Significance of Interval Valued Fuzzy Generator Graph

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ABSTRACT
The present paper is focusing on constructing the interval valued fuzzy graph from the generator graph associated with the modular additive and multiplicative cyclic groups. We focus on the K_3 graphs for which each vertex is a part and not a part with the total number of K_3 graphs to obtain theinterval valued fuzzy graph. We study the order and size of the IVFG of the generator graph of modular cyclic groups.

Key Words: Cyclic Group, Generator of a cyclic Group, Generator Graph of a cyclic group, membership of the vertex, interval valued fuzzy graph

1.INTRODUCTION

The study of groups is not considered easy due to its abstract nature. The study becomes far more interesting if the finite groups are represented in the form of graphs. In our paper [1] the properties of cyclic groups are studied using cyclic graphs. We call the graph as generator graph, since the main role in obtaining the graph is played by the generators of the cyclic group. First we will start with drawing cyclic graphs using the generators. Then we will consider additive and multiplicative modular group which are cyclic in nature and discuss how to draw generator graphs. Once we draw generator graphs, we can find the number of K_3 graphs in each of these generator graph.

Rosenfeild [11] developed the theory of fuzzy graphs, by giving membership values to nodes and edges of graphs. We see a lot of variety in choosing this membership values for edges and arcs of each graph. In this paper we are relating the member ship degree of the nodes and arcs in the generator graph to the number of K_3 graphs in it and hence trying to form a fuzzy graph. We have started from cyclic groups to generator graph and from there to fuzzy graph hoping to study more about the abstractness of the groups in general.

Definition 1

An algebraic structure (G,*) is said to form a group if G is a non empty set and * is a binary operation defined on G with the following properties satisfied.[2]

(i) $(a * (b * c) = (a * b) * c \; \forall \; a, b, c \in G$.

(ii) If $\forall \; a \in G \; \exists \; e \in G$ such that $a * e = a = e * a$.

(iii) If $\forall \; a \in G \; \exists \; a^{-1} \in G$ such that $a * a^{-1} = e = a^{-1} * a$.

Definition 2

An ordered pair (V, E) is called a graph if V represents the vertices and E represents the edges of the graph. A cyclic graph is a closed trail in which all the vertices are different.

Definition 3

A Group (G,$*$) is said to be a cyclic group if it possess a generator a ∈G.A cyclic group is of the form$G = \{a, a^2, \ldots \ldots.., a^n = e\}$where
e is the identity and a is the generator. It is possible for G to have many generators. If a∈G is a generator, then a^{-1} ∈ G is also a generator [3].

Definition 4

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\mu(u, v) = \sigma(u) \times \sigma(v)$, for all u, v in V where V is the vertex set.

2.Generator Graphs

The generator graphs can be defined with the generators as the vertices on the extreme left connected within themselves and to all the other non-generator elements on the right as vertices in the cyclic group. The adjacency is defined as x is adjacent with y if $x * x = y$ where 'x' is a generator. An edge connects a generator vertex to any other vertex, if and only if that particular element is generated by the generator. Consider the following examples of the generator graphs formed by the additive modular groups $(Z_n, +_n)$ when n is odd or even. We have proved that if $(\mathrm{Z_n}, +_\mathrm{n})$ be a cyclic group of order n, then the generator graph will have $[[\mathrm{n} - ⍰(n)] \times \sum(⍰(n) - 1)]$ K$_3$ graphs where $⍰(n)$ is the number of generators in the cyclic group[1].

Example 1 Figure 1 shows the generator graph of $(Z_{11}, +_{11})$

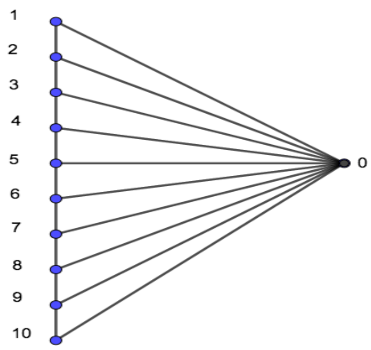

Fig 1

The number of K$_3$ graphs in figure 1 is 45. i.e$1 \times \sum(10 - 1)$ where there is 1 non generator and 10 generators in $(Z_{11}, +_{11})$.

Example 2

Figure 2 and Figure 3 shows the additive modular groups of$(Z_8, +_8)$ and $(Z_{12}, +_{12})$respectively.

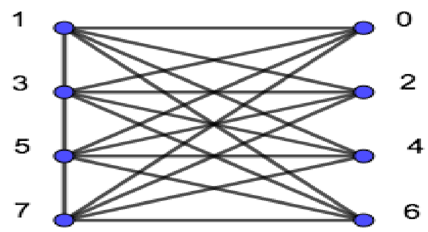

Fig 2. $(Z_8, +_8)$

The number of K$_3$ graphs in figure 2 is 24.

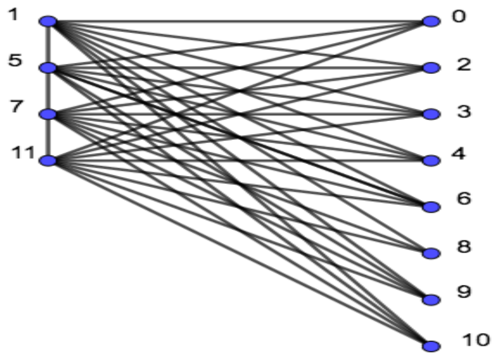

Fig 3. $(Z_{12}, +_{12})$

The number of K_3 graphs in figure 3 is 48. i.e$8 \times \sum(4-1)$ where there are 8 non generators and 4 generators in $(Z_{12}, +_{12})$.

3.Construction of an interval valued fuzzy graph from the generator graph of a cyclic group

Let $G^* = (V, E)$be a generator graph of a cyclic group, with 'n' nodes and 'm' arcs. Let the nodes be named as $v_1, v_2, v_3 \dots \dots v_n$ .Let v_i be any arbitrary node of G^*.We define the membership interval as $\mu^-_A(v_i) = \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex}}{k}$, $\mu^+_A(v_i) = \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is not a vertex}}{k}$ $\forall\ v_i \epsilon V$ for the vertices and for the edges$\mu^-_B(v_i v_j) = \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex } \times No.of\ k_3 \text{ graphs in which } v_j \text{is a vertex}}{k^2}$, $\mu^+_B(v_i v_j) = \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is not a vertex} \times No.\ of\ k_3 \text{ graphs in which } v_j \text{is not a vertex}}{k^2} \forall\ v_i\, v_j \epsilon\, E$

The above construction yields an interval valued fuzzy graph $G = (A, B)$from the generator graph of a cyclic group.

Example 1Figure 4 shows the construction of IVFG from the generator graph of $(Z_6 +_6)$.

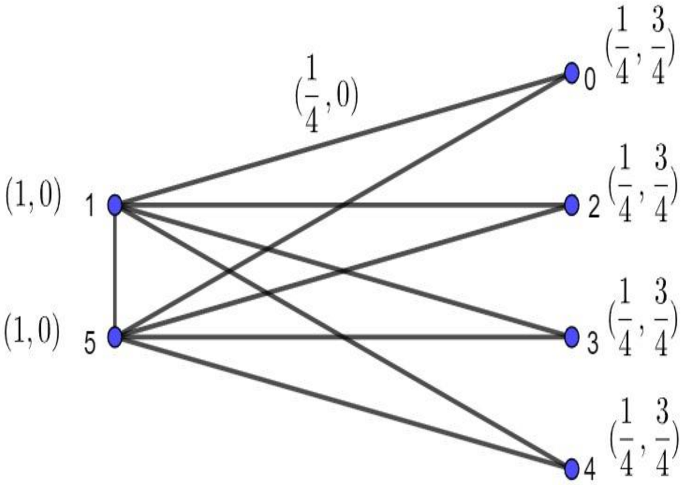

Fig 4. $(Z_6, +_6)$

Example 2 Figure5 shows the construction of IVFG from the generator graph of $(Z_{10} +_{10})$.

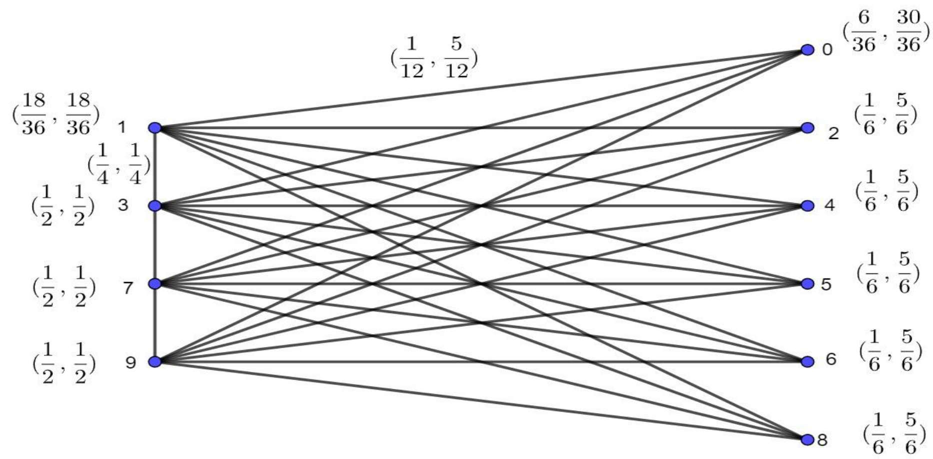

Fig 5. $(Z_{10}, +_{10})$

Theorem 1

Let $G^* = (V, E)$be a generator graph of a modular cyclic group with 'k' number of $k_3$graphs. We define the membership interval as $\mu^-_A(v_i) = \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex}}{k}$, $\mu^+_A(v_i) = \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is not a vertex}}{k}$ $\forall\ v_i \epsilon V$ for the vertices and for the edges$\mu^-_B(v_i v_j) =$

$\frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex } \times No. of\ k_3 \text{ graphs in which } v_j \text{is a vertex}}{k^2}$,

$\mu^+{}_B(v_i v_j) = \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is not a vertex} \times No.\ of\ k_3 \text{ graphs in which } v_j \text{ is not a vertex}}{k^2}$

$\forall\ v_i\ v_j \epsilon\ E$.Then $G = (A, B)$is a fuzzy graph.

Proof:

Let $G^* = (V, E)$be a generator graph of a modular cyclic group with $|V| = n, |E| = m$.

Let $A = \{(v_i , [\mu^-{}_A(v_i), \ \mu^+{}_A(v_i)]): v_i \epsilon V\}$ *and* $B = \{(v_i v_j , [\mu^-{}_B(v_i v_j), \ \mu^+{}_B(v_i v_j)]): v_i v_j \epsilon E\}$.We prove that A is an IVFS on V and B is an IVFS on E satisfying the requirement of an IVFG.

To show that A is an IVFS on V

From the construction we have,

$\mu^-{}_A(v_i) = \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex}}{k}$,$\mu^+{}_A(v_i) = \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is not a vertex}}{k}$

The number of k_3 graphs in which $v_i$is a part will definitely be less than the total k_3 graphs in the generator graph and hence$0 < \ \mu^-{}_A(v_i) \leq 1$. Similarly the number of k_3 graphs in which $v_i$is not a part will definitely be less than the total k_3 graphs in the generator graph and we get $0 < \ \mu^+{}_A(v_i) \leq 1$ .Hence A is an IVFS on V.

To show that B is an IVFS on E

We have, $\mu^-{}_B(v_i v_j) = \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex } \times No.\ of\ k_3 \text{ graphs in which } v_j \text{is a vertex}}{k^2}$,

And$\mu^+{}_B(v_i v_j) = \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is not a vertex} \times No. of\ k_3 \text{ graphs in which } v_j \text{ is not a vertex}}{k^2}$ $\forall\ v_i\ v_j \epsilon E$.

By definition of B,the product of number of k_3 graphs in which $v_i$is a vertex and the $n$umber of k_3 graphs in which v_j is a vertex will definitely be less k^2 as each quantity taken separately is less than k.The numerator being less than the denominator the fraction derived is a proper fraction.Hence $0 \leq \mu^-{}_B(v_i v_j) \leq 1$. Similarly the number of k_3 graphs in which $v_i$is not a vertex or v_j is not a vertex is less than the total k_3 graphs in the generator graph represented by k.As a result the numerator is less than the denominator and the proper fraction obtained follows the condition $0 \leq \mu^+{}_B(v_i v_j) \leq 1$. Since the member ship of all the nodes and the edges are in the interval [0, 1],$G = (A, B)$ is anIVFG.

4. Order of the interval valued fuzzy graph of the generator graph

We define the order of the interval valued fuzzy graph as the sum of all membership values of all the vertices.It is observed that a constant number independent of the number of generators and non-generators is obtainedas the order of the IVFG of the Generator graph.The difference in the number of generators in the additive and multiplicative modular groups is reflected in the order of the corresponding IVFG ‘s of additive and multiplicative generator graphs.

4.1 For additive modular groups

Example 3 Figure 6 shows generator graph and IVFG of $(Z_7 +_7)$.

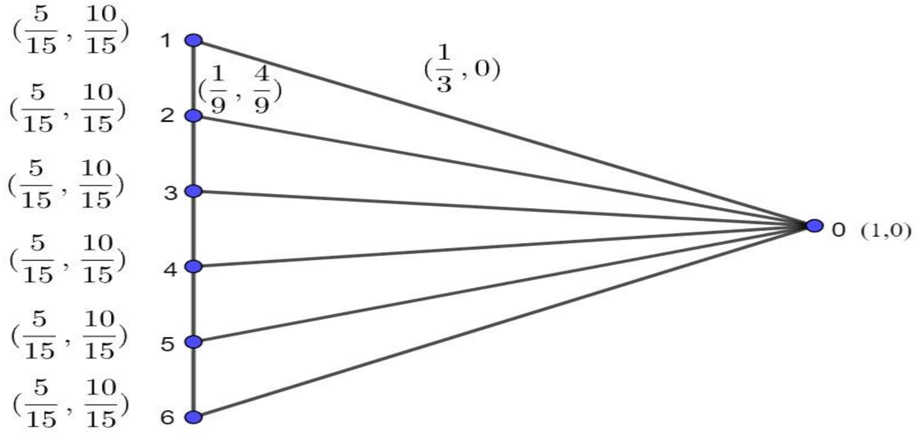

Fig 6. $(Z_7, +_7)$

The total number of k_3 triangles in $(Z_7, +_7)$ is 15. The vertex represented by generator 1 is part of only 5 k_3 triangles and not a part of remaining 10 k_3 triangles. Hence its membership value is $\left(\frac{5}{15}, \frac{10}{15}\right)$. The vertex represented by 0 is part of all the $15 k_3$ triangles. Hence its membership value is $\left(\frac{15}{15}, \frac{0}{15}\right) = (1,0)$ .It is evident that the graph has 6 generators and one non-generator and therefore 7 vertices. The order of IVFG of $(Z_7, +_7)$ is the sum of membership values of all the vertices in all the k_3 triangles. The O(G) of IVFG of generator graph of $(Z_7, +_7)$is calculated as

$$O(G) = 6 \times \left(\frac{5}{15}, \frac{10}{15}\right) + 1 \times \left(\frac{15}{15}, \frac{0}{15}\right) = (2,4) + (1,0) = (3,4)$$

Example 4 Figure 7 shows generator graph and IVFG of $(Z_9 +_9)$.

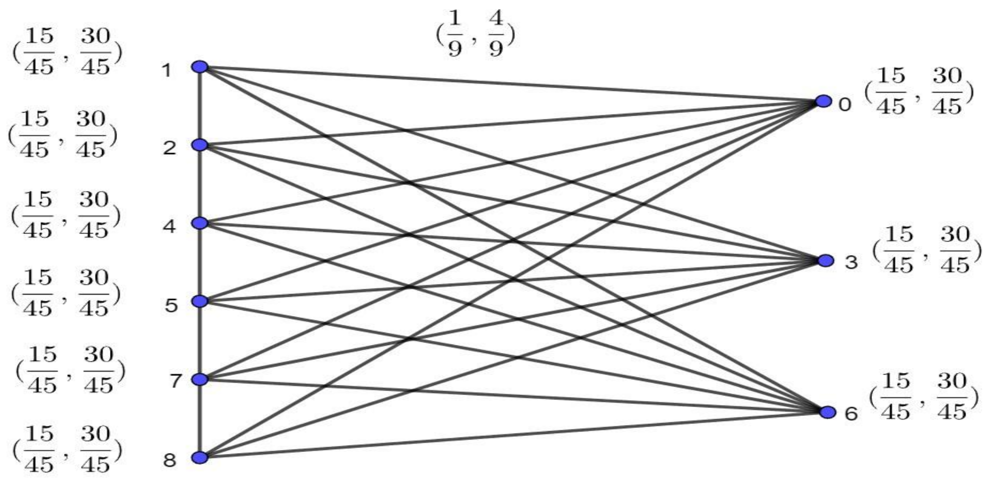

Fig 7. $(Z_9, +_9)$

The total number of k_3 triangles in $(Z_9, +_9)$ is 45. The vertex represented by generator 2 is part of only 15 k_3 triangles and not a part of remaining 30 k_3 triangles. Hence its membership value is $\left(\frac{15}{45}, \frac{30}{45}\right)$. The vertex represented by 0 is part of all the $15 k_3$ triangles and not a part of remaining 30 k_3 triangles. Hence its membership value is $\left(\frac{15}{45}, \frac{30}{45}\right))$ .The graph has 6 generators and 3 non-generator and therefore 9 vertices. The order of IVFG of $(Z_9, +_9)$ is is calculated as

$$O(G) = 6 \times \left(\frac{15}{45}, \frac{30}{45}\right) + 3 \times \left(\frac{15}{45}, \frac{30}{45}\right) = 9 \times \left(\frac{15}{45}, \frac{30}{45}\right) = (3,6) = (3, [n-3])$$

Theorem 2

Let $G^* = (V, E)$be a generator graph of an additive modular cyclic group $(Z_n +_n)$ with $[n - ▯(n)] \times \sum[▯(n) - 1]\ k_3$ graphs. Let $G = (A, B)$ be an IVFG associated with G^*,Then $O(G) = [3, (n-3)]$.

Proof

Let $G = (A, B)$ is a fuzzy graph in which$A = \{(v_i, [\mu^-{}_A(v_i), \ \mu^+{}_A(v_i)]) : v_i \epsilon V\}andB = \{(v_i v_j, [\mu^-{}_B(v_i v_j), \ \mu^+{}_B(v_i v_j)]) : v_i v_j \epsilon E\}$.Now if v_i is a generator or a non-generator$O(G) = [\sum_{i=1}^{n} \mu^-{}_A(v_i), \sum_{i=1}^{n} \mu^+{}_A(v_i))]$

Case(i) If v_i is a generator,

$$\frac{\text{The number of } k_3 \text{ graphs in which } v_i \text{is a vertex}}{Total\ k_3 \text{ graphs in the generator graph}}$$

$$= \frac{[n-▯(n)] \times [▯(n)-1]}{[n-▯(n)] \times \sum[▯(n)-1]}$$

$$= \frac{[▯(n)-1]}{[▯(n)-1]+[▯(n)-2]+\cdots\ldots\ldots\ldots+[▯(n)-(▯(n)-1)]}$$

$$= \frac{[▯(n)-1]}{\left[\frac{[▯(n)-1][▯(n)]}{2}\right]}$$

$$= \frac{2}{[▯(n)]}.$$

$$[\mu^-{}_A(v_i), \mu^+{}_A(v_i)] = \left[\frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{ is a vertex}}{k}, \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{ is not a vertex}}{k}\right]$$
$$= \left[\frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{ is a vertex}}{k}, 1 - \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{ is a vertex}}{k}\right]$$
$$= \left[\frac{2}{\phi(n)}, \quad 1 - \frac{2}{\phi(n)}\right]$$

Case(ii) If v_i is not a generator,

$$\frac{\text{The number of } k_3 \text{ graphs in which } v_i \text{ is a vertex}}{Total\ k_3 \text{ graphs in the generator graph}}$$
$$= \frac{\phi(n)C_2}{[n-\phi(n)]\times\sum[\phi(n)-1]}$$
$$= \frac{\left[\frac{\phi(n)\times[\phi(n)-1]}{2}\right]}{[n-\phi(n)]\times[[\phi(n)-1]+[\phi(n)-2]+\cdots\ldots\ldots+[\phi(n)-(\phi(n)-1)]]}$$
$$= \frac{\left[\frac{\phi(n)\times[\phi(n)-1]}{2}\right]}{[n-\phi(n)]\times\left[\frac{[\phi(n)-1][\phi(n)]}{2}\right]}$$
$$= \frac{1}{[n-\phi(n)]}$$

$$[\mu^-{}_A(v_i), \mu^+{}_A(v_i)] = \left[\frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{ is a vertex}}{k}, \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{ is not a vertex}}{k}\right]$$
$$= \left[\frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{ is a vertex}}{k}, 1 - \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{ is a vertex}}{k}\right]$$
$$= \left[\frac{1}{[n-\phi(n)]}, \quad 1 - \frac{1}{[n-\phi(n)]}\right]$$

Hence if v_i is a generator and v_j is not a generator.

$$O(G) = \left[\left(\sum \mu^-{}_A(v_i) + \sum \mu^-{}_A(v_j)\right), \left(\sum \mu^+{}_A(v_i) + \sum \mu^+{}_A(v_j)\right)\right]$$
$$= \phi(n) \times \left[\frac{2}{\phi(n)}, \quad 1 - \frac{2}{\phi(n)}\right] + [n-\phi(n)] \times \left[\frac{1}{[n-\phi(n)]}, \quad 1 - \frac{1}{[n-\phi(n)]}\right]$$
$$= \left[\phi(n) \times \frac{2}{\phi(n)} + [n-\phi(n)] \times \frac{1}{[n-\phi(n)]}, \phi(n) \times \left(1 - \frac{2}{\phi(n)}\right) + [n-\phi(n)](1 - \frac{1}{[n-\phi(n)]})\right]$$
$$= [(2+1), (\phi(n) - 2 + [n-\phi(n)] - 1)]$$
$$= [3, (n-3)].$$

4.2 For multiplicative modular groups

The number of generators for the cyclic group $(Z_n, \times_n)$ are different for different values of prime 'n'. The number of generators in $(Z_n, \times_n)$ is $\phi(n-1)$and non generators are $[(n-1) - \phi(n-1)]$.By corollary, the number of k_3 graphs in the generator graph of the cyclic group $(Z_n, \times_n)$ will be $[(n-1) - \phi(n-1) \times \sum(\phi(n-1) - 1)$.

Example Figure 8 shows the interval valued fuzzy graph of $(Z_7, \times_7)$size

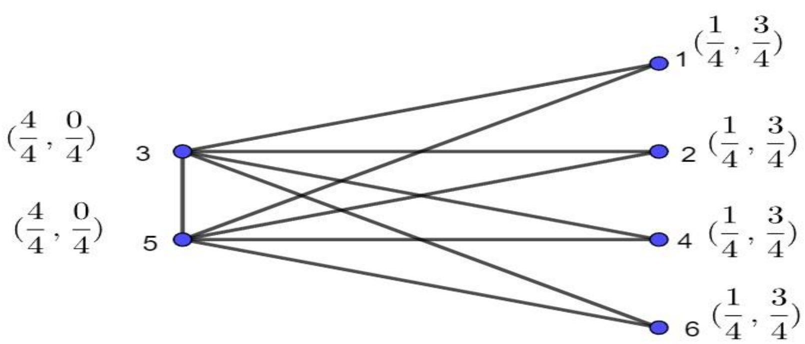

Fig 8. $(Z_7, \times_7)$

The total number of k_3 triangles in $(Z_7, \times_7)$ is 4. The vertex represented by generator 3 is part of all 4 k_3 triangles. Hence its membership value is $\left(\frac{4}{4}, \frac{0}{4}\right)$. The vertex represented by 1 is part of only one 0f the 4 k_3 triangles and not a part of remaining 3 k_3 triangles. Hence its membership value is $\left(\frac{1}{4}, \frac{3}{4}\right)$) .The graph has 2 generators and 4 non-generator and therefore 6 vertices. The order of IVFG of $(Z_7, \times_7)$ is the sum of membership values of all the vertices in all the k_3 triangles. The O(G) of IVFG of generator graph of $(Z_7, \times_7)$ is calculated as

$\text{O(G)} = 2 \times (1,0) + 4 \times \left(\frac{1}{4}, \frac{3}{4}\right) = (3,3)$

Example Figure 9 shows the interval valued fuzzy graph of $(Z_{11}, \times_{11})$

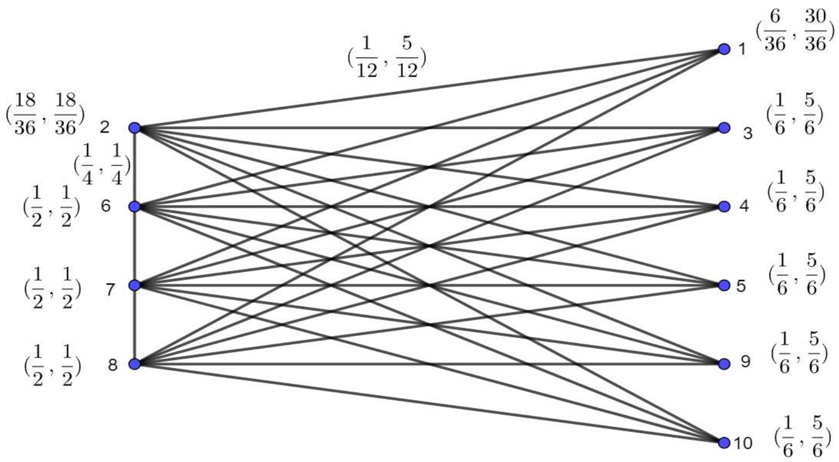

Fig 9. $(Z_{11}, \times_{11})$

The total number of k_3 triangles in $(Z_{11}, \times_{11})$ is 36 The vertex represented by generator 2 is part of 18 k_3 triangles and not a part ofpart of remaining 18 k_3 triangles. Hence its membership value is $\left(\frac{18}{36}, \frac{18}{36}\right)$. The vertex represented by 1 is part of only one 0f the 6 k_3 triangles and not a part of remaining 30 k_3 triangles. Hence its membership value is $\left(\frac{6}{36}, \frac{30}{36}\right)$) .The graph has 4 generators and 6 non-generator and therefore 10 vertices. The order of IVFG of $(Z_{11}, \times_{11})$ is the sum of membership values of all the vertices in all the k_3 triangles) and is calculated as

$\text{O(G)} = 4 \times \left(\frac{18}{36}, \frac{18}{36}\right) + 6 \times \left(\frac{6}{36}, \frac{30}{36}\right) = (3,7) = (3, [n-1] - 3)$

Theorem 3

Let $G^* = (V, E)$be a generator graph of an multiplicative modular cyclic group $(Z_n +_n)$ with $[(n-1) - \square(n-1)] \times \sum[\square(n-1) - 1]\, k_3$ graphs. Let $G = (A, B)$ be an IVFG associated with G^*,Then $O(G) = [3, ((n-1) - 3)]$.

Proof Let $G = (A, B)$ is a fuzzy graph in which$A = \{(v_i, [\mu^-{}_A(v_i), \ \mu^+{}_A(v_i)]) : v_i \epsilon V\}$and $B = \{(v_i v_j, [\mu^-{}_B(v_i v_j), \ \mu^+{}_B(v_i v_j)]) : v_i v_j \epsilon E\}$.Now if v_i is a generator or a non-generator$O(G) = [\sum_{i=1}^{n} \mu^-{}_A(v_i), \sum_{i=1}^{n} \mu^+{}_A(v_i))]$

Case(i) If v_i is a generator,

$$\frac{\text{The number of } k_3 \text{ graphs in which } v_i \text{is a vertex}}{Total\ k_3 \text{ graphs in the generator graph}}$$

$$= \frac{[(n-1) - \square(n-1)] \times [\square(n-1) - 1]}{[(n-1) - \square(n-1)] \times \sum[\square(n-1) - 1]}$$

$$= \frac{[\square(n-1) - 1]}{[\square(n-1) - 1] + [\square(n-1) - 2] + \cdots \ldots [\square(n-1) - (\square(n-1) - 1)]}$$

$$= \frac{[\square(n-1) - 1]}{[\square(n-1) - 1] + [\square(n-1) - 2] + \cdots \ldots [1]}$$

$$= \frac{[\square(n-1) - 1] \times 2}{[\square(n-1) - 1][\square(n-1)]} = \frac{2}{[\square(n-1)]}$$

$$[\mu^-{}_A(v_i), \mu^+{}_A(v_i)]$$

$$= \left[\frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex}}{k}, \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is not a vertex}}{k}\right]$$

$$= \left[\frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex}}{k}, 1 - \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex}}{k}\right]$$

$$= \left[\frac{2}{\square(n-1)},\quad 1 - \frac{2}{\square(n-1)}\right]$$

Case(ii) If v_i is not a generator,

$$\frac{\text{The number of } k_3 \text{ graphs in which } v_i \text{is a vertex}}{Total\ k_3 \text{ graphs in the generator graph}}$$

$$= \frac{\square(n-1)C_2}{[(n-1)-\square(n-1)]\times\sum[\square(n-1)-1]}$$

$$= \frac{\left[\frac{\square(n-1)\times[\square(n-1)-1]}{2}\right]}{[(n-1)-\square(n-1)]\times\left[[\square(n-1)-1]+[\square(n-1)-2]+\cdots\ldots\ldots\ldots+[\square(n-1)-(\square(n-1)-1)]\right]}$$

$$= \frac{\left[\frac{\square(n-1)\times[\square(n-1)-1]}{2}\right]}{[(n-1)-\square(n-1)]\times\left[\frac{[\square(n-1)-1][\square(n-1)]}{2}\right]}$$

$$= \frac{1}{[(n-1)-\square(n-1)]}$$

$$[\mu^-{}_A(v_i), \mu^+{}_A(v_i)]$$

$$= \left[\frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex}}{k}, \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is not a vertex}}{k}\right]$$

$$= \left[\frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex}}{k}, 1 - \frac{No.\ of\ k_3 \text{ graphs in which } v_i \text{is a vertex}}{k}\right]$$

$$= \left[\frac{1}{[(n-1)-\square(n-1)]},\quad 1 - \frac{1}{[(n-1)-\square(n-1)]}\right]$$

Hence if v_i is a generator and v_j is not a generator.

$$O(G) = \left[\left(\sum \mu^-{}_A(v_i) + \sum \mu^-{}_A(v_j)\right), \left(\sum \mu^+{}_A(v_i) + \sum \mu^+{}_A(v_j)\right)\right]$$

$$= \square(n-1) \times \left[\frac{2}{\square(n-1)},\quad 1 - \frac{2}{\square(n-1)}\right] + [(n-1)-\square(n-1)] \times \left[\frac{1}{[(n-1)-\square(n-1)]},\quad 1 - \frac{1}{[(n-1)-\square(n-1)]-1}\right]$$

$$= \left[\begin{array}{c} \square(n-1) \times \frac{2}{\square(n-1)} + [(n-1)-\square(n-1)] \times \frac{1}{[(n-1)-\square(n-1)]}, \\ \square(n-1) \times \left(1 - \frac{2}{\square(n-1)}\right) + [(n-1)-\square(n-1)](1 - \frac{1}{[(n-1)-\square(n-1)]}) \end{array}\right]$$

$$= [(2+1), (\square(n-1) + [(n-1)-\square(n-1)] - 2 - 1)]$$

$$= [3, ((n-1)-3)]\ .$$

5. Size of the IVFG of generator graphs for modular groups

We define the size of the interval valued fuzzy graph as the sum of all membership values of all the edges. The membership value of any edge is the product of the membership value of its corresponding vertices. We obtain a constant value for the size of the graph in case of interval valued fuzzy generator graph of additive and multiplicative modular groups.

5.1 For additive modular groups

Example Figure 10shows the interval valued fuzzy graph of$(Z_6, +_6)$

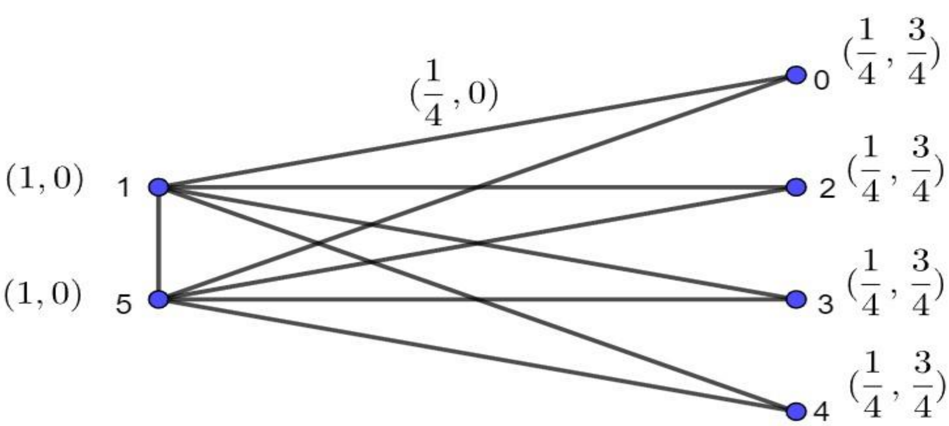

Fig 10. $(Z_6, +_6)$

The size of the graph is the sum of the membership values of all the 12 edges. In 4 triangles 4 edgesare between the generator vertices and the remaining 8 are between a generator vertex and a non-generator vertex. The product of the membership between the vertices which are generators is (1,0)(1,0) and the ones in which one is a generator and other is not a generator is $(1,0)\left(\frac{1}{4},\frac{3}{4}\right)$.The size of $(Z_6,+_6)$is calculated as

$S(G) = 4 \times (1,0)(1,0) + 4 \times (1,0)\left(\frac{1}{4},\frac{3}{4}\right) = (4,0) + (1,0) = (5,0)$

Example Figure 11 shows theinterval valued fuzzy graph of$(Z_9,+_9)$

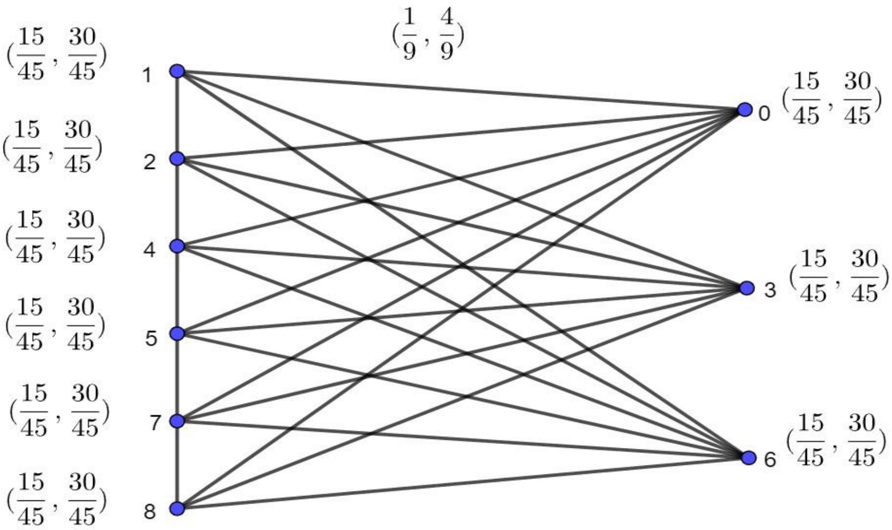

Fig 11. $(Z_9,+_9)$

In this graph there are 45 triangles and the number of edges will be 45×3 and the membership value for each edge is $\left(\frac{15}{45},\frac{30}{45}\right) \times \left(\frac{15}{45},\frac{30}{45}\right)$.Hence S(G) is calculated as

$S(G) = 45 \times 3 \times \left(\frac{15}{45},\frac{30}{45}\right) \times \left(\frac{15}{45},\frac{30}{45}\right) = (15,60)$

Theorem 4

Let $G^* = (V,E)$be a generator graph of an additive modular cyclic group (Z_n+_n) with $[n-⍰(n)] \times \sum[⍰(n)-1]\, k_3$ graphs. Let $G=(A,B)$ be a fuzzy graph associated with G^*,Then $S(G) = \left[\frac{2n[⍰(n)-1]}{⍰(n)}, [⍰(n)-1]C_2\left(3n - \frac{3}{⍰(n)} - \frac{2n}{⍰(n)}\right)\right]$

Proof: Let $A = \{\mu(v_i), v_i \in V\}$and $B = \{\mu(v_iv_j), v_i \in V, v_j \in V\}$in the generator graph $G=(A,B)$of the additive modular cyclic group(Z_n+_n). Now $S(G) = \sum_{i=1}^{n} \mu_B(v_iv_j)$where v_i ,v_j is a generator or a non-generator. Let the number of triangles in $G=(A,B)$ be k.

Hence $S(G)$

$$= \text{Number of } k_3 \text{ triangles} \times [\mu(v_i)]^2 + 2 \times \text{Number of } k_3 \text{ triangles} \times \mu(v_i) \times \mu(v_j)$$

$$= k \times \left[\frac{2}{⍰(n)}, 1-\frac{2}{⍰(n)}\right]\left[\frac{2}{⍰(n)}, 1-\frac{2}{⍰(n)}\right] + 2 \times k \times \left[\left(\frac{2}{⍰(n)}\right), \left(1-\frac{2}{⍰(n)}\right)\right]\left[\frac{1}{n-⍰(n)}, 1-\frac{1}{n-⍰(n)}\right]$$

$$= k \times \left[\frac{2}{⍰(n)}, 1-\frac{2}{⍰(n)}\right]^2 + 2 \times k \times \left[\left(\frac{2}{⍰(n)}\right)\left(\frac{1}{n-⍰(n)}\right), \left(1-\frac{2}{⍰(n)}\right)\left(1-\frac{1}{n-⍰(n)}\right)\right]$$

$$= \left[k \times \left(\frac{2}{⍰(n)}\right)\left[\frac{2}{⍰(n)} + \frac{2}{n-⍰(n)}\right], k \times [1-\frac{2}{⍰(n)}][\left(1-\frac{2}{⍰(n)}\right) + (2-\frac{2}{n-⍰(n)})]\right]$$

$$= \left[k \times \left(\frac{2}{⍰(n)}\right)\left[\frac{2n-2⍰(n)+2⍰(n)}{⍰(n)[n-⍰(n)]}\right], k \times [1-\frac{2}{⍰(n)}][\left(\frac{⍰(n)-2}{⍰(n)}\right) + (\frac{2(n-⍰(n))-2}{n-⍰(n)})]\right]$$

$$= \left[k \times \left[\frac{4n}{⍰(n)^2[n-⍰(n)]}\right], k \times [1-\frac{2}{⍰(n)}]\left(\frac{[⍰(n)-2][n-⍰(n)+⍰(n)[2(n-⍰(n))-2]]}{⍰(n)[n-⍰(n)]}\right)\right]$$

$$= \left[k \times \left[\frac{4n}{[⍰(n)]^2[n-⍰(n)]}\right], k \times [1-\frac{2}{⍰(n)}]\left(\frac{[3n\,⍰(n)-3[⍰(n)]^2-2n]}{⍰(n)[n-⍰(n)]}\right)\right]$$

$$= \left[[n-⍰(n)] \times \sum[⍰(n)-1] \times \left[\frac{4n}{[⍰(n)]^2[n-⍰(n)]}\right], [n-⍰(n)] \times \sum[⍰(n)-1] \times [1-\frac{2}{⍰(n)}]\left(\frac{[3n\,⍰(n)-3[⍰(n)]^2-2n]}{⍰(n)[n-⍰(n)]}\right)\right]$$

$= \left[\sum[\square(n)-1]\times\left[\frac{4n}{[\square(n)]^2}\right], \sum[\square(n)-1]\times[1-\frac{2}{\square(n)}]\left(\frac{[3n\,\square(n)-3[\square(n)]^2-2n]}{\square(n)}\right)\right]$

$= \left[\left[\frac{[\square(n)-1][\square(n)]}{2}\right]\times\left[\frac{4n}{[\square(n)]^2}\right], \left[\frac{[\square(n)-1][\square(n)]}{2}\right]\times\left[\frac{\square(n)-2}{\square(n)}\right]\left(\frac{[3n\,\square(n)-3[\square(n)]^2-2n]}{\square(n)}\right)\right]$

$=\left[\frac{2n[\square(n)-1]}{[\square(n)]}, \left(\frac{[\square(n)-1][\square(n)-2]}{2}\right)\times\left(\frac{[3n\,\square(n)-3[\square(n)]^2-2n]}{\square(n)}\right)\right]$

$=\left[\frac{2n[\square(n)-1]}{\square(n)}, (\square(n)-1)C_2\left(3n-\frac{3}{\square(n)}-\frac{2n}{\square(n)}\right)\right].$

5.1 For multiplicative modular groups

The number of generators in additive and multiplicative modular groups vary as the former is $\square(n)$ and the latter is $\square(n-1)$.Hence the calculation for size of the graph vary slightly for the multiplicative modular groups.

Example Figure 12 shows the interval valued fuzzy graph of $(Z_7,\times_7)$

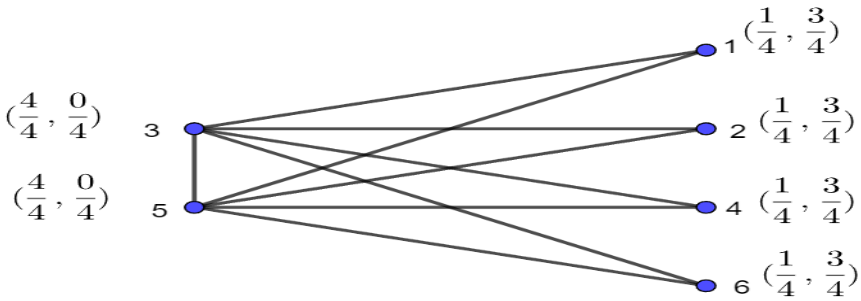

Fig 12. $(Z_7,\times_7)$

The number of $k_3$triangles in $(Z_7,\times_7)$is only 4 and it has 8 edges with membership values(1,0)(1,0) for edges between the generator vertices and $(1,0)\left(\frac{1}{4},\frac{3}{4}\right)$between thegenerator and non-generator vertices.The size of the graph is the sum of all its edge membership values and is calculated as

$S(G) = 4\times(1,0)(1,0)+4\times(1,0)\left(\frac{1}{4},\frac{3}{4}\right) = (4,0)+(1,0) = (5,0)$

Example Figure 13 shows the interval valued fuzzy graph of $(Z_{11},\times_{11})$

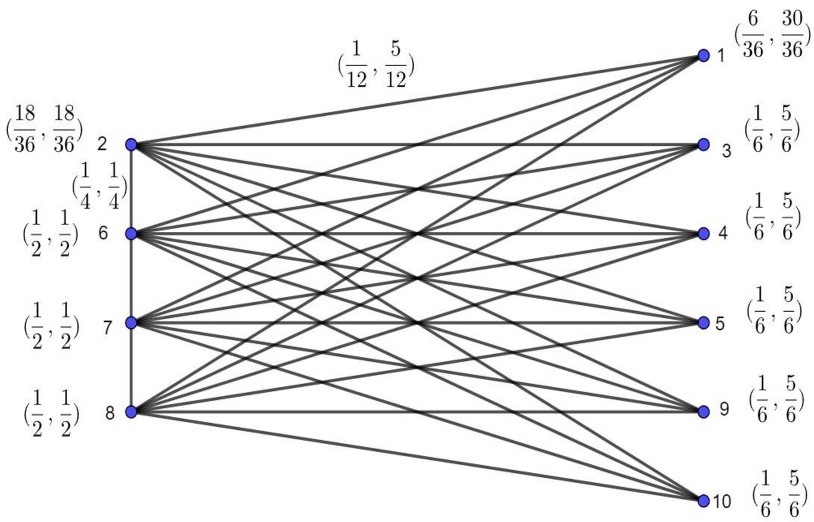

Fig 13. $(Z_{11},\times_{11})$

In this graph there are 36 triangles and the number of edges will be 36×3 and the membership value for each edge is $\left(\frac{15}{45},\frac{30}{45}\right)\times\left(\frac{15}{45},\frac{30}{45}\right)$.Hence S(G) is calculated as

$S(G) = 36\times\left(\frac{1}{2},\frac{1}{2}\right)\left(\frac{1}{2},\frac{1}{2}\right)+2\times36\times\left(\frac{1}{2},\frac{1}{2}\right)\left(\frac{1}{6},\frac{5}{6}\right) = (15,39)$

Theorem 5

Let $G^* = (V, E)$be a generator graph of a multiplicative modular cyclic group $(Z_n \times_n)$ with $[(n-1) - \square(n-1)] \times \sum[\square(n-1) - 1]\, k_3$ graphs. Let $G = (A, B)$ be a fuzzy graph associated with G^*,Then $S(G) = \left[\frac{\mathbf{2(n-1)[\square(n-1)-1]}}{\mathbf{\square(n-1)}}, \square(n-2)C_2\left(3(n-1) - \frac{3}{\square(n-1)} - \frac{2(n-1)}{\square(n-1)}\right)\right]$

Proof: Let $A = \{\mu(v_i), v_i \in V\}$and $B = \{\mu(v_i v_j), v_i \in V, v_j \in V\}$in the generator graph $G = (A, B)$of the multiplicative modular cyclic group$(Z_n \times_n)$. Now $S(G) = \sum_{i=1}^{n} \mu_B(v_i v_j)$where v_i ,v_j is a generator or a non-generator. Let the number of triangles in $G = (A, B)$ be k.

Hence $S(G)$

$$= \text{Number of } k_3 \text{ triangles} \times [\mu(v_i)]^2 + 2 \times \text{Number of } k_3 \text{ triangles} \times \mu(v_i) \times \mu(v_j)$$

$$= k \times \left[\frac{2}{\square(n-1)}, 1 - \frac{2}{\square(n-1)}\right]\left[\frac{2}{\square(n-1)}, 1 - \frac{2}{\square(n-1)}\right]$$

$$+2 \times k \times \left[\left(\frac{2}{\square(n-1)}\right), \left(1 - \frac{2}{\square(n-1)}\right)\right]\left[\frac{1}{(n-1) - \square(n-1)}, 1 - \frac{1}{(n-1) - \square(n-1)}\right]$$

$$= k \times \left[\frac{2}{\square(n-1)}, 1 - \frac{2}{\square(n-1)}\right]^2 + 2 \times k$$

$$\times \left[\left(\frac{2}{\square(n-1)}\right)\left(\frac{1}{(n-1) - \square(n-1)}\right), \left(1 - \frac{2}{\square(n-1)}\right)\left(1 - \frac{1}{(n-1) - \square(n-1)}\right)\right]$$

$$= \left[k \times \left(\frac{2}{\square(n-1)}\right)\left[\frac{2}{\square(n-1)} + \frac{2}{(n-1) - \square(n-1)}\right], k \times [1 - \frac{2}{\square(n-1)}][\left(1 - \frac{2}{\square(n-1)}\right) + (2 - \frac{2}{(n-1) - \square(n-1)})]\right]$$

$$=\left[k \times \left(\frac{2}{\square(n-1)}\right)\left[\frac{2(n-1) - 2\square(n-1) + 2\square(n-1)}{\square(n-1)[(n-1) - \square(n-1)]}\right], k \times [1 - \frac{2}{\square(n-1)}][\left(\frac{\square(n-1)-2}{\square(n-1)}\right) + (\frac{2((n-1) - \square(n-1)) - 2}{(n-1) - \square(n-1)})\right]$$

$$=\left[k \times \left[\frac{4(n-1)}{\square(n-1)^2[(n-1) - \square(n-1)]}\right], k \times [1 - \frac{2}{\square(n-1)}]\left(\frac{[\square(n-1)-2][(n-1) - \square(n-1) + \square(n-1)[2((n-1) - \square(n-1)) - 2]]}{\square(n-1)[(n-1) - \square(n-1)]}\right)\right]$$

$$= \left[k \times \left[\frac{4(n-1)}{[\square(n-1)]^2[(n-1) - \square(n-1)]}\right], k \times [1 - \frac{2}{\square(n-1)}]\left(\frac{[3(n-1)\square(n-1) - 3[\square(n-1)]^2 - 2(n-1)]}{\square(n-1)[(n-1) - \square(n-1)]}\right)\right]$$

$$= \left[[(n-1) - \square(n-1)] \times \sum[\square(n-1) - 1] \times \left[\frac{4(n-1)}{[\square(n-1)]^2[(n-1) - \square(n-1)]}\right], [(n-1) - \square(n-1)] \times \sum[\square(n-1) - 1] \times [1 - \frac{2}{\square(n-1)}]\left(\frac{[3(n-1)\square(n-1) - 3[\square(n-1)]^2 - 2(n-1)]}{\square(n-1)[(n-1) - \square(n-1)]}\right)\right]$$

$$= \left[\sum[\square(n-1) - 1] \times \left[\frac{4(n-1)}{[\square(n-1)]^2}\right], \sum[\square(n-1) - 1] \times [1 - \frac{2}{\square(n-1)}]\left(\frac{[3(n-1)\square(n-1) - 3[\square(n-1)]^2 - 2(n-1)]}{\square(n-1)}\right)\right]$$

$$= \left[\left[\frac{[\square(n-1)-1][\square(n-1)]}{2}\right] \times \left[\frac{4(n-1)}{[\square(n-1)]^2}\right], \left[\frac{[\square(n-1)-1][\square(n-1)]}{2}\right] \times \left[\frac{\square(n-1)-2}{\square(n-1)}\right]\left(\frac{[3(n-1)\square(n-1) - 3[\square(n-1)]^2 - 2(n-1)]}{\square(n-1)}\right)\right]$$

$$= \left[\frac{2(n-1)[\square(n-1)-1]}{[\square(n-1)]}, \left(\frac{[\square(n-1)-1][\square(n-1)-2]}{2}\right) \times \left(\frac{[3(n-1)\square(n-1) - 3[\square(n-1)]^2 - 2(n-1)]}{\square(n-1)}\right)\right]$$

$$=\left[\frac{2(n-1)[\square(n-1)-1]}{\square(n-1)}, (\square(n-1) - 1)C_2\left(3(n-1) - \frac{3}{\square(n-1)} - \frac{2(n-1)}{\square(n-1)}\right)\right].$$

CONCLUSION

In this paper we have constructed interval valued Fuzzy Graphs from the Generator graph of the cyclic group based on the membership values of the vertices as the number of k_3 graphs in the generator graph for which these vertices are a part divided by the total number of k_3 graphs in the generator graph. We have proved that the order and size of the of the interval valued fuzzy graph associated with both the generator graph of multiplicative and additive modular cyclic group is dependent on the number of generators and non- generators in a group. The future study of this paper will focus on applications of the interval valued fuzzy graph in the neural networks and data mining.

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