

Graphical Passwords in the Context of Contra Harmonic Mean Labeling

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Abstract

Graphical Passwords have proved to be efficient and user-friendly authentication method compared to usual text-based passwords. Topsnut-GPWs are new type of graphical passwords made using topological structures and number theory. In this article, we construct Topsnut-GPWs using a certain type of caterpillar tree and Alphabet graphs by means of Contra Harmonic Mean labeling. In addition, we explore the possibility of Alphabet graphs and caterpillar tree being the key-model and lock-model respectively.

1. Preliminaries

The increasing complexity of cyber threats has exposed the vulnerabilities of traditional text-based passwords (TB-paws), which are often susceptible to brute force attacks, phishing and shoulder surfing. This has led to significant researches in alternative authentication mechanisms. Among these, graphical passwords stands as a promising method due to human brain's ability to recognize and recall visual patterns when compared to abstract string of characters.

A novel class of graphical passwords known as Topsnut-GPWs was introduced by Wang et. al which uses the concept of creating graphical passwords using 'topological structure + number theory', later called as 'Topsnut-GPW' in [5] and [10]. Subsequent research has expanded on this concept, leading to variants like Hanzi-GPWs, which utilizes Chinese characters.

The mathematical foundation of many Topsnut-GPWs lie in graph labelings. The security and uniqueness of a Topsnut-GPW depend on the type of labeling employed. In this paper, we utilize the Contra Harmonic Mean labeling, a concept introduced by

Sandhya et al. In this article i) we establish that a certain class of caterpillar trees admits a Contra Harmonic Mean labeling, thereby qualifying them for use as key models ii) we introduce Alphabet graphs and obtain an algorithm to Super Contra Harmonic mean label the Alphabet graphs iii) we demonstrate the construction of Topsnut-GPWs using Alphabet graphs and a special class of caterpillar tree to create efficient lock and key models.

We provide a brief summary of definitions which are necessary for the present investigation.

Definition 1.1 [3]: A graph $G(V, E)$ with p vertices and q edges is said to be Contra Harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $0, 1, 2, \dots, q$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$ or $\left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$ with distinct edge labels. Then f is called Contra Harmonic mean labeling of G .

Definition 1.2 [3]: An injective function $V(G) \rightarrow \{1, 2, \dots, p + q\}$ is called a Super Contra Harmonic mean labeling if the induced edge labeling $f(e = uv)$ is defined as $f(e) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$ or $\left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$, so that $\{f(V(G))\} \cup \{f(e) : e \in E(G)\} = \{1, 2, \dots, p + q\}$. A graph which allows Super Contra Harmonic Mean labeling is called Super Contra Harmonic Mean graph.

Definition 1.3 [3]: A graph $G(V, E)$ with p vertices and q edges is said to be Odd Contra Harmonic Mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $0, 1, 2, \dots, 2q$ in such a way that each edge $e = uv$ labeled with $f(e = uv) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$ or $\left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$ gets distinct edge labels $1, 3, 5, \dots, 2q - 1$. Then f is called Odd Contra Harmonic Mean labeling of G .

Definition 1.4 : Let $G_1(a, q_1)$ and $G_2(a, q_2)$ be two graphs such that the vertex identified graph $G(p, q) = G_1 \circ G_2$ has a mapping $f: V(g) \rightarrow [0, q]$ such that i) $f(x) \neq f(y)$ for any pair of vertices of $x, y \in G$, ii) f is an Odd Contra Harmonic Mean labeling of G_1 iii) the set $f(E(G_i)) = [1, 2q_i - 1]^0$ with $i = 1, 2$. Then G_1 is said to be a source graph (key-model) and G_2 an associated graph (lock-model) of G_1 .

2. Main Results

2.1 Contra Harmonic Mean Labeling on Caterpillar Trees

Theorem 2.1.1: A caterpillar tree such that no vertices have degree > 4 is Contra Harmonic Mean iff it admits Super Contra Harmonic Mean labeling.

Proof

Let $T(p, q)$ be a caterpillar tree such that no vertices have degree > 4 .

Suppose that the tree T has a Contra Harmonic Mean labeling defined as

$f(x_i) = i - 1, x_i \in V(T), i \in [1, V(T)]$ such that

$$f(x_i x_j) = \left\lfloor \frac{f(x_i)^2 + f(x_j)^2}{f(x_i) + f(x_j)} \right\rfloor \text{ or } \left\lceil \frac{f(x_i)^2 + f(x_j)^2}{f(x_i) + f(x_j)} \right\rceil \in [1, E(T)]$$

We construct a new labeling $g: V(T) \rightarrow \{1, 2, \dots, p + q\}$ of T by

$$g(x_i) = 2f(x_i) + 1, g(x_i x_j) = f(x_i x_j)$$

Hence $g(x_i x_j)$ and $g(x_i)$ are distinct and $g[V(T) \cup E(T)] = [1, V(T) + E(T)]$.

Hence g is Super Contra Harmonic Mean labeling.

Conversely, assume g' be the Super Contra Harmonic Mean of T such that

$$g'(x_i) = 2i - 1, i \in [1, V(T)] \text{ and}$$

$$g'(x_i x_j) = \left\lfloor \frac{g'(x_i)^2 + g'(x_j)^2}{g'(x_i) + g'(x_j)} \right\rfloor \text{ or } \left\lceil \frac{g'(x_i)^2 + g'(x_j)^2}{g'(x_i) + g'(x_j)} \right\rceil \in [1, V(T) + E(T)]$$

Now, construct a labeling $f': V(T) \rightarrow \{1, 2, \dots, p\}$ defined as

$$f'(x_i) = g'(x_i) - 1 \text{ and } f'(x_i x_j) = g'(x_i x_j)$$

Clearly, $f'(V(T)) = [0, |E(T)|]$ and $f'(E(T)) = [1, |E(T)|]$

Hence f' is Contra Harmonic Mean labeling.

Fig. 1 shows the illustration of theorem 2.1.1

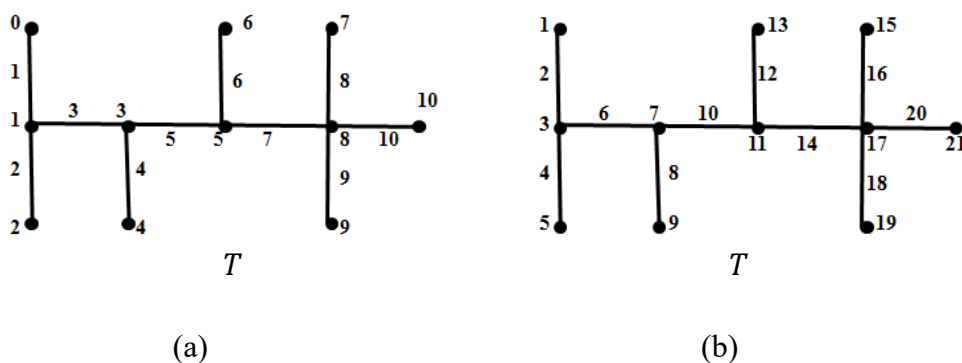


Fig.1: (a) Contra Harmonic Mean labeling of T and Fig.1(b) Super Contra Harmonic Mean labeling of T .

Next, we construct large scale caterpillar trees by using vertex identification operation as follows

Theorem 2.1.2: If the vertex-identification operation on two disjoint trees $T_1(p_1, q_1)$ and $T_2(p_2, q_2)$ is performed by combining u_i of T_1 and v_j of T_2 where $f(u_i) = q_1$ and $f(v_j) = 0$ or $f(u_i) = 0$ and $f(v_j) = q_2$ to form a new graph T , then T is a caterpillar tree.

Proof

Let T_1 be a tree such that $f(u_i) = q_1$ and $f(v_j) = 0$

We consider two cases,

Case (i): Let u_i be the vertex on the central path of T_1 .

Let $u_1, u_2, \dots, u_i$ be the vertices on the central path and let $v_1, v_2, \dots, v_h$ be the vertices on the central path of T_2 .

Then by adjoining u_i to v_j , we get a tree T with central path $u_1 u_2 \dots u_i v_j v_1 v_2 \dots v_h$.

Hence T is a caterpillar tree.

Case (ii): Let u_i be a leaf of T_1

Let $u_1, u_2, \dots, u_g$ be the vertices on the central path and let $v_1, v_2, \dots, v_h$ be the vertices on the central path of T_2 .

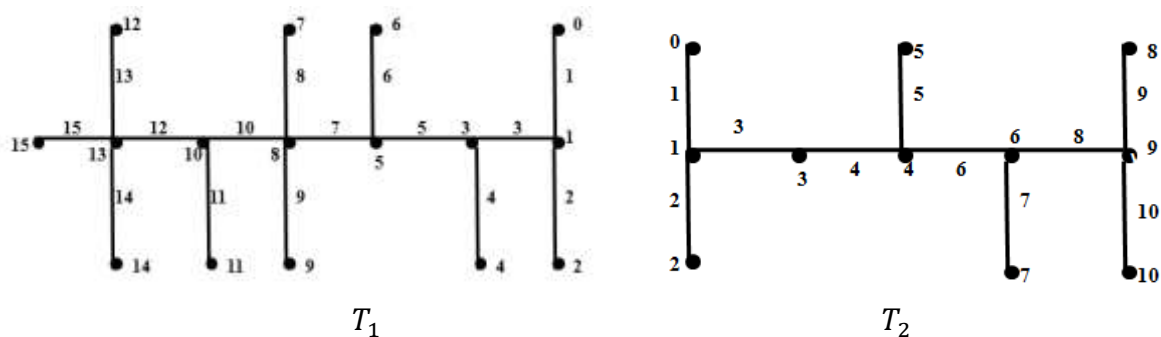
Then by adjoining u_i to v_j , we get a tree T with central path $u_1 u_2 \dots u_g u_i v_j v_1 v_2 \dots v_h$.

Hence T is a caterpillar tree.

Similarly, we can prove the result for $f(u_i) = 0$ and $f(v_j) = q_2$.

Next, we have the following remark

Remark 2.1.3: The theorem 2.1.1 and theorem 2.1.2 can be used to create large scale caterpillar trees. If $T_1, T_2, \dots, T_n$ are Contra Harmonic Meangraphs and applying the vertex-identification operation form a new tree having Super Contra Harmonic Mean labeling. Fig. 2 shows the illustration of vertex-identification operation.



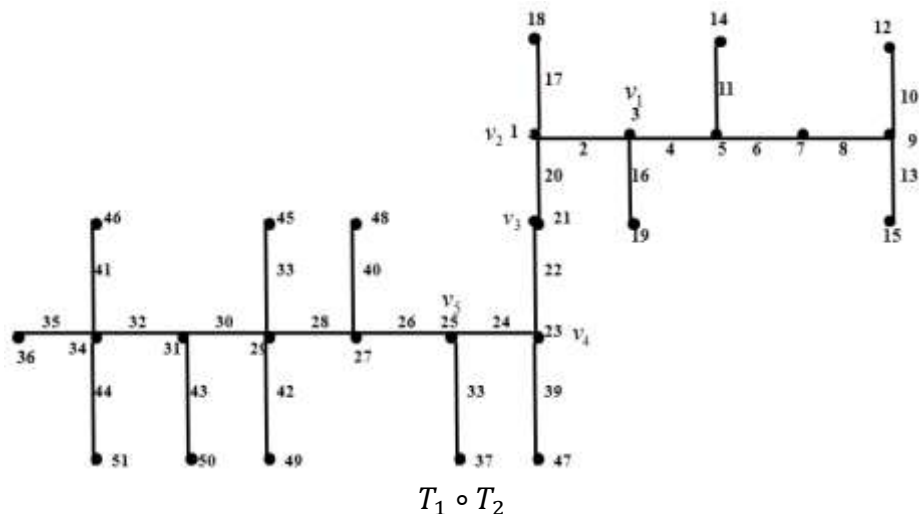


Fig.2: T_1 and T_2 are Contra Harmonic Mean labeled and
 $T_1 \circ T_2$ is Super Contra Harmonic Mean labeled

Using path-neighbour method in Fig. 2, we get vv-type TB-paw along path $p_1 = v_1 v_2 v_3 v_4 v_5$ as

$$P_{vv}(p_1) = 3151931318211211232123212547232523273725$$

2.2 Super Contra Harmonic Mean Labeling on Alphabet Graphs

The risk with Topsnut-GPW is that there is a risk of forgetting when not used for a longer time, whereas TB-paws have a greater chance of recalling later in time since users tend to use alphanumeric passwords that are associated with them in some way. Hence, we investigate the idea of converting text-based passwords to graphical structures. Alphabet graphs are English alphabets converted to graphs. Since, English is the most spoken global language, wide number of users benefit from Alphabet graphs.

Here we create an algorithm to construct Super Contra Harmonic Mean labeled Alphabet graphs.

Algorithm 2.2.1: Algorithm to Construct Super Contra Harmonic Mean labeled Alphabet Graphs

Let A denote Alphabet graphs

The vertices and edges of A are labeled as $v_1 e_2 v_3 e_4 v_5 \dots$ such that e_2 is the edge connecting v_1 and v_3 .

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Step 1: First, we consider Tree like alphabets C, E, F, G, H, I, J, K, L, M, N, S, T, U, V, W, X, Y, Z. This is further subdivided into Alphabets that are Paths and Trees with at least one branch.

Subcase (i): First, we consider path like alphabets C, G, I, L, M, N, S, U, V, W, Z. The vertices and edges are labeled in the same way as labeling paths using consecutive numbers, i.e., $f(v_1) = 1, f(v_3) = 3, \dots, f(v_n) = n$, where $n = p + q$

Subcase (ii): Next we consider the alphabets E, F, H, J, K, T, X, Y. The labeling is defined as follows:

$$f(v_i) = i, \text{ if } d(v_i) \leq 3, \forall v_i \in A$$

$$f(v_i) = i, f(v_n) = n, f(v_{n-2}) = n - 1, \text{ where } n = p + q, p = |V(A)| \text{ and } q = |E(A)|$$

Step 2: Next we consider alphabets that are either cycles or contain cycles which include A, B, D, O, P, Q, R.

Subcase (i): First we define labeling in cyclic alphabets D and O

$$f(v_j) = j, \text{ where } v_j \text{ denotes the initial vertex in } A,$$

$$f(v_{n-1}) = n \text{ where } n = p' + q', p' \text{ and } q' \text{ are number of vertices and edges in the cycle.}$$

Now the remaining vertices are labeled in such a way that,

$$f(v_{i+2}) = \begin{cases} f(v_i) + 3, & \text{if either } f(v_{i+2}) = f(e_n) \text{ or } f(v_i v_{i+2}) = f(e_n) \\ f(v_i) + 2, & \text{otherwise} \end{cases}$$

Subcase (ii): Here we consider the alphabets that contain cycle which include A, B, P, R, Q. These alphabets have at most 6 vertices and there exist at least one vertex v_i in A, such that $d(v_i) \geq 3$, then we define

$$f(v_i) = i, f(v_{i-2}) = i - 2, f(v_{i-2}) = i - 2,$$

$$f(v_{i+4}) = p + q, f(v_{i+6}) = f(v_{i+4}) - 2,$$

$$\text{The remaining vertex is labeled as } f(v_{i+8}) = f(v_{i+2}) + 2$$

Next, we have the edge-series operation process

Remark 2.2.2: The Alphabet graphs are converted to Word graphs by adding an edge between u_i of A_1 and v_j of A_2 where $f(u_i) = p_1 + q_1$, where $p_1 = |V(A_1)|$ and $q_1 = |E(A_1)|$ and v_j is the initial vertex in A_2 .

Obtaining a Word graph from Alphabet graphs is illustrated in Fig. 3

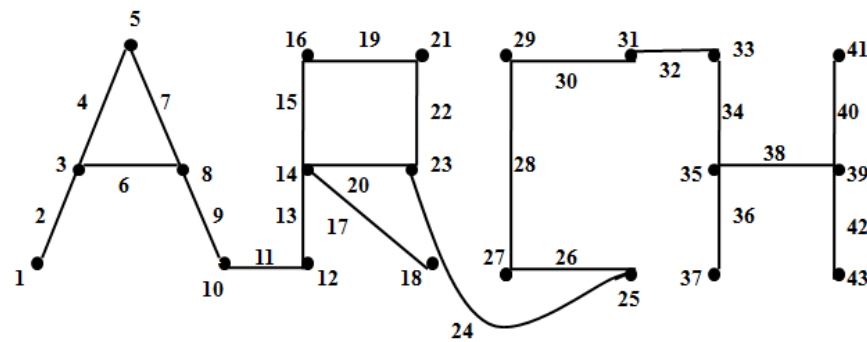


Fig. 3: Word graph obtained from the Alphabets A, R, C and H

2.3 Key and Lock Models using Caterpillar trees and Alphabet graphs

To create efficient key models, we use Odd Contra harmonic Mean labelling to label Alphabet graphs.

Theorem 2.3.1: Alphabet graphs are the source graphs of caterpillar trees with no vertices having degree >4 .

Proof:

Let A denote Alphabet graph and let T be a Caterpillar tree with no vertices of degree >4

Let v_i be the vertices of A and u_i be the vertices of T

Let q_1 and q_2 denote the size of A and T respectively

The source graphs are defined in the following three cases

Case 1: Alphabets that are cycles or contain cycles

Label the vertices in the cycle in such a way that all vertices in the cycle contain the labels 1, 3, 7, where vertex labeled 1 is attached to the vertex labeled 3 and the vertex labeled 3 is adjacent to the vertex labeled 7 and then label the vertex adjacent to 1 as 0.

Thus, we label the vertices as

$$f(v_1) = 0, f(v_2) = 1, f(v_3) = 3, f(v_4) = 7, f(v_5) = 9$$

If the graph has 6 vertices, then $f(v_6) = 2e - 1, e = |E(A)|$

Then we get distinct edge labels $[1, 2q_1 - 1]^o$

Case 2: Tree like alphabets C, E, F, G, H, I, J, K, L, M, N, S, T, U, V, W, X, Y, Z. This is further subdivided into Alphabets that are Paths and Trees with at least one branch.

Subcase (i): Trees with atleast one branch

Let v_1 be the vertex with the highest degree and let v_2 be the vertex adjacent to v_1

The labeling is defined as

$$f(v_1) = 0, f(v_2) = 1, f(v_i) = 2i - 3, i \geq 3$$

Then we get distinct edge labels $[1, 2q_1 - 1]^o$

Subcase (ii): Path like Alphabets

The labeling is defined as

$$f(v_1) = 0, f(v_2) = 1, f(v_i) = 2i - 3, i \geq 3$$

Then we get distinct edge labels $[1, 2q_1 - 1]^o$

Now the associated graph T is defined as a function $f: V_2 \rightarrow [0, 2q_2]$ such that

$$f(u_2) = 1, f(u_1) = 2, f(u_i) = 2i - 2, \text{ for all } i \geq 3$$

Then we get distinct edge labels $[1, 2q_2 - 1]^o$

Hence, A and T can be Odd Contra Harmonic Mean labeled

Clearly, $f(u_i) \neq f(v_i)$, for all $i \neq 2$

Thus A is a source graph of associated graph T

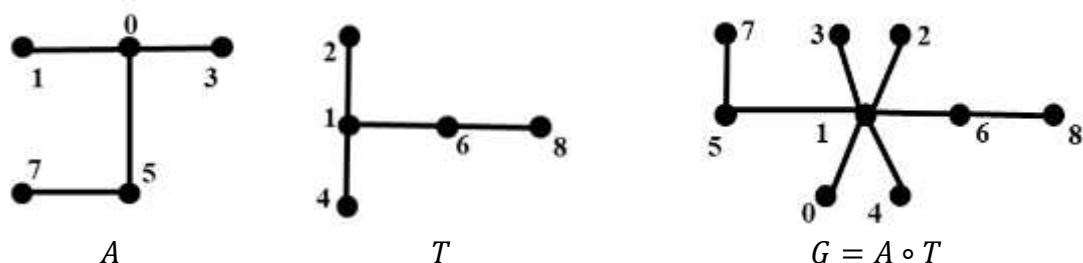


Fig 4: An identifying graph $G = A \circ T$ with source graph A and associated graph T

Conclusion and scope

In this paper we have investigated the role of Contra Harmonic Mean labeling for information security. While the Super Contra Harmonic Mean labeled graphs can be used for simple user logins, Odd Contra Harmonic Mean labeled graphs can be used to develop lock and key-models. Further research can be done on applying labeling on more user friendly graphs like Eulerian graphs.

References

1. Gallian J. A., A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 1.Dynamic Surveys (2018), DS6
2. Harary F, *Graph Theory*, Narosa Publishing House, New Delhi, (1988).

10.48047/jocaaa.2024.33.05.31

3. Rajeshni Golda. J, *Contra Harmonic Mean Labeling of Graphs*, Thesis submitted to Manonmaniam Sundaranar University, (2019)
4. Wang H, Xu J, Ma M, and Zhang H, A New Type of Graphical Passwords Based on Odd-Elegant Labelled Graphs, *Security and Communication Networks*, (2018), <https://doi.org/10.1155/2018/9482345>
5. Wang H, Xu J, Yao B, Exploring new cryptographical construction of complex network data, *IEEE First International Conference on Data Science in Cyberspace(DSC)*, (2016), 155–160.
6. Wang H, Xu J, Yao B, The key-models and their lock-models for designing new labellings of networks, *2016 IEEE Advanced Information Management, Communicates, electronic and Automation Control Conference*, (2016).
7. Wang H, Xu J, Yao B, Twin odd-graceful trees towards information security, *Procedia Computer Science*, 107 (2017), 15-20.
8. Yao B, Sun H, Wang H, Su J, Xu J, Graph theory towards new graphical passwords in information networks, *arXiv:1806.02929v1*, (2018).
9. Yao B, Sun H, Wang H, Zhang X, Mu Y, Xu J, New-type graphical passwords made by chinese characters with their topological structures, *2018 2nd IEEE Advanced Information Management, Communicates,, Electronics and Automation Control Conference(IMCEC)*, (2018), 1606-1610.
10. Yao B, Zhang X, Sun H, Mu Y, Sun Y, Wang X, Wang H, Ma F, Su J, Yang C, Yang S, Zhang M, Text-based passwords generated from topological graphic passwords, *arXiv preprint arXiv:1809.04727*,(2018).