Solving multi-choice solid stochastic multi objective transportation problem with supply, demand and conveyance capacity involving Newton divided difference interpolations

Vishwas Deep Joshi[1], Medha Sharma[1] and Jagdev Singh[1,2,*]

[1] Department of Mathematics, JECRC University Jaipur, Rajasthan, INDIA

[2]Department of Computer Science and Mathematics,
Lebanese American University, Beirut, Lebanon

Email: vdjoshi.or@gmail.com, medhasharma1997@gmail.com,
jagdevsinghrathor@gmail.com

[*]Corresponding author

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Abstract

The main concern is the uncertainty in the real-world solid transportation problem. This study examines a supply, demand, and conveyance capacity-based multi-choice solid stochastic multi-objective transportation problem (MCSS-MOTP). Due to uncertainty, the concrete objective function coefficients of the proposed model are of multivariate type. Furthermore, the parameters of the constraints are treated as independent multivariate random variables with normal distribution. First, a Newton divided difference method-based interpolation polynomial is described that extends an interpolation polynomial using practical properties at non-negative integer nodes to deal with any multiple-choice parameter. Second, the probabilistic constraints are converted into precise ones utilizing a stochastic programming approach. In the end, ranking procedure was used to compare the existing approach with the old models. The proposed model's applicability was confirmed using a numerical example.

Keywords- Solid transportation problem; Newton divided difference; Stochastic programming; multi-choice random parameter; Ranking of solutions

1 Introduction

The first and most significant use of the linear programming problem is in transportation [20]. It has numerous applications in inventory control, supply management, logistics systems, and production planning, among others. By taking into account the standard transportation problem's parameters are cost, supply, and demand. However, given the level of market competition today, it's possible that the criteria aren't presented precisely. The price of the product may change occasionally or it may depend on how the product is made. Additionally, because information on the shipping goods is unavailable, supply and demand may be ambiguous in nature. For these reasons and to deal with ambiguous information, Zadeh[16]developed the idea of ambiguity.

In numerous fields including Economics, Psychology, Philosophy, Mathematics, and Statistics, decision-making is crucial. The necessity of transportation as a component of distribution networks must be acknowledged. The main objective of the transportation problem (TP) is to reduce the price of transferring goods between consumers and producers so that manufacturers may more easily satisfy consumers' demands. The TP's parameters are price, supply, and demand. We may transfer goods from sources to destinations using different modes of transportation even though there are many modes of transportation accessible for shipments of commodities in a transportation system if we want to save money or meet deadlines. The fundamental TP was first expressed by Hitchcock [13] and later, according to the literature, it was widely discussed by many authors.

When there are random parameters involved in an optimization problem, stochastic programming (SP) techniques are applied. This indicates that some of the parameters in the model coefficients have known probability distributions that indicate they are known with uncertainty. Typically, SP arises frequently in a wide range of real-world management science, engineering, and technology challenges that contain some stochastic factors, i.e., uncertain input data, and models built on unreliable information. Because of the rapid advancement of computers and contemporary optimization techniques over the past five decades, there have been an increasing number of stochastic optimization applications to various challenging real-world decision-making situations. SP models have been effectively used to a number of applications, including supply chain management, environmental planning, telecommunications, transportation, and planning for energy and financial resources.

A mathematical method called stochastic programming is used to resolve optimization problem with uncertainty. Stochastic programming considers the randomness or variability of these values as opposed to conventional optimization techniques, which assume deterministic values for variables. By taking into account a variety of potential outcomes and the corresponding probabilities, it enables decision-makers to make educated decisions. For instance, stochastic programming in finance can be used to choose the best investment portfolio by taking into account various market conditions and their probabilities. It can be used in supply chain management to optimize inventory levels by taking uncertain demand and supply disruptions into account. A potent tool for making decisions in complicated and uncertain contexts is stochastic programming.

The solid transportation problem (STP), also known as three-dimensional TP or three-dimensional TP, is a developed version of the well-known TP that was first modelled by Schell [11]and developed by Haley [15]. The objective of STP is to transport homogeneous goods from their origin to their final destination using different modes of transportation to minimize the total cost of transportation. A three-dimensional TP's parameters include the product's availability at source points, the product's needs at destination points, and the carrying capacity of different modes of transportation (such as trucks, cargo planes, goods trains, ships, etc.) used to move the product from sources to destinations. Due to the inclusion of multiple variables, such as equipment failure and labor concerns for manufacturing, market mode, road condition, and weather conditions for transportation, the problem's parameters are not deterministic in real life. Random variables are occasionally used to describe these uncertainties, particularly stochastic ones. When formulating a real-world STP, we must take into account the optimization of a number of goals, including minimizing transportation time, minimizing loss during transit, and minimizing transportation cost. This knowledge prompts us to take into account a stochastic multi-objective STP. The STP is a significant study area from both a theoretical and a practical standpoint. In this field of study, numerous researchers have made substantial contributions. Supply, demand, transportation capacity, direct costs, and fixed charges are all unknown variables in the fixed charge STP that Zhang et al.[9] discussed.

An urgent situation in the transportation sector that needs immediate attention and a solution is referred to as a "solid transportation problem." When there is a lack of dependable and effective transportation infrastructure, it can cause delays, traffic, or poor connectivity. For instance, if a city's public transportation infrastructure is out of date and unable to handle the rising demand, the city may have a serious transportation issue. As a result, travellers may experience crowded buses, protracted waits, and frustration. To ensure a smooth and efficient movement of people and commodities, solving solid transportation issues needs thoughtful planning, investment in infrastructure development, and competent management.

The majority of real-world, practical decision-making issues are modelled using multiple choices. The use of multi-choice optimization techniques has grown in importance in a variety of fields, including technology, business, transportation, and military applications. The price indices the objective function's C_{ijk} might stand in for the price of moving a unit of production from source i to destination j by conveyance k . Due to rising fuel prices and other important factors, let us present a multiple-choice version of the cost coefficient of the objective function for the transportation problem. Supply and demand parameters should also be multi-choice in order to account for market price fluctuations for all items. Multiple choice programming, which Healy [25] initially invented, is a method for solving linear programming problems with zero-one variables.

Mathematicians and computer scientists utilize Newton's divided difference interpolation as a numerical technique to approximate a function from a collection of data points. Its foundation is the idea of divided differences, which entails figuring out the variations between related data points. This method enables the construction of a polynomial function that traverses each of the provided data points. Newton's Divided Difference a multi-choice fractional stochastic transport problem can be solved using interpolation by transforming it into a deterministic model [14]. A method for solving MCFS-MOTPs by interpolating multi-choice parameters, transforming probabilistic constraints, linearizing the problem, and solving using fuzzy goal programming and ϵ-constraint method [4]. A method for solving MOSSTP under uncertainty by formulating it as a chance-constrained programming problem and using global criterion method and fuzzy goal programming approach to find good solutions in a reasonable amount of time [17]. A new approach for analysing STP by combining multi-choice programming and stochastic programming, and using a transformation technique to find an optimal solution [18]. A weighted goal programming approach for multi-objective transportation problems that can obtain compromise solutions according to the decision-maker's priorities [2]. A weighted goal programming approach for multi-objective transportation problems that finds compromise solutions according to the decision-maker's priorities, illustrated with a numerical example [21]. A method for solving multi-choice stochastic transportation problems by using Lagrange's interpolating polynomial to select an appropriate choice and transforming stochastic supply constraints into deterministic constraints [24].

A new transformation technique for solving multi-choice stochastic transportation problems with exponential distribution by introducing binary variables for each aspiration level of each cost coefficient, transforming probabilistic constraints into deterministic constraints, and formulating a non-linear deterministic model [8]. A method for solving multi-choice transportation problems by using Lagrange's interpolating polynomial and chance technique to select an appropriate choice and formulate a non-linear mathematical model [7, 23]. A mathematical model for a transportation problem with nonlinear cost and multi-choice demand is proposed by developing a general transformation technique and formulating a multi-objective decision making model [19]. A solution procedure for multi-choice stochastic transportation problem with extreme value distribution by transforming probabilistic constraints into deterministic constraints, handling multi-choice type cost coefficients using binary variables [6].

Table 1: Comparison of the approach to the present models

Reference	S	D	C	MO	MC	Methodology
Joshi[21]	✓	✓		✓		**GP using WS**
Agrawal[14]	✓	✓			✓	**NDD**
Das[17]	✓	✓	✓	✓		**WD**
Roy[18]	✓	✓			✓	**CD**
Sayed[4]	✓	✓		✓	✓	**NDD**
Roy[22]	✓	✓			✓	**WD**
Proposed Approach	✓	✓	✓	✓	✓	**NDD**

S*= Supply, **D*= Demand**,**C*= Conveyance**, **MO*=Multi-Objective**, **MC*= Multi-Choice**, **GP*= Goal programming**,**WS*= Weighted Sum**,**WD*= Weibull Distribution**,**CD*= Cauchy' Distribution**,**NDD*= Newton's divided difference**,

A solution procedure for multi-objective stochastic unbalanced transportation problem by changing the problem into deterministic scenario using fuzzy theory [5]. A solution procedure for multi-choice stochastic transportation problem with Weibull distribution by transforming probabilistic constraints into deterministic [10]. A solution procedure for multi-objective capacitated transportation problem with uncertain input information by transforming the uncertain information into deterministic form and solving the resultant MOCTP for the compromise solution [25]. A method for solving linear programming problems with multi-choice parameters by interpolating technique [1]. A multi-choice stochastic transportation problem with extreme value distribution is solved by transforming probabilistic constraints into deterministic constraints [6]. A two-phase solution procedure for multi-objective capacitated transportation problem with uncertain input information is proposed [3]. A solution methodology for multi-choice stochastic transportation problem with Weibull distribution and multi-choice cost coefficients is proposed [22].

The paper is organized as follows. Section 1 presents a review of the relevant literature and introduction. Basic definitions that are related to this article presents in section 2. This paper's notation is covered in section 3. Section 4 presents the exhaustive problem statement. Section 5 illustrates the process for solving the given problem. Section 6 proposes a new solution method for the problem. Section 7 evaluates the performance of the proposed solution method on a set of numerical examples. Section 8 discusses the theoretical and practical implications of the proposed method. Section 9 concludes the paper and suggests directions for future research.

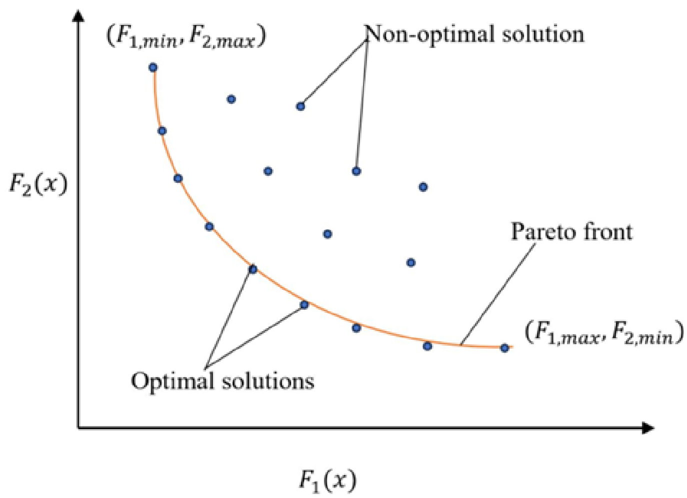

Figure 1: Pareto optimal solution

2 Basic definitions

2.1 Feasible solution:

A feasible solution to an optimization problem is a set of values for the decision variables that satisfies all of the constraints of the problem.

2.2 Pareto optimal solution:

A set of "non-inferior" solutions in the objective space that specify a limit beyond which none of the objectives can be improved without compromising at least one of the other objectives is known as a pareto optimum solution.

2.3 Compromise solution:

A compromise solution is a balanced outcome that takes into account multiple conflicting factors or goals. It involves finding a middle ground that satisfies different objectives without fully favouring one over the others. It's like reaching a fair agreement that considers everyone's preferences. Decision-makers prioritize the compromise option over all other solutions when taking into account all the criteria in the multi-objective.

2.4 Ideal solution:

When a problem involves minimization, the ideal solution is one in which each objective function achieves its optimal minimum.

2.5 Anti-ideal solution:

When a problem involves minimization, the anti-ideal solution is one in which each objective function achieves its maximum value.

3 Notations

- R**:** number of objective functions
- m**:** number of supply sources
- n**:** number of demand destinations
- l**:** number of conveyances
- x_{ijk}**:** amount of shipment from i^{th} supply source to j^{th} demand destination using k^{th} transportation mode
- Z_r**:** r^{th} objective functions
- c^r_{ijk}**:** unit cost in the r^{th} objective function
- a_i**:** amount of supply at the i^{th} supply source
- b_j**:** amount of demand at the j^{th}demand destination
- e_k**:** amount of conveyance capacity of the k^{th} transportation mode
- ϕ**:** the cumulative distribution functions
- θ_i**:** probability for a_i
- δ_j**:** probability forb_j
- σ_k**:** probability for e_k
- g_{θ_i}**:** the value of standard normal variable for a_i
- g_{δ_j}**:** the value of standard normal variable for b_j
- g_{σ_k}**:** the value of standard normal variable for e_k
- $E(F_{a_i}(w_{a_i}))$**:** the mean of supply of interpolating polynomial $F_{a_i}(w_{a_i})$
- $E(F_{b_j}(w_{b_j}))$**:** the mean of demand of interpolating polynomial $F_{b_j}(w_{b_j})$
- $E(F_{e_k}(w_{e_k}))$**:** the mean of conveyance of interpolating polynomial $F_{e_k}(w_{e_k})$
- $V(F_{a_i}(w_{a_i}))$**:** the variance of supply of interpolating polynomial $F_{a_i}(w_{a_i})$
- $V(F_{b_j}(w_{b_j}))$**:** the variance of demand of interpolating polynomial $F_{b_j}(w_{b_j})$
- $V(F_{e_k}(w_{e_k}))$**:** the variance of conveyance of interpolating polynomial $F_{e_k}(w_{e_k})$

4 Problem Statement:

A transportation company must convey its products from numerous production facilities to numerous retail locations. There are m production houses, n retail stores, and l vehicles, assuming that a homogeneous product is conveyed from the i^{th} production house to the j^{th} retail store by the k^{th} vehicle. Let x_{ijk} serve as a representation of the product's unit quantity. The parameters for supplies, demand, and conveyance capacity are thought of as multi-choice random parameters since the values of the parameters are not always set due to the environment's uncertainty and variety of possibilities. As a result, the defined problem's constraints are probabilistic with regard to their degree of want. The mathematical formulation of the aforementioned problem is as follows because the objective function is in linear form and the transportation cost is considered to be of the multi-choice variety:

$$Min\ Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}(c_{ijk}^1, c_{ijk}^2, \dots, c_{ijk}^R)x_{ijk}, \tag{1}$$

Subject to:

$$P\{\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq (a_i^1, a_i^2, \dots, a_i^u)\} \geq 1-\theta_i, \qquad i = 1,2,\dots,m \tag{2}$$

$$P\{\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq (b_j^1, b_j^2, \dots, b_j^v)\} \geq 1-\delta_j, \qquad j = 1,2,\dots,n \tag{3}$$

$$P\{\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq (e_k^1, e_k^2, \dots, e_k^q)\} \geq 1-\sigma_k, \qquad k = 1,2,\dots,l \tag{4}$$

$$x_{ijk} \geq 0, \forall\ i,j\ and\ k \tag{5}$$

Where the multi-choice random parameters for the total availability a_i at the i^{th} manufacturing house, regarded as an independent random variable, are $(a_i^1, a_i^2, \dots, a_i^u)$.The multi-choice random parameters $(b_j^1, b_j^2, \dots, b_j^v)$ for the overall quantity b_j of the product at the j^{th} retail outlets are regarded as independent random variables. The multi-choice random parameters for the total capacity e_k of the conveyance at the k^{th} vehicle, which is regarded as an independent random variable, are $(e_k^1, e_k^2, \dots, e_k^q)$. The probability of meeting the constraints is represented by the values θ_i, δ_j and σ_k.

5 Solutions Methodology

5.1 Newton's divided difference interpolating polynomial for multi-choice parameters

The Newton's divided Difference Interpolation numerical approximation technique is used to convert the multi-choice parameter into the best option. Introduce an integer variable so that the interpolating polynomial can be defined for each option of a multi-choice parameter. The integer variables $w^t_{c_{ijk}}, (t = 0, 1, \ldots, s-1)$ are used since there are s possible cost options in the problem above.

For each alternative, the integer variables $w^p_{a_i}(p = 0, 1, \ldots, u-1)$, $w^h_{b_j}(h = 0, 1, \ldots, v-1)$ and $w^g_{e_k}(g = 0, 1, \ldots, q-1)$ are introduced since supplies, demands, and conveyance capacity are multi-choice random parameters. Each multi-choice parameter has a different divided difference that is determined based on the alternatives. Using Table 2, which lists various divided difference orders, Newton's divided difference (NDD) interpolation polynomial is created for the cost parameter in equation (6).

Table 2: Divided difference (DD)

$w^t_{c_{ij1}}$	$F_{c_{ijk}}(w^t_{c_{ijk}}$	First DD	Second DD	Third DD
0	c^1_{ijk}			
		$f[w^0_{c_{ijk}}, w^1_{c_{ijk}}]$		
1	c^2_{ijk}		$f[w^0_{c_{ijk}}, w^1_{c_{ijk}}, w^1_{c_{ijk}}]$	
		$f[w^1_{c_{ijk}}, w^2_{c_{ijk}}]$		$f[w^0_{c_{ijk}}, w^1_{c_{ijk}}, w^2_{c_{ijk}}, w^3_{c_{ijk}}]$
2	c^3_{ijk}		$f[w^1_{c_{ijk}}, w^2_{c_{ijk}}, w^3_{c_{ijk}}]$	
		$f[w^2_{c_{ijk}}, w^3_{c_{ijk}}]$		
3	c^4_{ijk}			

$$F_{c_{ijk}}(w_{c_{ijk}}) = f[w^0_{c_{ijk}}] + (w_{c_{ijk}} - w^0_{c_{ijk}})f[w^0_{c_{ijk}}, w^1_{c_{ijk}}] + (w_{c_{ijk}} - w^0_{c_{ijk}}) (w_{c_{ijk}} - w^1_{c_{ijk}})f[w^0_{c_{ijk}}, w^1_{c_{ijk}}, w^2_{c_{ijk}}] +(w_{c_{ijk}} - w^0_{c_{ijk}})(w_{c_{ijk}} - w^1_{c_{ijk}}), \ldots, (w_{c_{ijk}} - w^{s-1}_{c_{ijk}}) f[w^0_{c_{ijk}}, w^1_{c_{ijk}}, \ldots, w^{s-1}_{c_{ijk}}] \quad (6)$$

$$F_{c_{ijk}} = c^1_{ijk} + (w_{c_{ijk}} - w^0_{c_{ijk}})(c^2_{ijk} - c^1_{ijk}) + (w_{c_{ijk}} - w^0_{c_{ijk}})(w_{c_{ijk}} - w^1_{c_{ijk}}) (\frac{c^3_{ijk} - 2c^2_{ijk} + c^1_{ijk}}{(w^2_{c_{ijk}} - w^0_{c_{ijk}})} + \cdots + \sum_{t=1}^{s} \frac{c^t_{ijk}}{\prod_{t \neq p+1, p=0}^{s-1}(w^{t-1}_{c_{ijk}} - w^p_{c_{ijk}})} \quad (7)$$

Similarly, by replacing the multiple choice parameters in the program with its interpolated polynomials for supply, demand, and transportation capacity,

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represented by $F_{a_i}(w_{a_i}), F_{b_j}(w_{b_j})$ and $F_{e_k}(w_{e_k})$ Respectively, the mathematical model can be formulated as follows.

$$Min\ Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} F_{ijk}(w_{ijk})x_{ijk}, \tag{8}$$

Subject to:

$$P\{\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq F_{a_i}(w_{a_i})\} \geq 1-\theta_i, \qquad i=1,2,\ldots,m \tag{9}$$

$$P\{\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq F_{b_j}(w_{b_j})\} \geq 1-\delta_j, \qquad j=1,2,\ldots,n \tag{10}$$

$$P\{\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq F_{e_k}(w_{e_k})\} \geq 1-\sigma_k, \qquad k=1,2,\ldots,l \tag{11}$$

$$x_{ijk} \geq 0, \forall\ i, j\ and\ k \tag{12}$$

5.2 The transformation of probabilistic constraints

The multi-choice parameters were transformed into their interpolating polynomials so that the resulting probabilistic constraints would be transformed into their deterministic form. To transform its deterministic restrictions into probabilistic ones, we consider the supply's constraints.

Consider the constraint (9) for every, $i = 1, 2, \ldots, m$

$$P\{\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq F_{a_i}(w_{a_i})\} \geq 1-\theta_i$$

or

$$1 - P\{\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq F_{a_i}(w_{a_i})\} \geq 1-\theta_i$$

Applying Chance constrained technique, this implies

$$P\{\frac{F_{a_i}(w_{a_i}) - E(F_{a_i}(w_{a_i}))}{\sqrt{V(F_{a_i}(w_{a_i}))}} \leq \frac{\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} - E(F_{a_i}(w_{a_i}))}{\sqrt{V(F_{a_i}(w_{a_i}))}}\} \leq \theta_i$$

$$P\{\xi_{a_i} \leq \frac{\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} - E(F_{a_i}(w_{a_i}))}{\sqrt{V(F_{a_i}(w_{a_i}))}}\} \leq \theta_i$$

$$\phi\{\frac{\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} - E(F_{a_i}(w_{a_i}))}{\sqrt{V(F_{a_i}(w_{a_i}))}}\} \leq \phi(-g_{\theta_i})$$

$$\{\frac{\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} - E(F_{a_i}(w_{a_i}))}{\sqrt{V(F_{a_i}(w_{a_i}))}}\} \leq -g_{\theta_i}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq E(F_{a_i}(w_{a_i})) - g_{\theta_i}\sqrt{V(F_{a_i}(w_{a_i}))} \tag{13}$$

The mean and variance of the interpolating polynomial $F_{a_i}(w_{a_i})$ are, respectively, denoted by $E(F_{a_i})(w_{a_i})$ and $V(F_{a_i})(w_{a_i})$ accordingly. Additionally, let ϕ be the standard normal distribution's cumulative distribution function and g_{θ_i} stand for the standard normal variable's value. Equation (13) thus expresses the deterministic constraint of the probabilistic constraint (9).

The analogous deterministic constraint for every $j = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, l$ is as follows. In a similar manner, using the same method to the demand and conveyance capacity constraints

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq E(F_{b_j}(w_{b_j})) + g_{\delta_j}\sqrt{V(F_{b_j}(w_{b_j}))} \tag{14}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{m} x_{ijk} \leq E(F_{e_k}(w_{e_k})) - g_{\sigma_k}\sqrt{V(F_{e_k}(w_{e_k}))} \tag{15}$$

where, $E(F_{b_j}(w_{b_j}), E(F_{e_k}(w_{e_k})$ and $V(F_{b_j}(w_{b_j}), V(F_{e_k}(w_{e_k})$ denotes the mean and the variance of interpolating polynomial $F_{b_j}(w_{b_j})$ and $F_{e_k}(w_{e_k})$ respectively g_{δ_j} and g_{σ_k} denotes the value of standard normal variable. We compute the random interpolating polynomial's mean and variance as

$$\mathrm{E}(\mathrm{F}_{a_i}(w_{a_i})) = E\{a_i^1 + (w_{a_i} - w_{a_i}^0)(a_i^2 - a_i^1) + (w_{a_i} - w_{a_i}^0)(w_{a_i} - w_{a_i}^1)\frac{a_i^3 - 2a_i^2 + a_i^1}{w_{a_i}^2 - w_{a_i}^0} + \cdots + \sum_{t=1}^{s}\frac{a_i^t}{\prod_{t\neq p+1, p=0}^{s-1}(w_{c_{ijk}}^{t-1} - w_{c_{ijk}}^{p})}\}$$

$$= \{E(a_i^1) + (w_{a_i} - w_{a_i}^0)(E(a_i^2) - E(a_i^1)) + (w_{a_i} - w_{a_i}^0)(w_{a_i} - w_{a_i}^1)\frac{E(a_i^3) - 2E(a_i^2) + E(a_i^1)}{w_{a_i}^2 - w_{a_i}^0} + \ldots + \sum_{t=1}^{s}\frac{E(a_i^t)}{\prod_{t\neq p+1, p=0}^{s-1}(w_{c_{ijk}}^{t-1} - w_{c_{ijk}}^{p})}\} \tag{16}$$

$$\mathrm{V}(\mathrm{F}_{a_i}(w_{a_i})) = V\{a_i^1 + (w_{a_i} - w_{a_i}^0)(a_i^2 - a_i^1) + (w_{a_i} - w_{a_i}^0)(w_{a_i} - w_{a_i}^1)\frac{a_i^3 - 2a_i^2 + a_i^1}{w_{a_i}^2 - w_{a_i}^0} + \cdots + \sum_{t=1}^{s}\frac{a_i^t}{\prod_{t\neq p+1, p=0}^{s-1}(w_{c_{ijk}}^{t-1} - w_{c_{ijk}}^{p})}\}$$

$$= \{V(a_i^1) + (w_{a_i} - w_{a_i}^0)(V(a_i^2) - V(a_i^1)) + (w_{a_i} - w_{a_i}^0)(w_{a_i} - w_{a_i}^1)\frac{V(a_i^3) - 2V(a_i^2) + V(a_i^1)}{w_{a_i}^2 - w_{a_i}^0} + \ldots + \sum_{t=1}^{s}\frac{V(a_i^t)}{\prod_{t\neq p+1, p=0}^{s-1}(w_{c_{ijk}}^{t-1} - w_{c_{ijk}}^{p})}\} \tag{17}$$

The $F_{a_i}(w_{a_i})$ mean and variance are shown in equations (16) and (17). Equations (16) and (17) can also be used to calculate the mean and variance of the

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interpolating polynomial for demand and conveyance capacity.

The deterministic model is implemented with chance constraints and Newton's Divided Difference Interpolation.

$$Min\ Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} F_{ijk}(w_{ijk})x_{ijk}, \qquad r = 1, 2 \ldots, R$$

Subject to:

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq E(F_{a_i}(w_{a_i})) - g_{\theta_i}\sqrt{V(F_{a_i}(w_{a_i}))}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq E(F_{b_j}(w_{b_j})) + g_{\delta_j}\sqrt{V(F_{b_j}(w_{b_j}))}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{m} x_{ijk} \leq E(F_{e_k}(w_{e_k})) - g_{\sigma_k}\sqrt{V(F_{e_k}(w_{e_k}))}$$

$$x_{ijk} \geq 0, \forall\ i, j\ and\ k$$

The multi-choice solid stochastic multi-objective transportation problem (MCSS-MOTP) can be applied to a variety of real-world problems, such as:

- **Supply chain management:** The MCSS-MOTP can be used to optimize the transportation of goods and materials in a supply chain, where the cost coefficients are uncertain and the objective is to minimize the total transportation cost and satisfy the demand at each destination with a specified probability.
- **Project management:** The MCSS-MOTP can be used to optimize the allocation of resources in a project, where the cost coefficients are uncertain and the objective is to minimize the total cost and complete the project on time with a specified probability.
- **Financial planning:** The MCSS-MOTP can be used to optimize the allocation of funds in a financial portfolio, where the return on investment is uncertain and the objective is to maximize the expected return and minimize the risk with a specified probability.
- **Energy management:** The MCSS-MOTP can be used to optimize the generation and distribution of energy in a power grid, where the cost of energy is uncertain and the objective is to minimize the total cost and meet the demand at each node with a specified probability.

The MCSS-MOTP is a powerful tool that can be used to solve a variety of real-world problems. However, it is important to note that the problem may be difficult to solve, especially if the number of sources, destinations, and probabilistic constraints are large.

Here are some of the challenges in solving the MCSS-MOTP:

- The problem may be computationally expensive to solve, especially if the number of sources, destinations, and probabilistic constraints are large.
- The problem may be non-convex, which means that there may be multiple local optima.
- The problem may be NP-hard, which means that it may not be possible to find an optimal solution in polynomial time.
- Despite these challenges, the MCSS-MOTP is a valuable tool that can be used to solve a variety of real-world problems.

6 Approaches to solve the MCSS-MOTP

6.1 First approach

In this we have used the weighted sum method to convert multiple objectives into a single objective. In which the multi-choice cost parameter is reduced to a single choice using Newton's divided difference method. The mathematical formulation is as follows:

$$Min\ Z = \sum_{r=1}^{R} d_r Z_r$$

Subject to:

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq E(F_{a_i}(w_{a_i})) - g_{\theta_i}\sqrt{V(F_{a_i}(w_{a_i}))}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq E(F_{b_j}(w_{b_j})) + g_{\delta_j}\sqrt{V(F_{b_j}(w_{b_j}))}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{m} x_{ijk} \leq E(F_{e_k}(w_{e_k})) - g_{\sigma_k}\sqrt{V(F_{e_k}(w_{e_k}))}$$

$$x_{ijk} \geq 0, \forall\ i, j\ and\ k$$

Where Z_r = individual objectives that converted into single choice using Newton's divided difference approach

6.2 Second approach

In this we have used the Joshi's method to convert multiple objectives into a single objective in which each multi choice objective converted into single choice using NDD approach. The mathematical formulation is as follows:

$$Min\ \mu' = \sum_{r=1}^{R} \mu(1-d_r)$$

Subject to:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} F_{ijk}(w_{ijk})x_{ijk} \leq Z_r^* + \frac{\mu(1-d_r)}{Z_r^U - Z_r^L},\ r = 1, 2 \ldots, R$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq E(F_{a_i}(w_{a_i})) - g_{\theta_i}\sqrt{V(F_{a_i}(w_{a_i}))}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq E(F_{b_j}(w_{b_j})) + g_{\delta_j}\sqrt{V(F_{b_j}(w_{b_j}))}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{m} x_{ijk} \leq E(F_{e_k}(w_{e_k})) - g_{\sigma_k}\sqrt{V(F_{e_k}(w_{e_k}))}$$

$$x_{ijk} \geq 0, \forall\ i, j\ and\ k$$

Where Z_r^* = individual objectives that converted into single choice using Newton's divided difference approach

6.3 Third approach

Again, we convert multichoice into single choice using NDD approach and solved the converted problem using Nomani's method. The mathematical formulation is as follows:

$$Min\ \mu' = \sum_{r=1}^{R} \mu(1-d_r)$$

Subject to:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} F_{ijk}(w_{ijk})x_{ijk} \leq Z_r^* + \mu(1-d_r),\ r = 1, 2 \ldots, R$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq E(F_{a_i}(w_{a_i})) - g_{\theta_i}\sqrt{V(F_{a_i}(w_{a_i}))}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \le E(F_{b_j}(w_{b_j})) + g_{\delta_j}\sqrt{V(F_{b_j}(w_{b_j}))}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{m} x_{ijk} \le E(F_{e_k}(w_{e_k})) - g_{\sigma_k}\sqrt{V(F_{e_k}(w_{e_k}))}$$

$$x_{ijk} \ge 0, \forall\, i, j \; and \; k$$

Where Z_r^* = individual objectives that converted into single choice using Newton's divided difference approach

This introduces the need to rank these methods due to the variety of approaches available for handling multi-objective transportation problems. To address this, a tool is required to assist in ranking and selecting the most suitable method. It is at this point that the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [12] is useful. TOPSIS helps rank different methods based on their optimal solutions' performances. In this situation, the criteria are objective functions, and the alternatives are the best solutions. In essence, TOPSIS helps us determine which method is the most effective in terms of achieving optimal solutions for the problem at hand.

7 Numerical Example

Let's consider the attached MCSS-MOTP:

$$\text{Min } Z_1 = c^1_{111}x_{111} + c^1_{121}x_{121} + c^1_{211}x_{211} + c^1_{221}x_{221} + c^1_{112}x_{112} + c^1_{122}x_{122} + c^1_{212}x_{212} + c^1_{222}x_{222}$$

$$Min\ Z_2 = c^2_{111}x_{111} + c^2_{121}x_{121} + c^2_{211}x_{211} + c^2_{221}x_{221} + c^2_{112}x_{112} + c^2_{122}x_{122} + c^2_{212}x_{212} + c^2_{222}x_{222}$$

Subject to:

Supply constraints

$$P\{\ x_{111} + x_{112} + x_{121} + x_{122} \le (a^1_1, a^2_1, a^3_1)\} \ge 1 - \theta_1,$$

$$P\{\ x_{211} + x_{212} + x_{221} + x_{222} \le (a^1_2, a^2_2, a^3_2)\} \ge 1 - \theta_2,$$

Demand constraints

$$P\{\ x_{111} + x_{112} + x_{211} + x_{212} \le b^1_1\} \ge 1 - \delta_1,$$

$$P\{\ x_{121} + x_{122} + x_{221} + x_{222} \le b^1_2\} \ge 1 - \delta_2,$$

Conveyance capacity constraints

$$P\{\ x_{111} + x_{121} + x_{211} + x_{221} \le (e^1_1, e^2_1, e^3_1)\} \ge 1 - \sigma_1,$$

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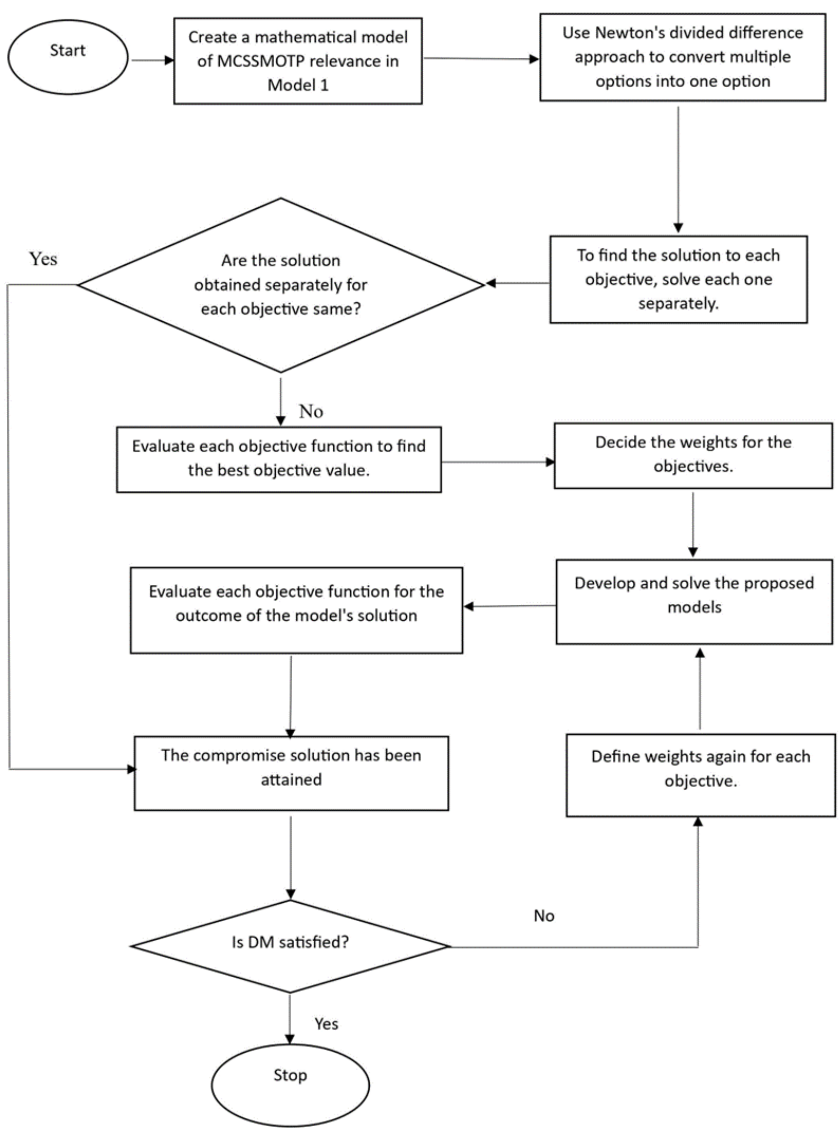

Figure 2: Flow chart for the proposed method

$$P\{\ x_{111}+x_{122}+x_{212}+x_{222} \le (e_2^1, e_2^2, e_2^3)\} \ge 1-\sigma_2,$$
$$x_{ijk} \ge 0, \forall\ i, j\ and\ k$$

where the multi-choice criteria are described as

Table 3: Transportation cost for first objective

		b_1	b_2
c^1_{ij1}	a_1	$15, 18, 20$	$15, 16, 18, 19$
	a_2	$16, 19, 22$	$25, 27, 28, 30$
c^1_{ij2}	a_1	$14, 17, 19, 22$	$12, 15, 19$
	a_2	$11, 18, 20$	$20, 21, 22, 25$

Table 4: Transportation cost for second objective

		b_1	b_2
c^2_{ij1}	a_1	$8, 10, 13$	$11, 13$
	a_2	$9, 12, 15$	$7, 9, 11, 14$
c^2_{ij2}	a_1	$5, 8, 9, 11$	$9, 11$
	a_2	$13, 17$	$11, 13, 14$

Table 5: Supply; mean, variance and significance level

RV	$E(a^1_i)$	$Var(a^1_i)$	$E(a^2_i)$	$Var(a^2_i)$	$E(a^3_i)$	$Var(a^3_i)$	**SL**
a_1	11.28	0.194	5.2	0.25	13.673	0.7712	0.89
a_2	10.1	0.17	5	0.24	13.647	0.489	0.97

RV^*=**Random Variable**,SL^*=**Significance Level**

Table 6: Demand; mean, variance and significance level

RV	$E(b^1_j)$	$Var(b^1_j)$	**SL**
b_1	10	3	0.15
b_2	9	2	0.2

Table 7:conveyance capacity; mean, variance and significance level

RV	$E(e^1_k)$	$Var(e^1_k)$	$E(e^2_k)$	$Var(e^2_k)$	$E(e^3_k)$	$Var(e^3_k)$	**SL**
e_1	11.28	0.18	6.2	0.24	13.07041	0.542	0.97
e_2	10.1	0.16	6	0.25	13.673	0.7412	0.96

$$
\begin{aligned}
\text{Min } \ \mathrm{Z}_1 = {} & [15 + 3w_{111} - 0.5w_{111}(w_{111} - 1)]x_{111} + [15 + w_{121} + 0.5w_{121} \\
& (w_{121} - 1) - \frac{1}{3}w_{121}(w_{121} - 1)(w_{121} - 2)]x_{121} + \\
& [16 + 3w_{211}]x_{211} + [25 + 2w_{221} - 0.5w_{221}(w_{221} - 1) \\
& + \frac{1}{3}w_{221}(w_{221} - 1)(w_{221} - 2)]x_{221} + [14 + 3w_{112} \\
& -0.5w_{112}(w_{112} - 1) + \frac{1}{3}w_{112}(w_{112} - 1)(w_{112} - 2)]x_{112} + \\
& [12 + 3w_{122} - 0.5w_{122}(w_{122} - 1)]x_{122} + [11 + 7w_{212} \\
& - \frac{5}{2}w_{212}(w_{212} - 1)(w_{212} - 2)]x_{212} + [20 + w_{222} \\
& + \frac{1}{3}w_{222}(w_{222} - 1)(w_{222} - 2)]x_{222}
\end{aligned}
$$

$$
\begin{aligned}
\text{Min } \ \mathrm{Z}_2 = {} & [8 + 3v_{111} - 0.5v_{111}(v_{111} - 1)]x_{111} + [11 + 2v_{121}]x_{121} + \\
& [9 + 3v_{211}]x_{211} + [7 + 2v_{221} + \frac{1}{6}v_{221}(v_{221} - 1) \\
& (v_{221} - 2)]x_{221} + [5 + 3v_{112} - v_{112}(v_{112} - 1) \\
& + \frac{1}{2}v_{112}(v_{112} - 1) \\
& (v_{112} - 2)]x_{112} + [9 + 2v_{122}]x_{122} + \\
& [13 + 4v_{212}]x_{212} + [11 + 2v_{222} \\
& - \frac{1}{2}v_{222}(v_{222} - 1)(v_{222} - 2)]x_{222}
\end{aligned}
$$

Subject to:

Supply constraints

$$
\begin{aligned}
\mathrm{x}_{111} + x_{112} + x_{121} + x_{122} \leq {} & 11.28 - 6.08r_1 + 7.277r_1(r_1 - 1) \\
& + \phi^{-1}(0.89)\sqrt{(0.194 + 0.444r_1^2 + 0.491r_1^2(r_1 - 1)^2)}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{x}_{211} + x_{212} + x_{221} + x_{222} \leq {} & 10.1 - 5.1r_2 + 6.874r_2(r_2 - 1) \\
& + \phi^{-1}(0.97)\sqrt{(0.17 + 0.41r_2^2 + 0.404r_2^2(r_2 - 1)^2)}
\end{aligned}
$$

Demand constraints

$$x_{111} + x_{112} + x_{211} + x_{212} \geq 10 + \phi^{-1}(1 - 0.15)\sqrt{3}$$

$$x_{121} + x_{122} + x_{221} + x_{222} \geq 9 + \phi^{-1}(1 - 0.20)\sqrt{2}$$

Conveyance capacity constraints

$$
\begin{aligned}
\mathrm{x}_{111} + x_{121} + x_{211} + x_{221} \leq {} & 11.28 - 5.08r_3 + 5.9752r_3(r_3 - 1) + \\
& \phi^{-1}(0.97)\sqrt{(0.18 + 0.42r_3^2 + 0.4205r_3^2(r_3 - 1)^2)}
\end{aligned}
$$

$$x_{112} + x_{122} + x_{212} + x_{222} \leq 10.1 - 4r_4 + 5.8865r_4(r_4 - 1) + \phi^{-1}(0.96)\sqrt{(0.16 + 0.41r_4^2 + 0.4753r_4^2(r_4 - 1)^2)}$$

$$0 \leq w_{111} \leq 2; 0 \leq w_{121} \leq 3; 0 \leq w_{211} \leq 2; 0 \leq w_{221} \leq 3;$$

$$0 \leq w_{112} \leq 3; 0 \leq w_{122} \leq 2; 0 \leq w_{212} \leq 2; 0 \leq w_{222} \leq 3;$$

$$0 \leq v_{111} \leq 2; 0 \leq v_{121} \leq 1; 0 \leq v_{211} \leq 2; 0 \leq v_{221} \leq 3;$$

$$0 \leq v_{112} \leq 3; 0 \leq v_{122} \leq 1; 0 \leq v_{212} \leq 1; 0 \leq v_{222} \leq 2;$$

$$0 \leq r_1 \leq 2; 0 \leq r_2 \leq 2; 0 \leq r_3 \leq 2; 0 \leq r_3 \leq 2, \quad s = 1, 2, 3, 4$$

$$x_{ijk} \geq 0, \forall\, i, j \text{ and } k \qquad r_s, w_{ijk}, v_{ijk} \in Z^+$$

8 Results and Discussion

Using supply as a multi-choice random parameter, demands, and conveyance as random variables with Normal Distribution, the numerical examples demonstrate the multi-objective function in solid form with constraints. LINGO 18.0 software was used to generate the solutions.

Table 8: The solutions obtained for both the objectives separately, ignoring other objectives, are follows:

S.No.	$Z_1(IS)$	$Z_2(AIS)$	$Z_1(IS)$	$Z_2(AIS)$	X_1	X_2
1					$w_{111} = 1$	$v_{111} = 1$
2					$w_{121} = 0$	$v_{121} = 0$
3					$w_{211} = 0$	$v_{211} = 0$
4					$w_{221} = 0$	$v_{221} = 0$
5					$w_{112} = 0$	$v_{112} = 0$
6					$w_{122} = 0$	$v_{122} = 0$
7					$w_{212} = 0$	$v_{212} = 0$
8	287.6861	140.3208	455.6261	270.2812	$w_{222} = 0$	$v_{222} = 0$
9					$x_{111} = 0$	$x_{111} = 0$
10					$x_{121} = 6.1587$	$x_{121} = 0$
11					$x_{211} = 0$	$x_{211} = 0$
12					$x_{221} = 0$	$x_{221} = 11.6163$
13					$x_{112} = 0$	$x_{112} = 11.8013$
14					$x_{122} = 5.4576$	$x_{122} = 0$
15					$x_{212} = 11.8013$	$x_{212} = 0$
16					$x_{222} = 0$	$x_{222} = 0$

IS^*=**Ideal Solution**, AIS^*= **Anti-Ideal Solution**

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Table 9: Comparison of proposed method

S.N.	W	$M1$	$M2$	$M3$	$R(M1)$	$R(M2)$	$R(M3)$
1	$n_1 = 0.1$	455.626	388.333	393.3888	0.4363	0.5563	0.5674
	$n_2 = 0.9$	140.321	155.073	455.626			
2	$n_1 = 0.2$	399.566	367.338	374.718	0.5436	0.6001	0.619
	$n_2 = 0.8$	148.945	166.524	399.566			
3	$n_1 = 0.3$	399.566	350.568	358.668	0.5436	0.6416	0.6621
	$n_2 = 0.7$	148.945	175.672	399.566			
4	$n_1 = 0.4$	319.222	336.863	344.723	0.7194	0.6762	0.6931
	$n_2 = 0.6$	192.769	183.147	319.222			
5	$n_1 = 0.5$	313.496	325.454	347.125	0.7185	0.6043	0.7123
	$n_2 = 0.5$	197.351	189.370	313.496			
6	$n_1 = 0.6$	313.496	322.961	321.683	0.7185	0.7169	0.6492
	$n_2 = 0.4$	197.351	210.589	313.496			
7	$n_1 = 0.7$	313.496	309.836	312.714	0. 7185	0.7156	0.6931
	$n_2 = 0.3$	197.351	207.108	313.496			
8	$n_1 = 0.8$	287.686	303.748	306.564	0.5637	0.6709	0.6528
	$n_2 = 0.2$	270.281	223.344	287.686			
9	$n_1 = 0.9$	288.786	296.569	298.516	0.5597	0.6206	0.6087
	$n_2 = 0.1$	271.581	243.632	288.786			
10	w.p.	313.496	305.454	347.125	0.7185	0.6043	0.707
		197.351	205.695	313.496			

W^*=**Weights**, M^*=**Method**, R^*=**Ranking**, **w.p.***=**Without preference**

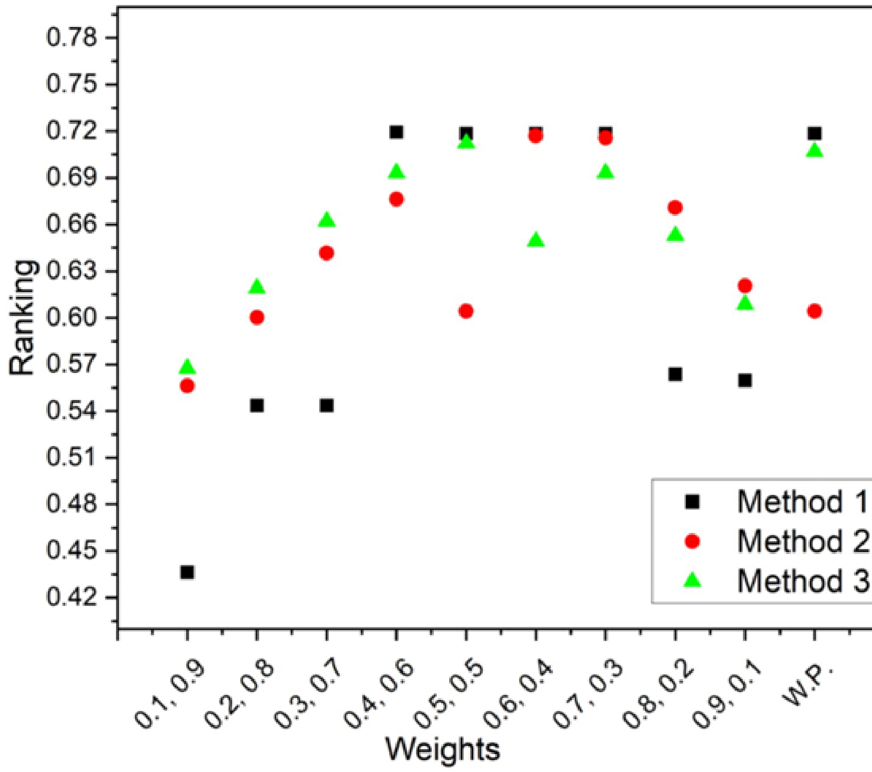

Figure 3: Graphical representation of a comparison of the consistency of the method 1,2 and 3.

A graph that shows the comparison between the method 1, 2 and 3. This graph is showing the rank. We can see that Method 3 is better than method 1 and 2. It's like a race, and our method is winning by being closer to what we want. The graph is like a storyteller that tells us method 3 is good at finding the right answers. Graph is 2 objective of solid stochastic transportation problem.

9 Conclusion

The MCSS-MOTP has been discussed in this research. Solid multi-choice parameters support the provided model's objective function. The transportation problem can be solved most effectively by combining three different ways (the stochastic approach, normal randomness, and Newton's divided difference approach). The constraints parameters are random multi-option parameters. Supply, demand, and conveyance are considered to be random variables with a normal distribution. The deterministic constraints are obtained by applying the chance constrained programming to the probabilistic constraints. The multi-choice parameters were reduced to a single choice with the use of Newton's Divided Difference Interpolation, ensuring that the resulting solution would be ideal. LINGO 18.0 software are applied to solve the above MCSS-MOTP. In this you can work on MCSS-MOTP with fractional objective in future. In the real world, transportation problems are often characterized by uncertainty. For example, the demands at the destinations may be uncertain, or the cost of transportation may fluctuate due to changes in fuel prices. Stochastic programming is a programming approach that can be used to deal with uncertainty in transportation problems.

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