

Some results of weak 2-vertex duplication self-switching graphs

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Abstract

Let G be a graph and let $\sigma \subseteq V$ be a non-empty subset of V . σ is said to be a self switching if $G \cong G^\sigma$ where G^σ is obtained from G by removing all edges between σ and $V - \sigma$ and adding edges between all non-adjacent vertices of σ and $V - \sigma$. We also call it as $|\sigma|$ -vertex self switching. When $|\sigma| = 2$, we call it as 2-vertex self switching. Duplication of a vertex v of a graph G produces a new graph G by adding a new vertex v such that all vertices which are adjacent to v in G are also adjacent to v in G . In this paper, we characterize path, minimum and maximum degree vertices of weak 2-vertex duplication self switching.

Keywords: Switching, 2-vertex self switching, duplication 2-vertex self switching, $dss_2(G)$, $\delta(G)$, $\Delta(G)$.

1. Introduction

For a finite undirected simple graph $G(V, E)$ with $|V(G)| = p$ and a non-empty set $\sigma \subseteq V$, the switching of G by σ is defined as the graph $G^\sigma(V, E')$ which is obtained from G by removing all edges between σ and its complement, $V - \sigma$ and adding as edges all non-edges between σ and $V - \sigma$. Switching has been defined by Seidel [8] and is also referred to as Seidel switching. When $\sigma = \{v\} \subset V$, the corresponding switching G^v is called a *vertex switching* and is denoted by G^v . A non-empty set $\sigma \subseteq V$ is said to be self switching if $G \cong G^\sigma$. We also call it as $|\sigma|$ -vertex self switching. The set of all k -vertex self switchings of G each with cardinality k is denoted by $SS_k(G)$ and its cardinality by $ss_k(G)$. If $k = 1$, then we call the corresponding self switching as self vertex switching. The survey on self vertex switchings of graphs by Jayasekaran [1] motivated us to study on graphs with $|\sigma| = 2$. Jayasekaran and Prabavathy [3, 4] introduced the concept of duplication self vertex switching. A vertex v is said to be the duplication of v if all the vertices which are adjacent to v in G are also adjacent to v in G . The new graph obtained after duplication of a vertex v is denoted as $D(vG)$.

For any vertex $v \in V(G)$, the open neighbourhood $N(v)$ of v is the set of all vertices adjacent to v . That is $N(v) = \{u \in V(G) / uv \in E(G)\}$. The minimum degree among

the vertices of G is denoted by $\delta(G)$ and is defined as $\min \{d(v)/v \in V(G)\}$ and the maximum degree represented by $\Delta(G)$ and is defined as $\max \{d(v)/v \in V(G)\}$.

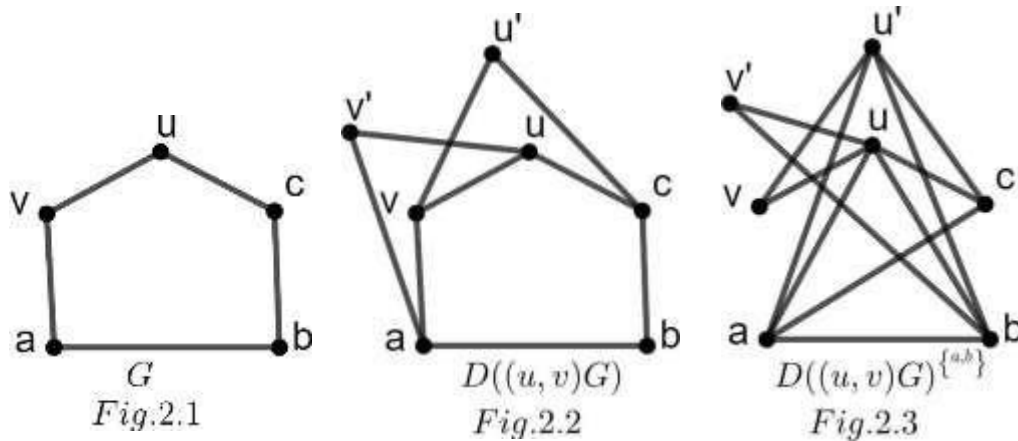
G.Sumathy and K.S Shruthi [10,12] introduced the concept of 2-vertex duplication self switching. 2 – vertex duplication of a graph G is the duplication of any two vertices $u, v \in V(G)$ is u', v' such that x', y' are adjacent to all the vertices to u & v . It is denoted as $D((u, v)G)$.

2. Weak 2 –vertex duplication self switching of graphs

Definition 2.1: The **2-vertex duplication switching** of G by λ is the graph obtained by duplicating any two vertices u, v and by removing all edges between λ and its complement $V - \lambda$ also by adding all edges between λ and $V - \lambda$ which are not in G , without affecting the adjacency and non-adjacency of vertices in λ .

It is denoted by $D((u, v)G)^\lambda$ or $D((u, v)G)^{\{x,y\}}$.

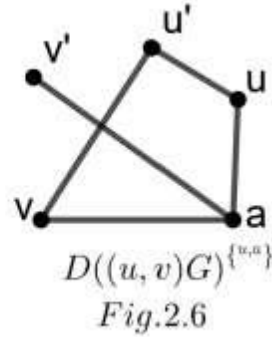
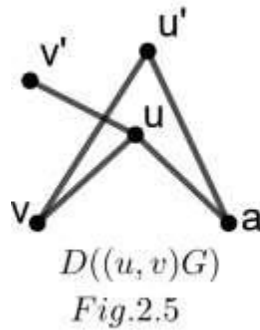
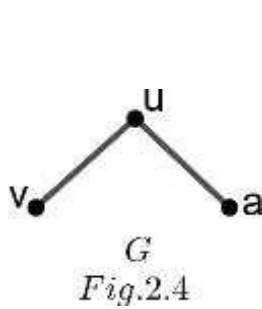
Example 2.2 : 2-vertex duplication switching of the graph G is given in the figures 2.1 to 2.3.



Definition 2.3: If $\lambda = \{\{x, y\}/\{x, y\} \subseteq V(G)\}$ is called a **2 – vertex duplication self switching** of a graph G if the resultant graph obtained after duplication of $\{u, v\}$ has λ as a self vertex switching. (ie) $D((u, v)G) \cong D((u, v)G)^{\{x,y\}}$.

If $\lambda = \{\{x, y\}/\{x, y\} \neq \{u, v\} \subseteq V(G)\}$, then λ is called the **weak 2 –vertex duplication self switching** of G .

Example 2.4: Weak 2-vertex duplication self switching of a graph is given in the figures 2.4 to 2.6.



Theorem 2.5: If $\lambda = \{x, y\} \subseteq V$ is a weak 2 –vertex duplication self switching of a graph G , then $d_G(x) + d_G(y) = p$ if $xy \in E(G)$.

Proof:

Let $\lambda = \{x, y\} \subseteq V$ be a weak 2 –vertex duplication self switching of a graph G . By the definition, $D((u, v)G) \cong D((u, v)G)^\lambda$ and hence $|E(D(u, v)G)| = |E(D(u, v)G)^\lambda|$.

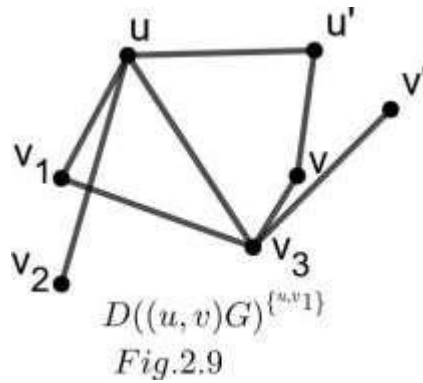
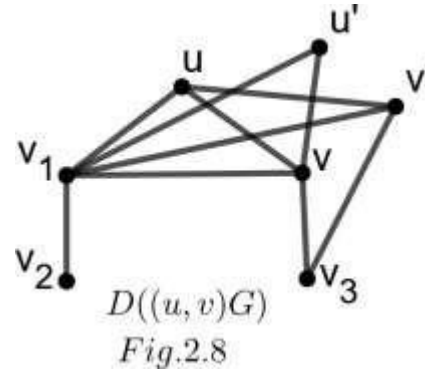
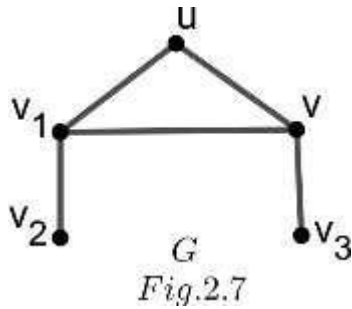
This implies that,

$$\begin{aligned} q + d_G(x) + d_G(y) &= q + d_G(x) + d_G(y) + [p + 2 - 1 - d_{D((u,v)G)}(x)] - \\ &\quad d_{D((u,v)G)}(x) + [p + 2 - 1 - d_{D((u,v)G)}(y)] - d_{D((u,v)G)}(y) + 2. \\ &= p + 1 - [d_G(x) + 1] - (d_G(x) + 1) + p + 1 - [d_G(y) + 1] - (d_G(y) + 1) + 2. \\ &= 2p + 2 - 2d_G(x) - 2 - 2d_G(y) - 2 + 2. \\ &= 2p - 2d_G(x) - 2d_G(y). \end{aligned}$$

$$p = d_G(x) + d_G(y).$$

0

Note 2.6: The converse of the theorem need not be true. For example, let us consider the graph G with 5 vertices given in figure 2.1. In this graph $uv_1 \in E(G)$ and $d_G(u) + d_G(v_1) = 5$ but $D((u, v)G) \not\cong D((u, v)G)^{[u,v]_1}$.



$2, \quad \text{if } p = 3$

Theorem 2.7: For $p \geq 2$, $dss2(P_p) = \begin{cases} 2, & \text{if } p = 4 \\ 0, & \text{otherwise.} \end{cases}$

Proof:

Case (i): If $p = 3$

Let $\lambda = \{u_i u_{i+1}, u_i u_{i+2} / 1 \leq i \leq 2\}$.

Then $d(u_i) + dG(u_{i+1}) = 1 + 2 = 3 = p$ and $d(u_i) + dG(u_{i+2}) = 1 + 2 = 3 = p$.

Hence by Theorem 2.5, λ is a weak 2 –vertex duplication self switching of G .

Therefore, $dss2(P_3) = 2$.

Case (ii): If $p = 4$

Let $\lambda = \{u_2, u_3\}$. Then, $dG(u_2) + dG(u_3) = 2 + 2 = 4 = p$.

Hence by Theorem 2.5, λ is a weak 2 –vertex duplication self switching of G

Therefore, $dss2(P_4) = 1$.

Case (iii): If $p > 4$.

Let u, v be any two adjacent vertices in $V(Pp)$.

Then, $dG(u) + dG(v) \neq p$. Hence by the Theorem 2.5, λ is not a weak 2-vertex duplication self switching of Cp .

Therefore, $dss2(Pp) = 0$.

Hence the theorem.

Theorem 2.8: In a non-regular graph G , $\lambda = \{\{x, y\} / \text{either } x \text{ or } y \text{ is adjacent to any minimum degree vertex in } D((u, v)G)\}$ is a weak 2-vertex duplication self switching .

Proof:

Let w be the minimum degree vertex in $D((u, v)G)$. Let $\lambda = \{\{x, y\} / \text{either } x \text{ or } y \text{ is adjacent to any minimum degree vertex in } D((u, v)G)\}$ and x be the vertex adjacent to w in $D((u, v)G)^\lambda$. Then $d(D(u, v)G)^\lambda(v_i) = d_{(D(u, v)G)}(v_i)$ for all $v_i \in N(x)$ and $v_i \neq y$ and $d(D(u, v)G)^\lambda(w) = \delta(D(u, v)G) = d_{(D(u, v)G)}(w)$. This implies that $D((u, v)G) \cong D((u, v)G)^\lambda$.

Hence λ is a weak 2-vertex duplication self switching.

Theorem 2.9: In a non-regular graph G , $\lambda = \{\{x, y\} / \text{either } x \text{ or } y \text{ is adjacent to any maximum degree vertex in } D((u, v)G)\}$ is a weak 2-vertex duplication self switching .

Proof:

Let w be the maximum degree vertex in G . Let $\lambda = \{\{x, y\} / \text{either } x \text{ or } y \text{ is adjacent to any maximum degree vertex in } D((u, v)G)\}$ and x be the vertex adjacent to w in $D((u, v)G)^\lambda$. Then $d(D(u, v)G)^\lambda(v_i) = d_{(D(u, v)G)}(v_i)$ for all $v_i \in N(x)$ and $v_i \neq y$ and $d(D(u, v)G)^\lambda(w) = \Delta(D(u, v)G) = d_{(D(u, v)G)}(w)$. This implies that $D((u, v)G) \cong D((u, v)G)^\lambda$. Hence λ is a weak 2-vertex duplication self switching.

3. Conclusion

In this paper, we have discussed about path, minimum and maximum of 2-vertex duplication self switching graphs. Further we are analyzing more graphs.

References

- [1] C.Jayasekaran, “A study on Self vertex switching of graphs”, Ph.D thesis Manonmaniam Sundaranar University, Tirunelveli, India, 2007.
- [2] C. Jayasekaran, and G. Sumathy, “Self vertex switchings of connected two- Cyclic graphs”, *Journal of Discrete Mathematical Sciences and Cryptograph*, Vol.17, No.2, April 2014,pp 157-179,ISSN 0972-0529.
- [3] C. Jayasekaran &V. Prabavathy, ‘Some results on Duplication self vertex switchings’, *International Journal of Pure and Applied Mathematics*, vol. 116, No. 2, 427 – 435.
- [4] C. Jayasekaran &V. Prabavathy, ‘A characterization of duplication self vertex switching in graphs’, *International Journal of Pure and Applied Mathematics*, vol. 118, No.2.
- [5] C. Jayasekaran, J. Christabel Sudha & M.Ashwin Shijo, ‘2 – vertex self switchings of some special graphs’,*International Journal of Scientific Research and Review*, vol.7, Issue 12, 2018. ISSN No:2279 – 543X
- [6] C. Jayasekaran, J. Christabel Sudha, ‘ Some results on 2 – vertex self switching in joints’, *Italian Journal of Pure and Applied Mathematics*, communicated.
- [7] Harary.F,Graph Theory, Addition Wesley, 1972.
- [8] J.J Seidal,A survey of two graphs,Proceedings of the international Coll. Theorie Combinatorie (Rome 1973). Tomo I, Acca, Naz. Lincei pp. 481-511,1976.
- [9] Selvam Avdayppan and M. Bhuvaneshwari, More results on self vertex Switching , *International journal of Modern sciences and Engineering Technology*, Volume 1Issue 3, 2014, pp.10-17.
- [10] K.S. Shruthi and G. Sumathy, “~~ds~~of strong 2-vertex duplication self Switching of some special graphs ”,*Turkish journal of computer and Mathematics Education*,Vol.13,No.2(2022),591-594.
- [11] G. Sumathy, Self vertex switching of two-cyclic and bicyclic graphs, Ph.D thesis Manonmaniam Sundaranar University, Tirunelveli, India, 2014.
- [12] G. Sumathy and K.S. Shruthi , “Some results on strong 2-vertex duplication Self switching of some connected graphs”,*Malaya journal of Mathematik*, Vol.8,No.4,ISSN No.2319-3786,2020.