TRANSIENT MICROPOLAR FLUID FLOW PAST A SEMI-INFINITE VERTICAL PLATE WITH RADIATION IN SLIP FLOW REGIME

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ABSTRACT

This work examines the unstable magnetohydrodynamic free convective flow of a micropolar fluid adjacent to a semi-infinite vertical plate. A homogeneous magnetic field is applied orthogonally to the plate. The suction/injection on the plate and the permeability of the porous material are presumed to fluctuate over time. Disregarding the induced magnetic field, the solutions for velocity and temperature are derived using the regular perturbation technique. The graphical representation illustrates the impacts of the permeability parameter, coupling stress, microrotation parameter, slip flow parameter, and other relevant parameters involved in the problem.

Keywords: Micropolar fluid, MHD, Porous Medium, Radiation

1. INTRODUCTION

The micropolar fluid theory, formulated by Eringen [1,2,3], delineates certain physical systems that do not conform to the Navier-Stokes equations. This general theory of micropolar fluids deviates from that of Newtonian fluid by adding two more variables to the velocity. These variables are micro-rotation that is spin and microinertia tensor describing the distributions of atoms and molecules inside the microscopic fluid particles. This hypothesis may be applied to describe the phenomenon of the flow of colloidal fluids, liquid crystals, polymeric suspensions, animal blood etc. An exceptional research of micropolar fluids and their applications was reported by Ariman et. Al. [4]. Gorla et.al. [5] have evaluated the steady state heat transfer in a micropolar fluid flow over a semi-infinite plate utilizing similarity variables analytical variables. Rees and Pop [6] examined free convection boundary layer flow of a micropolar fluid from a vertical flat plate. Singh [7] have examined free convection movement of micropolar fluid using finite difference technique.

In the present stage of modern technologies, the study of flow and heat transfer for an electrically conducting micropolar fluid under the influence of a magnetic field has caught the interest of many investigators due to its vast diversity of applications. These applications include magnetohydrodynamic (MHD) generators, plasma investigations, nuclear reactors, oil exploration, geothermal energy extractions and the boundary layer control in the field of aerodynamics. Also, the porous media heat transfer problems have various practical engineering applications such as crude oil extraction, ground water contamination and another many practical uses. Hiremath and Patil [8] studied the effect on free convection currents on the oscillatory flow of polar fluid through a porous media, which is constrained by vertical flat surface of constant temperature. Unsteady hydromagnetic free convection flow of Newtonian and polar fluid has been examined by Helmy [9].

El-Hakien et.al. [10] studied the effects of viscous and Joule heating on MHD-free convection flow with variable plate temperature in a micropolar fluid. El-Amin [11] studied the MHD free-convection and mass transfer flow in a micropolar fluid across a stationary vertical plate with constant suction. Kim [12] examined unsteady free convection flow of micropolar fluid past a vertical plate embedded in porous media and extended his work [13] to study the implications of heat and mass transfer in MHD micropolar fluid flow past a vertical moving plate.

At the macroscopic level, it is widely acknowledged that the boundary condition for a viscous fluid at a solid wall is one of no-slip, meaning that the fluid velocity equals the solid boundary velocity. Although the no-slip condition has been shown to be accurate for a variety of macroscopic flows, it is still assumed that it is not founded on physical principles. In many real-world situations, a particle near a solid surface no longer absorbs the surface's velocity. The tangential velocity of the particle at the surface is constrained. It moves across the surface. This influence cannot be disregarded, and the flow regime is known as the slip flow regime. Power generators, refrigeration coils, transmission lines, electric transformers, and heating elements are only a few of the crucial technical applications for the study of magneto-micropolar fluid flows in the slip flow regime with heat transfer. The effects of permeability variation on MHD unsteady flow of polar fluid through a porous medium in slip flow regime over an infinite porous flat plate were investigated by Khandelwal et al. [14]. Sharma and Chaudhary [15] studied the effect of altering suction on transient free convective viscous incompressible flow via a vertical plate in slip-flow regime. In the slip-flow regime, Sharma [16] investigated the effects of periodic temperature and concentration on unsteady free convection flow through a vertical plate.

Khandelwal et al. [17] investigated how slip factors affected the radiative unsteady MHD free convection. Free magnetopolar fluid with a temperature-dependent heat source in the slip flow regime was studied by Kumar and Tak [18]. In the slip-flow regime, Chaudhary and Jha [19] talked about MHD micropolar fluid flow past a vertical plate. Singh [20] talked about the Free Convective Flow of Magneto-Polar Fluid in Non-Homogeneous Porous Medium in Slip Flow Regime. Unsteady magnetohydrodynamic boundary layer slip flow past a with thermal radiation was perturbed and analyzed by Pal and Talukdar [21]. Sengupta and Ahmed [22] have determined the free convective low of a dissipative fluid with thermal diffusion and changing wall temperature in velocity slip regime. In the presence of a transverse magnetic field, boundary layer flow nea with slip has been investigated by Yakubu and Makinde [23]. In the presence of heat radiation, Choudhary et al. [24] demonstrated MHD free convective slip flow of polar and Newtonian fluids across porous media.

2. FORMULATION OF THE PROBLEM

We examine the free convective unstable laminar flow of an electrically conducting, incompressible micropolar fluid in the slip flow regime, where a transverse magnetic field and radiation are present, across an infinite porous vertical flat plate buried in a non-homogeneous porous medium. We made the assumption that the plate was porous and that the suction/injection velocity was falling exponentially over time, with a constant mean that was not zero. In cartesian co-ordinate system, the x^*-axis is chosen along the porous plate, which moves with uniform velocity $u^* = u_0(1 + \varepsilon e^{-n^* t^*})$ and y^*-axis normal to it. Besides, the analysis is based on the following assumptions:

(i) The homogeneous magnetic field with modest magnetic Reynolds number works transversely to the direction of flow.

(ii) The induced magnetic field is insignificant since the Reynolds number of the magnetic field is low.

(iii) The viscous dissipation effect, the polarization effect, and the Hall effect are disregarded.

(iv) The size of holes in the porous plate is significantly greater than the characteristics tiny length scale of the micropolar fluid to ease formulation of the boundary conditions.

(v) Because of the infinite plate assumption, the flow variables are just functions of time $t^*$and normal distance y^* .

(vi) Ohmic heating and viscous dissipation effects are disregarded.

(vii) By simulating the Boussinesq approximation, density changes are only visible in the momentum equation's normal buoyancy parts.

(viii) It is assumed that the suction/injection velocity and the porous medium's permeability have the following forms: $K^*(t) = K_0^*\left(1 + \varepsilon e^{-n^*t^*}\right)$ and $v^*(t) = v_0^*\left(1 + \varepsilon e^{-n^*t^*}\right)$ respectively.

Under the present configuration, the equations governing the flow are:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} - v_0\left(1 + \varepsilon e^{-n^*t^*}\right)\frac{\partial u^*}{\partial y^*} = (\nu + \nu_r)\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - T_\infty^*) - \frac{\nu}{K_0^*\left(1+\varepsilon e^{-n^*t^*}\right)}u^* - \frac{\sigma}{\rho}B_0^2 u^* + 2\nu_r\frac{\partial \omega^*}{\partial y^*} \tag{2}$$

$$\frac{\partial \omega^*}{\partial t^*} - v_0\left(1 + \varepsilon e^{-n^*t^*}\right)\frac{\partial \omega^*}{\partial y^*} = \frac{\gamma}{\rho j}\frac{\partial^2 \omega^*}{\partial y^{*2}} \tag{3}$$

$$\frac{\partial T^*}{\partial t^*} - v_0\left(1 + \varepsilon e^{-n^*t^*}\right)\frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*} \tag{4}$$

The proper boundary conditions of the problem are: -

$$u^* = u_0\left(1 + \varepsilon e^{-n^*t^*}\right) + \frac{(2 - m_1)L}{m_1}\frac{\partial u^*}{\partial y^*}, \frac{\partial \omega^*}{\partial y^*} = -\frac{\partial^{2u^*}}{\partial y^{*2}},$$

$$T^* = T_w^* + \varepsilon(T_w^* - T_\infty^*)e^{-n^*t^*} + \frac{(2-m_1)}{m_1}\frac{2\gamma}{(\gamma+1)}\frac{L}{Pr}\left(\frac{\partial T^*}{\partial y^*}\right) \text{ at } y^* = 0$$

$$u^* \to 0, \quad \omega^* \to 0, \quad T^* \to 0 \text{ as } y^* \to \infty \tag{5}$$

The following non-dimensional quantities are introduced in order to solve the equations governing the flow:

$$u = \frac{u^*}{u_0}, v = \frac{v}{v_0}, y = \frac{v_0 y^*}{\nu}, t = \frac{t^* v_0^2}{\nu}, n = \frac{n^*\nu}{v_0^2}, \omega^* = \frac{\omega u_0 v_0}{\nu}, T = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}$$

Considering above specified non-dimensional quantities in equations (1)-(4), we get:

$$\frac{\partial u}{\partial t} - (1 + \varepsilon e^{-nt})\frac{\partial u}{\partial y} = (1 + \alpha)\frac{\partial^2 u}{\partial y^2} + GrT - \frac{1}{K_0(1+\varepsilon e^{-nt})}u - M^2 u + 2\alpha\frac{\partial \omega}{\partial y} \tag{6}$$

$$\frac{\partial \omega}{\partial t} - (1 + \varepsilon e^{-nt})\frac{\partial \omega}{\partial y} = \frac{1}{\lambda}\frac{\partial^2 \omega}{\partial y^2} \tag{7}$$

$$\frac{\partial T}{\partial t} - (1+\varepsilon e^{-nt})\frac{\partial T}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} - FT \qquad (8)$$

Where $\alpha = \frac{\nu_r}{\nu}$ (Kinematic rotational viscosity parameter)

$Gr = \frac{g\beta\nu(T_W^* - T_\infty^*)}{u_0 v_0^2}$ (Buoyancy parameter), $K_0 = \frac{K_0^* v_0^2}{\nu^2}$ (Permeability parameter)

$M^2 = \frac{\sigma\nu B_0^2}{\rho v_0^2}$ (Magnetic parameter), $\lambda = \frac{\mu j}{\gamma}$ (Microrotation parameter)

$Pr = \frac{\nu\rho C_p}{k}$ (Prandtl number), $F = \frac{4\nu I'}{\rho C_p v_0^2}$ (Radiation parameter)

$\frac{\partial q_r^*}{\partial y^*} = 4(T_W^* - T_\infty^*)I'$ where $I' = \int_0^\infty k_{\lambda w}\frac{\partial e_{b\lambda}}{\partial T^*}d\lambda$, $k_{\lambda w}$ is absorption coefficient at the wall and Planck's function is represented by $e_{b\lambda}$.

The non-dimensional form of the boundary conditions in (5) is as follows:

$$u = 1 + \varepsilon e^{-nt} + h_1\frac{\partial u}{\partial y}, \quad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad T = 1 + \varepsilon e^{-nt} + h_2\frac{\partial T}{\partial y} \quad at \quad y = 0$$

$$u \to 0, \quad \omega \to 0, \quad T \to 0 \quad as \quad y \to \infty \qquad (9)$$

Where $h_1 = \frac{(2-m_1)Lv_0}{m_1\nu}$ (Refraction parameter) and

$h_2 = \frac{(2-m_1)}{m_1}\frac{2\gamma}{(\gamma+1)}\frac{Lv_0}{Pr\nu}$ (Temperature jump parameter)

3. SOLUTION OF THE PROBLEM

In order to solve differential equations (6)-(8), we assume that u, ω and $T(\varepsilon \ll 1)$ have the following solutions.

$$\left.\begin{aligned} u(y,t) &= u_0(y) + \varepsilon u_1(y)e^{-nt} + 0(\varepsilon^2) \\ \omega(y,t) &= \omega_0(y) + \varepsilon \omega_1(y)e^{-nt} + 0(\varepsilon^2) \\ T(y,t) &= T_0(y) + \varepsilon T_1(y)e^{-nt} + 0(\varepsilon^2) \end{aligned}\right\} \qquad (10)$$

Introducing (10) into the non-dimensional differential equations (6)-(8) and comparing the coefficient of ε^0 and ε^1, neglecting the coefficient of ε^2 and higher powers, we obtain:

$$(1+\alpha)\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} - M_1 u_0 = -GrT_0 - 2\alpha\frac{\partial \omega_0}{\partial y} \qquad (11)$$

$$(1+\alpha)\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial u_1}{\partial y} - M_2 u_1 = -GrT_1 - 2\alpha\frac{\partial \omega_1}{\partial y} - \frac{\partial u_0}{\partial y} - \frac{1}{K_0}u_0 \qquad (12)$$

$$\frac{\partial^2\omega_0}{\partial y^2}+\lambda\frac{\partial\omega_0}{\partial y}=0 \tag{13}$$

$$\frac{\partial^2\omega_1}{\partial y^2}+\lambda\frac{\partial\omega_1}{\partial y}+n\lambda\omega_1=-\lambda\frac{\partial\omega_0}{\partial y} \tag{14}$$

$$\frac{\partial^2 T_0}{\partial y^2}+Pr\frac{\partial T_0}{\partial y}-FPrT_0=0 \tag{15}$$

$$\frac{\partial^2 T_1}{\partial y^2}+Pr\frac{\partial T_1}{\partial y}+(Prn-FPr)T_1=-Pr\frac{\partial T_0}{\partial y} \tag{16}$$

Introducing (10), the boundary conditions (9) transform to the following form:

$$u_0=1+h_1\frac{\partial u_0}{\partial y},u_1=1+h_1\frac{\partial u_1}{\partial y},\frac{\partial\omega_0}{\partial y}=-\frac{\partial^2 u_0}{\partial y^2},\frac{\partial\omega_1}{\partial y}=-\frac{\partial^2 u_1}{\partial y^2},$$

$$T_0=1+h_2\frac{\partial T_0}{\partial y},T_1=1+h_2\frac{\partial T_1}{\partial y}\quad at\quad y=0$$

$$u_0\to 0,u_1\to 0,\omega_0\to 0,\omega_1\to 0,T_0\to 0,T_1\to 0,\ as\ \ y\to\infty \tag{17}$$

Solutions of differential equations (11)-(16) satisfying boundary conditions (17)

are obtained as follows:

$$\omega_0=C_1e^{-\lambda y} \tag{18}$$

$$\omega_1=C_2e^{-m_1y}+\frac{\lambda}{n}C_1e^{-\lambda y} \tag{19}$$

$$T_0=C_0e^{-m_0y} \tag{20}$$

$$T_1=C_3e^{-m_2y}+Be^{-m_0y} \tag{21}$$

$$u_0=C_4e^{-m_3y}+A_1C_0e^{-m_0y}+A_2C_1e^{-\lambda y} \tag{22}$$

$$u_1=C_5e^{-m_4y}+A_3C_2e^{-m_1y}+A_4C_3e^{-m_2y}+A_5C_4e^{-m_3y}+A_6e^{-m_0y}$$
$$+A_7C_1e^{-\lambda y} \tag{23}$$

SKIN-FRICTION AND RATE OF HEAT TRANSFER

The non-dimensional skin-friction (τ) at the plate is given by:

$$\tau=(1+\alpha)\left(\frac{\partial u}{\partial y}\right)_{y=0}=(1+\alpha)\left(\frac{\partial u_0}{\partial y}\right)_{y=0}+\varepsilon(1+\alpha)\left(\frac{\partial u_1}{\partial y}\right)_{y=0}e^{-nt}$$

$$= (1+\alpha)(-m_3C_4 - m_0A_1C_0 - \lambda A_2C_1) + \varepsilon(1+\alpha)(-m_4C_5 - m_1A_3C_2$$

$$-m_2A_4C_3 - m_3A_5C_4 - m_0A_6 - \lambda A_7C_1)\, e^{-nt} \quad (24)$$

The non-dimensional rate of heat transfer in terms of Nusselt number (Nu) is given by:

$$Nu = \left(\frac{\partial T}{\partial y}\right)_{y=0} = \left(\frac{\partial T_0}{\partial y}\right)_{y=0} + \varepsilon\left(\frac{\partial T_1}{\partial y}\right)_{y=0} e^{-nt}$$

$$= -C_0m_0 + \varepsilon(-m_2C_3 - m_0B)\, e^{-nt} \quad (25)$$

4. RESULT AND DISCUSSION

In the preceding section, solutions for the microrotation velocity (ω), temperature (T) and the velocity (u) are derived and illustrated in (18)-(23). In order to acquire physical insight into the problem and establish the impacts of different parameters on the microrotation velocity, the flow field and temperature distribution of the micropolar fluid, numerical calculations are done and displayed graphically. These figures show that the stream-wise velocity and micro-rotation (which comprises entirely the rotations about the centroid of the mass element) as well as temperature profiles for the micropolar fluid with the fixed flow conditions ε= 0.01 n = 0.1 and t = 1 , while Grashof number (Gr), Prandtl number (Pr), magnetic parameter (M), permeability parameter (K_0), slip flow parameter (h_1), temperature jump parameter (h_2) and viscosity ratio (α) are varied over a range which are listed in figure captions. To be realistic, the values of Prandtl number (Pr) are chosen to be Pr = 0.71 (air), Pr = 0.025 (mercury), Pr = 1 (electrolyte solutions) and Pr = 7 (water at 20°C), at one atmospheric pressure. The values of the perturbation parameter (ε), frequency parameter (n) and time parameter (t) are fixed and non-zero, therefore in all situations, the unsteady flow of the micropolar fluid with suction velocity, variation in permeability, slip flow and jump in temperature is explored.

Figure 1 exhibits variations in the velocity profiles against y for varied numerical values of magnetic field parameter (M) and permeability parameter (K_0) and perturbation parameter (ε) for fixed values of n = 0.1, t = 1.0, α= 0.2, h_1 = 0.02, h_2 = 0.5 Pr = 1, γ= 2.0 and Gr = 5.0. It is noticed that the increasing values of magnetic field parameter leads in a decreased velocity dispersion throughout the boundary layer. Also, when K_0 grows, the velocity boundary layer tends to increase in the velocity of the plate and afterward decays gradually towards the y-axis. Furthermore, we note that in case of homogeneous porous medium (case of Kumar and

Tak [18]), the velocity attains peak value more rapidly as compared to non-homogeneous porous medium (present case), but for higher values of y-coordinate, the profiles fall more rapidly for homogeneous porous medium than non-homogeneous porous medium. Thus, to sustain the velocity, non-homogeneous porous material is more effective in comparison to homogeneous one.

fig.1:variations in velocity against y for different values of M ,K_0 and ϵ (Gr=5,t=1,n=0.1,F=0,Pr=1,α=0.2,λ=2,h_1=0.02,h_2=0.5)

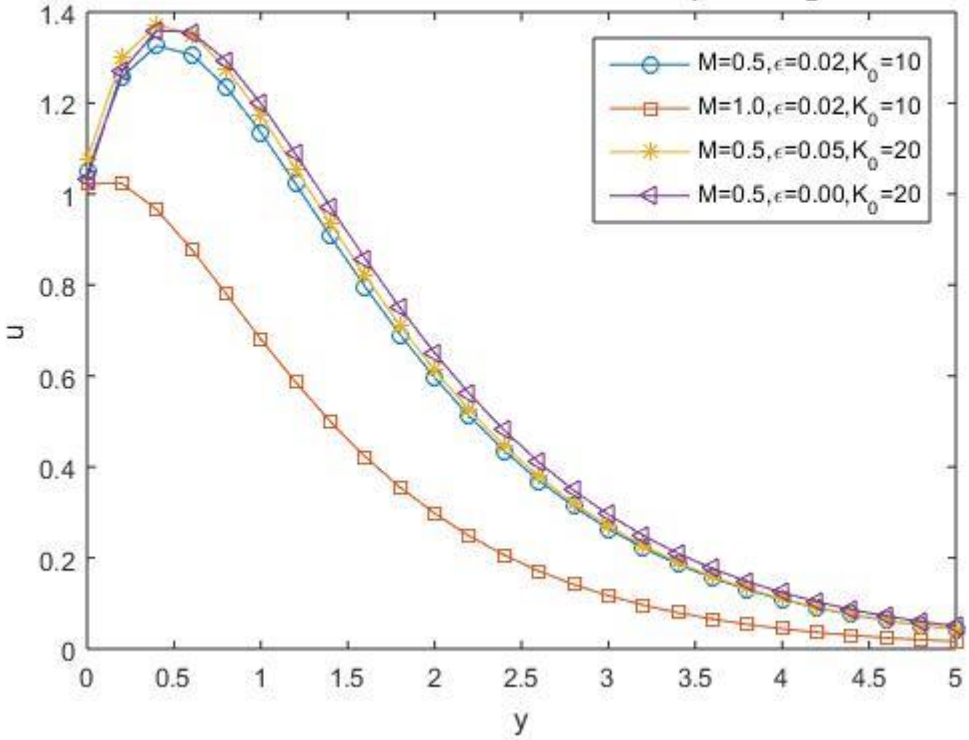


Figure 2 represents the variations in the velocity profiles for different values of Grashof number (Gr), kinematic rotational viscosity parameter (α) for the cooling case (Gr > 0) of the moving plate at the fixed values of n = 0.1, t = 1.0, ε= 0.02, λ= 2.0. h_2 = 0.5, h_1 = 0.02, Pr = 1.0, M = 0.5 and K_0= 10.0. Figure clearly shows that in the case of cooling of the plate, velocity increase with an increase of thermal Grashof number (Gr). From the numerical results, we also observed e that the velocity is less for the Newtonian fluid (α= 0) with the same flow conditions and fluid properties, compared to the micropolar fluid. When the kinematic rotational viscosity parameter is less than 1.0, the velocity increases in the vicinity of the plate and after attaining a peak value it decreases smoothly with increase in y. However, when a takes values greater than 1.0, i.e., the gyro-viscosity is larger than the translational viscosity, the velocity distribution shows a decelerating nature near the porous plate. It is observed that agreement with the result obtained by Jain and Gupta [25] is excellent.

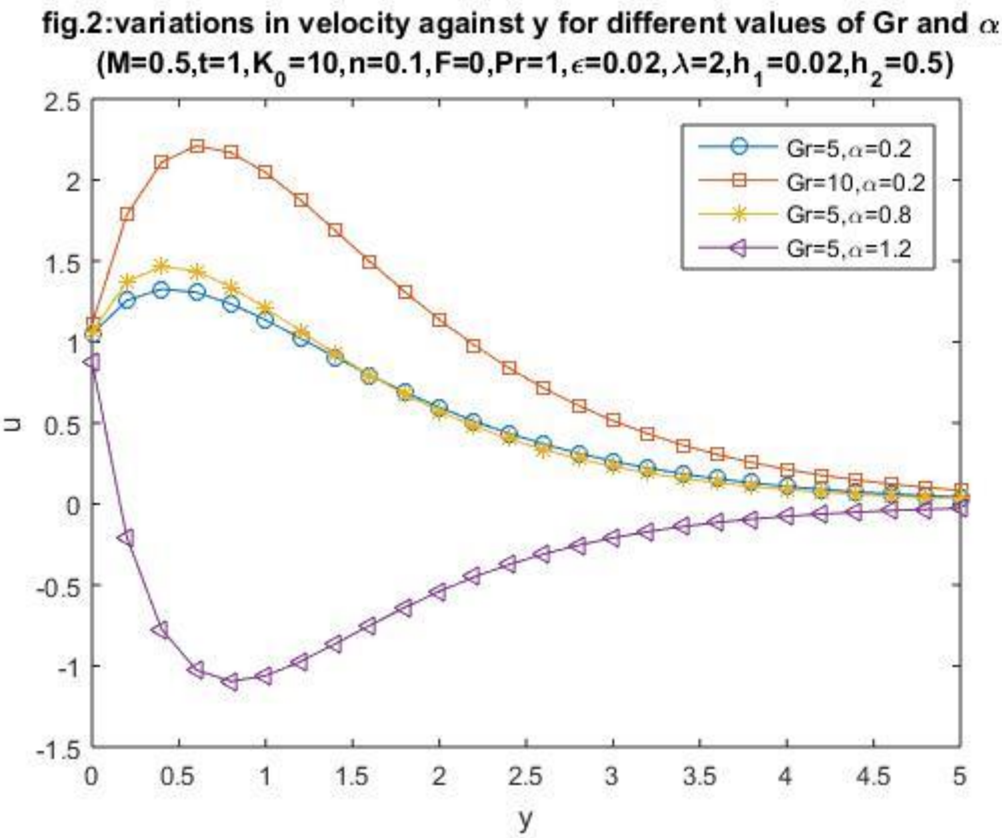


Figure 3 depicts the variations in the velocity profile for different values of slip flow parameter (h_1) and temperature jump parameter (h_2) with fixed values of ε= 0.02, n = 0.1, t=1.0, Pr = 1.0, M = 0.5, K_0 = 10.0, Gr = 5.0 and α= 0.2. Figure clearly shows that an increase in h_1 increases the velocity, while reverse phenomenon is observed for h_2. This is due to the fact that increase in the slip flow results in less friction at the porous plate, which enhances the velocity. Contrast to this fact, increase in temperature jump parameter increases the friction at the plate due to increased denseness of the fluid, which ultimately reduces the velocity. These findings are similar to those obtained by Khandelwal and Jain [17].

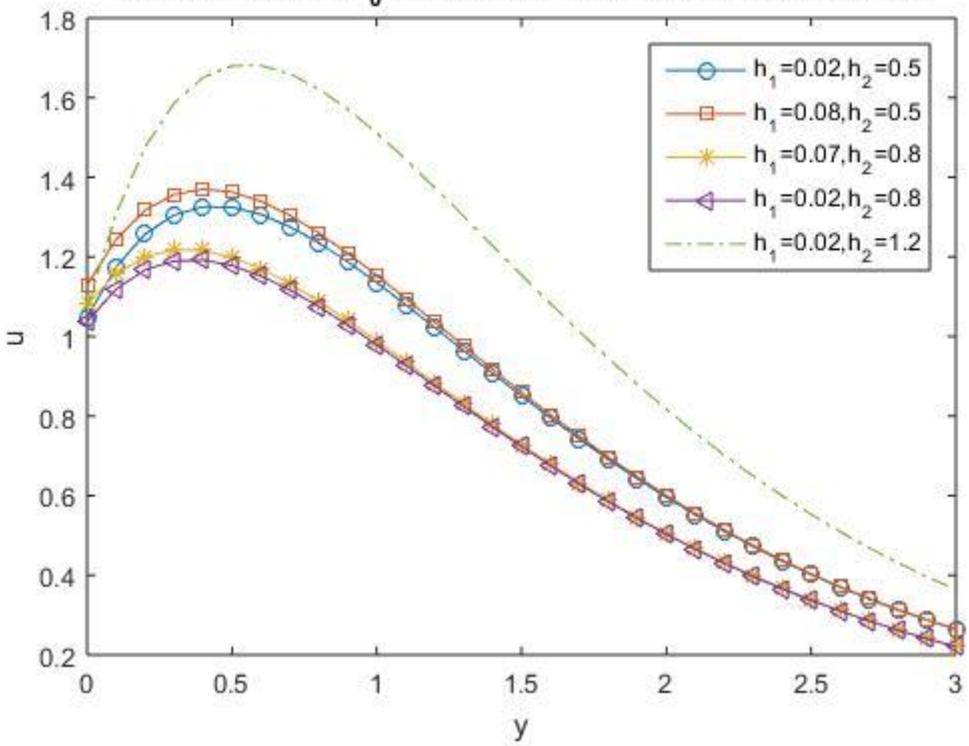
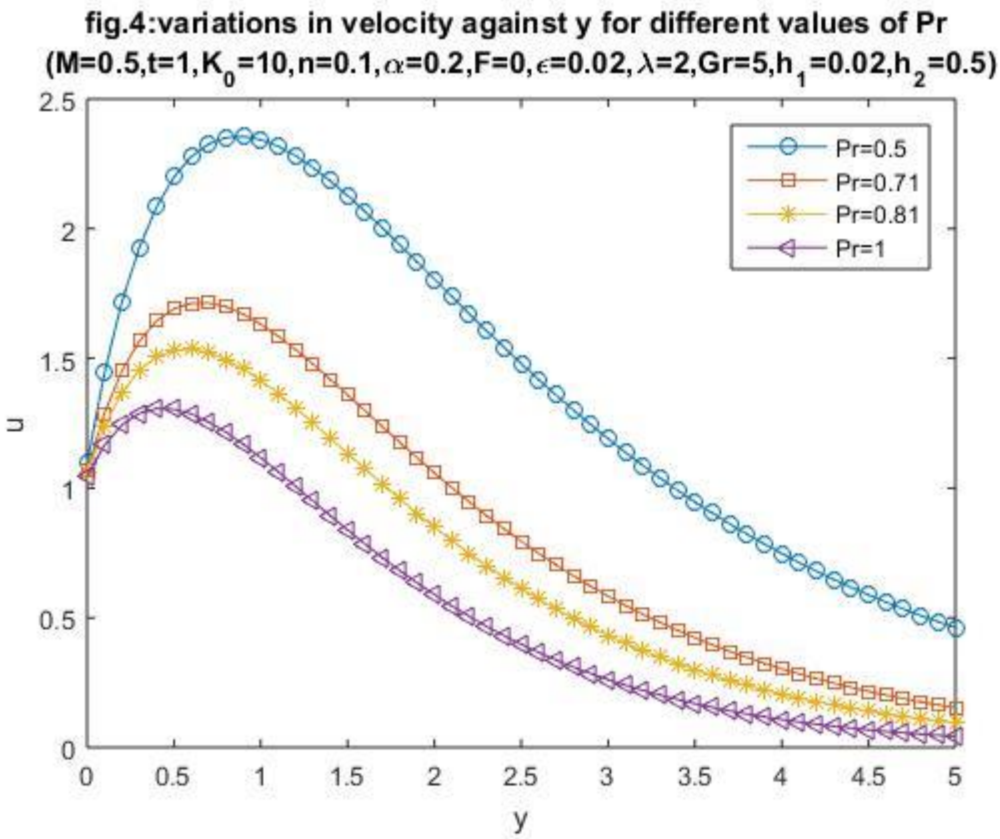
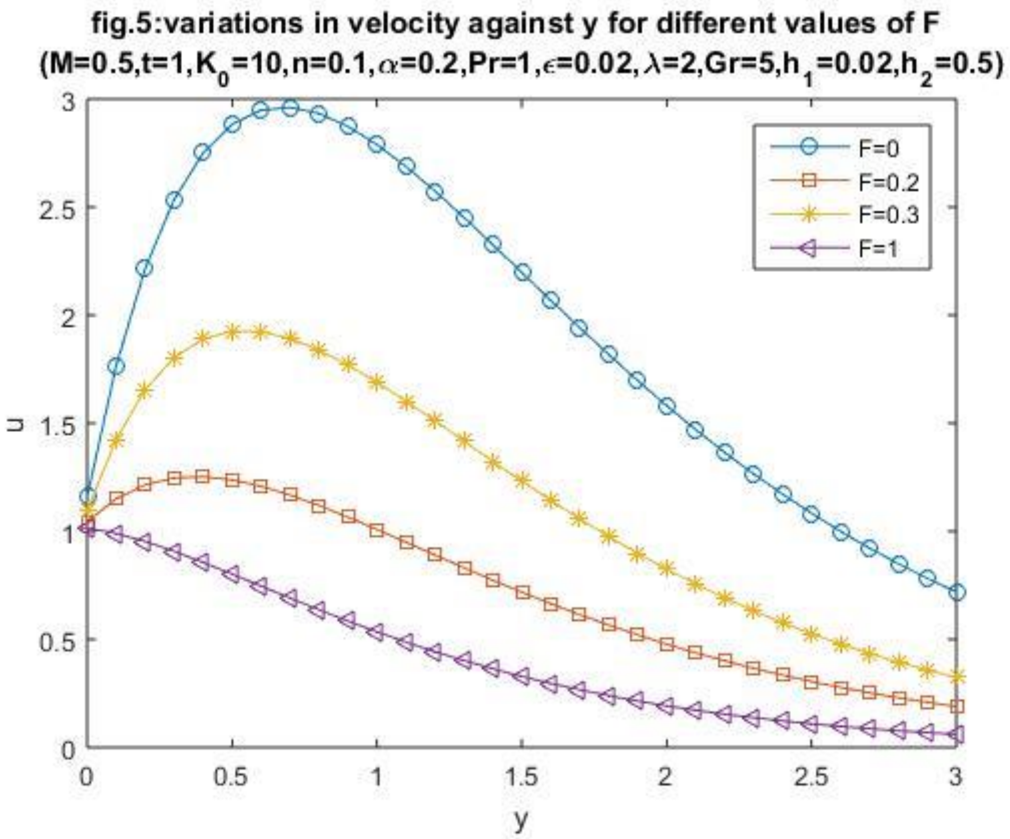


fig.3:variations in velocity against y for different values of h_1 and h_2 ($M=0.5, t=1, Pr=1, K_0=10, n=0.1, \alpha=0.2, F=0, \epsilon=0.02, \lambda=2, Gr=5$)

Figure 4 presents velocity profiles against span wise coordinate y for different values of Prandtl number (Pr) and at the fixed values of $\varepsilon= 0.02$, $n = 0.1$, $t = 1.0$, $h_1=0.02$, $h_2=0.5$, $M=0.5$, $K_0=10.0$, $Gr=5.0$ and $\alpha=0.2$. The results show that the effect of increasing value of Pr results in a decreasing velocity, which implies that the fluid with lower Pr values is favourable to reduce decay in the velocity distribution. In fact, in the light of the definition of Prandtl number, higher Pr-values fluid transfer heat less effectively in comparison with the lower Pr-value fluids. Therefore, the denseness of the fluid particles is more for higher Pr-value fluids than lower Pr-value fluids, which results in decreasing velocity with increasing Prandtl number.



In figure 5, the velocity profiles for different values of the radiation parameter are displayed for the fixed values of $\varepsilon = 0.02$, $n = 0.1$, $t = 1.0$, $h_1=0.02$, $h_2=0.5$, $M=0.5$, $K_0=10.0$, $Gr=5.0$ and $\alpha=0.2$. It is clear that velocity decreases with increasing value of radiation parameter. This is due to the fact that an increase in radiation parameter means increase in mean absorption coefficient.



The temperature profiles versus y for different values of Prandtl number (Pr) at fixed values of ε=0.02, n =0.1, t=1.0, and h_2 = 0.5 are shown in figure 6. It is observed that an increase of Prandtl number, leads to a decrease in temperature, which implies a decrease in thermal boundary layer thickness. This result explains the fact that smaller values of Pr are equivalent to increasing the thermal conductivities so that heat is able to diffuse away from the surface more rapidly. Hence, decay in much less in lower Pr-value fluids as compared to higher Pr-value fluid.

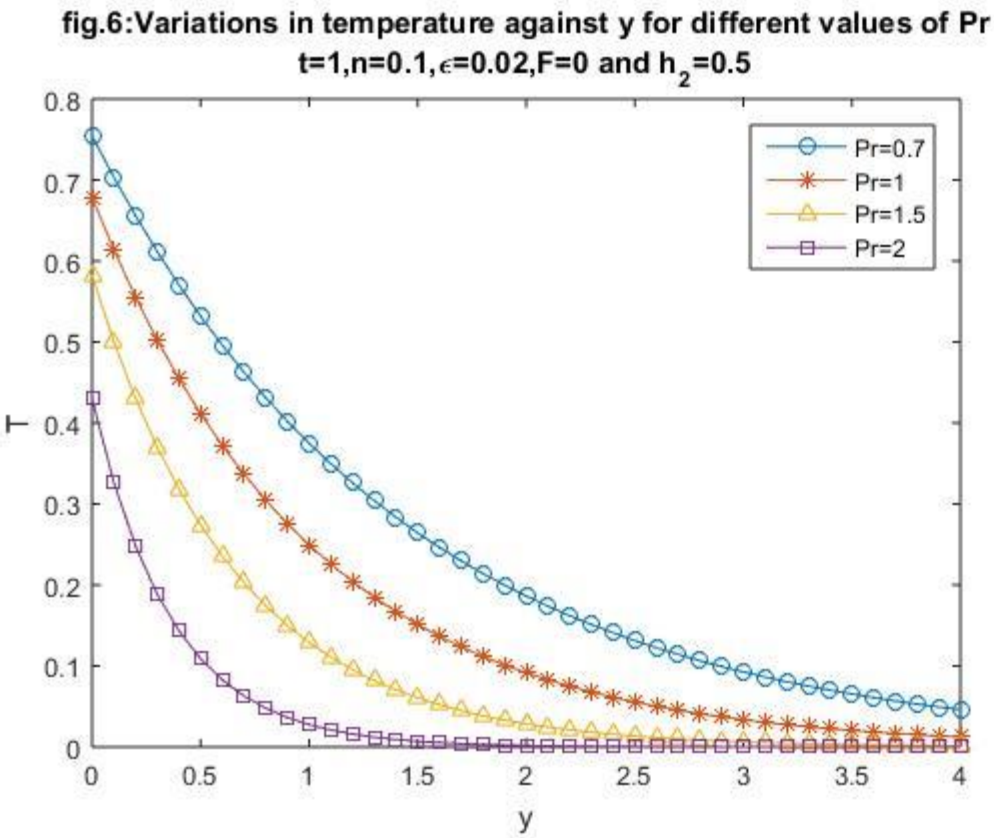
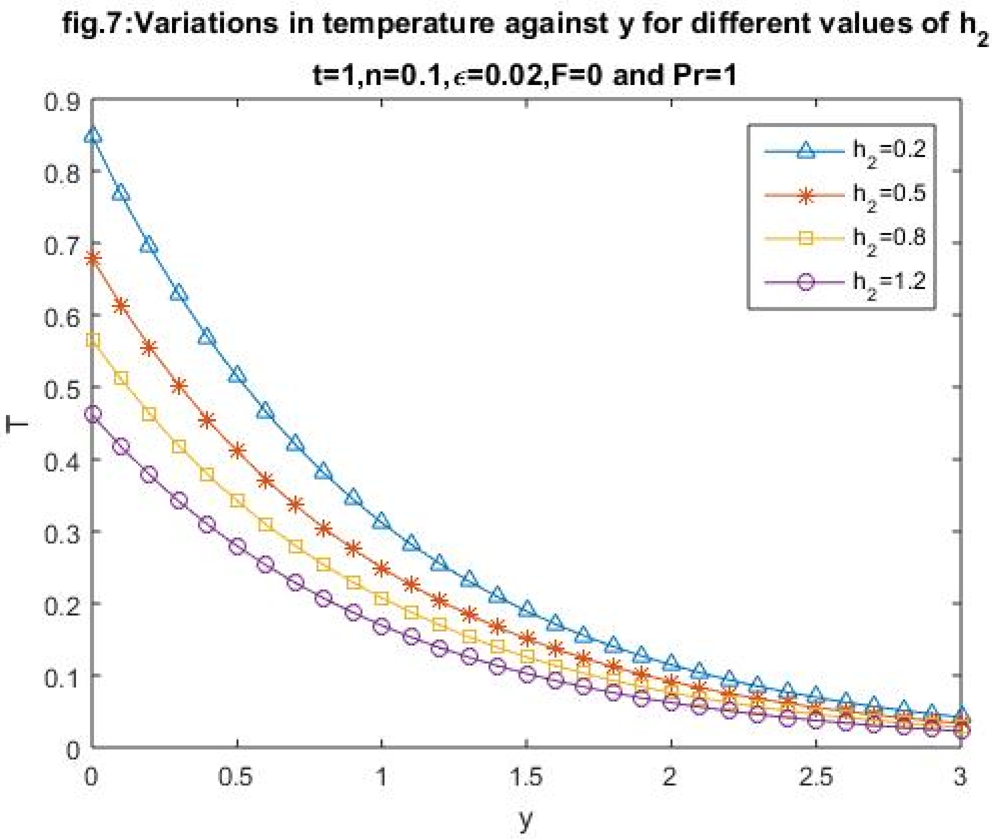


Figure 7 and 8 illustrates variations in the temperature distribution due to change in the temperature jump parameter (h_2) and radiation parameter at fixed values of ε=0.02, n=0.1, t=1.0 and Pr=1.From figure 7, We observed that an increase in temperature jump parameter (h_2) increases the temperature field with given flow conditions and material parameters because an increase in temperature jump parameter (h_2) results in a decreasing thermal boundary layer thickness and less uniform temperature distribution across the boundary layer, so that temperature decreases with increase in temperature jump parameter. Figure 8 clearly shows that temperature decreases with increases with increasing values of radiation parameter. The reason is that the thermal boundary layer was found to thicken in the presence of radiation.



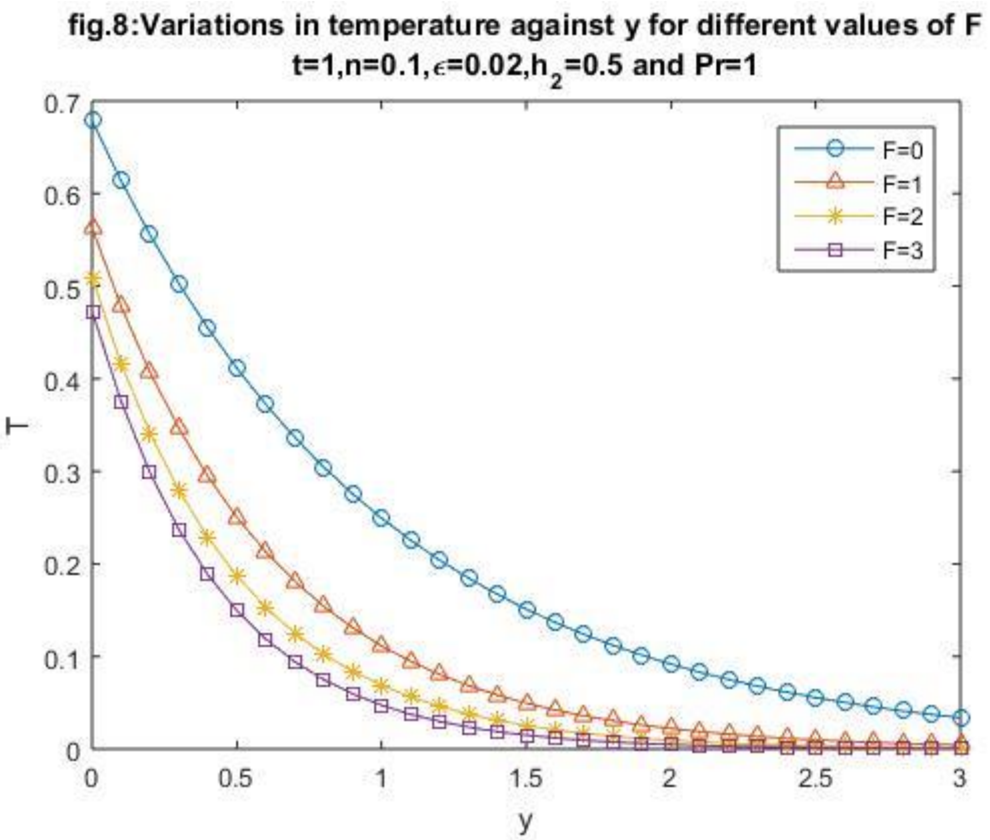


Figure 9 shows the effect of micro-rotation parameter (λ) on the microrotation velocity (ω) versus y for fixed captioned values of ε=0.02, n=0.1, t=1.0, h_1=0.02, h_2=0.5, Pr=1.0, Gr=5,

α=0.2, M=0.5, and K_0=10.0. It is observed that the microrotation velocity is less for laminar flow of Newtonian fluid (λ=0) for the same fixed values of the parameters. As the value of λ increases microrotation velocity increases near the plate and becomes asymptotic to y-axis as y increases. However, for λ >1, the microrotation velocity changes its nature and decreases in the vicinity of the plate and shows asymptotic nature towards y-axis with increase in y-coordinate.

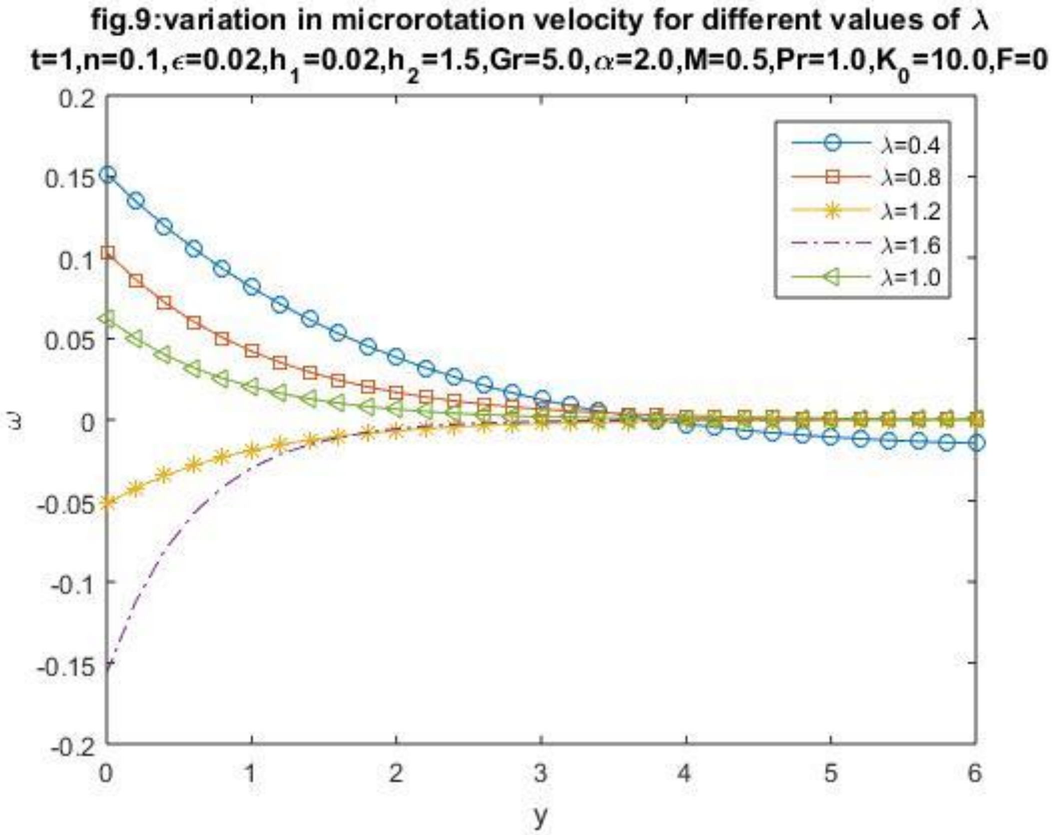


Figure 10 represents variations of the surface skin-friction (τ) versus slip velocity parameter (h_1) for various values of magnetic parameter (M). It is observed the effect of increasing values of slip velocity parameter (h_1) lead to an increasing surface skin-friction (τ) on the porous plate. It is also observed that at increase in increases the skin-friction.

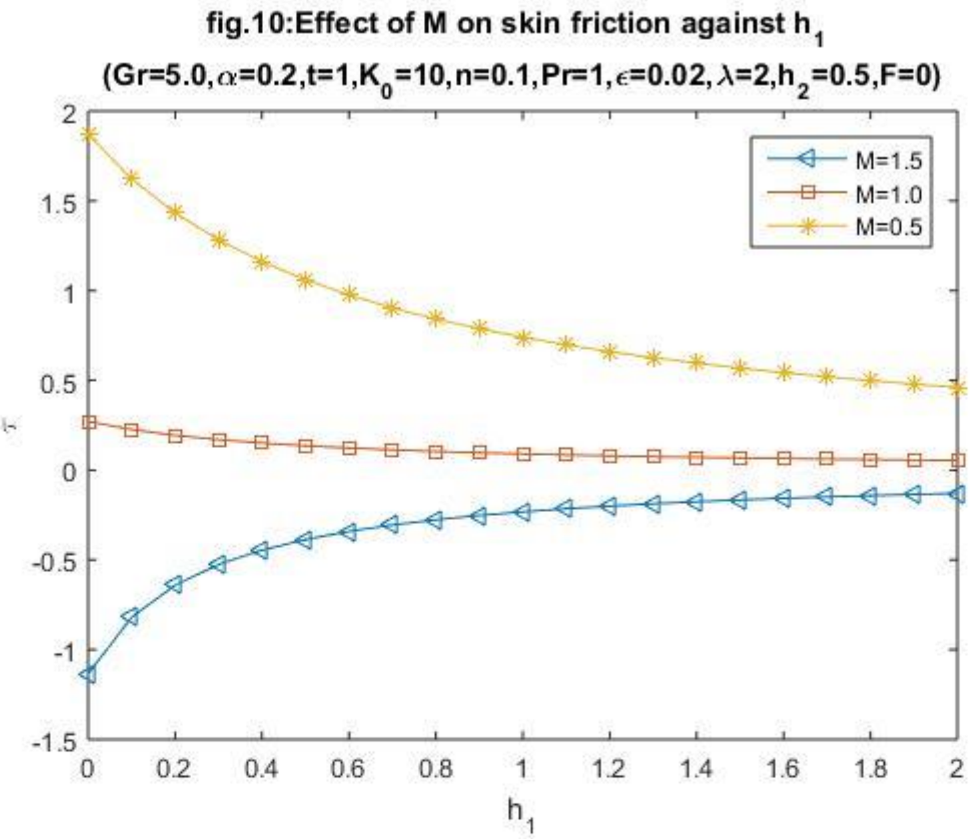
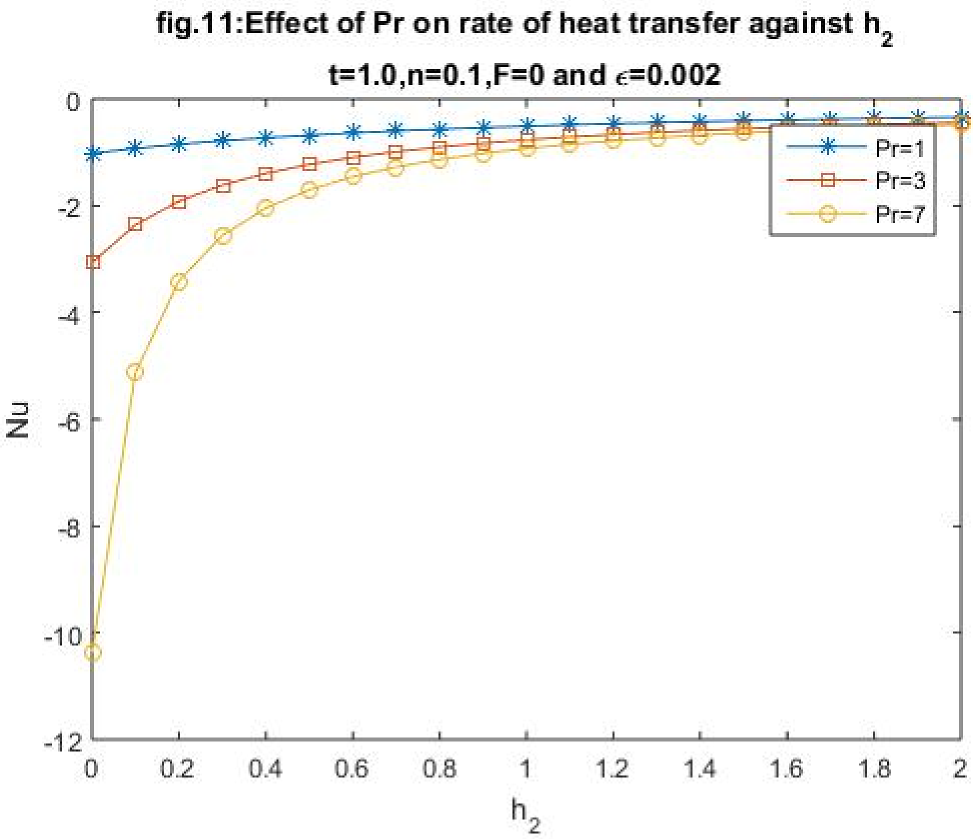


Figure 11 illustrates the variation of surface heat transfer (Nu) with the temperature jump parameter (h_2) for different values of Prandtl number (Pr). It is observed that, as Pr is increased, the surface heat transfer increases. Moreover, the surface heat transfer from the porous plate tends to decrease slightly on increasing the magnitude of temperature jump parameter.



5. CONCLUSIONS

The problem of velocity slip and temperature jump parameter on unsteady convective flow of a micropolar fluid along a uniformly moving vertical porous plate with time dependent suction velocity and non-homogeneous porous medium is investigated. Main conclusions of the study cooling case ($Gr > 0$) are as follows:

1. The velocity (u) increases rapidly near the plate and after attaining peak value and then starts decreasing uniformly.
2. The velocity (u) increases with increase in Grashof number (Gr), kinematical viscosity parameter (α) or permeability parameter (K_0) or slip flow parameter (h_1) while reverse effects are observed for magnetic parameter (M) or Prandtl number (Pr) or temperature jump parameter (h_2) increases.
3. The temperature (T) decreases as Prandtl number (Pr) increases but increases with increase in temperature jump parameter.
4. The microrotation velocity (ω) increases with increase in microrotation parameter (λ) and changes its nature when λ exceeds the numerical value unity.
5. Increasing values of slip velocity parameter leads to a decreasing (h_1) skin-friction on the porous plate but effect is noted for increasing magnetic induction.

6. The heat transfer rate (Nu) increases as Prandtl number (Pr) increases but decreases with increase in temperature jump parameter.
7. An increase in temperature jump parameter (h_2) decreases the temperature.

APPENDIX

$$M_1 = M^2 + \frac{1}{K_0},\ \ M_2 = M^2 + \frac{1}{K_0} - n,\ \ m_0 = \frac{Pr + \sqrt{Pr^2 + 4FPr}}{2},\ \ m_1 = \frac{\lambda + \sqrt{\lambda^2 - 4n\lambda}}{2}$$

$$m_2 = \frac{Pr + \sqrt{Pr^2 - 4Pr\,(n-F)}}{2},\ \ m_3 = \frac{1 + \sqrt{1 + 4(1+\alpha)M_1}}{2},\ \ m_4 = \frac{1 + \sqrt{1 + 4(1+\alpha)M_2}}{2}$$

$$A_1 = -\frac{Gr}{(1+\alpha)m_0^2 - m_0 - M_1},\ \ A_2 = \frac{2\alpha\lambda}{(1+\alpha)\lambda^2 - \lambda - M_1},\ \ A_3 = \frac{2\alpha m_1}{(1+\alpha)m_1^2 - m_1 - M_2}$$

$$A_4 = -\frac{Gr}{(1+\alpha)m_2^2 - m_2 - M_2},\ \ A_5 = \frac{m_3 - K_0^{-1}}{(1+\alpha)m_3^2 - m_3 - M_2},\ \ A_6 = \frac{A_1 C_0 (m_0 - K_0^{-1})}{(1+\alpha)m_0^2 - m_0 - M_2}$$

$$A_7 = \frac{A_2(\lambda - K_0^{-1}) + 2\alpha\lambda^2 n^{-1}}{(1+\alpha)\lambda^2 - \lambda - M_2},\ \ B = \frac{Prm_0}{1 + h_2 m_0\{m_0^2 - Prm_0 + Pr(n-F)\}},\ \ C_0 = \frac{1}{1 + h_2 m_0}$$

$$C_1 = \frac{a_3 a_4 - a_1 a_6}{a_2 a_4 - a_1 a_5},\ \ C_2 = \frac{a_9 a_{10} - a_7 a_{12}}{a_8 a_{10} - a_7 a_{11}},\ \ C_3 = \frac{1 - B(1 + h_2 m_0)}{1 + h_2 m_2},\ \ C_4 = \frac{a_2 a_6 - a_3 a_5}{a_2 a_4 - a_1 a_5}$$

$$C_5 = \frac{a_8 a_{12} - a_9 a_{11}}{a_8 a_{10} - a_7 a_{11}},\qquad a_1 = 1 + m_3 h_1,\qquad a_2 = (1 + \lambda h_1)A_2$$

$$a_3 = 1 - (1 + h_1 m_0)A_1 C_0,\qquad a_4 = m_3^2,\qquad a_5 = \lambda(A_2\lambda - \lambda)$$

$$a_6 = -A_1 C_0 m_0^2,\qquad a_7 = 1 + h_1 m_4,\qquad a_8 = A_3(1 + m_1 h_1)$$

$$a_9 = 1 - A_4 C_3 (1 + h_1 m_2) - A_5 C_4 (1 + h_1 m_3) - A_6(1 + h_1 m_0) - A_7 C_1 (1 + h_1\lambda)$$

$$a_{10} = m_4^2,\qquad a_{11} = m_1(A_3 m_1 - m_1)$$

$$a_{12} = \lambda^2 C_1 (n^{-1} - A_7) - m_2^2 A_4 C_3 - m_3^2 A_5 C_4 - m_0^2 A_6$$

NOMENCLATURE

B_0 - Uniform magnetic field

C_p - Specific heat at constant pressure

G_r - Grashof number

g - Acceleration due to gravity

j' - Dimensional micro-inertia

j - non-dimensional micro-inertia

k - Effective thermal conductivity

k_c - Absorption coefficient

K_0 - Mean permeability of the medium

L - Mean free path

m_1 - Maxwell's reflection coefficient

n^* - Dimensional exponential index

n - non-dimensional exponential index

Pr - Prandtl number

T^* - Dimensional temperature of the fluid

T - non-dimensional temperature of the fluid

T^*_∞ - Temperature of the free stream

T^*_w - Temperature of the plate

t^* - Dimensional time

t - non-dimensional time

u_0 Scale of free stream velocity

u^*, v^* - Dimensional components of velocity along x' and y'-directions

u, v - along non-dimensional components of velocity x' and y'-directions

v_0 - Scale of suction velocity

x^*, y^* - Dimensional spatial coordinates along and normal to the plate

x, y - non-dimensional spatial coordinates along and normal to the plate

GREEK SYMBOLS

α - Effective thermal diffusivity of the fluid

β - Coefficient of volumetric expansion of the fluid

γ_1 - Specific heat ratio

ε - Perturbation parameter ($<<1$)

ρ -Density of the fluid

σ - Electrical conductivity of the fluid

λ - Microrotation parameter

μ - Dynamic viscosity

ν - Kinematic viscosity of the fluid

ν_r - Kinematic rotational viscosity of the fluid

ω^* - Dimensional microrotation variable

ω - non-dimensional microrotation variable

γ^* -Dimensional spin gradient viscosity

γ - non-dimensional spin gradient viscosity

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