Application of the Probability Theory to Study of a Transportation Problem of an Essential Item from Various Origins to Different Destinations

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Received: 06.04.2024 Revised : 17.05.2024 Accepted: 24.05.2024

ABSTRACT
The transportation problem refers to find the optimal solution to the transportation matrix. The transportation matrix provides supplies from the given sources as well as demands of the given destinations with total supply equaling total demand. The provided supplies and demands are considered as relative frequencies or probabilities to the total supply or total demand. Using probabilities, we determine the initial basic feasible solution to the transportation problem. Further, the optimal solution is reached by the given optimality test or by MODI method, Stepping Stone method or WAM optimality test. This research paper provides the new approach of relative probability to the initial basic feasible solution of the transportation problem.

Keywords: probabilities, destinations, relative, transportation, solution.

1. INTRODUCTION

The fields of operations research and engineering science have extensively investigated transportation-related issues. It is a basic network flow problem that is typically used to minimize transportation costs when moving a single product between a predetermined number of sources (factories, for example) and a predetermined number of destinations (warehouses), all while meeting supply and demand constraints. Transportation models are crucial for reducing costs and improving services in logistics and supply-chain management. The transportation problem, which aims to move various quantities of the same processed goods from multiple sources to distinct destinations where the transportation cost is minimized, is both a real-world application of linear programming and an intriguing topic covered in operations research.
Therefore, it is possible to find the best strategies to reduce costs. The North-West Corner Rule (NWCM), Column Minima Method (CMM), Row Minima Method (RMM), Least Cost Method (LCM), Vogel's Approximation Method (VAM), and other methods search the first fundamental workable answers that are necessary to move on to the next phase. The best solution is then reached by continuously enhancing the first fundamental workable solution. This can be achieved by the MODI approach and the Stepping Stone (SS) method. There are two types of transportation problems: balanced transportation problems and unbalanced transportation problems. We are discussing a balanced transport problem when the total number of requests and the number of resources are equal. If not, there is a problem of unbalanced transportation.
FL Hitchcock originally brought up the problem of transportation in 1941 [6]. In this sector of the economy, minimizing transportation expenses is vital. In 1947, TC Koopmans [7] made additional contributions to the best possible transit system. TC Koopmans and G B Dantzig [4] further refined linear programming work in 1951 in terms of defining and solving it. The Stepping Stone method was created by C. Charnes in 1954 [3]. In 2012, Sudhakar et al. [12] and Abdul Quddoos et al. [1] distinguished two distinct methods for identifying the best solution.
In recent years, researchers have discovered linear programming issues using fuzzy numbers. To find the best solution, they have applied the simplex method. Reena Patel and colleagues have published a description of the optimality with less computation solution for transportation problems recently in 2017 [10].
More recently in 2021, Mona M. Gothi et al. [14] have developed new statistical method to find the optimal solution which is called the Weighted Arithmetic Mean (WAM) method. She has established an initial basic feasible solution and optimal solution or close to the optimal solution to the transportation problem.

The transportation problem is a part of linear programming problems. Practically, it is a real world problem dealing with the transport of different products from several sources to various destinations. The products are supplied in accordance with the supply and demand of the sources and destinations respectively in order to minimize the total transportation cost. It is feasible to find the solution considering the supply and demand as in the method of North West Corner Method (NWCM) of the transportation problem. It involves some degree of randomness depending on the supply and demand.
The probability deals with the randomness of the events. This theory gives the mathematical understanding of the chance processes like, tossing a coin, rolling a dice or playing cards. It has the purpose of providing mathematical models of situations governed by chance effects. For instances, in weather forecasting, life insurance, traffic problems and transportation problems.
In transportation problem, the supplies from the sources and the demands of the destinations are relative to the total supply or total demand. We consider the ratio of individual supply to the total supply as relative frequency or probability of the particular source. Similarly, consider the ratio of individual demand to the total demand to be relative frequency or probability of the particular destination.
The probability deals with random experiment whose possible outcomes are known. The set of all possible outcomes is referred to as the sample space. A subset of the sample space is known as an event. If an event E occurs m times in an n-trial experiment, then the probability, P(E), of happening of the event E is defined as
$P(E) = \lim_{n\to\infty}\frac{m}{n}$.
This definition implies that if the experiment is repeated indefinitely$(n \to \infty)$, then the desired probability is represented by $\frac{m}{n}$.
The mathematical or classical definition of the probability gives the chance of happening of the event A. It reads as if a trial results in n exhaustive, mutually exclusive and equally likely cases and m of them remains favourable to the happening of an event A then the probability of happening of A is given by
$P(A) = \frac{\text{Number of favourable cases}}{\text{Exhaustive number of cases}} = \frac{n(A)}{n(S)} = \frac{m}{n}$.
As an illustration, in the experiment of rolling a die, the possible cases are:
$S = \{1,2,3,4,5,6\} \Longrightarrow n(S) = 6$.
If A is an event of getting a number 5 in this experiment then
$A = \{5\} \Longrightarrow n(A) = 1$
Henceforth, applying the above definition the probability of the event Ais $P(A = 5) = \frac{1}{6}$.
The supplies from the different sources are considered to be favorable cases to those sources and the total supply of all those sources is considered as exhaustive cases. The requirements of the various locations are thought of as advantageous instances for those locations, while the combined demands of all those locations are viewed as exhaustive cases. With the aid of the definition of the probability, we calculate the relative frequency or the probability of all sources and destinations from the given transportation problem.
In this paper, we developed new methodology employing the concept of the probability to achieve the initial basic feasible solution. We utilized the concept of relative frequency or probability to reach to the basic feasible solution. The optimal solution can be checked by the provided optimality test, MODI method, Stepping Stone method or WAM optimality test [14].

Algorithm

(1) Construct the transportation matrix from the given data.(2) Consider the relative frequency or probability for each source and destination with the help of total supply and total demand.
(3) Choose the maximum probability which corresponds to a particular row or a column. Select minimum cost from that particular row or column and allocate minimum of supply and demand to that cost cell. If there occurs a tie between two or more costs then take maximum allocation into consideration.
Note: We obtain two or more alternate initial basic feasible solutions from which the optimal solution will be considered.
(4) Exhaust the satisfied row or column from the matrix. Compute the relative frequency or probability for remaining sources and destinations with the help of remaining total supply and total demand.
(5) Repeat step-3 and step-4 untill the supplies and demands are annihilated.
(6) Calculate the total cost of the transportation problem by summing up the product of the cost and its allocated value.
Remark: While allocating the values to the cells, the probability reduces from 1 to 0 gradually. When supplies and demands are full the probability is 1. When supplies and demands are nullified the probability becomes 0.

(7) Select the minimum unoccupied cost cell and trace a closed path beginning from that cell with alternately plus (+) sign and minus (-) sign.
(8) Choose the minimum allocated value from the cells with minus (-) sign and add that allocated value to the cells with plus (+) sign and subtract the same value from the cells with minus (-) sign.
(9) Calculate the total cost of the transportation problem by summing up the product of the cost and its allocated value.
(10) Repeat step-7 and step-8 till we obtain minimal transportation cost.

Optimality Test
The optimality appears to be related with the maximum supply and maximum demand as per the algorithm. The maximum supply of any source must be distributed among exactly n-1 destinations and the maximum demand of any destination must be fulfilled/satisfied by exactly m-1 sources.
This test provides the optimality depending on the maximum supply and the maximum demand of sources and destinations.
Or alternatively, check the optimality by MODI method, Stepping Stone method or WAM optimality test.
Numerical example-1: Consider the following transportation cost matrix:

3×4	**D1**	**D2**	**D3**	**D4**	**Supply**
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand	250	350	400	200	1200

After Applying the algorithm for initial basic feasible solution, the allocations to the transportation cost matrix are obtained as follows:

3×4	**D1**	**D2**	**D3**	**D4**	**Supply**
O1	3	1 [300]	7	4	300
O2	2 [250]	6 [50]	5 [100]	9	400
O3	8	3	3 [300]	2 [200]	500
Demand	250	350	400	200	1200

Total cost $= 1 * 300 + 2 * 250 + 6 * 50 + 5 * 100 + 3 * 300 + 2 * 200 = 2900$.
Further, after selecting the minimum unoccupied cost cell and tracing a path in (O3, D2) cell, we obtain the allocations to the transportation cost matrix as follows:

3×4	**D1**	**D2**	**D3**	**D4**	**Supply**
O1	3	1 [300]	7	4	300
O2	2 [250]	6	5 [150]	9	400
O3	8	3 [50]	3 [250]	2 [200]	500
Demand	250	350	400	200	1200

Total cost $= 1 * 300 + 2 * 250 + 5 * 150 + 3 * 50 + 3 * 250 + 2 * 200 = 2850$.
This calculated total transportation cost is far lesser than the cost obtained by North West Corner Method (NWCM). So, this algorithm produces the better initial basic feasible solution to the transportation cost matrix. This solution can be checked for optimality by the provided optimality test or MODI method, Stepping Stone method or WAM optimality test.
Numerical example-2: Let the cost matrix from four sources 1,2,3& 4 and three destinations A,B & C be given as follows:

4×3	**A**	**B**	**C**	**Supply**
1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	34

The algorithm for initial basic feasible solution is applied to this cost matrix and the allocations to the cost matrix are determined as follows:

4 × 3	**A**	**B**	**C**	**Supply**
1	2	7 [2]	4 [3]	5
2	3	3	1 [8]	8
3	5	4 [7]	7	7
4	1 [7]	6	2 [7]	14
Demand	7	9	18	34

Total cost $= 7 * 2 + 4 * 3 + 1 * 8 + 4 * 7 + 1 * 7 + 2 * 7 = 83$
Further, after selecting the minimum unoccupied cost cell and tracing a path in (1, A) cell, we obtain the allocations to the transportation cost matrix as follows:

4 × 3	**A**	**B**	**C**	**Supply**
1	2 [3]	7 [2]	4	5
2	3	3	1 [8]	8
3	5	4 [7]	7	7
4	1 [4]	6	2 [10]	14
Demand	7	9	18	34

Total cost $= 2 * 3 + 7 * 2 + 1 * 8 + 4 * 7 + 1 * 4 + 2 * 10 = 80$
Numerical example-3: Consider the given transportation cost matrix as follows:

3 × 4	**D1**	**D2**	**D3**	**D4**	**Supply**
O1	19	30	50	10	7
O2	70	30	40	60	9
O3	40	8	70	20	18
Demand	5	8	7	14	34

After Applying the algorithm for initial basic feasible solution, the allocations to the transportation cost matrix are obtained as follows:

3 × 4	**D1**	**D2**	**D3**	**D4**	**Supply**
O1	19	30	50	10 [7]	7
O2	70 [2]	30	40 [7]	60	9
O3	40 [3]	8 [8]	70	20 [7]	18
Demand	5	8	7	14	34

Total cost $= 10 * 7 + 70 * 2 + 40 * 7 + 40 * 3 + 8 * 8 + 20 * 7 = 814$.
Further, after selecting the minimum unoccupied cost cell and tracing a path in (O1, D1) cell, we obtain the newly found allocations to the transportation cost matrix as follows:

3 × 4	**D1**	**D2**	**D3**	**D4**	**Supply**
O1	19 [3]	30	50	10 [4]	7
O2	70 [2]	30	40 [7]	60	9
O3	40	8 [8]	70	20 [10]	18
Demand	5	8	7	14	34

Total cost $= 19 * 3 + 10 * 4 + 70 * 2 + 40 * 7 + 8 * 8 + 20 * 10 = 781$.
The distinction between the North West Corner Method (NWCM) of the transportation problem and the new method using probability needs to be discussed here. North West Corner Method starts at the North West (upper left) corner cell of the transportation problem. Adjusting the associated amounts of supply and demand of the corresponding cells and crossing out the rows and/or columns which are exhausted in the process, we reach the initial basic feasible solution.
The new method starts with the maximum probability in the transportation problem. It then selects the minimum cost from particular row or column and allocates minimum of supply and demand to the cost

corresponding cell. This process is then repeated with the help of remaining total supply and total demand till the initial basic feasible solution is reached.
The new method provides far lesser transportation cost than the North West Corner Method which shows the effective application of the probability to the transportation problem. The optimality test presented in this research paper produces the best (optimal) solution as illustrated in above examples.

2. CONCLUSION

This research paper develops the new algorithm to obtain the initial basic feasible solution of the transportation cost matrix using probabilities. The maximum probability chooses the particular row or column and that particular row or column provides the minimum cost for allocation. In this process, we finally achieve the initial basic feasible solution to the matrix which is much lesser than North-West Corner Method (NWCM) of the transportation problem. The optimal (best) solution to the matrix will be produced by the provided optimality test, MODI method, Stepping Stone method or WAM optimality test.

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