Comparison of some survival uniform exponential Weibull distribution estimate methods

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ABSTRACT
The primary aim of this study is to determine which estimation method is most effective in estimating parameters for the newly proposed survival uniform exponential Weibull distribution of(SU-EW). This will be achieved through a comparison of various estimation methods, including maximum likelihood, least squares, maximum product distance, and Method of Percentiles Estimators. In order to compare estimation methods based on the standard mean square error (MSE) of the parameters (n = 25, 50, 100, 150), Monte Carlo simulations were conducted with varying sample sizes. As determined by the simulation outcomes, the partial estimators (PE) method provided the most accurate estimates across all sample sizes, with the (MLE) method coming in second.

Keywords: uniform distribution; exponential distribution; survival function.

1. INTRODUCTION

The purpose of this study is to determine the preferred estimation method for the proposed new survival uniform exponential Weibull distribution by comparing the maximum likelihood estimator, least squares method, maximum product of divergence estimation method, and percentile estimators' method.

In pursuit of this interest, they have devised various approaches to present families of probability distributions, showcased the efficacy of these distributions, and estimated distribution parameters through the utilisation of numerous estimation methods. In addition, they have employed simulation methods to ascertain the superiority of these methods and implement them on real-world data.Alkhairy et al. [1] they studied some estimation methods. Such that the cumulative distribution function:

$$F(x) = \int_{a}^{G(K(x,\lambda))} m(t)dt \quad (1)$$

With P.D.F is

$f(x) = m(G(K(x,\lambda))\frac{d}{dx}G(k(x,\lambda))$

Teimouri el at.[2] A novel approach for approximating the parameters of the Weibull distribution, known as the L-moment estimator, was examined in comparison to four alternative estimation methods. Simulation results demonstrated that the (LM) method outperformed the others.

Sajid et al. [3] Various techniques for approximating the parameters of the elastic Weibull distribution were examined, and simulation was employed to demonstrate that the Bayesian method is superior to the alternatives. Al Mutairi et al. [4] analysed studied six approved estimation methods were presented, and their efficacy was assessed and compared via a simulation study. Alomari, H. M. [5]The discussion included four distinct approaches. A simulation was conducted to prove the superiority of the Bayesian method over other methods

Noori, and Mohammed Salih. [6] The researcher estimated the reliability function of the inverse Weibull distribution for the three parameters utilising three estimation methods(α,β,θ). Louzada, F. et al [7] Various estimators were introduced, facilitating the examination of distinct estimation methods for the unknown parameters of the extended geometric exponential distribution. The optimal method was then ascertained through simulation. Nwobi, and Ugomma, [8] The researcher investigated various techniques for approximating the Weibull distribution's parameters. Determining the optimal method through a comparison of approaches using the Kolmogorov-Smirnov and mean square error (MSE) metrics.

2. The Survival Uniform Exponential Weibull Distribution (SU-EWD)

Now a new way to find the lifetime distribution can be presented.

According to Alkhairy et al. [1], the DF G(x)of the T-X family is as follows:

$$\boldsymbol{G(x)} = \int_{\pi_1}^{F_2(F_1(x))} \boldsymbol{m(t)dt} \qquad (2)$$

That is there are two distributions, the baseline distribution with PDF m(t) and another distribution with $F_2(F_1(x))$.
Now our proposed T-X distribution using a survival function which can defined as follows:
Let S_1 (x),S_2 (x) be to survival functions of X_1,X_2 respectively, m(t) is a baseline PDF of a random variable T with [a,b], -∞<a<b<∞
Thus $S_2(S_1(x))$ satisfies the following condition by taking advantage the above condition.

1) $S_2(S_1(x)) \epsilon [a,b]$
2) $S_2(S_1(x))$differentiable and monotonically increasing function
3) $S_2(S_1(x)) \to a, \quad as\ x \to -\infty, \quad and \quad S_2(S_1(x)) \to b, \quad as\ x \to \infty$

So that new proposed T-X distribution is.

$$Z(x) = \int_{S_2(S_1(x))}^{b} m(t)dt \qquad (3)$$

$S_2(S_1(x)) = \exp[-\alpha(e^{(-\lambda x)})^{\beta}]$

$$Z(x) = \frac{b - \exp[-\alpha(e^{(-\lambda x)})^{\beta}]}{(b-a)} \qquad (4)$$

Where Z(x) the survival function of the survival uniform exponential Weibull distribution (SU-EW) such as.
α and λ: Scale parameter.
β: Shape parameter.

$\frac{\ln(\frac{-\ln a}{\alpha})^{(-1/\beta)}}{\lambda} < x < \frac{\ln(\frac{-\ln b}{\alpha})^{(-1/\beta)}}{\lambda}, \quad 0 < a \ \&\ b < 1$

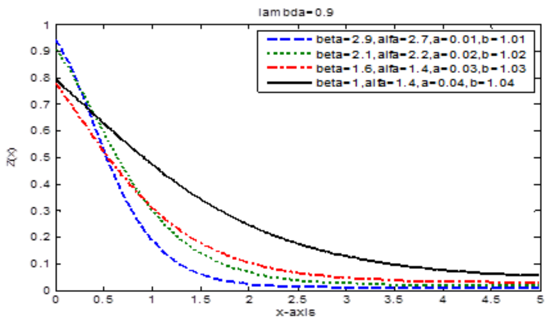

Figure 1. The survival function of SUEWD at various values of a,b,α,β,λ.

2. The CDF and PDF of The SU-EW Distribution

The PDF of the SU-EWD in eq(5)

$$W(x,a,b,\alpha,\lambda,\beta) = 1 - Z(x) = \frac{-a + \exp[-\alpha(e^{(-\lambda x)})^{\beta}]}{b-a} \qquad (5)$$

$\alpha, \beta, \lambda, a, b > 0, \quad a < b$

$\frac{\ln(\frac{-\ln b}{\alpha})^{(-1/\beta)}}{\lambda} < x < \frac{\ln(\frac{-\ln a}{\alpha})^{(-1/\beta)}}{\lambda}$

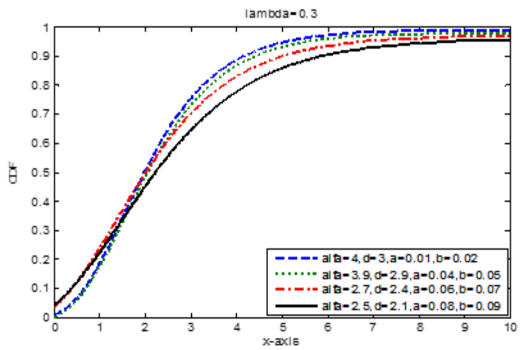

Figure 2. The CDF of SU-EWD at various values of a,b,α,β,λ.

The PDF of the SUEWD in eq(6)

$$w(x,a,b,\alpha,\beta,\lambda)=\frac{\alpha\beta\lambda\,(e^{-\lambda x})^{\beta}\,exp\,[-\alpha(e^{\,-\lambda x})^{\beta}\,]}{b-a} \tag{6}$$

We have

$$\int_{\frac{ln\left(\frac{-lna}{\alpha}\right)^{-1/\beta}}{\lambda}}^{\frac{ln\left(\frac{-lnb}{\alpha}\right)^{-1/\beta}}{\lambda}} f(x,a,b,\alpha,\beta,\lambda)dx=1 \tag{7}$$

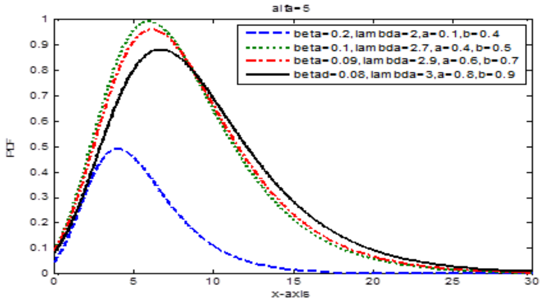

Figure 3. The PDF of SU-EWD at various values of a,b,α,β,λ.

4. Estimation modalities

Estimation serves as a critical foundation for statistical inference, particularly when it comes to determining the parameters of a coordinated community model using the statistics derived from its samples. We shall estimate the parameters of the novel model (SU-EWD) utilizing a variety of techniques and ascertain which one is more feasible from a research standpoint.

4.1. Maximum Likelihood Estimators (MLE):

Fisher introduced this estimation method in 1920. It has since become one of the most significant and widely used due to its numerous benefits and qualities, the most prominent of which are consistency and adequacy, as well as its precise measurement capabilities. In comparison to alternative approaches, estimation becomes more significant as the sample size expands, as denoted by (L).

If it was (x_i) a random variable follows a distribution (SU-EWD) the maximum likelihood function represents the common function for independent random variables $(x_1,x_2,\ldots\ldots,x_n)$ my agencies:

$$ML(x_i,\theta)=\prod_{i=1}^{n} f(x\_i,\alpha,\beta,\lambda,a,b)=\prod_{i=1}^{n}\left[\frac{\alpha\beta\lambda\left(e^{-\lambda x_i}\right)^{\beta}\exp[-\alpha\left(e^{-\lambda x_i}\right)^{\beta}]}{b-a}\right] \tag{8}$$

$$lnML(x_i,\theta)=nln\alpha+nln\beta+nln\lambda\;-\lambda\beta\sum_{i=1}^{n}x_i\;-\alpha\sum_{i=1}^{n}\left(e^{-\lambda x_i}\right)^{\beta}\;-nln(b-a) \tag{9}$$

$$\hat{\alpha}=\frac{n}{\sum_{i=1}^{n}exp\,(-\lambda x_i\,\beta)} \tag{10}$$

$$\frac{n}{\hat{\beta}}-\lambda\sum_{i=1}^{n}x_i\;+\alpha\lambda\sum_{i=1}^{n}x_i(\,e^{-\lambda x_i})^{\hat{\beta}}\;=0 \tag{11}$$

$$\frac{n}{\hat{\lambda}}-\beta\sum_{i=1}^{n}x_i\;+\;\alpha\beta\sum_{i=1}^{n}x_i\,(e^{-\hat{\lambda}x_i}\,)^{\beta}\;=0 \tag{12}$$

$$\frac{n}{(b-\hat{a}\,)}=0 \tag{13}$$

$$\frac{(-n)}{(\hat{b}\,-a)}=0 \tag{14}$$

$\hat{a}\;=min(X_i)$
$\hat{b}\;=max(X_i)$

4.2. Maximum Product of Spacing Estimation Method (MPS)

As an alternative to the method (MLE) for estimating unknown parameters for continuous univariate distributions, this technique is regarded as highly effective. The approach outlined in this method (MPS) was initially proposed by Cheng and Amin in 1938. To derive parameter estimates for the new distribution (SU-EW) using this method, the following steps are taken:

If it is an ordered random sample $(x_1, x_2, \dots, x_{n-1})$ and spaced regularly between its vocabulary and taken from a population that follows a probability distribution (SU-EW) which owns a function probability density (PDF) and a cumulative function (CDF), the maximum output of the divergence estimator is obtained from by maximizing the geometric mean distances, as shown as follows:

$$P(\alpha,\beta,\lambda,a,b) = [\prod_{i=1}^{n+1} D_i(\alpha,\beta,\lambda,a,b)]^{n+1} \tag{15}$$

As that

$D_i(\alpha,\beta,\lambda,a,b) = W(x_i,\alpha,\beta,\lambda,a,b) - W(x_{i-1},\alpha,\beta,\lambda,a,b)$

Such that

$\sum_{i=1}^{n+1} D_i\,(\alpha,\beta,\lambda,a,b) = 1 \qquad\qquad i = 1,2,\dots,n+1$

And when we take the ln of equation (15) results.

$l\,n\,P(\alpha,\beta,\lambda,a,b) = \frac{1}{(n+1)} \sum_{i=1}^{n+1} lnd_i(\,\alpha,\beta,\lambda,a,b)$

$$lnM(\alpha,\beta,\lambda,a,b) = \frac{1}{(n+1)}\,[lnW(x_1) + \sum_{i=2}^{n} ln(W(x_i\,) - W(x_{(i-1)}\,)) + ln(1 - W(x_n\,))] \tag{16}$$

And when substituting for the cumulative distribution function into equation (16), it results.

$$\ln M(\alpha,\beta,\lambda,a,b) = 1/(m+1)\,[\ln(\frac{-a + e^{-\alpha(e^{-\lambda x_1})^{\beta}}}{b-a}) + \sum_{i=2}^{n} \ln(\frac{-a + e^{-\alpha(e^{-\lambda x_i})^{\beta}}}{b-a} - \frac{-a + e^{-\alpha(e^{-\lambda x_{i-1}})^{\beta}}}{b-a}) + \ln(1 - \frac{-a + e^{-\alpha(e^{-\lambda x_n})^{\beta}}}{b-a})] \tag{17}$$

We take the partial derivative of equation (17) for the parameters$(\alpha,\beta,\lambda,a,b)$ let's get five equations and after equality the five equations to zero and solving them using (Newton-Raphson) method because they are non-linear, we get parameters are estimated using the maximum product of spacing estimation method$(\hat{\alpha},\hat{\beta},\hat{\lambda},\hat{a},\hat{b})$for the new distribution (SU-EW).

4.3. Method of Percentiles Estimators (PE)

It is one of the methods of appreciation that I suggested before the world (Kao), Assuming that (q_i) represents an estimate of a cumulative distribution function *W(x,α,λ,β,a,b).*

$W(x,\alpha,\lambda,\beta,a,b) = \frac{-a + exp\,(-\alpha(e^{(-\lambda x)})^{\beta})}{b-a}$

The parameter estimator $(\hat{\alpha},\hat{\lambda},\hat{\beta},\hat{a},\hat{b})$ is obtained by performing the partial derivation of the formula below with respect to for parameter.

$$Q = \sum_{i=1}^{n} [q_i - W(x,\alpha,\lambda,\beta,a,b)\,]^2 \tag{18}$$

$$Q = \sum_{i=1}^{n} [\frac{i-0.3}{n+0.25} + \frac{a + exp\,(-\alpha(e^{-\lambda x}\,)^{\beta}\,)}{b-a}\,]^2 \tag{19}$$

We take the partial derivative of equation (19) for the parameters (*α, β, λ, a, b*) and solving the resulting equations by (Newton-Raphson) method, we get the value of the estimator parameters $(\hat{\alpha},\hat{\beta},\hat{\lambda},\hat{a},\hat{b})$by the method of percentiles.

4. 4. Least Square Method (LS)

Least squares estimation is among the most significant and extensively applied estimation techniques. By minimizing the sum of the error squares, which determines the optimal data distribution, the following can be said about the bias and consistency of the resulting estimations:

$$L = \sum_{i=1}^{n} [W(x_i\,) - \frac{i}{n+1}]^2 \tag{20}$$

$W(x_i)$: The cumulative distribution function of the survival uniform Exponential Weibull distribution represents (SU-EW) i/(n+1): A non-parametric expression, which corresponds to the unbiased expression of the pooled distribution function.

$$L = \sum_{i=1}^{n} [\frac{-a + exp(-\alpha(e^{-\lambda x})^{\beta})}{b-a} - \frac{i}{n+1}]^2 \quad (21)$$

And by performing the partial derivation of equation (21) with respect to the parameters of the distribution (α,λ,β,a,b) to get the equations below after setting them equal to zero, and solving the resulting equations by (Newton-Raphson) method, we get the value of the estimator parameters ($\hat{\alpha},\hat{\beta},\hat{\lambda},\hat{a},\hat{b}$) by the method of percentiles estimators.

5. Discuss the simulation results

The parameter estimates of the proposed model (SU-EW) for the four estimation methods were obtained subsequent to the execution of the simulation process utilising the Monte Carlo method and the (MATLAB) programing with varying sample sizes (30, 50, 100, 150). Estimative tables and graphs have been generated from these results. The ranks method was utilised to compare the estimation methods employed during the estimation process, and the estimates were subsequently assessed using the statistical criterion of mean square error (MSE) in order to obtain the most accurate results.

Table 1. The ranks of the parameter estimators for the proposed distribution SU-EW for the experimental data for the eight hypothesized models and for the chosen sample sizes (30), (50), (100) and (150).

Models	n	MLE	PE	LS	MPS
MODEL 1	30	19	8	10	13
	50	18	7	11	14
	100	17	6	9	18
	150	15	14	6	15
	$\sum$ **Rank**	69	35	36	60
MODEL 2	30	19	8	10	13
	50	19	7	11	13
	100	18	8	10	14
	150	18	8	9	15
	$\sum$ **Rank**	74	31	40	55
MODEL 3	30	18	10	9	13
	50	13	9	12	16
	100	17	10	7	16
	150	18	8	9	15
	$\sum$ **Rank**	66	38	37	60
MODEL4	30	12	9	16	13
	50	18	8	9	15
	100	18	7	9	16
	150	18	9	10	13
	$\sum$ **Rank**	76	33	42	57
MODEL5	30	16	8	11	16
	50	17	8	11	14
	100	19	8	10	13
	150	18	8	9	15
	$\sum$ **Rank**	70	32	41	58
MODEL6	30	18	7	12	13
	50	19	9	9	13
	100	19	9	9	13
	150	16	9	11	14

	$\sum$ **Rank**	72	34	41	53
MODEL7	30	17	9	8	16
	50	13	12	7	18
	100	13	11	8	18
	150	14	12	6	18
	$\sum$ **Rank**	57	44	29	70
MODEL8	30	18	9	7	16
	50	11	12	9	18
	100	17	7	8	18
	150	17	6	9	16
	$\sum$ **Rank**	63	34	33	70
	$\sum$ **Ranks**	547	290	299	483

From the Table (1),the results of the simulation experiment can be summarized in the following points:
1) It is noted that the best method is the PE method because it obtained the lowest total of the ranks, and the LS method comes after it.
2) The best sample size for the best model and the rest of the models is the sample size (150) because it has the least mean square error.
3) The first model is the best model from the rest of the models because it obtained the lowest mean square error 8.85E-08.

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