New Heuristic Techniques for Solving Capacitated Vehicles Routing Problem

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ABSTRACT
The vehicle routing problem (VRP) is part from a combinatorial optimization problem (COP) with huge numbers of applications. One of its important applications is Capacitated VRP (CVRP) which is it generalizes the travelling salesman problem (TSP).This research divided into two parts, the first part is the theoretical part; which includes introducing the mathematical formulation of the CVRP and some special cases. While the second part is the practical part.In this partwe introduced new heuristic methods for solving the CVRP. The results of these techniques are compared with exact methods (such as complete enumeration method). Also, they compared with optimal datasets. The comparison results proved the efficiency and speed in CPU-time of the suggested techniques.

Keywords: Vehicles Routing Problems, Capacitated Vehicles Routing Problems, Complete Enumeration Method, Branch and Bound Method, Nearest Neighbours Method.

1. INTRODUCTION

The vehicle routing problems (VRPs) is an NP-hard combinatorial optimization (CO) and integer programming problem with main goal to construct an optimal route(or Best route) among set of routes that deliver goods from depot center (distributions center) to set of locations (cities) with known given data (demands of each locations, distance between each cities and distribution center and each cities to other, vehicles capacity, etc) that aim to minimizing the main objective function (cost, distance, travel time, etc)[1].

VRPs consider among the most studied problems in the space of CO.It first appeared in a paper by Dantzig and Ramser in 1959,although the Traveling Salesman Problem (TSP) could be seen as belonging to the class, the most basic and studied problem is the CVRP, where cities, each with a known certain demand, delivered by number of identical vehicles. Each vehicle has to go only one route. All routes start from the distribution center and return to it. The main goal to be considered involve the assignment of cities to vehicles and the sequencing vehicles in such a way that the total routing distance is minimized. In general, the VRPs is used when the cities to be visited are assigned to numbers of vehicles.[2]

The VRP consists of several branch which includes; single TSP, Multiple TSP, CVRP, VRP with time windows (VRPTW), dynamic VRP (DVRP), pickup and delivery VRP (PDVRP), periodic VRP (PVRP), single CVRP and so on. These variants have several applications[3]. Some of the classifications of VRP is shown in figure(1).

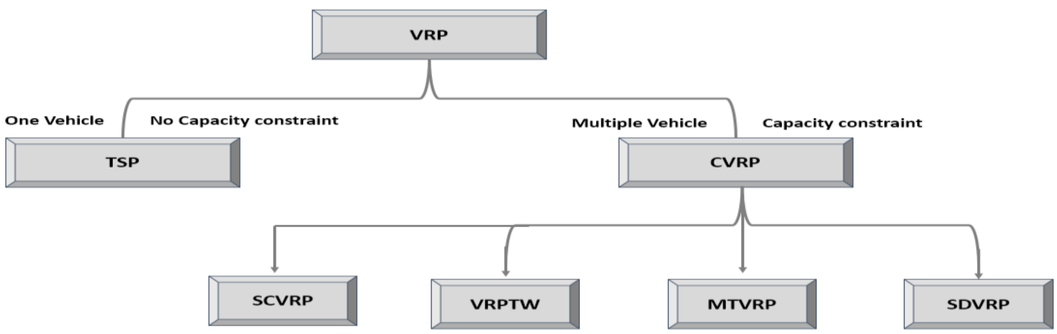

Figure 1. VRP Classification.

VRP is a type of COP with numerous application possibilities. There are currently many methods for solving VRPs, which can be divided into two categories: exact methods and heuristic (meta-heuristic) methods. However, due to the complexity of VRPs, exact methods are limited for solving large-number of nodes VRPs. Many successful attempts to solve VRP have been made in recent years, depending on computer science and applications such as MATLAB[4].
As Literature Survey we have many applications of VRP.In 2018, Qiu et al.[5]apply the Tabu search (TS) algorithm to solve VRP. The experimental results show the accuracy of the TS. In 2019, Lu et al.[6]using machine learning ML-based techniques to solve VRP. They show that the applied exercise has improved the solution at the same computational cost. In2020, Adnan et al.[7]introducing a graph representing the roads is created for VRP, then Dijkstra's algorithm is used and show the improvement through results. In 2021 Abdoul-Hafaret al.[8]presented newparticle swarm algorithm (PSO) for the CVRP and prove its efficiency from other given techniques. In 2022, Xiaodong et al.[9]Using differential evolutionary (DE)based onwhale optimization algorithm (WOA)to solve VRP. The result of the proposed model is faster than basic WOA and the overall optimization is improved by 23%. In 2023, Kangye et al.[10]takethe novel coronavirus pandemic is a major global public health emergency, which is considered as VRP based on the problems of insufficient timeliness and high total system cost of emergency logistics distribution in major epidemic situations. The total cost was reduced by 20.1%by using improved PSO algorithm compared with the basic PSO algorithm.
In this paper we will focus the general CVRP and introduce new heuristic methods to solve the Problem. In section (2) we introduce the basic concepts and definitions of the Graph Theory. In section (3) we give the definition of CVRP and its mathematical model and their assumptions. In section (4) discuss some of CVRP special cases. In section (5) we present an overview of the methods used for the solution of the CVRP.In section (6) we introduce our new methods for solving CVRP.In section (7) applied examples to the new proposed methods. In section (8) introduce the discussions and analysis of results.

2. Graph Theory[11],[12]

In mathematics, graph theory (GT) is the study of graphs, which are mathematical structures used to model pairwise relations between objects,a graph G(V; E) is a set V of vertices and a set E of edges. In an undirected graph, an edge is an unordered pair of vertices,A distinction is made between undirected graphs, where edges link two vertices symmetrically, and directed graphs, where edges link two vertices asymmetrically.
A graph is an ordered pair $G = (V, E)$ comprising:

- V a set of vertices (also called nodes or points)
- $E \subseteq \{\{x, y\} \mid x, y \text{ and } x \neq y\}$, a set of edges (also called links or lines) which are unordered pairs of vertices (that is, an edge is associated with two distinct vertices).
- As shown below figure (2-a) show a graph with three vertices and three edge and figure (2-b) a drawing of a graph.
- A graph $G = (V, E)$ consists of a set V of vertices (also called nodes) and a set E of edges.
- If an edge connects to a vertex, we say the edge is incident **to** the vertex and say the vertex is an endpoint of the edge.
- Two vertices that are joined by an edge are called adjacent vertices.
- If an edge has only one endpoint, then it is called a loop edge.
- If two or more edges have the same endpoints then they are called multiple or parallel edges.

In mathematics, the Euclidean Distance (ED) between two points in Euclidean space are the length of the line segment between them. It can be calculated from the Cartesian coordinates of the points using the Pythagorean theorem, and therefore, is occasionally called the Pythagorean distance.

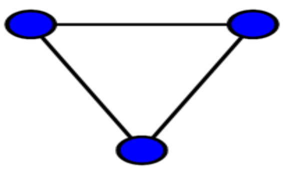

Figure (2-a)

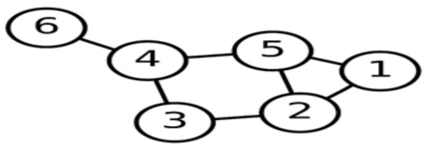

Figure (2-b)

In the Euclidean plane, let point p have Cartesian coordinates $(p_1, p_2)$and let point q have coordinates (q_1, q_2). Then the distance between p and q is given by:

$$d(p,q) = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2} \qquad (1)$$

3. Capacitated Vehicles Routing Problem

The capacitated vehicle routing problem (CVRP) refers to a VRP in which vehicles with known limited carrying capacity pick up or deliver items at a variety of locations. The vehicles have a maximum carrying capacity. The challenge is to deliver the items over the shortest distance while never exceeding the capacity of the vehicles.

Dantzigand and Ramser (1959)[13]define the CVRP as follows: A central depot houses a large number of similar vehicles with a specific capacity. They are available to service a specific set of city orders (either all deliveries or all pickups). Each client's order has a particular location and size. Costs for traveling between every point are provided. The goal is to generate a low-cost set of vehicle routes that visit all customers once while adhering to vehicle capacity.

3.1 Mathematical Model of CVRP[14]

I. The Objective Function of CVRP

The main objective of CVRP is finding the minimum scheduling total route length (shortestpath length can be used to obtain the minimum time for resource distribution).

II. CVRP Requirements

1. Suppose that the position of the Distribution Centre (DC) is known, and the distance between the DC and every city in the graph network is known as well between them.
2. The resource at each point orcity is delivered by just one vehicle.
3. At any demand point or city,the load capacity of every vehicle mustsatisfy the demand for cities.
4. Supposing that every vehicle has the same capacity and speed.
5. there is no priority level,all level of cities demand is the same in each point.

III. Variable Definitions

$\boldsymbol{n}$ **:** number of cities.
$\boldsymbol{m}$ **:** number of vehicles.
$\boldsymbol{b_k}$ **:** Load of each vehicle, $k = 1,2, \dots, m$.
$\boldsymbol{r_i}$ **:** Resource requirements of each point, $i = 1,2, \dots, n$.
$\boldsymbol{d_{ij}}$ **:** Distance from demand city$i$ to j.
$\boldsymbol{d_{0i}}$,$\boldsymbol{d_{i0}}$ **:** Represents distance from DC to demand point i.

$$v_{ik} = \begin{cases} 1 & if\ k-th\ vehicle\ goes\ to\ the\ demand\ point\ i \\ 0 & otherwise \end{cases}$$

$$x_{ijk} = \begin{cases} 1 & if\ kth\ vehicle\ passes\ through\ point\ i\ \text{and}\ reaches\ point\ j \\ 0 & otherwise \end{cases}$$

IV. Model Construction

We have n cities not including the DC with m vehicles.

$$min\ F = min \sum_{k=1}^{m} \sum_{j=0}^{n} \sum_{i=0}^{n} d_{ij} x_{ijk} \quad \textbf{(CVR-1)}$$

Subject to:

$$\sum_{k=1}^{m} v_{ik} = 1, i = 1,2, \dots, n \quad \textbf{(CVR-2)}$$

$$\sum_{j=1}^{n} x_{0jk} = \sum_{j=1}^{n} x_{j0k} = 1, k = 1,2, \dots, m \quad \textbf{(CVR-3)}$$

$$\sum_{j=0}^{n} x_{jik} = v_{ik}, i = 1,2, \dots, n; k = 1,2, \dots, m \quad \textbf{(CVR-4)}$$

$$\sum_{i=1}^{n} r_i v_{ik} \le b_k, k = 1,2, \dots, m \quad \textbf{(CVR-5)}$$

$$d_{ij} = d_{ji}, i, j = 0,1,2, \dots, n \text{ (Symmetric)} \quad \textbf{(CVR-6)}$$

$$x_{ijk} \in \{0,1\}, i.j = 0,1,2, \dots, n; k = 1,2, \dots, m \quad \textbf{(CVR-7)}$$

$$v_{ik} \in \{0,1\}, i = 1,2, \dots, n; k = 1,2, \dots, m \quad \textbf{(CVR-8)}$$

$$d_{ij} > 0, i, j = 0,1, \dots, n \quad \textbf{(CVR-9)}$$

$$r_i > 0, b_k > 0, i = 1,2, \dots, n; k = 1,2, \dots, m \quad \textbf{(CVR-10)}$$

(CVR)

(CVR-1): is the objective function of CVRP, which assign the shortest route can be graphed to minimize the total distance moved by all the specific vehicles.
(CVR-2):means that each city demand point is only receive just one vehicle.
(CVR-3):represents that each vehicle starts from the DC and moved via each point in specific route.
(CVR-4):means that there one vehicle passing via each resource demand point.
(CVR-5):means no overload of vehicles, in another word, that the load capacity of distribution vehicles will not be less than that demand points visited.
(CVR-6): denotes that the rout length from point i to point $j(i \to j)$ is the same as that from $j \to i$. That mean our CVRP is symmetric.
(CVR-(7,8,9,10)):describe the range of values of the CVRPvariable.
Example (1):let say we have $n = 6$ cities with given coordinate (x, y)as shown in table (1), and vehicles capacity is 100.

Table 1. (x, y) Coordinates for example (1).

	DC	C1	C2	C3	C4	C5	C6
Demands	0	15	13	14	12	9	10
x	0	7	-1	8	-2	6	-2
y	0	6	5	-3	-6	6	9
(x, y)	(0,0)	(7,6)	(-1,5)	(8, -3)	(-2, -6)	(6,6)	(-2,9)

Then by using ED we can transforms table (1) from (x, y) coordinates into distance between citiesusing relation (1)(see in table (2)).

Table 2. Distance table using ED.

Distance	DC	C1	C2	C3	C4	C5	C6
DC	0	9.2195	5.099	8.544	6.3246	8.4853	9.2195
C1	9.2195	0	8.0623	9.0554	15	1	9.4868
C2	5.099	8.0623	0	12.0416	11.0454	7.0711	4.1231
C3	8.544	9.0554	12.0416	0	10.4403	9.2195	15.6205
C4	6.3246	15	11.0454	10.4403	0	14.4222	15
C5	8.4853	1	7.0711	9.2195	14.4222	0	8.544
C6	9.2195	9.4868	4.1231	15.6205	15	8.544	0

Finally, by applying GT we can obtain the graph as shown in figure (3).

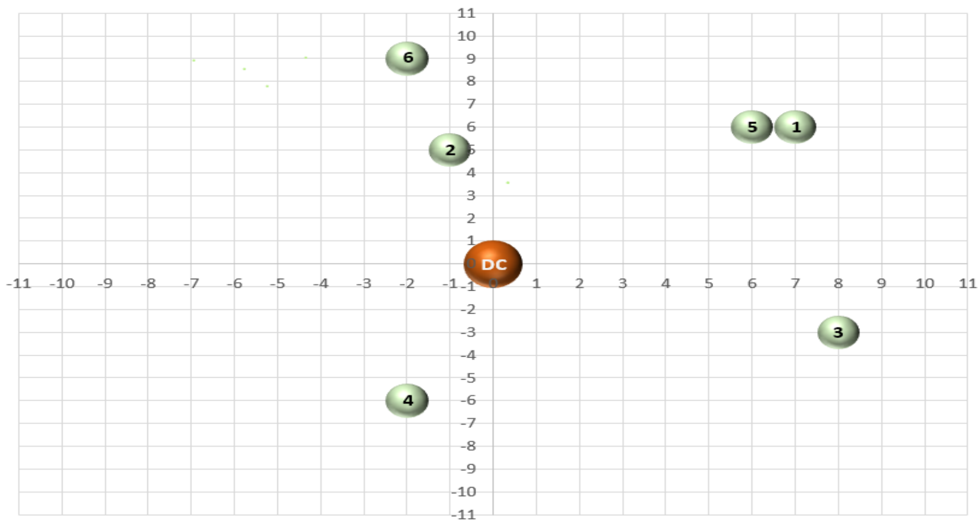

Figure 3. graph of distances of cites of Example (1).

Now if we have 2 vehicles available then we can apply any method to get the feasiblesolution path for each vehicle V_1 and V_2, the pathsare as follows:

$V_1 = DC \rightarrow C_3 \rightarrow C_1 \rightarrow C_5 \rightarrow C_6 \rightarrow C_2 \rightarrow DC$
$V_2 = DC \rightarrow C_4 \rightarrow DC$
And by taking the distance from table (2) we can calculate the cost for each vehicle:
$F(V_1) = d_{03} + d_{31} + d_{15} + d_{56} + d_{62} + d_{20} = 36.3655$
$F(V_2) = d_{04} + d_{40} = 12.6492$
Then the objective function OF is calculate by sum the cost of all vehicles$F(V_k)$, $k = 1,2$.

$$OF = \sum_{k=1}^{2} F(V_k) = 49.0147 \ , k = 1,2.$$

where all the constraints are satisfied:

4. Special Cases for VRP

In this section we will prove some facts and special cases for CVRP.
Case (4.1): For CVRP, if $m = 1$where capacity constraint is satisfied, then CVRP convertedtosingle CVRP (SCVRP) with multi-shift.
Proof: For $m = 1$, this means we uses one vehicle to fulfil demand over a scheduling period of serval work shifts.
The objective function will be:

$$min \sum_{k=1}^{1} \sum_{j=0}^{n} \sum_{i=0}^{n} d_{ij} x_{ijk} = min \sum_{j=0}^{n} \sum_{i=0}^{n} d_{ij} x_{ij} \quad (2)$$

for (n+1) cites:

$$x_{ij} \in \{0,1\} \quad , i.j = 0,1,2, \dots, n \quad (3)$$

and

$$v_i \in \{0,1\}, i = 1,2, \dots, n \quad (4)$$

And we have only one $b_1 > 0$ which satisfied capacity constraint.
This means we have SCVRP.

Case (4.2): For VRP, if m=1, without capacity constraint, then VRP converted tobasic TSP.
Proof: For $m = 1$, since relation (2) is satisfied, then we have $(n + 1)$ cities where DC is the starting and finishing city, since the capacity constraints is not satisfied, that means the (CVR-2), (CVR-4), (CVR-5) and (CVR-8) not important constraints.
Then the mathematical formulation (CVR) will be:

$$min \sum_{j=0}^{n} \sum_{i=0}^{n} d_{ij} x_{ij}$$

$s.t.$

$$\sum_{i=0}^{n} x_{ij} = 1 \ , \ j = 0,1, \dots, n. \quad (5)$$

$$\sum_{j=0}^{n} x_{ij} = 1 \ , \ i = 0,1, \dots, n. \quad (6)$$

$$x_{ij} \in \{0,1\} \quad , i.j = 0,1,2, \dots, n. \quad (7)$$

And that is the mathematical formation of TSP.

Case (4.3): For CVRP, if $m = n$ then CVRP has only one unique solution with $n!$ paths.
Proof: Since $m = n$, this means we can assign one vehicle for each city, since the capacityconstraints are satisfied for each vehicle. Then it's not important that which vehicle will travel to any city.So, we have $n!$ paths with unique objective function and CVRP will convert toAssignment Problem (AP) with equal assigning values, which calculated by relation (8).

$$OF = 2n \sum_{k=0}^{m} d_{0k} \quad (8)$$

Case (4.4): For CVRP, if $d = d_{ij}$ then CVRP has only one unique solution $F = (d + 1)(n + 1)\, m$ with $n!$ paths.
Proof: From objective function (CVR-1) of CVRP.

$$min \sum_{k=1}^{m} \sum_{j=0}^{n} \sum_{i=0}^{n} d_{ij} x_{ijk} = \sum_{k=1}^{m} (d + 1) \sum_{j=0}^{n} \sum_{i=0}^{n} x_{ijk}$$

$$= (d + 1) \sum_{k=1}^{m} \sum_{j=0}^{n} \sum_{i=0}^{n} x_{ijk}$$

$$= (d + 1) \sum_{k=1}^{m} (n + 1) = (d + 1)(n + 1)\, m$$

Which has $n!$ paths.

5. Solving Methods for Capacitated Vehicles Routing Problem

In this section we present an overview of some methods used for solving the CVRP.

5.1 Exact Methods

Complete enumeration method (CEM) sampling is a method used mainly in surveys and data collect and analysis to explore all possible elements in a finite set. It involves the selection, acquisition, and quantification of a part of the solutions space, with the aim of providing a representative sample based on certain criteria. This approach is particularly useful when dealing with small problem, as it allows for an exact test to be conducted by considering the complete distribution of the test statistic[15].
The branchand bound (BAB) method is another exact method has been used extensively in recent decades to solve the CVRP and its main variants [16].

5.2 Heuristics Methods

In recent decades, academics and researchers have become interested in general heuristic approximations, which can improve specific heuristics in the field. The researchers have vested and fed the related literature. Many methods have been developed, and it is extremely difficult to establish a systematic and widely accepted classification. A attainable grouping is as follows[17]:

- **Constructive heuristics**:Its family of strategies can be that applies when the solution can be found by choosing the most suitable subset of a given set, starting from a blank set and iteratively adding a single aspect to the solution according to some specific criterion.
- **Meta-heuristics:**Meta-heuristics, also known as local search methods (LSMs), are multi-goal methods or, more accurately, algorithmic schemes that arise independently of a specific COP. They introduced some components and their interactions, allowing them to create effective solutions. LSM include simulated annealing (SA), ariable neighbourhood search (VNS), tabu search (TS), greedy randomized adaptive search techniques (GRAST), stochastic local search (SLS), particle swarm optimization (PSO), and bee's algorithm (BA) and genetic algorithm (GA) among others[18].

5.3 Nearest Neighbours Method (NNM)

One of the important heuristic methods is Nearest Neighbours Method (NNM) which is aneffective and fast algorithm for solving optimization problems as CVRP. The NNM is operating by firstly select a point that represents the starting position. Then select the nearest position to be go next and so on. if all positions have been connected, then the sequence is finish and must return to starting point[19].
NNM has a greedy nature, which means that the node closest to it will be served next. The identified route will be followed by various vehicles to meet the demands of various nodes. The first vehicle with everything it can carry starts from 0 and serves the nodes in the specified sequence of the route identified until its capacity is exhausted, when it returns to 0, the next vehicle starts from 0 and serves the balance demand of the last worked node and proceeds on the route until all the nodes are served.
The NNM algorithm can be summarized follows[20]:
Step1: Initialization; Read $(n, m, d_{ij}, r_i, b_k). i, j = 1,2, \dots, n$; $k = 2,3, \dots, m$.
Step 2: Find route using nearest neighbour, starting from Positions 0 ($P0$) and passing all other positions then return to $P0$.
Step 3: Calculate the cost of the determined route.
Step 4: Take in the consideration of the total demand and find the number of vehicles required.
Step 5: While (total demand >0)
Step 5.1: Route new vehicle on giant route delivering the different positions till its capacity exhausted.
Step 5.2: Vehicles Return to $P0$.
Step 5.3: Calculate the total cost for the vehicles.
Step 5.4:End while.
Step 6: Output the minimized objective function.
In this paper we will use the NNM to compare its results with the results of new proposed methods for solving CVRP.

6. New Techniques for Solving CVRP

In this section we introduce new methods for solving CVRP.

6.1 Enhanced nearest Neighbours Method (ENNM)

We introduce a new constructive heuristic method to solved CVRP that give best solution to minimizing travel time (distance) in short time period comparing to the exact method. We called the new method as Enhanced nearest Neighbours Method (ENNM). This method based on NNM method and split the cities on every possible position on certain track.
The ENNM algorithm steps are as follows:

Step 1:Input data (n, m, d_{ij}, r_i, b_k), $i, j = 1,2, \dots, n$; $k = 2,3, \dots, m$.
Step 2: calculate full route for all cities using nearest neighbour, starting from DC passing every city and return to it.
Step 3: Get the total demand and calculate number of vehicles required.
Step 4:While (total demand >0)
Step 4.1: For the obtain route from step (2) with minimum number of vehicles try all possible assignment to each vehicle with same order as in the obtain route.
Step 4.2: Route new vehicle on giant route delivering the different positions till its capacity exhausted.
Step 4.3: Calculate the total cost for each vehicle$F(V_k)$.
Step 4.4: End While.
Step (5): Outputs Calculate the objective function. Then take the minimum:
$F = \sum_{k=1}^{m} F(V_k)$.

6.2NearestPairs Sequence Cut (NPSC)

We introduce a new constructive heuristic method to solve CVRP that give best solution to minimizing travel time (distance) in short time period comparing to the exact method.We called the new method as Nearest Pair Sequence Cut (NPSC) the technique of this method based on combine nearest point together as pair of (2,3,…) and sequence these pair as one point with all other ones. The NPSC algorithm steps are as follows:
Step 1:Input data (n, m, d_{ij}, r_i, b_k),$i, j = 1,2, \dots, n$; $k = 2,3, \dots, m$.
Step 2: Formulate pairs table its elementP'_r from (P_i, P_j) with nearest position in distance and order it ascending.
$P'_1 \leq P'_2 \leq \dots \leq P'_r$;$r = C_2^n = \frac{n^2-n}{2}$.
Step 3: Take the minimum first pair P′ say (P_1, P_2) in step (2) and start comparing the minimum distance from P_1 to $P_i (i \neq 1)$ and P_2 to $P_j (j \neq 2)$ then sequence the minimum one among them.
Step 4: Remove selected pair P' in Step (3) from the pairs table and remove every pair in that sequence from step (3) except first and last city.
Step 5: If all cites $Pi$are sequenced then go to step (7) otherwise return tostep (3).
Step 6: break the obtained sequence into m-subsequence $(\beta 1, \beta 2, \dots, \beta m)$,from (start-end) position with respect to not exceed vehicles capacity.
Step 7: Calculate the sub-objective function for each subsequence:
$F(\beta_k)$, $k = 1,2, \dots, m$.
Step 8: Outputs Calculate the main objective function. Then take the minimum.
$F = \sum_{k=1}^{m} F(\beta k)$.

6.3 Divided Tree Method (DTM)

Another new heuristic method introduced to solve CVRP that minimizing travel time (distance),We called the new method as Divided Tree Method (DTM) they based on new rules for choosing the parent node that branching from their (demands requirements – far city from the center) then removing all sub trees that not include useful nodes base on the constraints. The DTM algorithm steps are as follows:
Step 1:Input data (n, m, d_{ij}, r_i, b_k) and Initialization.
Step 2: For k-vehicles let $G(u) = (V(u, Ł), E(u, Ł))$, $u = 1,2, .. k$; $Ł = A, B$ be undirected graphconsists of finite set of V vertices and a set E of Pairs of vertices.
Step 3: For each $G(u)$ in step (2) we first calculate the parent node (say$P\alpha$) where is represent the following:
Approach (A) – AP1: Pα is the city with highest demands.
Approach (B) – AP2: Pα is the city with biggest distance from the DC.
While demands ≤b_k
Step 4: Calculate the nearest two cities to the parent node (Pα) and consider them as the edge $E = (E(1, A), E(1, B))$. [1st level of branching].
Step 5: Now set $P\alpha = E(1, A)$ then set $P\alpha = E(1, B)$. [2nd level of branching].
Step 6: Calculate distance for the fourth branching in level 2 and take the minimum one and remove the others.
Step 7: if all cities are sequenced go to step (7) otherwise go to step (4).
Step 8: Connect the vertices V1(A) and V2(B) from both sides of divided tree.

$$V1(A) = \{Pi\};\ V1(B) = \{Pj\};\ i = 1,2, \dots, s ;\ j = 1,2, \dots, s;\ s \leq n$$

$$V1 = [V1(A)\ V1(B)] \text{ with } F(V1) = Obj(V1(A)) + Obj(V1(B)).$$

Step (9): Outputs Calculate the main objective function by sum the objective function for each vehicle.

$$F = \sum_{k=1}^{m} F(Vk).$$

Remark (1): To improve the solutions of CVRP for all proposed methods, when one vehicle has $n \leq 10$ cities in its path we apply one of the exact methods (like CEM), else we apply the specific methods as it's described in sections (6.1) step (4.2), (6.2) step (6) and (6.3) step (8).

7. Applied the Proposed Techniques toSolve CVRP

To applied the proposed techniques, firstly we take 5 random sets ofexamples for each $n = 4:10$ (positions coordinates X, Y between [-10,10]) and test it with all vehicles range ($m = 2, .., n-1$) and compare the result with the optimal solutions obtained from CEM.

Note: For all tables we give the following notations:

OP: Optimal value of objective function.

BV: Best value of objective functions.

T: CPU-time in seconds.

POP: Percentage of BV for OP, s.t. POP $= \frac{BV}{OP} \times 100\%$.

R: Time less than 1 second, $R \in (0,1)$.

TM: Total mean.

U: Unknown.

Table (3) show comparing the results of the sets of simulation random examples between CEM with NNM and ENNM.Table (4) introduce the comparing results of the simulation sets between CEM with NPSC and DTM.

Table 3. Comparing results of NNMand ENNM with CEM for n=4:10.

n	m	CEM		NNM		ENNM		POP	
		OP	T	BV	T	BV	T	NNM	ENNM
4	2	47	R	47	R	47	R	100%	100%
	3	57		57		57		100%	100%
Mean		51.9	R	52	R	52.0	R	100%	100%
5	2	48	R	50	R	49	R	95%	98%
	3	56		58		56		96%	99%
	4	67		69		67		98%	100%
Mean		57.0	R	59	R	57.3	R	97%	99%
6	2	55	R	57	R	56	R	97%	99%
	3	62		63		62		98%	100%
	4	70		72		70		98%	100%
	5	83		84		83		99%	100%
Mean		67.5	R	69	R	67.8	R	98%	100%
7	2	57	R	58	R	57	R	98%	99%
	3	62		64		63		97%	99%
	4	70		71		70		98%	100%
	5	80		85		80		94%	100%
	6	92		98		92		94%	100%
Mean		72.2	R	75	R	72.5	R	96%	100%
8	2	58	4.2	61	R	59	R	95%	98%
	3	63		67		64		94%	98%
	4	70		77		70		91%	100%
	5	79		85		79		94%	101%
	6	91		92		91		99%	100%
	7	106		107		106		99%	100%
Mean		77.9	4.2	81	R	78.2	R	96%	100%

	2	56		61		60		92%	94%
	3	62		70		64		88%	97%
	4	71		79		71		89%	99%
9	5	81	201.7	87	R	81	R	93%	100%
	6	92		98		92		94%	100%
	7	105		108		105		98%	100%
	8	119		123		119		97%	100%
Mean		83.8	201.7	89	R	84.6	R	94%	99%
	2	69		75		72		92%	95%
	3	76		84		79		91%	96%
	4	85		92		86		92%	98%
	5	94		104		96		91%	99%
10	6	106	1861.3	113	R	107	R	93%	99%
	7	118		123		118		95%	99%
	8	131		134		131		97%	100%
	9	145		149		148		98%	98%
Mean		102.8	1861.3	109	R	104.6	R	94%	98%
TM		79.5	689.1	83	R	80.2	R	95%	99%

Table 4. Comparing results of NPSC, DTM with CEM for n=4:10

n	**m**	**CEM**		**NPSC**		**DTM**		**POP**	
		OP	**T**	**BV**	**T**	**BV**	**T**	**NPSC**	**DTM**
4	2	47	R	47	R	47	R	100%	100%
	3	57		57		57		100%	100%
Mean		51.9	R	52	R	52.0	R	100%	100%
	2	48		50		51		95%	94%
5	3	56	R	58	R	57	R	96%	98%
	4	67		69		68		98%	99%
Mean		57.0	R	59	R	58.7	R	97%	97%
	2	55		55		58		100%	96%
6	3	62	R	62	R	67	R	100%	92%
	4	70		70		70		100%	100%
	5	83		83		83		100%	100%
Mean		67.5	R	68	R	69.5	R	100%	97%
	2	57		57		59		99%	96%
	3	62		63		64		99%	98%
7	4	70	R	70	R	72	R	100%	98%
	5	80		80		80		100%	100%
	6	92		92		92		100%	100%
Mean		72.2	R	72	R	73.4	R	100%	98%
	2	58		60		62		97%	93%
	3	63		64		63		98%	100%
8	4	70	4.2	71	2.1	70	R	99%	100%
	5	79		80		81		99%	98%
	6	91		91		91		100%	100%

	7	106		106		106		100%	100%
Mean		77.9	4.2	79	2.1	78.9	R	99%	99%
9	2	56	201.7	59	2.3	60	R	96%	94%
	3	62		65		67		95%	93%
	4	71		73		71		96%	100%
	5	81		83		81		98%	100%
	6	92		94		95		99%	97%
	7	105		105		105		100%	100%
	8	119		119		119		100%	100%
Mean		83.8	201.7	86	2.3	85.5	R	98%	98%
10	2	69	1861.3	72	2.5	75	R	95%	92%
	3	76		81		82		94%	92%
	4	85		88		89		96%	95%
	5	94		97		99		97%	95%
	6	106		107		108		99%	98%
	7	118		119		120		99%	98%
	8	131		132		134		99%	97%
	9	145		146		148		99%	98%
Mean		102.8	1861.3	105	2.50000	106.9	R	98%	96%

Table (5) shows the summary of results in table (3) and (4) by taking the mean time only for n=4:10 for CEM, NNM, ENNM, NPSC and DTM.

Table 5. the summary of results in table (3) and (4) for n = 4: 10 for CEM, NNM, ENNM, NPSC and DTM.

n	**CEM**		**NNM**		**ENNM**		**NPSC**		**DTM**		**POP**			
	OP	**T**	**BV**	**T**	**BV**	**T**	**BV**	**T**	**BV**	**T**	**NNM**	**ENNM**	**NPSC**	**DTM**
4	51.9	R	51.9	R	52.0	R	51.9	R	52.0	R	100 %	100%	100 %	100 %
5	57.0	R	58.9	R	57.3	R	58.9	R	58.7	R	97%	99%	97%	97%
6	67.5	R	68.9	R	67.8	R	67.5	R	69.5	R	98%	100%	100 %	97%
7	72.2	R	75.3	R	72.5	R	72.4	R	73.4	R	96%	100%	100 %	98%
8	77.9	4.2	81.5	R	78.2	R	78.7	2.1	78.9	R	96%	100%	99%	99%
9	83.8	201.7	89.5	R	84.6	R	85.5	2.3	85.5	R	94%	99%	98%	98%
10	102. 8	1861. 3	109.2	R	104.6	R	105.3	2.5	106.9	R	94%	98%	98%	96%
TM	79.5	689.1	83.5	R	80.2	R	80.7	2.3	81.5	R	95%	99%	98%	98%

Figure (5) shows the comparison results of table (5) for n = 4: 10.

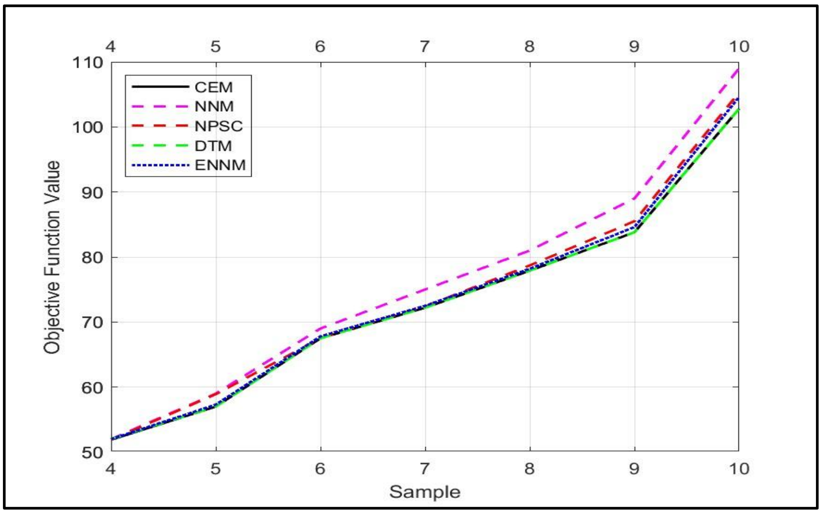

Figure 5. the comparison results of table (5) for n = 4: 10.

Table (6) shows the comparison results between the optimal solution obtain from external dataset obtain from Augerat et al[21] with results of proposed methods ENNM, NPSC and DTM (n = 21,31,52,60,79,100) for choosing m.

Table 6. comparison results between the optimal solution of external dataset with results of proposed methods ENNM, NPSC and DTM (n = 21,31,52,60,79,100) for choosing m

n	m	CEM	NNM		ENNM		NPSC		DTM - AP1		DTM - AP2		POP				
		OP	BV	T	BV	T	BV	T	BV	T	BV	T	NNM	ENNM	NPSC	DTM-AP1	DTM-AP2
21	4	375	493	R	383	R	380	4.6	375	2.2	383	3.2	76.1%	97.9%	98.7%	100.0%	97.9%
31	5	784	1024	R	842	R	792	6.2	797	2.2	817	2.1	76.6%	93.1%	99.0%	98.4%	96.0%
52	7	1010	1162	1.9	1103	1.9	1035	15.9	1067	2.4	1053	2.0	86.9%	91.6%	97.6%	94.7%	95.9%
60	9	1034	1286	1.6	1107	R	1095	22.2	1114	3.0	1071	2.3	80.4%	93.4%	94.4%	92.8%	96.5%
79	10	1763	2034	1.7	1949	1.6	1854	45.7	1958	3.2	1904	2.3	86.7%	90.5%	95.1%	90.0%	92.6%
100	10	820	1095	1.7	820	1.9	827	88.2	827	309.0	820	2.2	74.9%	100.0%	99.2%	99.2%	100.0%
TM		964.3	1182.3	1.1	1034.0	0.9	997.2	30.5	1023.0	53.7	1008.0	2.3	81.6%	93.3%	96.7%	94.3%	95.7%

Figure (6) show the comparison results of table (6) for n = 21,31,52,60,79,100.

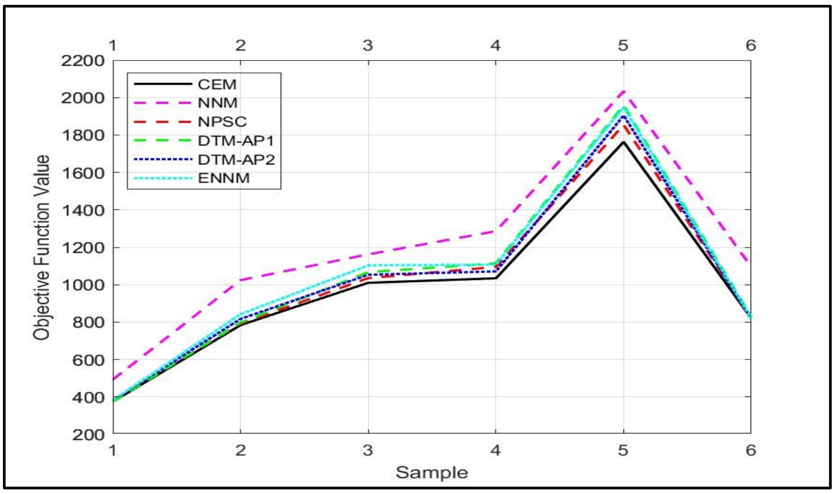

Figure 6. the comparison results of table (6) for n = 21,31,52,60,79,100.

Table (7) shows Accuracy percentage of CEM with NNM, ENNM NPSC and DTM

Table 7. Accuracy percentage of CEM with NNM, ENNM NPSC and DTM

Method	Accuracy percentage							
	n= 4:10	n= 21	n=31	n=52	n=60	n=79	n=100	TM
CEM	100%	100%	100%	100%	100%	100%	100%	100%
NNM	95%	76%	77%	87%	80%	87%	75%	82%
ENNM	99%	98%	93%	92%	93%	91%	100%	95%
NPSC	98%	99%	99%	98%	94%	95%	99%	97%
DTM-AP1	98%	100%	98%	95%	93%	90%	99%	96%
DTM-AP2	98%	98%	96%	96%	97%	93%	100%	97%

7. Discussions and Analysis of Result

1. The results in table (3) and (6) show that our new ENNM has superiors over the classic NNM and increase mean accuracy of the objective function by (4%) and (12%) respectively.
2. After comparing CPU-time of methods NNM and ENNM it's obvious that the two methods achieve same results according to their results CPU-time (R).
3. When comparing results of table (4) and (6) it's obvious that the best methods with respect to accuracy is DTM-AP2 at (98%) and (96.7) respectively.
4. While DTM give advanced time speed of around (R) and (2.3 Sec) respectively.
5. Over all when comparing all the proposed methods results (Simulations, External Data) we get two facts:

a- The Best method among them with respect to its accuracy is equal between NPSC and DTM-AP2.
b- According to CPU-time the best one of the introduced methods is ENNM given CPU-time (R).

8. Conclusions and Future Work

Based on the recent real problem and computational results and analysis, from the various applications and operational perspective, leads to serval facts:

1. Althoughthat the exact method such as CEM and BAB give optimal solutions, still sometimes can be expensive due to its high completion time for large scale problem such as in different disasters (earthquake, hurricane, health pandemic, emergency logistics, etc) where vehicles are required to delivered the supplies of the affected sites in a shortest time possible.
2. Here comes the importance of heuristic methods that give best solutions (near optimal solutions or even sometimes an optimal solutions) that obtain in much shorter time as soon as possible at fair results.
3. There are hundreds of heuristic methods to the present day that give best solutions and the attempts continue to improve this result to minimizing cost and distance for the CVRP problem.
4. The proposed method for solving CVRP prove their effecting through their results in solving CVRP.
5. Notice that the NPSC and DTM-AP2 are the best in accuracy while ENNM is the best in CPU-time in solving CVRP.
6. We suggest using exact methods to solve CVRP like BAB method to obtain optimal solutions.
7. We suggest usingLSMs like GA, PSO,BA,…, etc to solve CVRP.

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