SIMULATION ANALYSIS OF TRAFFIC CONGESTION AT UNSIGNALIZED CIRCULAR ROUNDABOUT USING MICROSCOPIC TRAFFIC FLOW MODEL

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Abstract

The Effective management of roundabout is very important as a part of the current traffic infrastructures in developing countries. A comprehensive understanding of its dynamics especially at unsignalized roundabouts is imperative. In this paper, we study and present a Microscopic Simulation of traffic in an unsignalised roundabout using a Microscopic Car Following model based on the Intelligent Driver Model (IDM). At an unsignalized roundabout, traffic flow relies heavily on the principles of yielding and right-of-way. Vehicles entering the roundabout must yield to circulating traffic already inside. This dynamic often leads to a smoother flow compared to traditional intersections with traffic lights. However, during peak hours or when drivers fail to yield properly, congestion can occur, causing delays. Proper understanding and adherence to roundabout rules are essential for maintaining efficient traffic flow and preventing accidents. Hence, this paper delves into the mathematical modeling of traffic dynamics specifically tailored for unsignalized circular roundabouts. we meticulously examine mathematical models governing traffic flow, treating it as a continuous phenomenon, while delving into their inherent limitations. The impacts of the additional straight and turning movements in the Intelligent Driver Model (IDM) on traffic dynamics inside the circular roundabout are numerically analyzed using MATLAB, employing a fourth-order Runge-Kutta method after Fast Fourier Transform(FFT). Four Vehicles are put in a simulation environment over a specified period of time over which they are considered for analysis and the simulated results culminate in the determination of the range of velocities for three different types of movement considered in the analysis and different types of manoeuvers serving as a gauge for roundabout capacity and management. This study furnishes invaluable insights for scientist and road engineers involved in roundabout design in developing countries.

Key Words: Microscopic model,Intelligent Driver Model (IDM), Simulation analysis, Equation of traffic flow, Unsignalized Circular Roundabout.

1 Introduction

Roundabouts are essential in developing countries for several reasons. Firstly, they help improve traffic flow by minimizing congestion and reducing the need for traffic signals, thus enhancingoverall efficiency. Secondly, they are cost-effective solutions compared to traditional intersectionswith traffic lights, making them more feasible for countries with limited resources. Thirdly,roundabouts enhance road safety by reducing the risk of severe accidents, as they encourageslower speeds and provide clearer traffic patterns. Lastly, they contribute to urban planning and development by accommodating increased traffic volumes as cities expand, offering sustainable solutions for transportation infrastructure.

Developing countries have been relaying on roundabout as a major intersection of many roads[21, 1], However the auto-control of this infrastructure is a big issue due to the cost of energy and technological capacity in many developing countries[8, 4]. One of the major indicators of the development of any nation is the effectiveness,safety and the ability to move people and goods[18, 22]. Unsignalized roundabout are examples of bottlenecks that are interesting in the study of traffic congestion since they are not directly controlled in many developing countries. More recently, many researchers suggested models on networks that are able to describe the dynamics at junctions.[7, 11] and the roundabout were not left behind.[2, 9]. Circular roundabouts have become very popular in recent years due to its management flexibility, safety and mobility [16]. Traffic congestion is a major problem of transport in many countries and it's not expected to end soon [6, 13], Recent researchers in civil engineering and mathematical modeling such as [15, 19] have formulated different microscopic and macroscopic models which described the source of traffic congestion and have in common findings the poor management of different types of intersections such as T-Junctions and Roundabouts[12, 17], The cities have been developed in a disintegrated urban form spreading along major traffic corridors [20]. At the unsignalized Roundabout, we have three different type of manoeuvers, two type of straight movements and a lane changing movement [2, 24].

This research analyses the traffic congestion evolution process using simulation analysis and proposes a theory to analyze roundabout traffic flows and a strategy to determine the effective use of a unsignalized roundabout focusing on three different types of movement inside the roundabout. The generated microscopic model adding lane change in Intelligent Driver Model (IDM) is developed to address the problem of congestion in the roundabout which is the source of accident and source of traffic congestion and delay in most roads in Africa [10, 3]. The objective is to suggest safety measures and Planning improvements in transport infrastructure in many developping countries. Therefore, it is very important to redevelop procedures for integrating various local traffic characteristics for a thorough analysis as planning expanding the roads and resources to achieve it are quite expensive and unsustainable hence, cannot be relied upon in African Countries. we assure that All the parameters and variables chosen for the study are well evaluated, and Our research consists of five steps consecutively,namely: mathematical modeling, model discretisation, computer programming,numerical simulation, and observation of simulation results.

In a real world road network, there are a few specific events that occur as vehicles move around the system: Arrivals In a simulation, only a restricted area of the road system is included and there must be a model that controls the arrival of vehicles to the network. Traffic Flow Once vehicles are in the network, their behavior as they move along roads should be as realistic as possible. Turning When vehicles arrive at intersections, they must choose a direction to continue their travel. In our system we have formulated several essential models that govern the movement, arrival, and decisions of vehicles in the network. A model for the flow of traffic and the behavior of vehicles as they either drive freely or follow other vehicles is given. Also, models for vehicle arrivals and turning behavior,both of which are based on real-world vehicle count data, are presented.

2 MODEL PROBLEM

The Model developed in this research is generated based on Intelligent Driver Model (IDM) one of the types of car-following models. To determine the simulation analysis of traffic congestion by adding lane change to the straight movement inside the roundabout is the major work of this research. In this research we considered the common roundabout of four entering and four exiting roads as one that allows the best potential vehicles throughput while reducing vehicles delay and that has the best use in many developing countries. The major goal is to discover the optimum range of velocities allowed in order to simulate the traffic congestion analysis,time needed for action to clear the roundabout for straight and turning movements. We considered a roundabout in the following geometric forms for mathematical analysis, the two unsignalized T-junctions Combined formed our unsignalized circular roundabout for analysis.

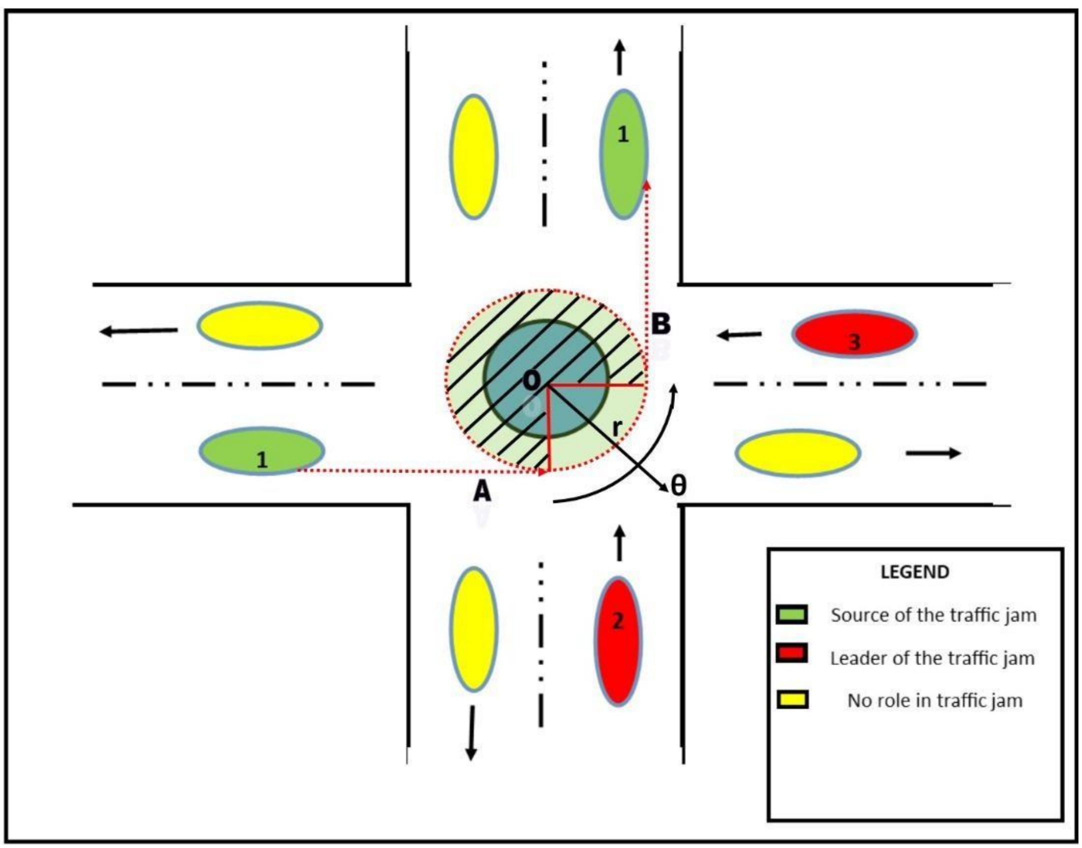

Figure 1: Turning Movement along circular roundabout Image from [14]

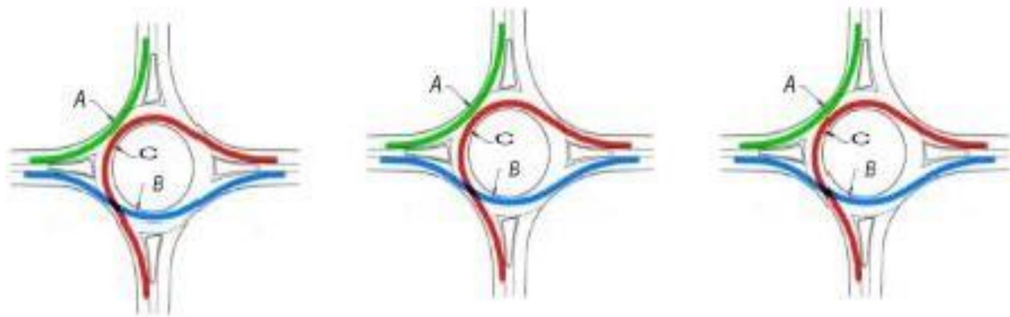

Figure 2: A
Straight Movement

Figure 3: B
First type of turning Movement

Figure 4: C
Second type of turning Movement

Figure 5: The three considered Movements inside the roundabout

3 EQUATIONS OF THE MODEL

A microscopic model of a traffic describes the vehicle (car) following behavior as well as the lane-changing behavior of every vehicle in the traffic. A modern roundabout gives priority to a circulating flow; however, a conventional roundabout gives priority to traffic that enters the roundabout [5, 23], this theory is used to describe all my following equations.The initial condition for motion through the path **A,B or C Paths** (5) is given by $0 < \theta < 2\pi$. The initial condition will keep changing depending on the direction of the turn. For the motion described by AB, a vehicle moves from the straight path and to the connecting path. This motion makes an arc like path. A parametrization of the formed arc is done to help us in describing the motion. If we continue this path, the vehicle(s) moves in such a way that it forms part of a circle with a center somewhere and with radius r which is always constant throughout the motion while θ as the vehicle moves along path AB. The movement is occurring on a two-dimensional plane on the x and y-axis where considering x and y in terms of the polar coordinates; $x = 2\pi r\cos\theta$ and $y = 2\pi r\sin\theta$. Looked this way, r is a vector and is given as $\vec{r}(\theta) = 2\pi r(\cos\theta_i e_1 + \sin\theta_j e_2)$. A velocity v_i is applied on the vehicle as it travels through the desired distance xy or the r path., The effects of the radius,the number of different paths and the gradient used to reach every path are the three parameters to be incorporated in the equation of the traffic motion in order to analyze their effects on the flow in and out of the roundabout.

According to the two parts of the roundabout A,B and C (5) entering and exiting which form the entering and getting out parties of the roundabout the equation of continuity changes to the following equation as they are cars entering and exiting the roundabout,α is characterizing the cars entering the roundabout when β is for vehicles leaving the roundabout over a specified period of time

$$\frac{\partial\rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = \alpha_j(x,t) - \beta_i(x,t) \tag{1}$$

. α_j : The leading car β_i : The leading car The right part of equation (1) is the difference of distance between the leading cars and the following cars exiting in one of the three considered types of movement inside the roundabout which we considered to be constant due to time and position $\gamma(x,t)$,hence we have the final equation of the movement.

$$\frac{\partial \rho(x,t)}{\partial t}+\frac{\partial q(x,t)}{\partial x}=\gamma_k(x,t) \tag{2}$$

The equation of a fluid flow is described by $\rho(x,\ t)$ the density of the traffic stream at a given point in the roundabout x at a given time t and q be the flux of traffic through the point (x,t).

Now we consider a dynamical system of n vehicles, translated as follows;

$$x(t)=\begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(n) \end{pmatrix} \tag{3}$$

The i^{th} component of the $x(t)$ is $x_i(t)$ $i \in \{1,\dots,n\}$ and $x(t)$ is again the vector position. By differentiation, we obtain;

$$v(t,x)=\frac{dx}{dt}=\begin{pmatrix} \frac{d}{dt}x_1(t) \\ \frac{d}{dt}x_2(t) \\ \vdots \\ \frac{d}{dt}x_n(t) \end{pmatrix} \tag{4}$$

It can be seen that $\frac{dx}{dt}=f(t,x)$ where $f(t,x)$ is the vector field of the system. In our case, $f(t,x)=v(t,x)$. By integration;

$\int \frac{dx}{dt}dt=\int v(t,x)dt$ For the turning movement, we now use approximation for x which is given by; $x(t)=\int v(t,\ x)dt$. This approximation will give us a formula for the turning angles θ. For each vehicle i, its position $x_i(t)$ is given by;

$$x_i(t)=\int v_i(t,x)dt; \text{ where } i=1,2,\dots\dots\dots,n \tag{5}$$

Looking at the Fig (2) (3) (4) bellow, there will be three types of movements to be analyzed.
We will make our equation using the idea that no cars appear or vanish. So, any car that enters a part of the roundabout at A will eventually leave at B. The way they go depends on which way they turn and where they exit the roundabout in the three considered movement.

Origin \ Destination	Zone 1	Zone 2	Zone 3	Zone 4
Zone 1	0	r, $0° < \theta < 90°$	r_2, $90° < \theta < 180°$	r_3, $180° < \theta < 270°$
Zone 2	r, $0° < \theta < 90°$	0	r, $0° < \theta < 90°$	r_2, $90° < \theta < 180°$
Zone 3	r_2, $90° < \theta < 180°$	r, $0° < \theta < 90°$	0	r, $0 < \theta < 90°$
Zone 4	r_3, $180° < \theta < 270°$	r_2, $90° < \theta < 180°$	r, $0° < \theta < 90°$	0

$$\dot{x}_i = v_i(t)$$
$$\dot{x}_i(t) = 2\pi r_1[\cos\theta_i(t)e_1 + \sin\theta_i(t)e_2]$$
$$\dot{x}_i(t) = 2\pi r_2[\cos\theta_i(t)e_1 + \sin\theta_i(t)e_2]$$
$$\dot{x}_i(t) = 2\pi r_3[\cos\theta_i(t)e_1 + \sin\theta_i(t)e_2]$$

Where; $\theta_i(t) \cong \int v_i(t)dt$ and e_1 and e_2 are unit orthogonal vectors,the final function of the motion $F(M)$ in the whole roundabout is finally given by a microscopic model of traffic flow that describes the behavior of individual vehicles within the traffic stream. One common microscopic model is the car-following model the Intelligent Driver Model (IDM) which represents the interactions between a vehicle and the vehicle immediately in front of it .

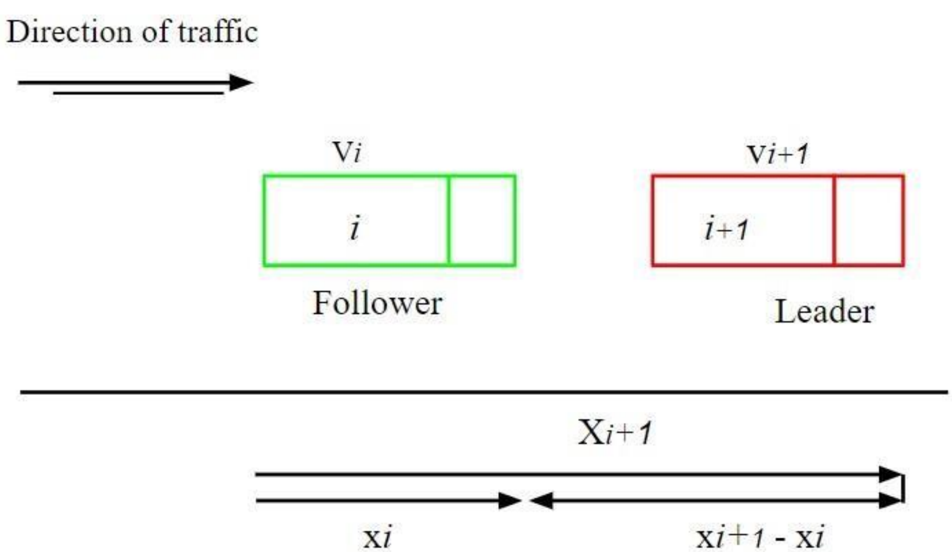

Figure 3: Illustration of Car following Model in straight Movement

The IDM is expressed by the Four following equations:

$$\frac{\partial\rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0 \quad (6)$$

$$v(t + \Delta t) = v(t) + \frac{dv}{dt}\Delta t \quad (7)$$

$$a(t + T) = cv^m \frac{\Delta v}{(\Delta x)^l} \quad (8)$$

$$\frac{dv}{dt} = a\left(1 - \left(\frac{v}{v_0}\right)^\delta - \left(\frac{s^*}{s}\right)^2\right) \quad (9)$$

Where: v is the velocity of the vehicle, a is the maximum acceleration, v_0 is the desired velocity, s is the spacing between the current vehicle and the vehicle in front, s^* is the desired minimum spacing, δ is the exponent determining the sensitivity to velocity differences.

Update Equation of Velocity adding the change of position:

$$x(t+\Delta t) = x(t) + v(t)\Delta t + \frac{1}{2}\frac{dv}{dt}(\Delta t)^2 \tag{10}$$

Final equation for Position in circular roundabout:

$$x(t+\Delta t) = x(t) + v(t)\Delta t + \frac{1}{2}\frac{dv}{dt}(\Delta t)^2 \tag{11}$$

For circular Roundabout the turning angle is given by the following equations:

$$\theta(t+\Delta t) = \theta(t) + \frac{v(t)}{R}\Delta t$$

Where: - θ is the angular position of the vehicle on the roundabout. - R is the radius of the roundabout/circular path.

adding the turning movement to the above equations we have the following equations:

$$x(t+\Delta t) = x(t) + v(t) * r_i[\cos\theta_i(t)e_1 + \sin\theta_i(t)e_2]\Delta t + \frac{1}{2}\frac{dv}{dt}(\Delta t)^2 \tag{12}$$

Where; $\theta_i(t) \cong \int v_i(t)dt$ and e_1 and e_2 are unit orthogonal vectors, the final function of the motion $F(M)$ in the whole roundabout. Which helps to get the final update equation for the position of vehicle i in Microscopic Model inside the circular roundabout motion.

For straight Movement:

$$F(Str_{Mov}) = \begin{cases} v_1 = \dot{x}_i(t+\Delta t) = \dot{x}(t) + v(t)\Delta t + \frac{1}{2}\frac{dv}{dt}(\Delta t)^2 \\ v_2 = \dot{x}_i(t+\Delta t) = \dot{x}(t) + v(t)\Delta t + \frac{1}{2}\frac{dv}{dt}(\Delta t)^2 \\ v_3 = \dot{x}_i(t+\Delta t) = \dot{x}(t) + v(t)\Delta t + \frac{1}{2}\frac{dv}{dt}(\Delta t)^2 \\ \frac{\partial\rho(x,t+\Delta t)}{\partial t} + v_{\max}\frac{\partial\rho(x,t+\Delta t)}{\partial x} - \frac{v_{\max}}{\rho_{\max}}\frac{\partial\rho^2(x,t+\Delta t)}{\partial x} = \gamma_1(x,t+\Delta t) \\ \frac{\partial\rho(x,t+\Delta t)}{\partial t} + v_{\max}\frac{\partial\rho(x,t+\Delta t)}{\partial x} - \frac{v_{\max}}{\rho_{\max}}\frac{\partial\rho^2(x,t+\Delta t)}{\partial x} = \gamma_2(x,t+\Delta t) \\ \frac{\partial\rho(x,t+\Delta t)}{\partial t} + v_{\max}\frac{\partial\rho(x,t+\Delta t)}{\partial x} - \frac{v_{\max}}{\rho_{\max}}\frac{\partial\rho^2(x,t+\Delta t)}{\partial x} = \gamma_3(x,t+\Delta t) \end{cases} \tag{13}$$

For two types of turning movement we have the following system:

$$F(Turn_{Mov}) = \begin{cases} v_1 = \dot{x}_i(t+\Delta t) = \dot{x}(t) + v(t) * r_1[\cos\theta_i(t)e_1 + \sin\theta_i(t)e_2]\Delta t + \frac{1}{2}\frac{dv}{dt}(\Delta t)^2 \\ v_2 = \dot{x}_i(t+\Delta t) = \dot{x}(t) + v(t) * r_2[\cos\theta_i(t)e_1 + \sin\theta_i(t)e_2]\Delta t + \frac{1}{2}\frac{dv}{dt}(\Delta t)^2 \\ v_3 = \dot{x}_i(t+\Delta t) = \dot{x}(t) + v(t) * r_3[\cos\theta_i(t)e_1 + \sin\theta_i(t)e_2]\Delta t + \frac{1}{2}\frac{dv}{dt}(\Delta t)^2 \\ \frac{\partial\rho(x,t+\Delta t)}{\partial t} + v_{\max}\frac{\partial\rho(x,t+\Delta t)}{\partial x} - \frac{v_{\max}}{\rho_{\max}}\frac{\partial\rho^2(x,t+\Delta t)}{\partial x} = \gamma_1(r_1[\cos\theta_i(t)e_1 + \sin\theta_i(t)e_2], t+\Delta t) \\ \frac{\partial\rho(x,t+\Delta t)}{\partial t} + v_{\max}\frac{\partial\rho(x,t+\Delta t)}{\partial x} - \frac{v_{\max}}{\rho_{\max}}\frac{\partial\rho^2(x,t+\Delta t)}{\partial x} = \gamma_2(r_2[\cos\theta_i(t)e_1 + \sin\theta_i(t)e_2], t+\Delta t) \\ \frac{\partial\rho(x,t+\Delta t)}{\partial t} + v_{\max}\frac{\partial\rho(x,t+\Delta t)}{\partial x} - \frac{v_{\max}}{\rho_{\max}}\frac{\partial\rho^2(x,t+\Delta t)}{\partial x} = \gamma_3(r_3[\cos\theta_i(t)e_1 + \sin\theta_i(t)e_2], t+\Delta t) \end{cases} \tag{14}$$

These equations describe how the velocity and position of each vehicle change over time based on their interactions with neighboring vehicles. The IDM captures the tendency of drivers to maintain a desired velocity while keeping a safe distance from the vehicle ahead, while also considering the ability to accelerate and decelerate.

In practice, various modifications and extensions to the IDM exist to account for additional factors such as driver heterogeneity, traffic conditions, and road geometry. Microscopic traffic flow models like the IDM are fundamental tools in understanding and simulating traffic dynamics at the individual vehicle level.

4 Method of Solution

The obtained non-linear first order partial differential equation in the model will be transformed to first order ordinary differential equation using fast fourier transform(FFT). For the next step I fixed the initial parameters for initial conditions for the specific roundabout and solve for ρ in equation numerically in MATLAB using a fourth order Runge-kutta by assuming periodic boundary conditions of maximum velocity and maximum density, in order to determine the movement of each considered vehicles.

5 NUMERICAL ANALYSIS

5.1 Discretisation using Fast Fourier Transform(FFT)

By Fast Fourier Transform (FFT) we transform our partial differential equation(PDE) to ordinary differential equation(ODE)

$$(t+\Delta t) \neq 0:$$

$$\begin{cases} F\left(\frac{\partial\rho}{\partial t}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\partial\rho}{\partial t}e^{i\omega x}dx = \frac{\partial\hat{\rho}(k,t+\Delta t)}{\partial t} \\ F\left(\frac{\partial\rho}{\partial x}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\partial\rho}{\partial x}e^{i\omega x}dx = i\omega\hat{\rho}(x,t+\Delta t) \\ F\left(\frac{\partial\rho^2}{\partial x}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\partial\rho^2}{\partial x}e^{i\omega\pi}dx = i\omega\hat{\rho}^2(x,t+\Delta t) \\ F(\gamma(x,t+\Delta t)) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\gamma(x,t+\Delta t)e^{i\omega x}dx = \gamma(x,t+\Delta t)\times\frac{1}{2\pi}\delta(t+\Delta t) \\ F(v_{\max}) = \frac{1}{2\pi}\int_{-\infty}^{\infty}v_{\max}e^{i\omega x}dx = v_{\max}\times 2\pi\delta(t+\Delta t) \\ F\left(\frac{v_{\max}}{\rho_{\max}}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{v_{\max}}{\rho_{\max}}e^{i\omega x}dx = \frac{v_{\max}}{\rho_{\max}}\times 2\pi\delta(t+\Delta t) \end{cases} \quad (15)$$

By replacing in the equation of continuity I have the following equation,

$$\frac{\partial\hat{\rho}(K,t+\Delta t)}{\partial t} + v_{\max}\times\frac{1}{2\pi}\delta(K)iK\hat{\rho}(x,t+\Delta t) - \frac{v_{\max}}{\rho_{\max}}\times\frac{1}{2\pi}\delta(K)iK\hat{\rho}^2(K,t+\Delta t) = \gamma(x,t+\Delta t)\times\frac{1}{2\pi}\delta(K) \quad (16)$$

I have the final equation of continuity in the ordinary differential form:

For (t+Δ t) ≠ 0 :

$$\frac{d\hat{\rho}}{dt} + v_{\max} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho} - \frac{v_{\max}}{\rho_{\max}} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho}^2 = \gamma(x, t+\Delta t) \times \frac{1}{2\pi}\delta(K) \qquad (17)$$

we have the final model which will be analyzed using simulated data for one straight movement and two turning movement:

$$F(Str_{Mov}) = \begin{cases} v_1 = \dot{x}_i(t+\Delta t) \\ v_2 = \dot{x}_i(t+\Delta t) \\ v_3 = \dot{x}_i(t+\Delta t) \\ \frac{d\hat{\rho}}{dt} + v_{\max} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho} - \frac{v_{\max}}{\rho_{\max}} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho}^2 = \gamma_1(x, t+\Delta t) \times \frac{1}{2\pi}\delta(K) \\ \frac{d\hat{\rho}}{dt} + v_{\max} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho} - \frac{v_{\max}}{\rho_{\max}} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho}^2 = \gamma_2(x, t+\Delta t) \times \frac{1}{2\pi}\delta(K) \\ \frac{d\hat{\rho}}{dt} + v_{\max} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho} - \frac{v_{\max}}{\rho_{\max}} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho}^2 = \gamma_3(x, t+\Delta t) \times \frac{1}{2\pi}\delta(K) \end{cases} \qquad (18)$$

For first turning Movement we incorporate the gradient of turning in different directions inside the roundabout:

$$F(Fir_{T}urn_{Mov}) = \begin{cases} v_1 = \dot{x}_i(t+\Delta t) = r_1[\cos\theta_i(t+\Delta t)e_1 + \sin\theta_i(t+\Delta t)e_2] \\ v_2 = \dot{x}_i(t+\Delta t) = r_2[\cos\theta_i(t+\Delta t)e_1 + \sin\theta_i(t+\Delta t)e_2] \\ v_3 = \dot{x}_i(t+\Delta t) = r_3[\cos\theta_i(t+\Delta t)e_1 + \sin\theta_i(t+\Delta t)e_2] \\ \frac{d\hat{\rho}}{dt} + v_{\max} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho} - \frac{v_{\max}}{\rho_{\max}} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho}^2 = \gamma_1(x, t+\Delta t) \times \frac{1}{2\pi}\delta(K) \\ \frac{d\hat{\rho}}{dt} + v_{\max} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho} - \frac{v_{\max}}{\rho_{\max}} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho}^2 = \gamma_2(x, t+\Delta t) \times \frac{1}{2\pi}\delta(K) \\ \frac{d\hat{\rho}}{dt} + v_{\max} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho} - \frac{v_{\max}}{\rho_{\max}} \times \frac{1}{2\pi}\delta(K)iK\hat{\rho}^2 = \gamma_3(x, t+\Delta t) \times \frac{1}{2\pi}\delta(K) \end{cases}$$

(19)

For second type of turning Movement we incorporate the gradient of turning in different directions inside the roundabout and the radius becomes a conference of the circle around the radius of the turning movement:

$$F(Sec_{T}urn_{Mov}) =$$

$$\begin{cases} v_1 = \dot{x}_i(t+\Delta t) = 2\pi r_1[\cos\theta_i(t+\Delta t)e_1 + \sin\theta_i(t+\Delta t)e_2] \\ v_2 = \dot{x}_i(t+\Delta t) = 2\pi r_2[\cos\theta_i(t+\Delta t)e_1 + \sin\theta_i(t+\Delta t)e_2] \\ v_3 = \dot{x}_i(t+\Delta t) = 2\pi r_3[\cos\theta_i(t+\Delta t)e_1 + \sin\theta_i(t+\Delta t)e_2] \\ \frac{d\hat{\rho}}{dt} + v_{\max} \times \frac{1}{2\pi}\delta(K) iK\hat{\rho} - \frac{v_{\max}}{\rho_{\max}} \times \frac{1}{2\pi}\delta(K) iK\hat{\rho}^2 = \gamma_1(x, t+\Delta t) \times \frac{1}{2\pi}\delta(K) \\ \frac{d\hat{\rho}}{dt} + v_{\max} \times \frac{1}{2\pi}\delta(K) iK\hat{\rho} - \frac{v_{\max}}{\rho_{\max}} \times \frac{1}{2\pi}\delta(K) iK\hat{\rho}^2 = \gamma_2(x, t+\Delta t) \times \frac{1}{2\pi}\delta(K) \\ \frac{d\hat{\rho}}{dt} + v_{\max} \times \frac{1}{2\pi}\delta(K) iK\hat{\rho} - \frac{v_{\max}}{\rho_{\max}} \times \frac{1}{2\pi}\delta(K) iK\hat{\rho}^2 = \gamma_3(x, t+\Delta t) \times \frac{1}{2\pi}\delta(K) \end{cases}$$

(20)

6 Results and Discusions

In this section, I will present the results obtained from a simulation analysis involving four cars engaged in various types of movements within a roundabout. The study considers three distinct scenarios: straight movement inside the roundabout (case A), and two turning movements inside the roundabout (case B and Case C). Furthermore, we extend our analysis to include simulations involving multiple vehicles executing the same movement, and the outcomes of these simulations will also be discussed. The simulation employs the Fast Fourier Transform (FFT) method in conjunction with the fourth-order Runge-Kutta scheme, implemented using the MATLAB programming language. This approach allows for the generation of code and graphical representations within the MATLAB environment. The simulation setup involves the random generation of cars, each assigned specific movement patterns within the roundabout located in Bujumbura [14].

The study investigates the dynamics of vehicle movements within the roundabout under different scenarios. For instance, when two cars approach the roundabout, with one intending to turn right and the other left, the car making the left turn is given priority, while the right-turning vehicle waits until the intersection clears. Similarly, when multiple movements are occurring simultaneously, cars must yield to others until the roundabout is free for passage.
The analysis yields insights into the efficiency and safety of vehicle movements within the roundabout. Graphical representations, such as the Red, Blue, and Green lines, depict the trajectories of turning vehicles, highlighting their paths and interactions with other vehicles. The simulation results shed light on various factors influencing traffic flow, including the number of vehicles, their respective movements, and the sequencing of their actions within the roundabout. Understanding the dynamics of vehicle movements within roundabouts is crucial for optimizing traffic flow and ensuring road safety. The insights gained from this simulation study can inform urban planning and traffic management strategies aimed at improving transportation systems in cities. By identifying potential bottlenecks and optimizing traffic flow patterns, policymakers can enhance the overall efficiency and safety of road networks, benefiting both motorists and pedestrians alike.

6.1 Simulation Analysis for the straight Movement(Case A (2))

In our analysis, we focused on roundabouts with radii ranging from 15 meters to 45 meters. Over a period of 5 minutes, we meticulously assessed the velocities of vehicles along their designated paths, particularly in segments devoid of conflicts within the roundabout. Our evaluation involved adjusting velocities based on several factors, including the distance to the roundabout, the time required to navigate it, and the velocities needed upon exiting the roundabout. To accomplish this, we integrated an Intelligent Drive Model (IDM), which incorporates algorithms for adjusting velocities according to changing conditions. Specifically, we concentrated on the straight movement path within the roundabout's conflict-free zone.

Our simulation results revealed intriguing insights. For instance, we observed that drivers typically begin to decelerate approximately 25 meters before approaching the roundabout. Furthermore, it takes an average of 2 to 3 minutes for vehicles to complete their passage through the roundabout, maintaining a minimum velocity of 45 kilometers per hour upon exit. These findings underscore the nuanced dynamics of vehicle movement within roundabouts and highlight the importance of incorporating intelligent driving models to optimize traffic flow and ensure safe navigation.

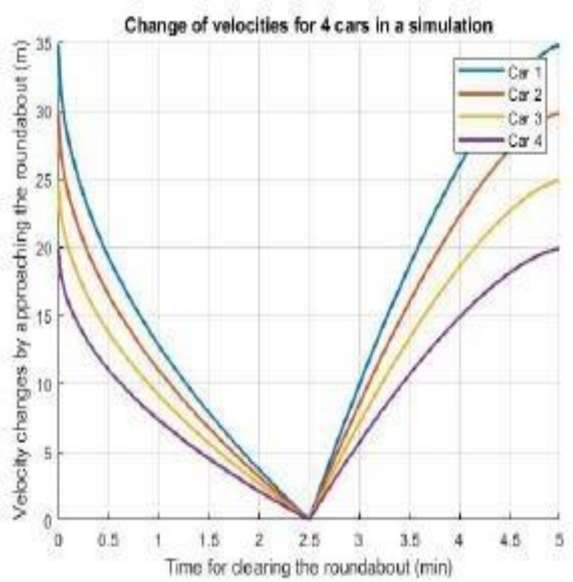

Figure 7: Four Cars for Simulation Analysis Straight Movement

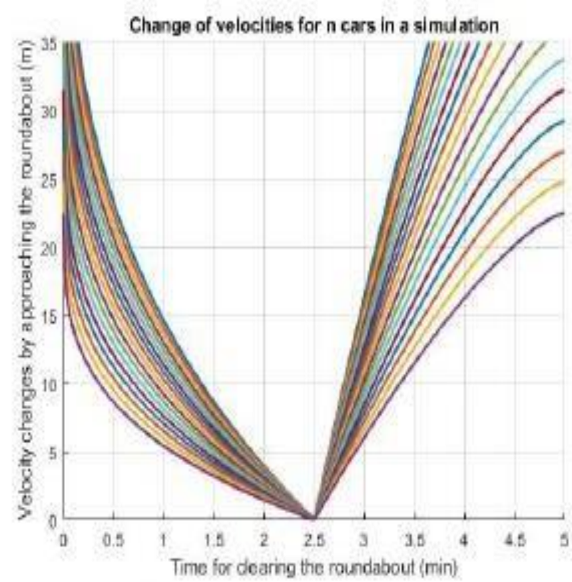

Figure 8: n Cars for Simulation Analysis Straight Movement

In the context of an unsignalized roundabout, a V-shaped pattern in velocities and accelerations typically indicates a rapid increase followed by a sharp decrease in both parameters as vehicles navigate the roundabout. As vehicles approach the entry point, velocities and accelerations rise as drivers accelerate to merge into circulating traffic. Once within the roundabout, velocities and accelerations decline as vehicles negotiate the curvature and adjust speed to safely exit. This V-shaped pattern reflects the dynamic nature of roundabout traffic flow, with acceleration and deceleration occurring in response to changing geometric and traffic conditions, promoting efficient and safe movement through the intersection. As vehicles approach the roundabout entry, velocities and accelerations typically increase as drivers accelerate to merge into the circulating flow of traffic. This acceleration phase forms the upward slope of the V-shape. Once within the roundabout, velocities remain relatively high but begin to decrease gradually as vehicles negotiate the curvature of the roadway. Simultaneously, accelerations decrease as drivers maintain a steady speed or slightly decelerate to navigate the roundabout safely. This deceleration phase constitutes the downward slope of the V-shape. The V-shaped profile in velocities and accelerations thus reflects the transition from entry acceleration to circulating speed maintenance and exit deceleration. Understanding this pattern aids in designing efficient roundabouts and implementing traffic management strategies to ensure smooth and safe traffic flow through these intersections.

6.2 Simulation Analysis for the first turning Movement (Case B (3))

The considered roundabout should have the radius varying from **15 m** to **45 m**. within a period of **5 min**, we evaluated the velocities along the movements for the movement without conflict part in our simulation Analysis. Changing the velocities according to the distance approaching the roundabout and the required time to clear the roundabout and the velocities of New path after Leaving the roundabout while the lane changing is incorporated in intellignet drive model(IDM), we considered the straight movement that path in the roundabout without conflict zone. The simulation analysis show that it takes around from 25m for a driver with a stimulus of straight movement to start slowing down in approachinf the roundabout and from 2 to three minutes to exist the roundabout with a minimum velocity of 45km/h to exist the roundabout.

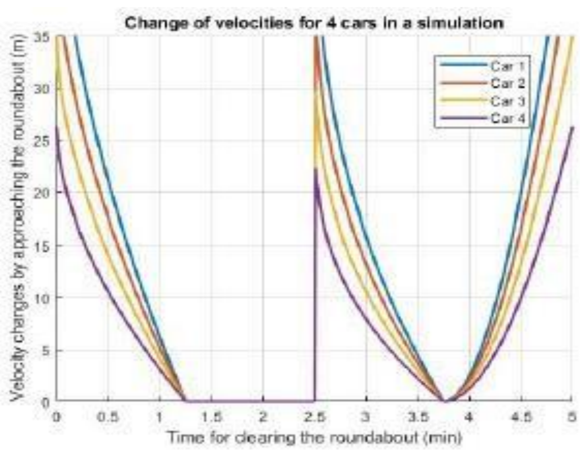

Figure 9: Four Cars for Simulation Analysis First type of turning Movement

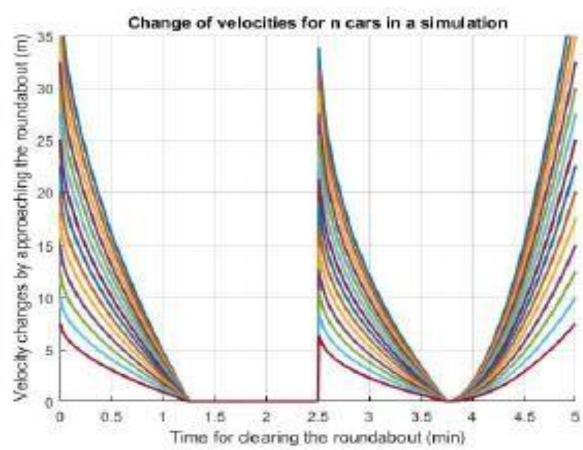

Figure 10: n Cars for Simulation Analysis First type of turning Movement

In the context of turning movements within an unsignalized roundabout, a W-shaped pattern in velocities and accelerations may occur due to the interaction between multiple vehicles navigating the roundabout. Initially, as vehicles approach the entry point, velocities and accelerations increase as they accelerate to merge into the circulating flow of traffic, forming the first peak of the W-shape. Subsequently, as vehicles negotiate the curvature of the roundabout, velocities and accelerations may decrease as drivers adjust their speed to safely navigate the turn, resulting in the first trough of the W-shape. As vehicles continue through the roundabout, velocities and accelerations rise again as they straighten their trajectory and exit the roundabout, forming the second peak of the W-shape. Finally, as vehicles complete the turn and exit the roundabout, velocities and accelerations decrease once more, returning to a lower level, forming the second trough of the W-shape. This complex pattern illustrates the dynamic nature of turning movements within unsignalized roundabouts, influenced by factors such as traffic volume, roundabout geometry, and driver behavior.

6.3 Simulation Analysis for the Second turning Movement(Case C (4))

The considered roundabout should have the radius varying from **15 m** to **45 m**. within a period of **5 min**, we evaluated the velocities along the movements for the movement without conflict part in our simulation Analysis. Changing the velocities according to the distance approaching the roundabout and the required time to clear the roundabout and the velocities of New path after Leaving the roundabout while the lane changing is incorporated in intelligent drive model(IDM), we considered the straight movement that path in the roundabout without conflict zone. The simulation analysis show that it takes around from 25m for a driver with a stimulus of straight movement to start slowing down in approaching the roundabout and from 2 to three minutes to exist the roundabout with a minimum velocity of 45km/h to exist the roundabout.

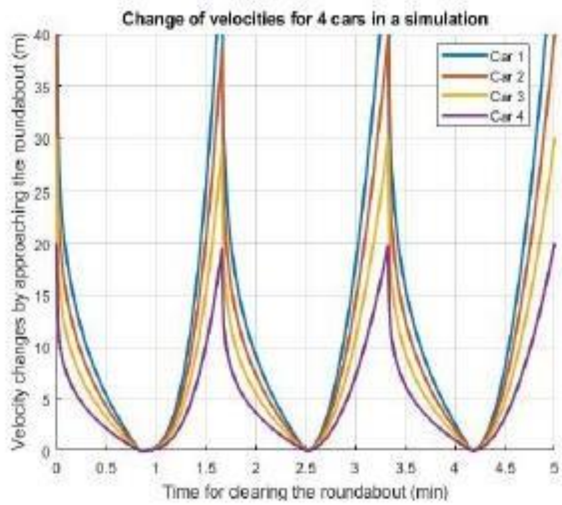

Figure 11: Four Cars for Simulation Analysis Second type of turning Movement

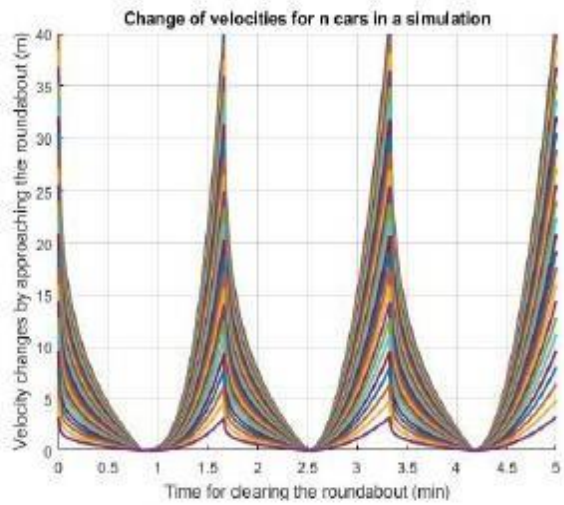

Figure 12: n Cars for Simulation Analysis Second type of turning Movement

In the context of turning movements within an unsignalized roundabout, a combined W and V-shaped pattern in velocities and accelerations may illustrate the intricate dynamics of vehicle interactions and navigation through the roundabout. Initially, as vehicles approach the entry point, velocities and accelerations increase as drivers accelerate to merge into the circulating flow of traffic, forming the upward slope of the V-shape. Once within the roundabout, velocities and accelerations may fluctuate due to adjustments made by drivers to navigate curves and yield to other vehicles, resulting in periodic decreases and increases, akin to the peaks and troughs of a W-shape. As vehicles exit the roundabout, velocities and accelerations typically decrease as drivers decelerate to transition back onto the main road, forming the downward slope of the V-shape. This combined pattern reflects the complex interplay of acceleration, deceleration, and navigation within unsignalized roundabouts, influenced by factors such as traffic flow, geometry, and driver behavior. Understanding these patterns is crucial for optimizing roundabout design and enhancing traffic flow efficiency and safety.

7 Conclusions and Future Work

The main objectif of this work was to examine how the radius (r), the gradient of the turning movement (θ) and the number of lanes (n) affect throughput velocities in the roundabout. We build the mathematical model for the specific type of the roundabout and test the output by considering the effects of any considered parameter on velocities to all scenarios of all getting out movements in the roundabout.The simulation analysis presented in this study offers valuable insights into the behavior of vehicles within a roundabout environment. By employing advanced simulation techniques and considering various scenarios, we gain a better understanding of traffic dynamics and can identify opportunities for enhancing transportation systems. Moving forward, further research and experimentation can build upon these findings to develop more comprehensive models for traffic analysis and urban planning.

In order to determine the optimal roundabout design, he simulation results show that the proposed model and method were effective and feasible.The proposed method has been implemented to this area and has a good effect.The followed research will be the collaborative research on the road channelization and the traffic information optimization and the study of the real time traffic simulation system using real data.To validate the model with real data on roundabout would be the future work.

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Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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