Journal of Computational Analysis and Applications **VOL. 33, NO. 6, 2024**
10.48047/jocaaa.2024.33.06.124

MULTIFACTORIZATION OF COMPLETE GRAPHS INTO EQUAL COPIES OF CYCLES AND PATHS.

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Received: 11.12.2023 Revised : 16.01.2024 Accepted: 22.03.2024

ABSTRACT
Let K_n (ƛ) denote a complete graph with n vertices with edge multiplicity ƛ. Factorization of a graph is a partition of the given graph into isomorphic spanning subgraphs. A (G,H) – Multifactorization of a graph is a factorization of given graph into G and H with at least one copy of G and H. In this paper, we studied about the (G,H) – Multifactorization of K_n with equal copies of G and H, where G is a Cycle factor and H is a Path factor.

Keyword: Complete Graphs, Multifactorization, Path and Cycle.
2000 Mathematics Subject Classification Number : 05C70,05C38.

1. INTRODUCTION

Let G be a simple and finite graph. Let K_n denote a complete graph on n vertices. G(ƛ) denote a graph G with edge multiplicity ƛ. Let C_n denote a cycle and P_n denote a path on n vertices respectively. Let K_n (ƛ) denote a complete graph on n vertices with edge multiplicity ƛ. Decomposition of a graph is a partition of the given graph into isomorphic subgraphs. Factorization of a graph is a partition of the given graph into isomorphic spanning subgraphs. A (G,H) – Multidecomposition of a graph is a decomposition of given graph into G and H with at least one copy of G and H. A (G,H) – Multifactorization of a graph is a factorization of given graph into G- factors and H- factors with at least one copy of G and H. The study of multidecomposition was introduced by Atif Abeiuda and Mike Daven in 2003 [1]. Priyadharsini and Muthusamy extended the study of Multidecomposition and Multifactorization for many cases [3]. More studies on multidecomposition can be found in book [5].

Hurd and Sarvate [4] studied a decomposition of a K_n (ƛ), (ƛ≥2) into H_3 graphs with equal copies. They gave a necessary and sufficient condition for the decomposition of K_n (ƛ) into equal copies of C_3 and P_3. Now we rewrite the above problem as (G,H) – Multidecomposition of K_n (ƛ) with equal copies of G and H. In this paper we extend the above problem as (G,H) – Multifactorization of K_n (ƛ) with equal copies of (C_n,P_n).

To prove our results, we need the following results.

Walecki Construction [2]:

(i) The graph K_{2n+1} has a Hamilton cycle decomposition.
(ii) The graph K_{2n} has a Hamilton Path decomposition.

Lemma [1.1]:

(i) For even m>2, the graph $2k_{2m}$ has a C_{2m} decomposition.

(ii) The graph $2K_{2m+1}$ has a Hamilton path decomposition.

Corollary [1.2]:

The graph K_{2n} has a Hamilton path decomposition.

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2.(P_n, C_n) – Multifactorization of $K_n(\lambda)$ with equal copies of P_n and C_n.

Lemma 2.1:[3]

For n ≥ 3, K_n (ƛ) has a (P_n,C_n) – multifactorization if and only if

(i)|E(K($_n$) (ƛ))| = r(n-1)+ sn , r,s > 0 and

(ii) ƛ >2.

The above theorem gave a necessary and sufficient conditions for multifactorization of P_n and C_n.

Theorem 2.2:

For n≥ 3, K_n (ƛ) has a (P_n,C_n)r – multifactoriazation into equal copies (r copies) if

(i) |E(K($_{n)}$ (ƛ))| = r(2n-1), r > 0 and

(ii) ƛ Ξ 0 (mod (2n-1).

Proof:

Assume that K_n (ƛ) has a (Pn,Cn)r – multifactoriazation. Then there exists an integer 'r' such that

K_n (ƛ) = rP_n⊕ rC_n. Hence |E(K(n) (ƛ))| = r(n-1) + sn = r(2n-1), r > 0

(i.e), ƛn(n-1)/2 = r (2n-1). This is possible only ƛ /2(2n-2), as (n,n-1) = 1, (n,2n-1) = 1 and (n-1, 2n-1) = 1.

Hence ƛ Ξ 0 (mod (2n-1).

Conversely, Assume conditions (i) and(ii) are satisfied.

We give the construction for ƛ=(2n-1). Consider (2n-1) K_n

Case(a): n is odd

(2n-1) K_n = (2n-3+2) K_n = (n+n-3+2) K_n

= (n-3)/2 nP_n ⊕ n(n-1)/2 C_n ⊕ nP_n.

= n(n-1)/2 P_n ⊕ n(n-1)/2 C_n. [By Waleckis Construction and Lemma1.1]

Case(b): n is even

(2n-1) K_n =(2n-3+2) K_n = (n+n-3+2) K_n

= (n-3) K_n ⊕ n K_n ⊕ 2 K_n.

= (n-3)n/2 P_n ⊕ n/2 (n-1) C_n ⊕ nP_n

= n(n-1)/2 P_n ⊕ n(n-1)/2 C_n. [By Waleckis Construction and Corollary 1.2]

Hence the Proof for all ƛ Ξ 0 (mod (2n-1)).

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