,n}$) is reverse super edge magic for positive integer n≥ 1

Proof. Let v_1, v_2,, ... ,v_n be the pendent vertices of $K_{1,n}$ and let v_0 be the central vertex of $K_{1,n.}$ In $addition$ let w_i be the pendant vertex of G adjacent to v_i for i=1,2, ... ,n.

Case I. Let n be an odd positive integer.Then n=2m+1 wher m is some positive integer. We define a labeling function f: VUE→{1,2,…,p+q} as follows:

f(v_0) = m+2

f(v_i) = i+1 if $1 \leq$ i $\leq$ m

= i+2 if m+1 $\leq$ i $\leq$ 2m

=2m+3 if i = 2m +1

$f(w_i) = 4m - 2i + 5$ if $1 \leq$ i $\leq$ m

= 6m-2i+4 if m+1 $\leq$ i $\leq$ 2m

= 1 if i = 2m +1

$f(v_o v_i)$= 4m+3+i if $1 \leq$ i $\leq$ m

= 4m+4+i if m+1 $\leq$ i $\leq$ 2m

= 6m+5 if i = 2m +1

f($v_i w_i$)= 7m+6-i if $1 \leq$ i $\leq$ m

=9m+6-i if m+1 $\leq$ i $\leq$ 2m

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=5m+4 if i = 2m +1

It is easy to see that f is a revesre super edge magic labeling of G with the revesre super edge magic number 3m

Now we consider the case when n is even. Then n=2m for some positive integer m. We consider two subcases:

Sub$case$ 1.1. Let m be even positive integer. We define a labeling f: VUE→{1,2,…,p+q} as follows:

f(v_0) = m+2

f(v_i) = i+1 if $1 \leq$ i $\leq$ m

= i+3 if m+1 $\leq$ i $\leq$ 2m-1

=2m+4 if i = 2m

$f(w_i) = 3m + i + 2$ if $1 \leq$ i $\leq$ m-1

= m+3 if i=m

= m+i+5 if i = m +1, m+3, ,…….,2m-3

= m+i+1 if i=m+2, m+4,……,2m-2

=3m+1 if i= 2m-1

=1 if i= 2m

$f(v_o v_i)$= 4m+1+i if $1 \leq$ i $\leq$ m

= 4m+3+i if m+1 $\leq$ i $\leq$ 2m-1

= 6m+4 if i = 2m

f($v_i w_i$)= 6m+2i +1 if $1 \leq$ i $\leq$ m-1

=5m+2 if i=m

=4m+6+2i if i = m +1, m+3, ,…….,2m-3

=4m+2+2i if i=m+2, m+4,……,2m-2

=8m+1 if i=2m-1

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=5m+3 if i=2m

Since m is even m +1, m+3, ,…….,2m-3 and m+2, m+4,……,2m-2 both are AP's with common difference 2 and so f is well defined. Then f is a reverse super edge magic labeling of G with reverse super edge magic number 3m-2 for m even **Sub*case* 1.2.** Let m be odd positive integer. We define a labeling f: VUE→{1,2,…,p+q} as follows:

$f(v_0)$ = m+2

$f(v_i)$ = i+1 if $1 \leq i \leq m$

= i+3 if $m+1 \leq i \leq 2m-1$

=2m+4 if i = 2m

$f(w_i) = 2m + i + 2$ if $1 \leq i \leq m-1$

= m+3 if i=m

= m+i+5 if i = m +1, m+3, ,…….,2m-4

= m+i+1 if i=m+2,m+4, ……..,2m-3

=3m+2 if i= 2m-2

=3m if i=2m-1

=1 if i= 2m

$f(v_o v_i)$= 4m+1+i if $1 \leq i \leq m$

= 4m+3+i if $m+1 \leq i \leq 2m-1$

= 6m+4 if i = 2m

$f(v_i w_i)$ =6m+1 +2i if $1 \leq i \leq m-1$

=5m+2 if i=m

=4m+2i +6 if i = m +1, m+3, ,…….,2m-4

=4m+2i+2 if i=m+2, m+4,……,2m-3

=8m+1 if i=2m-2

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=8m if i=2m-1

=5m+3 if i=2m

Since m is odd m +1, m+3, ,…….,2m-4 and m+2, m+4,……,2m-3 both are AP's with common difference 2 and so f is well defined. Then f is a reverse edge magic labeling of G with reverse edge magic number 3m-2 for m odd.This completes the proof of the Theorem

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