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SOME APPLICATIONS OF INTERVAL-VALUED SUBSETHOOD MEASURES WHICH ARE DEFINED BY INTERVAL-VALUED CHOQUET INTEGRALS IN TRADE EXPORTS BETWEEN KOREA AND ITS TRADING PARTNERS

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Abstract. In this paper, we consider subsethood measures introduced by Fan et al. [3] and the interval-valued Choquet integral with respect to a fuzzy measure of interval-valued fuzzy sets. Based on such a focus, we define three types of interval-valued subsethood measures and provide four interval-valued fuzzy sets to animal product exports between Korea and four selected trading partners.

In particular, we investigate a strong interval-valued subsethood measure defined by the interval-valued Choquet integral which represents the degree of trade surplus between Korea and three trading partners in terms of the model of trade transactions with the United States and Korea

1. Introduction

Zadeh[18] first developed fuzzy sets and Murofushi-Sugeno [11] have studied fuzzy measures and Choquet integrals. Subsequently, using set-valued analysis theory developed by Aumann[1], we studied interval-valued Choquet integrals and their related applications(see[5, 6, 7, 8, 9]). In particular, through the restudy of the interval-valued Choquet integral in 2004 by Zhang-Guo-Liu[21], this research has been developed in a much more systematical

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manner. Xuechang [16], Zeng-Li[19] have examined fuzzy entropy, distance and similarity measures, the likes of which form three key concepts of fuzzy set theory. Ruan-Kerre [12] also introduced various fuzzy implication operators and the Choquet integral were suggested for the first time by Choquet [2]. Further studied by Murofushi-Sugeno [9], Jang-Kwon [10], and Jang [12] provide some interesting interpretations of fuzzy measures and the Choquet integral. Subjective probability and Choquet expected utility were studied as an application of Choquet integral and form another pivotal component of fuzzy sets and information theories(see[13, 14, 15, 20]).

A subsethood measure refers to the degree to which a fuzzy set is a subset of another fuzzy set. Many researchers have contributed to the area of a fuzzy subsethood measure that is closely related to the various tools introduced above (see[6, 7, 8, 11]). Their efforts have considered axiomatizing the properties of a subsethoods measure.

In this paper, we consider subsethood measures introduced by Fan et al. [3] and the interval-valued Choquet integral with respect to a fuzzy measure of interval-valued fuzzy sets. Based on such a focus, we define three types of interval-valued subsethood measures and provide four interval-valued fuzzy sets to animal product exports between Korea and four selected trading partners. In configuring the four interval-valued fuzzy sets, the original data(see[4]) used had to be slightly modified to produce Table $A4$. In order to calculate the interval-valued Choquet integral, the rules (46) and (48) were introduced.

Furthermore, we also investigate a strong interval-valued subsethood measure defined by an interval-valued Choquet integral which represents the degree of trade surplus between Korea and 3 trading partners in terms of the model of trade transactions with the United States and Korea. The information for the above degree of surplus is of great significance in providing accurate comparative figures on the size of trade that exists between the four countries that trade with Korea.

2. Preliminaries and definitions

Throughout this paper, we write X to denote a set,

$$F(X) = \{A | A = \{(x, m_A(x)) |\ x \in X\},\ m_A : X \longrightarrow [0,1] \text{ is a function}\} \tag{1}$$

stands for the set of fuzzy sets in X(see[18]). We note that m_A expresses the membership of a fuzzy set A, A^c is the complement of A, that is,

$$A^c = \{(x, m_{A^c}(x)) |\ m_{A^c}(x) = 1 - m_A(x),\ x \in X\}. \tag{2}$$

Recall that for $A, B \in F(X)$, $A \subset B$ if and only if $m_A(x) \leq m_B(x)$, for all $x \in X$, and for $A \in F(X)$, $[A] = \{x \in X |\ m_A(x) > 0\}$, $n(A)$ is the cardinal number of crisp set $[A]$, and $M(A)$ is the fuzzy cardinal of A, that is, $M(A) = \sum_{x \in X} m_A(x)$. Now, we introduce three types of subsethood measure in Fan et al. [3].

Definition 2.1. ([3]) Let $c : F(X) \times F(X) \longrightarrow [0,1]$ be a function.

(1) c is called a strong subsethood measure if c has the following properties;

(S_1) if $A \subset B$, then $c(A, B) = 1$;
(S_2) if $A \neq \emptyset$ then $c(A, B)$

$$(\mathrm{S}_3) \quad \text{if } A \subset B \subset C, \text{ then } c(C,A) \leq c(B,A) \text{ and } c(C,A) \leq c(C,B). \tag{3}$$

(2) c is called a subsethood measure if c has the following properties;

$$\begin{array}{ll} (\mathrm{C}_1) & \text{if } A \subset B, \text{ then } c(A,B)=1; \\ (\mathrm{C}_2) & c(X,\emptyset)=0 \\ (\mathrm{C}_3) & \text{if } A \subset B \subset C, \text{ then } c(C,A) \leq c(B,A) \text{ and } c(C,A) \leq c(C,B). \end{array} \tag{4}$$

(3) c is called a weak subsethood measure if c has the following properties;

$$\begin{array}{ll} (\mathrm{W}_1) & c(\emptyset,\emptyset)=1, \ c(\emptyset,\emptyset)=1, \ c(A,B)=1; \text{and } c(X,X)=1 \\ (\mathrm{W}_2) & \text{if } A \neq \emptyset \; rmand \; A \cap B = \emptyset, \text{ then } c(A,B)=0; \\ (\mathrm{W}_3) & \text{if } A \subset B \subset C, \text{ then } c(C,A) \leq c(B,A) \text{ and } c(C,A) \leq c(C,B). \end{array} \tag{5}$$

We also list the set-theoretical arithmetic operators for the set of subintervals of an unit interval $[0,1]$ in $\mathbb{R}$. We denote

$$I([0,1]) = \{\overline{a} = [a^-, a^+] |\ a^-, a^+ \in [0,1] \text{ and } a^- \leq a^+\}. \tag{6}$$

For any $a \in [0,1]$, we define $a = [a,a]$.

Definition 2.2. ([5, 6, 7, 8, 9]) If $\overline{a} = [a^-, a^+], \overline{b} = [b^-, b^+] \in I([0,1])$, and $k \in [0,1]$, then the addition, scalar multiplication, minimum, maximum, inequality, subset, multiplication, and division as follows;
(1) $\overline{a} + \overline{b} = [a^- + b^-, a^+ + b^+]$,
(2) $k\overline{a} = [ka^-, ka^+]$,
(3) $\overline{a} \wedge \overline{b} = [a^- \wedge b^-, a^+ \wedge b^+]$,
(4) $\overline{a} \vee \overline{b} = [a^- \vee b^-, a^+ \vee b^+]$,
(5) $\overline{a} \leq \overline{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$,
(6) $\overline{a} < \overline{b}$ if and only if $\overline{a} \leq \overline{b}$ and $\overline{a} \neq \overline{b}$,
(7) $\overline{a} \subset \overline{b}$ if and only if $b^- \leq a^-$ and $a^+ \leq b^+$,
(8) $\overline{a} \otimes \overline{b} = [a^- b^-, a^+ b^+]$, and
(9) $\overline{a} \oslash \overline{b} = [a^-/b^- \wedge a^+/b^+, a^-/b^- \vee a^+/b^+]$.

From Definition 2.1 (9), the following theorem can be easily obtained.

Theorem 2.1. (1) *If* $\overline{a} = [a^-, a^+] \in I([0,1])$*, then* $\overline{a} \oslash \overline{a} = 1$.
(2) *If* $\overline{b} = [b^-, b^+] \in I([0,1])$ *and* $b^- > 0$*, then* $1 \oslash \overline{b} = [1/b^+, 1/b^-]$.

Definition 2.3. ([5, 6, 8, 9, 21]) Let (X, Ω) be a measurable space. (1) A fuzzy measure on X is a real-valued function $\mu : \Omega \longrightarrow [0,1]$ satisfies

$$\begin{array}{ll} (\mathrm{i}) & \mu(\emptyset)=0 \\ (\mathrm{ii}) & \mu(E_1) \leq \mu(E_2) \ \text{ whenever } \ E_1, E_2 \in \Omega \ \text{ and } \ E_1 \subset E_2. \end{array} \tag{7}$$

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(2) A fuzzy measure μ is said to be continuous from below if for any sequence $\{E_n\} \subset \Omega$ and $E \in \Omega$, such that

$$\text{if } E_n \uparrow E, \quad \text{then} \quad \lim_{n\to\infty} \mu(E_n) = \mu(E). \tag{8}$$

(3) A fuzzy measure μ is said to be continuous from above if for any sequence $\{E_n\} \subset \Omega$ and $E \in \Omega$ such that

$$\text{if} \quad E_n \downarrow E, \quad \text{then} \quad \lim_{n\to\infty} \mu(E_n) = \mu(E). \tag{9}$$

(5) A fuzzy measure μ is said to be continuous if it is continuous from below and continuous from above.

Definition 2.4. ([5, 6, 8, 9, 21]) (1) Let $A \in F(X)$. The Choquet integrals with respect to a fuzzy measure μ of a fuzzy set A on a set $E \in \Omega$ is defined by

$$C_{\mu,E}(A) = (C)\int_E m_A d\mu = \int_0^1 \mu_{E,m_A}(r)dr, \tag{10}$$

where $\mu_{E,m_A}(r) = \mu(\{x \in X|\ m_A(x) > r\} \cap E)$ and the integral on the right-hand side is an ordinary one.

(2) A measurable function is said to be integrable if $C_\mu(A) = C_{\mu,X}(A)$ exists.

It is well known that if X is a finite set, that is, $X = \{x_1, x_2, \cdots, x_n\}$, and $A \in F(X)$, then we have

$$C_\mu(A) = \sum_{i=1}^n m_A(x_{(i)}) \left(\mu(E_{(i)}) - \mu(E_{(i+1)})\right), \tag{11}$$

where $(\cdot)$ indicate a permutation on $\{1, 2, \cdots, n\}$ such that $m_A(x_{(1)}) \leq m_A(x_{(2)}) \leq \cdots \leq m_A(x_{(n)})$ and also $E_{(i)} = \{(i), (i+1), \cdots, (n)\}$ and $E_{(n+1)} = \emptyset$.

Theorem 2.2. *Let $A, B \in F(X)$. (1) If $A \leq B$, then $C_\mu(A) \leq C_\mu(B)$.*

(2) If we define $(m_A \vee m_B)(x) = m_A(x) \vee m_B(x)$ for all $x \in X$, then $C_\mu(A \vee B) \geq C_\mu(A) \vee C_\mu(B)$.

(3) If we define $(m_A \wedge m_B)(x) = m_A(x) \wedge m_B(x)$ for all $x \in X$, then $C_\mu(A \wedge B) \geq C_\mu(A) \wedge C_\mu(B)$.

3. Three types of interval-valued subsethood measures defined by interval-valued Choquet integral

In this section, we consider the interval-valued Choquet integral and list some properties of them.

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Definition 3.1. ([5, 6, 8, 9, 21]) (1) The interval-valued Choquet integral of an interval-valued measurable function $\overline{f} = [f^-, f^+]$ on $E \in \Omega$ is defined by

$$\overline{C}_{\mu,E}(\overline{f}) = (C)\int_E \overline{f}d\mu = \left\{C_{\mu,E}(f) \mid f \in S(\overline{f})\right\}, \tag{12}$$

where $S(\overline{f})$ is the family of measurable selection of $\overline{f}$.

(2) $\overline{f}$ is said to be integrable if $\overline{C}_\mu(\overline{f}) = \overline{C}_{\mu,X}(\overline{f}) \neq \emptyset$.

(3) $\overline{f}$ is said to be Choquet integrably bounded if there is an integrable function g such that

$$||\overline{f}(x)|| = sup_{r\in\overline{f}(x)}|r| \leq g(x), \quad \text{for all } x \in X. \tag{13}$$

Theorem 3.1. *([5, 6, 21]) (1) If a closed set-valued measurable function $\overline{f}$ is integralble and if $E_1 \subset E_2$ and $E_1, E_2 \in \Omega$, then $\overline{C}_{\mu,E_1}(\overline{f}) \leq \overline{C}_{\mu,E_2}(\overline{f})$.*

(2) If a fuzzy measure μ is continuous , and a closed set-valued measurable function $\overline{f}$ is Choquet integrably bounded, then $\overline{C}_\mu(\overline{f})$ is a closed set.

(3) If the fuzzy measure μ is continuous, and an interval-valued measurable function $\overline{f} = [f^-, f^+]$ is Choquet integrably bounded, then we have

$$\overline{C}_\mu(\overline{f}) = [C_\mu(f^-), C_\mu(f^+)]. \tag{14}$$

Let $IF(X)$ be the set of all interval-valued fuzzy sets which are defined by

$$\overline{A} = \left\{(x, \overline{m}_{\overline{A}}) | \overline{m}_{\overline{A}} : X \longrightarrow I([0,1])\right\}. \tag{15}$$

By using Theorem 2.2 and Theorem 3.1(3), we easily obtain the following theorem.

Theorem 3.2. *Let $\overline{A}, \overline{B} \in IF(X)$. (1) If $\overline{A} \leq \overline{B}$, then $\overline{C}_\mu(\overline{A}) \leq \overline{C}_\mu(\overline{B})$.*

(2) If we define $(\overline{m}_{\overline{A}} \vee \overline{m}_{\overline{B}})(x) = \overline{m}_{\overline{A}}(x) \vee \overline{m}_{\overline{B}}(x)$ for all $x \in X$, then $\overline{C}_{\overline{\mu}}(\overline{A} \vee \overline{B}) \geq \overline{C}_\mu(\overline{A}) \vee \overline{C}_\mu(\overline{B})$.

(3) If we define $(\overline{m}_{\overline{A}} \wedge \overline{m}_{\overline{B}})(x) = \overline{m}_{\overline{A}}(x) \wedge \overline{m}_{\overline{B}}(x)$ for all $x \in X$, then $\overline{C}_\mu(\overline{A} \wedge \overline{B}) \geq \overline{C}_\mu(\overline{A}) \wedge \overline{C}_\mu(\overline{B})$.

We denote $\overline{m}_{\overline{A}} = [m_{A^-}, m_{A^+}]$ and define three types of interval-valued subsethood measures on $IF(X) \times IF(X)$ as follows:

Definition 3.2. Let $\overline{c} : IF(X) \times IF(X) \longrightarrow I([0,1])$ be a function.

(1) $\overline{c}$ is called a strong interval-valued subsethood measure if $\overline{c}$ has the following properties;

$$\begin{array}{ll} (\mathrm{IS}_1) & \text{if } \overline{A} \subset \overline{B}, \text{ then } \overline{c}(\overline{A}, \overline{B}) = 1; \\ (\mathrm{IS}_2) & \text{if } \overline{A} \neq \emptyset \text{ then } \overline{c}(\overline{A}, \overline{B}) \\ (\mathrm{IS}_3) & \text{if } \overline{A} \subset \overline{B} \subset \overline{C}, \text{ then } \overline{c}(\overline{C}, \overline{A}) \leq \overline{c}(\overline{B}, \overline{A}) \text{ and } \overline{c}(\overline{C}, \overline{A}) \leq \overline{c}(\overline{C}, \overline{B}). \end{array} \tag{16}$$

(2) c is called an interval-valued subsethood measure if c has the following properties;

$$\begin{array}{ll} (\mathrm{IC}_1) & \text{if } \overline{A} \subset \overline{B}, \text{ then } \overline{c}(\overline{A}, \overline{B}) = 1; \\ (\mathrm{IC}_2) & \overline{c}(\overline{X}, \overline{\emptyset}) = 0 \\ (\mathrm{IC}_3) & \text{if } \overline{A} \subset \overline{B} \subset \overline{C}, \text{ then } \overline{c}(\overline{C}, \overline{A}) \leq \overline{c}(\overline{B}, \overline{A}) \text{ and } \overline{c}(\overline{C}, \overline{A}) \leq \overline{c}(\overline{C}, \overline{B}). \end{array} \tag{17}$$

(3) c is called a weak interval-valued subsethood measure if c has the following properties;

$$\begin{array}{ll} (\mathrm{IW}_1) & c(\emptyset, \emptyset) = 1,\ c(\emptyset, \emptyset) = 1,\ c(A, B) = 1; \text{and } c(X, X) = 1 \\ (\mathrm{IW}_2) & \text{if } A \neq \emptyset\ rmand\ A \cap B = \emptyset, \text{ then } c(A, B) = 0; \\ (\mathrm{IW}_3) & \text{if } A \subset B \subset C, \text{ then } c(C, A) \leq c(B, A) \text{ and } c(C, A) \leq c(C, B). \end{array} \tag{18}$$

Let $IF^*(X) = \{\overline{A} \in IF(X) |\ \overline{A}$ has the integrably bounded funstion $\overline{m}_{\overline{A}}\}$. Note that if X is a finite set, then $IF(X) = IF^*(X)$. Finally, we give three types of interval-valued subsethood measures defined by the Choquet integral with respect to a fuzzy measure on $IF^*(X)$. By Theorem 3.1 (3), we note that for $\overline{A} = [A^-, A^+], \overline{B} = [B^-, B^+] \in IF^*(X)$, $\overline{C}_\mu(\overline{A}) = [C_\mu(A^-), C_\mu(A^+)]$, $\overline{C}_\mu(\overline{B}) = [C_\mu(B^-), C_\mu(B^+)]$, and $\overline{C}_\mu(\overline{A} \wedge \overline{B}) = [C_\mu(A^- \wedge B^-), C_\mu(A^+ \wedge B^+)]$.

Theorem 3.3. *Let X be a set. If we define an interval-valued function $\overline{c}_1 : IF^*(X) \times IF^*(X) \longrightarrow I([0,1])$,*

$$\overline{c}_1(\overline{A}, \overline{B}) = \begin{cases} 1, & \text{if} \overline{A} = \overline{B} = \overline{\emptyset}, \\ \frac{\overline{C}_\mu(\overline{A} \wedge \overline{B})}{\overline{C}_\mu(\overline{A})}, & \text{if not}, \end{cases} \tag{19}$$

then $\overline{c}_1$ is a strong interval-valued subsethood measure on $IF^*(X)$.

Proof. (IS_1) If $\overline{A} \leq \overline{B}$ and $\overline{B} = \emptyset$, that is, $\overline{A} = \overline{B} = \emptyset$, then by the definition of $\overline{c_1}$, we have $\overline{c_1}(\overline{A}\overline{B}) = 1$. If $\overline{A} \leq \overline{B}$ and $\overline{B} \neq \emptyset$, then $\overline{m}_{\overline{A}} \leq \overline{m}_{\overline{B}}$. Thus, we have $m_{A^-} \leq m_{B^-}$ and $m_{A^+} \leq m_{B^+}$. Hence, we get

$$\begin{aligned} \overline{c}_1(\overline{A}, \overline{B}) &= \frac{\overline{C}_\mu(\overline{A} \wedge \overline{B})}{\overline{C}_\mu(\overline{A})} \\ &= \frac{[C_\mu(A^- \wedge B^-), C_\mu(A^+ \wedge B^+)]}{[C_\mu(A^-), C_\mu(A^+)]} \\ &= \frac{[C_\mu(A^-), C_\mu(A^+)]}{[C_\mu(A^-), C_\mu(A^+)]} = 1. \end{aligned} \tag{20}$$

(IS_2) If $\overline{A} \neq \overline{\emptyset}$ and $\overline{A} \wedge \overline{B} = \overline{\overline{\emptyset}}$, then we get $0 = \overline{m}_{\overline{\emptyset}} = \overline{m}_{\overline{A} \wedge \overline{B}}$ and hence $\overline{c}_1(\overline{A}, \overline{B}) = \frac{\overline{C}_\mu(\overline{A} \wedge \overline{B})}{\overline{C}_\mu(\overline{A})} = 0$.

(IS_3) If $\overline{A} \leq \overline{B} \leq \overline{C}$, then we have

$$\overline{m}_{\overline{A}} \leq \overline{m}_{\overline{B}} \leq \overline{m}_{\overline{C}} \tag{21}$$

and hence, by (21), we have

$$\overline{m}_{\overline{C}} \wedge \overline{m}_{\overline{A}} \leq \overline{m}_{\overline{B}} \wedge \overline{m}_{\overline{A}}. \tag{22}$$

Thus, by (21) and (22), we get

$$\overline{C}_\mu(\overline{B}) \leq \overline{C}_\mu(\overline{C}), \text{ and } \overline{C}_\mu(\overline{C} \wedge \overline{A})) \leq \overline{C}_\mu(\overline{B} \wedge \overline{A}). \tag{23}$$

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Note that if $\overline{C} = \overline{\emptyset}$, then $\overline{B} = \overline{\emptyset}$. By using (23), we have

$$\begin{aligned}\overline{c}_1(\overline{C}, \overline{A}) &= \begin{cases} 1, & \text{if}\, \overline{C} = \overline{\emptyset}, \\ \frac{\overline{C}_\mu(\overline{C} \wedge \overline{A})}{\overline{C}_\mu(\overline{C})}, & \text{if}\, \overline{C} \neq \overline{\emptyset} \end{cases} \\ &= \begin{cases} 1, & \text{if}\, \overline{B} = \overline{\emptyset}, \\ \frac{\overline{C}_\mu(\overline{B} \wedge \overline{A})}{\overline{C}_\mu(\overline{B})}, & \text{if}\, \overline{B} \neq \overline{\emptyset} \end{cases} \\ &= \overline{c}_1(\overline{B}, \overline{A}). \end{aligned} \tag{24}$$

From (21), we also get

$$\overline{C}_\mu(\overline{C} \wedge \overline{A})) \leq \overline{C}_\mu(\overline{B} \wedge \overline{B}). \tag{25}$$

By using (25), we also have

$$\begin{aligned}\overline{c}_1(\overline{C}, \overline{A}) &= \begin{cases} 1, & \text{if}\, \overline{C} = \overline{\emptyset}, \\ \frac{\overline{C}_\mu(\overline{C} \wedge \overline{A})}{\overline{C}_\mu(\overline{C})}, & \text{if}\, \overline{C} \neq \overline{\emptyset} \end{cases} \\ &= \begin{cases} 1, & \text{if}\, \overline{B} = \overline{\emptyset}, \\ \frac{\overline{C}_\mu(\overline{C} \wedge \overline{B})}{\overline{C}_\mu(\overline{C})}, & \text{if}\, \overline{B} \neq \overline{\emptyset} \end{cases} \\ &= \overline{c}_1(\overline{C}, \overline{B}). \end{aligned} \tag{26}$$

Therefore, $\overline{c}_1$ is a strong interval-valued subsethood measure.

Theorem 3.4. *Let X be a set.If we define an interval-valued function $\overline{c}_2 : IF^*(X) \times IF^*(X) \longrightarrow I([0,1])$,*

$$\overline{c}_2(\overline{A}, \overline{B}) = \begin{cases} 1, & \text{if}\, \overline{A} = \overline{B} = \overline{\emptyset}, \\ \frac{\overline{C}_\mu(\overline{B})}{\overline{C}_\mu(\overline{A} \vee \wedge \overline{B})}, & \text{if not}, \end{cases} \tag{27}$$

then $\overline{c}_2$ is an interval-valued subsethood measure on $IF^*(X)$.

Proof. (IC_1) If $\overline{A} = \overline{B} = \overline{\emptyset}$, then $\overline{c}_2(\overline{A}, \overline{B}) = 1$. Since $\overline{A} \leq \overline{B}$, we have $\overline{m}_{\overline{A}} \leq \overline{m}_{\overline{B}}$. Thus, we get

$$\overline{c}_2(\overline{A}, \overline{B}) = \frac{\overline{C}_\mu(\overline{B})}{\overline{C}_\mu(\overline{A} \vee \overline{B})} = 1. \tag{28}$$

(IC_2) By the definition of $\overline{c}_2$, we have

$$\overline{c}_2(\overline{X}, \overline{\emptyset}) = \frac{\overline{C}_\mu(\overline{\emptyset})}{\overline{C}_\mu(\overline{X} \vee \overline{\emptyset})} = 0. \tag{29}$$

(IC_3) If $\overline{A} \leq \overline{B} \leq \overline{C}$, then we have

$$\overline{m}_{\overline{A}} \leq \overline{m}_{\overline{B}} \leq \overline{m}_{\overline{C}} \tag{30}$$

and hence, by (30), we hsve

$$\overline{C}_\mu(\overline{C}) \geq \overline{C}_\mu(\overline{B}),\ \ \overline{C}_\mu(\overline{C} \vee \overline{A}) = \overline{C}_\mu(\overline{C}),\ and\, \overline{C}_\mu(\overline{B} \vee \overline{A}) = \overline{C}_\mu(\overline{B}). \tag{31}$$

Therefore by using (31), we have

$$\overline{c}_2(\overline{C}, \overline{A}) = \frac{\overline{C}_\mu(\overline{A})}{\overline{C}_\mu(\overline{C} \vee \overline{A})}$$

$$\begin{aligned} &= \frac{\overline{C}_\mu(\overline{A})}{\overline{C}_\mu(\overline{C})} \\ &\le \frac{\overline{C}_\mu(\overline{A})}{\overline{C}_\mu(\overline{B})} \\ &= \frac{\overline{C}_\mu(\overline{A})}{\overline{C}_\mu(\overline{B} \vee \overline{A})} = \overline{c}_2(\overline{B}, \overline{A}), \end{aligned} \tag{32}$$

and

$$\begin{aligned} \overline{c}_2(\overline{C}, \overline{A}) &= \frac{\overline{C}_\mu(\overline{A})}{\overline{C}_\mu(\overline{C} \vee \overline{A})} \\ &= \frac{\overline{C}_\mu(\overline{A})}{\overline{C}_\mu(\overline{C})} \\ &\le \frac{\overline{C}_\mu(\overline{B})}{\overline{C}_\mu(\overline{C})} \\ &= \frac{\overline{C}_\mu(\overline{B})}{\overline{C}_\mu(\overline{C} \vee \overline{B})} = \overline{c}_2(\overline{C}, \overline{B}). \end{aligned} \tag{33}$$

Therefore, $\overline{c}_2$ is an interval-valued subsethood measure.

The following definition $\overline{c}_3$ has some problem because of the definition of a complement of interval-valued fuzzy set. So, we note that for interval-valued fuzzy sets $\overline{A} = [A^-, A^+]$, the modified complement $\overline{A}^{mc}$ of $\overline{A}$ is defined by

$$\overline{m}_{\overline{A}^{mc}}(x) = [m_{A^+}(x), 1]. \tag{34}$$

Through this definition $\overline{A}^{mc}$, we can take note of the followings:

$$\begin{aligned} &\text{(i)} \quad \overline{m}_{\overline{A}^{mc}} = [m_{A^+}, 1] \\ &\text{(ii)} \quad \overline{m}_{\overline{A} \vee \overline{A}^{mc}} = [m_{A^-}, 1] \\ &\text{(iii)} \quad \text{if } \overline{A} \le \overline{B}, \text{ then } \overline{B}^{mc} \le \overline{A}^{mc}. \end{aligned} \tag{35}$$

Theorem 3.5. *Let X be a set. If we define an interval-valued function $\overline{c}_3 : IF^*(X) \times IF^*(X) \longrightarrow I([0,1])$,*

$$\overline{c}_3(\overline{A}, \overline{B}) = \frac{\overline{C}_\mu(\overline{A}^{mc}) \vee \overline{C}_\mu(\overline{B})}{\overline{C}_\mu(\overline{A} \vee \overline{A}^{mc} \vee \overline{B} \vee \overline{B}^{mc})} \tag{36}$$

then $\overline{c}_3$ is an interval-valued subsethood measure on $IF^*(X)$.

Proof. (IW_1) By the definition of $\overline{c}_3$, we get

$$\begin{aligned} \overline{c}_3(\overline{\emptyset}, \overline{\emptyset}) &= \frac{\overline{C}_\mu(\overline{\emptyset}) \vee \overline{C}_\mu(\emptyset)}{\overline{C}_\mu(\overline{\emptyset} \vee \overline{\emptyset}^{mc} \vee \overline{\emptyset} \vee \overline{\emptyset}^{mc})} \\ &= \frac{\overline{C}_\mu(\overline{X}) \vee \overline{C}_\mu(\overline{\emptyset})}{\overline{C}_\mu(\overline{\emptyset} \vee \overline{X} \vee \overline{\emptyset} \vee \overline{X})} \\ &= \frac{\overline{C}_\mu(\overline{X})}{\overline{C}_\mu(\overline{X})} = 1. \end{aligned} \tag{37}$$

Similarly, we have $\overline{c}_3(\overline{\emptyset}, \overline{X}) = 1$ and $\overline{c}_3(\overline{X}, \overline{X}) = 1$.

(IW_2) By the definition of $\bar{c}_3$, we have

$$\begin{aligned}\bar{c}_3(\overline{X},\overline{\emptyset}) &= \frac{\overline{C}_\mu(\overline{X}^{mc}) \vee \overline{C}_\mu(\overline{\emptyset})}{\overline{C}_\mu(\overline{X} \vee \overline{X}^{mc} \vee \overline{\emptyset} \vee \overline{\emptyset}^{mc})} \\ &= \frac{\overline{C}_\mu(\overline{\emptyset})}{\overline{C}_\mu(\overline{X})} = \frac{0}{1} = 0. \end{aligned} \tag{38}$$

(IW_3) If $\overline{A} \le \overline{B} \le \overline{C}$, then we have

$$\overline{m}_{\overline{A}} \le \overline{m}_{\overline{B}} \le \overline{m}_{\overline{C}}. \tag{39}$$

Thus, by (35)(i) and (39), we have

$$\overline{m}_{\overline{C}^{mc}} \le \overline{m}_{\overline{B}^{mc}} \le \overline{m}_{\overline{A}^{mc}}. \tag{40}$$

From (40), we get

$$\begin{aligned}\overline{C}_\mu(\overline{C} \vee \overline{C}^{mc} \vee \overline{A} \vee \overline{A}^{mc}) &= \overline{C}_\mu(\overline{C} \vee \overline{A}^{mc}) \\ &\ge \overline{C}_\mu(\overline{B} \vee \overline{A}^{mc}) \\ &= \overline{C}_\mu(\overline{B} \vee \overline{B}^{mc} \vee \overline{A} \vee \overline{A}^{mc}) \end{aligned} \tag{41}$$

Therefore by using (41), we have

$$\begin{aligned}\bar{c}_3(\overline{C},\overline{A}) &= \frac{\overline{C}_\mu(\overline{C}^{mc}) \vee \overline{C}_\mu(\overline{A})}{\overline{C}_\mu(\overline{C} \vee \overline{C}^{mc} \vee \overline{A} \vee \overline{A}^{cm})} \\ &\le \frac{\overline{C}_\mu(\overline{B}^{mc}) \vee \overline{C}_\mu(\overline{A})}{\overline{C}_\mu(\overline{B}^{cm} \vee \overline{B}^{cm} \vee \overline{A} \vee \overline{A}^{cm})} \\ &= \bar{c}_3(\overline{B},\overline{A}). \end{aligned} \tag{42}$$

Similarly, we have

$$\bar{c}_3(\overline{C},\overline{A}) \le \bar{c}_3(\overline{C},\overline{B}). \tag{43}$$

Therefore, $\bar{c}_3$ is a weak interval-valued subsethood measure.

4. Applications

In this section, by using the of Harmonized system (HS) product code data for product categories $(s_1, \ldots, s_5)$ between Korea and its trading partners (that is, Korea-United States, Korea-New Zealand, Korea-Turkey, and Korea-Indea) over the 2010-2013 period, we construct four interval-valued fuzzy sets related with four countries and calculate a strong interval-valued subsethood measure $\bar{c}_1$.

Note that the product code definitions have been provided by the UN Comtrade's online data base(see[22]) and the relevant categories are defined as follows:

s_1. Live animals: animal products.

s_2. Meat and edible meat offal.

s_3. Fish and crustacreans, mollusks and other aguatic invertebrates.

s_4. Dairy produce: bird's eggs; natural honey; edible products of animal origin, not elsewhere specified or included.

s_5. Products of animal origin; not elsewhere specified or included.

Firstly, we denote that s is year, $a(s)$ is trade value, and $u(a(s))$ is the utility of $a(s)$. By using the $\overline{u}(\overline{a}(s))$ for the trade values of animal product exports between Korea and selected trading partners for HS Product Codes $i = 1, 2, 3, 4, 5$ in Table A1 in [4], we can calculate the Choquet integral of an utility on the set of trade values (in USD) that represent Korea's trading relationship with a particular country for years $2010, 2012, 2012, 2013$. Let $S = \{s_1, s_2, s_3, s_4, s_5\}$ and $\overline{a}(s)$ be the interval-valued trade value of s during four years and

$$\overline{u}(\overline{a}(s)) = \left[\sqrt{\frac{a^-(s)}{100141401}}, \sqrt{\frac{a^+(s)}{100141401}}\right] \tag{44}$$

be an interval-valued utility of $\overline{a}(s)$. The following table $A1$ is used to create four interval-valued fuzzy sets required to draw a strong subsethood measure $\overline{c}_1$ defined by the interval-valued Choquet integral.

Table A1: The $\overline{u}(\overline{a}(s))$ for the trade value of animal product exports between Korea and selected trading partners for HS Product Codes s_i for $i = 1, 2, 3, 4, 5$.

TP	s	$\bar{a}(s)$(USD)	$\bar{u}(\bar{a}(s))$
USA	s_1	$[144949, 364918] = \bar{a}(s^{(1)})$	[0.03542, 0.06037]
	s_2	$[144949, 997539] = \bar{a}(s^{(3)})$	[0.03542, 0.09981]
	s_3	$[74866073, 100141401] = \bar{a}(s^{(5)})$	[0.86464, 1.00000]
	s_4	$[3722326, 5016833] = \bar{a}(s^{(4)})$	[0.19280, 0.22382]
	s_5	$[1017895, 863858] = \bar{a}(s^{(2)})$	[0.09288, 0.10082]
NZ	s_1	$[1589, 6650] = \bar{a}(s^{(2)})$	[0.00398, 0.00815]
	s_2	$[0, 0] = \bar{a}(s^{(1)})$	[0.00000, 0.00000]
	s_3	$[46632301, 91263506] = \bar{a}(s^{(5)})$	[0.68240, 0.95464]
	s_4	$[113751, 277350] = \bar{a}(s^{(3)})$	[0.03370, 0.05263]
	s_5	$[218022, 393025] = \bar{a}(s^{(4)})$	[0.04666, 0.06265]
TR	s_1	$[150, 6900] = \bar{a}(s^{(4)})$	[0.00122, 0.00830]
	s_2	$[0, 0] = \bar{a}(s^{(1)})$	[0.00000, 0.00000]
	s_3	$[199874, 2532837] = \bar{a}(s^{(5)})$	[0.04468, 0.15904]
	s_4	$[0, 0] = \bar{a}(s^{(2)})$	[0.00000, 0.00000]
	s_4	$[0, 0] = \bar{a}(s^{(3)})$	[0.00000, 0.00000]
IND	s_1	$[450, 1300] = \bar{a}(s^{(2)})$	[0.00212, 0.00360]
	s_2	$[12135, 50630] = \bar{a}(s^{(5)})$	[0.00992, 0.05551]
	s_3	$[1865, 8695] = \bar{a}(s^{(3)})$	[0.00432, 0.00932]
	s_4	$[12135, 30938] = \bar{a}(s^{(4)})$	[0.00992,0.02249]
	s_5	$[0, 0] = \bar{a}(s^{(2)})$	[0.00000, 0.00000]

We remark that in order to calculate the interval-valued Choquet integrals for four interval-valued fuzzy sets, we modified four interval-valued trading values for the United States and the India (see Table 5 in [4]) as follows;

$$[286892, 364918] = \overline{a}(s_1) \text{ and } [30005, 997539] = \overline{a}(s_2) \tag{45}$$

are changed by

$$\left[\frac{286892 + 3005}{2}, 364918\right] = \overline{a}(s_2) \text{ and } \left[\frac{286892 + 3005}{2}, 997539\right] = \overline{a}(s_2), \tag{46}$$

and

$$[2656, 50630] = \overline{a}(s_2) \text{ and } [21614, 30938] = \overline{a}(s_4) \tag{47}$$

are changed by

$$\left[\frac{2656+21614}{2}, 50630\right] = \overline{a}(s_2) \text{ and } \left[\frac{2656+21614}{2}, 30938\right] = \overline{a}(s_4), \tag{48}$$

From Table $A1$, we construct four interval-valued fuzzy sets from S to $I([0,1])$, $\overline{U} = \overline{U}(USA), \overline{N} = \overline{N}(NZ), \overline{T} = \overline{T}(TR)$, and $\overline{I} = \overline{I}(ID)$ as follows;

$$\begin{aligned} \overline{U} &= \{(s_1, [0.03542, 0.06037], (s_2, [0.03542, 0.09981], (s_3, [0.86464, 1.00000]), \\ &\quad (s_4, [0, 19280, 0.22382]), (s_5, [0.09288, 0.10082])\}, \end{aligned} \tag{49}$$

$$\begin{aligned} \overline{N} &= \{(s_1, [0.00398, 0.00815], (s_2, [0.00000, 0.00000], (s_3, [0.68240, 0.95464]), \\ &\quad (s_4, [0, 03370, 0.05263]), (s_5, [0.04666, 0.06265])\}, \end{aligned} \tag{50}$$

$$\begin{aligned} \overline{T} &= \{(s_1, [0.00122, 0.00830], (s_2, [0.00000, 0.00000], (s_3, [0.04468, 0.15904]), \\ &\quad (s_4, [0, 00000, 0.00000]), (s_5, [0.00000, 0.00000])\}, \end{aligned} \tag{51}$$

and

$$\begin{aligned} \overline{I} &= \{(s_1, [0.00212, 0.00360], (s_2, [0.00992, 0.05551], (s_3, [0.00432, 0.00932]), \\ &\quad (s_4, [0, 00992, 0.02249]), (s_5, [0.00000, 0.00000])\}, \end{aligned} \tag{52}$$

In order to calculate the interval-valued Choquet integral, the four interval-valued fuzzy sets((49), (50), (51), (52)) were made to be increasing interval-valued fuzzy sets as follows:

$$\begin{aligned} \overline{U} &= \{(s_{(1)}, [0.03542, 0.06037]), (s_{(2)}, [0.03542, 0.09981]), (s_{(3)}, [0.09288, 0.10082]), \\ &\quad (s_{(4)}, [0, 19280, 0.22382]), (s_{(5)}, [0.86464, 1.00000])\}, \end{aligned} \tag{53}$$

$$\begin{aligned} \overline{N} &= \{(s_{(1)}, [0.00000, 0.00000]), (s_{(2)}, [0.00398, 0.00815]), (s_{(3)}, [0, 03370, 0.05263]), \\ &\quad (s_{(4)}, [0.04666, 0.06265]), (s_{(5)}, [0.68240, 0.95464])\}, \end{aligned} \tag{54}$$

$$\begin{aligned} \overline{T} &= \{(s_{(1)}, [0.00000, 0.00000], (s_{(2)}, [0, 00000, 0.00000], (s_{(3)}, [0.00000, 0.00000]), \\ &\quad (s_{(4)}, [0.00122, 0.00830]), (s_{(5)}, [0.04468, 0.15904])\}, \end{aligned} \tag{55}$$

and

$$\begin{aligned} \overline{I} &= \{(s_{(1)}, [0.00000, 0.00000], (s_{(2)}, [0.00212, 0.00360], (s_{(3)}, [0.00432, 0.00932]), \\ &\quad (s_{(4)}, [0, 00992, 0.02249]), (s_{(5)}, [0.00992, 0.05551])\}, \end{aligned} \tag{56}$$

Now, the more diversified export items, the higher fuzzy measure are defined as follows(see[4]):

$$\begin{aligned} &\mu(E_{(6)}) = \mu(\emptyset) = 0, \mu(E_{(5)}) = \mu_1(\{s_{(5)}\}) = 0.1, \;\; \mu(E_{(4)}) = \mu_1(\{s_{(4)}, s_{(5)}\}) = 0.2, \\ &\mu(E_{(3)}) = \mu_1(\{s_{(3)}, s_{(4)}, s_{(5)}\}) = 0.4, \;\; \mu(E_{(2)}) = \mu_1(\{s_{(2)}, s_{(3)}, s_{(4)}, s_{(5)}\}) = 0.7, \\ &\mu(E_{(1)}) = \mu_1(\{s_{(1)}, s_{(2)}, s_{(3)}, s_{(4)}, s_{(5)}\}) = 1. \end{aligned} \tag{57}$$

By using two interval-valued fuzzy sets (53), (54) and the above fuzzy measure (57), we can calculate a strong interval-valued subsethood measure for $\overline{c}_1(\overline{U}, \overline{N})$ as follows:

$$\begin{aligned} \overline{C}_\mu(\overline{U}) &= [C_\mu(U^-), C_\mu(U^+)] \\ \overline{C}_\mu(\overline{U} \wedge \overline{N}) &= [C_\mu(U^- \wedge N^-), C_\mu(U^+ \wedge N^+)], \end{aligned} \tag{58}$$

and

$$\begin{aligned} \overline{c}_1(\overline{U}, \overline{N}) &= \frac{\overline{C}_\mu(\overline{U} \wedge \overline{N})}{\overline{C}_\mu(\overline{U})} \\ &= \left[\frac{C_\mu(U^- \wedge N^-)}{C_\mu(U^-)} \wedge \frac{C_\mu(U^+ \wedge N^+)}{C_\mu(U^+)}, \frac{C_\mu(U^- \wedge N^-)}{C_\mu(U^-)} \vee \frac{C_\mu(U^+ \wedge N^+)}{C_\mu(U^+)}\right], \end{aligned} \tag{59}$$

where

$$C_\mu(U^-) = m_{U^-}(s_{(1)})(\mu(E_{(1)}) - \mu(E_{(2)}))$$

$$\begin{aligned}
&+m_{U^-}(s_{(2)})(\mu(E_{(2)})-\mu(E_{(3)}))\\
&+m_{U^-}(s_{(3)})(\mu(E_{(3)})-\mu(E_{(4)}))\\
&+m_{U^-}(s_{(4)})(\mu(E_{(4)})-\mu(E_{(5)}))\\
&+m_{U^-}(s_{(5)})(\mu(E_{(5)}))-\mu(E_{(6)}),
\end{aligned} \tag{60}$$

$$\begin{aligned}
C_\mu(U^+) = \ & m_{U^+}(s_{(1)})(\mu(E_{(1)})-\mu(E_{(2)}))\\
&+m_{U^+}(s_{(2)})(\mu(E_{(2)})-\mu(E_{(3)}))\\
&+m_{U^+}(s_{(3)})(\mu(E_{(3)})-\mu(E_{(4)}))\\
&+m_{U^+}(s_{(4)})(\mu(E_{(4)})-\mu(E_{(5)}))\\
&+m_{U^+}(s_{(5)})(\mu(E_{(5)})-\mu(E_{(6)})),
\end{aligned} \tag{61}$$

and

$$\begin{aligned}
C_\mu(U^- \wedge N^-) &= m_{U^-\wedge N^-}(s_{(1)})(\mu(E_{(1)})-\mu(E_{(2)}))\\
&\quad +m_{U^-\wedge N^-}(s_{(2)})(\mu(E_{(2)})-\mu(E_{(3)}))\\
&\quad +m_{U^-\wedge N^-}(s_{(3)})(\mu(E_{(3)})-\mu(E_{(4)}))\\
&\quad +m_{U^-\wedge N^-}(s_{(4)})(\mu(E_{(4)})-\mu(E_{(5)}))\\
&\quad +m_{U^-\wedge N^-}(s_{(5)})(\mu(E_{(5)})-\mu(E_{(6)}))\\
&= (m_{U^-}(s_{(1)}) \wedge \mu_{N^-}(s_{(1)}))(\mu(E_{(1)}-)\mu(E_{(2)}))\\
&\quad +(m_{U^-}(s_{(2)}) \wedge m_{N^-}(s_{(2)}))(\mu(E_{(2)})-\mu(E_{(3)}))\\
&\quad +(m_{U^-}(s_{(3)}) \wedge m_{N^-}(s_{(3)}))(\mu(E_{(3)})-\mu(E_{(4)}))\\
&\quad +(m_{U^-}(s_{(4)}) \wedge m_{N^-}(s_{(4)}))(\mu(E_{(4)})-\mu(E_{(5)}))\\
&\quad +(m_{U^-}(s_{(5)}) \wedge m_{N^-}(s_{(5)}))(\mu(E_{(5)})-\mu(E_{(6)})),
\end{aligned} \tag{62}$$

$$\begin{aligned}
C_\mu(U^+ \wedge N^+) &= m_{U^+\wedge N^+}(s_{(1)})(\mu(E_{(1)})-\mu(E_{(2)}))\\
&\quad +m_{U^+\wedge N^+}(s_{(2)})(\mu(E_{(2)})-\mu(E_{(3)}))\\
&\quad +m_{U^+\wedge N^+}(s_{(3)})(\mu(E_{(3)})-\mu(E_{(4)}))\\
&\quad +m_{U^+\wedge N^+}(s_{(4)})(\mu(E_{(4)})-\mu(E_{(5)}))\\
&\quad +m_{U^+\wedge N^+}(s_{(5)})(\mu(E_{(5)})-\mu(E_{(6)}))\\
&= (m_{U^+}(s_{(1)}) \wedge m_{N^+}(s_{(1)}))(\mu(E_{(1)})-\mu(E_{(2)}))\\
&\quad +(m_{U^+}(s_{(2)}) \wedge m_{N^+}(s_{(2)}))(\mu(E_{(2)})-\mu(E_{(3)}))\\
&\quad +(m_{U^+}(s_{(3)}) \wedge m_{N^+}(s_{(3)}))(\mu(E_{(3)})-\mu(E_{(4)}))\\
&\quad +(m_{U^+}(s_{(4)}) \wedge m_{N^+}(s_{(4)}))(\mu(E_{(4)})-\mu(E_{(5)}))\\
&\quad +(m_{U^+}(s_{(5)}) \wedge m_{N^+}(s_{(5)}))(\mu(E_{(5)})-\mu(E_{(6)})).
\end{aligned} \tag{63}$$

Thus, by using (58) and (59), we get the following table A2 for the strong interval-valued subsethood measure between United States and New Zealand.

Table A2: The $\bar{c}_1(\overline{U},\overline{N})$ between United States and New Zealand.

$\overline{C}_\mu(\overline{U})$	$\overline{C}_\mu(\overline{U},\overline{N})$	$\bar{c}_1(\overline{U},\overline{N})$
$[0.14557, 0.19060]$	$[0.08084, 0.11470]$	$[0.55534, 0.60178]$

Given that $\bar{c}_1(\overline{U},\overline{V})$ represents the degree of trade surplus for the trading relationship for Korea and USA, and Korea and New Zealand.

Finally, we can calculate $\bar{c}_1(\overline{U},\overline{T})$ and $\bar{c}_1(\overline{U},\overline{I})$ in Table A1.

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Table A3: The $\bar{c}_1(\overline{U}, \overline{T})$ between United States and Turkey.

$\overline{C}_\mu(\overline{U})$	$\overline{C}_\mu(\overline{U}, \overline{T})$	$\bar{c}_1(\overline{U}, \overline{T})$
$[0.14557, 0.19060]$	$[0.00459, 0.01673]$	$[0.03153, 0.08778]$

Table A4: The $\bar{c}_1(\overline{U}, \overline{I})$ between United States and India.

$\overline{C}_\mu(\overline{U})$	$\overline{C}_\mu(\overline{U}, \overline{I})$	$\bar{c}_1(\overline{U}, \overline{I})$
$[0.14557, 0.19060]$	$[0.00348, 0.01074]$	$[0.02393, 0.05635]$

Tables A2, A3, and A4, demonstrate the results $\bar{c}_1(\overline{U}, \overline{N}), \bar{c}_1(\overline{U}, \overline{T}), \bar{c}_1(\overline{U}, \overline{I})$. They highlight the degree of trade surplus that exists with the three trading partners in terms of the model of trade transactions with United States and Korea.

5. Conclusions

Using the concept of intervals, we defined three types of interval-valued subsethood measures in Definitions 3.2, 3.3 and 3.4. From these definitions, we proposed three types of interval-valued subsethood measures defined by the interval-valued Choquet integrals with respect to a continuous fuzzy measure in Theorems 3.2, 3.3, and 3.4. The fuzzy measure μ in (57) means that if set E includes more categories between Korea and its trading partner, then $\mu(E)$ receives a higher score. Moreover, intervals are also a very useful tool to express the degree of trade surplus between Korea and its four trading partners analyzed over the 2010-2013 period.

In order to illustrate some applications of a strong interval-valued subsethood measure, we provided the four interval-valued fuzzy sets which were aggregated in (49), (50), (51), and (52) to animal product exports between Korea and four selected trading partners from 2010 to 2013. By using these interval-valued fuzzy sets, we obtained the strong interval-valued subsethood measure $\bar{c}_1(\overline{U}, \overline{N}), \bar{c}_1(\overline{U}, \overline{T}), \bar{c}_1(\overline{U}, \overline{I})$ which represent the degree of trade surplus between Korea and 3 trading partners in terms of the model of trade transactions with the United States and South Korea in Tables 2, 3, and 4. It was found that New Zealand was at least 0.55534 to 0.60178 times smaller than the United States, while Turkey and India were also smaller, with Turkey at least 0.03153 to 0.08778 times smaller and India being at least 0.2393 to 0.05635 times smaller than the United States between 2010 and 2013, in terms of the trade values of animal product exports that exists between Korea and selected trading partners.

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