Inverse Nonsplit Domination in Certain Interconnection Network Models

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Abstract

A graph with the server represented by its vertex and the link between its edges is commonly used to depict the topology of an interconnection network. An essential foundation for evaluating and assessing the dependability of interconnection networks is the dominating parameters. If <V-D> is connected, then a dominating set D ⊆ V(G) is considered nonsplit dominating set. A minimal nonsplit dominating set of G is denoted by D. With regard to D, let D' be the smallest inverse nonsplit dominating set of G. In the event that the induced subgraph <V-D'> is connected, D' is referred to be an inverse nonsplit dominating set of G. In this study, we define the inverse nonsplit domination number, give certain properties of mesh and torus networks, and calculate the inverse nonsplit domination number of two-dimensional mesh networks, generalized hypercube networks, and torus networks.

Keywords: Interconnection network, mesh, torus, generalized hypercube, nonsplit, inverse nonsplit.

AMS subject Classification (2020): 05C69, 05C12, 05C45.

1 Introduction:

A graph with the server represented by its vertex and the link between its edges is commonly used to depict the topology of an interconnection network. Let G be a connected graph with vertex set V(G) and edge set E(G) that is a finite graph.

In order to connect processors (or nodes) with a supercomputer, mesh and Torus networks are frequently utilised. These networks are usually distinguished by the way switches and nodes are arranged, which is crucial for reducing latency and optimising bandwidth throughout the network. Data center networks may also be designed using the generalized hypercube network architecture, which combines hypercube, mesh, and torus networks.

Graph domination theory is widely used in the research field of graph theory itself. The application of domination parameters as a tool in communication networks and monitoring systems is also developing and deepening. For example, literature [3,5,6] studies the domination of some interconnection networks.

A set of vertices is called independent in a graph where no two vertices in the set are connected by an edge. It is denoted by $\beta(G)$. A vertex cover in a graph is a set of vertices such that every edge in the graph has at least one endpoint in the set. The vertex cover number is the minimum size of any vertex cover in the graph. It is denoted by $\alpha(G)$. The chromatic number of a graph is the smallest number of colors needed to color the vertices of the graph so that no two adjacent vertices share the same color, often denoted as $\chi(G)$

In recent years, various domination concepts have been produced and the research results have been enriched. Here consider the problem of selecting two disjoint sets of transmitting stations D_1 (D_2) has a link with at least one station in D_1 (D_2), where $|D_1|$ and $|D_1 \cup D_2|$ are minimum among all the pairs of disjoint transmitting stations. This led Kulli et al. [7] to define the inverse domination number. K. Ameenal Bibi and R. Selva Kumar [2] introduced the concept of inverse nonsplit domination in graphs.

Since the problem of determining the domination parameters of graphs in an NPC problem, most scholars mainly study the upper and lower bounds of domination parameters and exact values of domination parameters of special graphs while the inverse domination numbers of many complex network topologies are less studied. In this paper, we found the inverse nonsplit domination number of mesh, torus and generalized hypercube network.

A set of vertices $D \subseteq V(G)$ is a dominating set of a graph G if each vertex in V is either in D or is adjacent to at least one vertex in D. Let D be a minimum dominating set of G. If V-D contains a dominating set say D' of G, then D' is called an inverse dominating set of G with respect to D. The inverse domination number $\gamma'(G)$ is the order of a smallest inverse dominating set of G. A dominating set $D \subseteq V(G)$ is said to be nonsplit dominating set if $\langle V-D \rangle$ is connected.

One of the important and simple ways to construct a topological network is the Cartesian product, where both generalized and hypercube networks derive two important classes of Cayley graphs via Cartesian products.

The cartesian product $G_1 \times G_2$ is defined as follows: Let $a = (a_1, a_2)$ and $b = (b_1, b_2)$ be in $V_1 \times V_2$. If ab is an edge in $G_1 \times G_2$, whenever $a_1 = b_1$ and a_2 is adjacent to b_2 or $a_2 = b_2$ and a_1 is adjacent to b_1.

Definition:1.1

Let D be a minimum nonsplit dominating set of G. Let D' be the minimum inverse nonsplit dominating set of G with respect to D. Then D' is called an inverse nonsplit dominating set of G if the induced subgraph $\langle V-D' \rangle$ is connected. The inverse nonsplit domination number $\gamma_{ns}'(G)$ is the order of a smallest inverse dominating set of G.

2 Main Results:

Definition:2.1

We denote n- dimensional generalized mesh network as M $(d_1, d_2, \ldots, d_n)$, d_i *is an integer*, $d_i \geq 2$, i = 1, 2, …., n. We can easily describe it as M $(d_1, d_2, \ldots, d_n) = P_{d_1} \times P_{d_2} \times \ldots \times P_{d_n}$. The vertex set V = $\{a_1 a_2 \ldots a_n : a_i \in \{0,1,\ldots,d_i - 1\}$. The vertices a = $a_1 a_2 \ldots a_n$ and $b_1 b_2 \ldots b_n$ are adjacent if and only if $\sum_{i=1}^{n} |a_i - b_i| = 1$.

Example:2.1.1

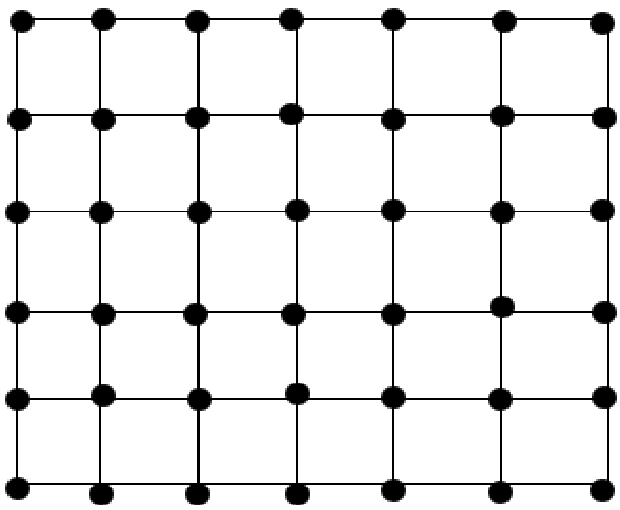

Figure 1: Two dimensional Mesh Network M(6,7)

Theorem:2.2

For n=2, the vertex cover number α (M(d_1, d_2)) = $\lfloor \frac{d_1 d_2}{2} \rfloor$.

Proof:

Let S be the vertex cover set of M(d_1, d_2). The Structural characteristics of the n- dimensional mesh network M(d_1, d_2) = $P_{d_1} \times P_{d_2}$. The vertex set is {0a: a ∈ $\{0,1,\ldots,(d_2 - 1)\}\}$ ∪ {1a: a ∈ $\{0,1,\ldots,(d_2 - 1)\}\}$ ∪……..∪ {$(d_1 - 1)$a: a ∈ $\{0,1,\ldots,(d_2 - 1)\}\}$. The vertex set can form a $d_1 \times d_2$ matrix.

It is clear that there is a second coordinate difference in each row's vertices. So each row can generate a path graph P_{d_2}. If $d_1 < d_2$, now to create a minimal vertex cover set S, it is sufficient to choose non adjacent vertices in each matrix row. Since the number of rows is minimum. |S| = $\bigcup_{i=0}^{d_1 - 1} \bigcup_{j=0}^{d_2 - 1} \{ij\} = \lfloor \frac{d_1 d_2}{2} \rfloor$.

It is clear that there is a first coordinate difference in each column's vertices. So each column can generate a path graph P_{d_1}. If $d_1 > d_2$, now to create a minimal vertex cover set S, it is sufficient

to choose non adjacent vertices in each matrix column. Since the number of column is minimum. |S| = $\bigcup_{i=0}^{d_1-1}\bigcup_{j=0}^{d_2-1}\{ij\} = \lfloor \frac{d_1 d_2}{2} \rfloor$.
If $d_1 = d_2$, choose non adjacent vertices from every row or column and collect the vertices in S such that |S| = $\bigcup_{i=0}^{d_1-1}\bigcup_{j=0}^{d_1-1}\{ij\} = \lfloor \frac{d_1{}^2}{2} \rfloor$.

Theorem:2.3

For n ≥ 2 , the chromatic Number of n- dimensional Mesh Network is $\chi($ M $(d_1 d_2, \ldots\ldots, d_n)$) = 2.

Proof:

Using mathematical induction we can prove this result. For n =2, M $(d_1, d_2) = P_{d_1} \times P_{d_2}$. Clearly, the vertex set can form a $d_1 \times d_2$ matrix. Suppose $d_1 \leq d_2$ or $d_1 > d_2$ both the cases , $\chi($ M (d_1, d_2)) = 2. For n = 3, M $(d_1, d_2, d_3) = P_{d_1} \times P_{d_2} \times P_{d_3}$. First consider the partition, $P_{d_2} \times P_{d_3}$. Previous case, $\chi($M (d_2, d_3)) = 2. Now V(M(d_1, d_2, d_3)) = {0a: a ∈ $\{0,1, \ldots , (d_3 - 1)\}$} ∪ {1a: a ∈ $\{0,1, \ldots , (d_3 - 1)\}$} ∪ ∪ {$(d_2 - 1)$a: a ∈ $\{0,1, \ldots , (d_3 - 1)\}$}. The vertex set can form d_1 times $d_2 \times d_3$ matrix. Since every row and every column form a path graph P_{d_3} and P_{d_2}. So Assign colour 1 and 2 to the non adjacent vertices of V(M(d_1, d_2, d_3)). Proceeding like this, we get $\chi($ M $(d_1 d_2, \ldots\ldots, d_n)$) = 2.

Lemma:2.4

Let D' be a minimum inverse nonsplit dominating set of M(d_1, d_2) with (a_i ,b_j) ∈ D' where i ∈ d_1, j ∈ d_2. Then D' contains at least |D'| - $\frac{d_2}{4}$ odd vertices in a_i.

Proof :

Case (i): d_1 is odd, d_2 is odd or even
In this case, consider $\bigcup_{i=1}^{d_1}\bigcup_{j=1}^{d_2}(a_{i\,,} b_j)$ ∈ D' where i is odd and j is even (or) odd.
Thus |D'| contains at most d_2 odd vertices.
Case (ii): d_1 is even, d_2 is odd (or) even
In this case, suppose |D'| = $\lceil \frac{d_2+1}{4} \rceil$ even vertices. Take a, b be any alternate vertices in D_1' where a is even and b is odd. Consider another inverse nonsplit dominating set
D'' = $\frac{D_1'}{b}$ which becomes a disconnected or is not a minimum. So D_1' > D'. which is a contradiction. Thus D' contains $\lceil \frac{d_2}{4} \rceil$ number of even vertices.

Theorem : 2.5

Let G be an 2 dimensional M(d_1, d_2) mesh network, Then

(i) γ_{ns}'(M (d_1, 3)) = d_1.

(ii) γ_{ns}'(M (d_1, 4)) = d_1 +1.

Proof:

(i) Let $V(M(d_1, d_2)) = \{(a_i, b_j) / 1 \le i \le d_1, 1 \le j \le 3\}$. The inverse nonsplit dominating set D' of $M(d_1, d_2)$ is $D' = \{(a_i, b_j) / 1 \le i \le d_1$, i is even $\} \cup \{(a_i, b_3) / 1 \le i \le d_1$, i is odd$\}$. And so $|D'| = d_1$. Thus $\gamma_{ns}'(M(d_1, 3)) = d_1$.

(ii)Let $V(M(d_1, d_2)) = \{(a_i, b_j) / 1 \le i \le d_1, 1 \le j \le 4\}$. By Previous Lemma,
Let $X = \{a_1, a_5, a_9, a_{13}, a_{17}, \ldots\}$ and $Y = \{a_3, a_7, a_{11}, a_{15}, \ldots\}$ are the set of odd vertices in D'.
If d_1 is odd, $D_{ns}'(M(d_1,4)) = \{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{4} (a_i, b_j) / a_i \in X, j \text{ is odd}\} \cup \{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{4} (a_i, b_j) / a_i \in Y, j \text{ is even}\}$. And so $|D'| = d_1 + 1$. Thus $\gamma_{ns}'(M(d_1, 4)) = d_1 + 1$.
If d_1 is even, $D_{ns}'(M(d_1, 4)) = \{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{4} (a_i, b_j) / a_i \in X, j \text{ is odd}\} \cup \{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{4} (a_i, b_j) / a_i \in Y, j \text{ is even}\} \cup \{(a_n, b_j) / a_{n-1} \in X \text{ and } j = 2\}$. And so $|D'| = d_1 + 1$.
Thus $\gamma_{ns}'(M(d_1, 4)) = d_1 + 1$.

Theorem :2.6

Let G be any $M(d_1, d_2)$ mesh network with $d_1, d_2 \ge 2$ and $d_1, d_2 \ne 3$. $1 \le i \le d_1$ Then

$$\gamma_{ns}'(M(d_1, d_2)) = \begin{cases} |X| \times \lfloor \frac{d_2}{2} \rfloor + |Y| \times \lceil \frac{d_2}{2} \rceil & \text{if } d_1 \text{ is odd} \\ |X| \times \lfloor \frac{d_2}{2} \rfloor + |Y| \times \lceil \frac{d_2}{2} \rceil + \lceil \frac{d_2 - 1}{4} \rceil & \text{if } d_1 \text{ is even } a_i \in A \\ |X| \times \lfloor \frac{d_2}{2} \rfloor + |Y| \times \lceil \frac{d_2}{2} \rceil + \lceil \frac{d_2}{4} \rceil & \text{if } d_1 \text{ is even } a_i \in B \end{cases}$$

Proof:

Let $V(M(d_1, d_2)) = \{(a_i, b_j) / 1 \le i, j \le d_1\}$. Consider $X = \{a_1, a_5, a_9, a_{13}, a_{17}, \ldots\}$ and $Y = \{a_3, a_7, a_{11}, a_{15}, \ldots\}$ are the collection of odd vertices in D'.

Case (i): d_1 is odd and d_2 is even or odd
Consider the inverse nonsplit dominating set D' of $M(d_1, d_2)$.
$D_{ns}'(M(d_1,d_2)) = \{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{d_2} (a_i, b_j) / a_i \in X, \text{ j is odd}\} \cup \{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{d_2} (a_i, b_j) / a_i \in Y$, j is even$\}$. Then $D' = |X| \times \{$odd vertices in $P_{d_1}\} + |Y| \times \{$even vertices in $P_{d_2}\}$
Therefore $D' = |X| \times \lfloor \frac{d_2}{2} \rfloor + |Y| \times \lceil \frac{d_2}{2} \rceil$.

Case (ii): d_1 is even and d_2 is even or odd
Without loss of generality, assume that $A = \{a_2, a_6, a_{10}, a_{14}, a_{18}, \ldots\}$ and $B = \{a_4, a_8, a_{12}, a_{16}, a_{20}, \ldots\}$ are the collection of even vertices in P_{d_1}.
subcase (i):
Suppose $A = \{a_2, a_6, a_{10}, a_{14}, a_{18}, \ldots\}$ and d_2 is odd then the inverse nonsplit dominating set D'.
$D_{ns}'(M(d_1,d_2)) = \{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{d_2} (a_I, b_j) / a_i \in X, \text{ j is odd}\} \cup \{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{d_2} (a_I, b_j) / a_i \in Y$, j is even$\} \cup \{(a_{d_1}, b_j) / a_{d_1} \in X \text{ and } j = 3,7,11,15, \ldots d_2 - 2\}$.. Then $D' = |X| \times \{$odd vertices in $P_{d_1}\} + |Y| \times \{$even vertices in $P_{d_2}\}$ .Therefore $D' = |X| \times \lfloor \frac{d_2}{2} \rfloor + |Y| \times \lceil \frac{d_2}{2} \rceil +$

$\lceil\frac{d_2-1}{4}\rceil$ .Thus γ_{ns}'(M(d1, d2)) $= |X| \times \lfloor\frac{d_2}{2}\rfloor + |Y| \times \lceil\frac{d_2}{2}\rceil + \lceil\frac{d_2-1}{4}\rceil$.

If d_2 is even then the inverse nonsplit dominating set D'. D_{ns}'(M(d1,d2)) = $\{\bigcup_{i=1}^{d_1}\bigcup_{j=1}^{d_2}(a_i, b_j)$ / $a_i \in$ X, j is odd$\}$ $\cup$ $\{\bigcup_{i=1}^{d_1}\bigcup_{j=1}^{d_2}(a_i, b_j) / a_i \in$ Y, j is even$\}$ $\cup$ $\{(a_n, b_j)/ a_n \in X \; and \; j = 3,7,11,\ldots.. d_2 - 1\}$. Then D' = |X| × {odd vertices in P_{d_1}} + |Y| × {even vertices in P_{d_2}} .Therefore D' $= |X| \times \lfloor\frac{d_2}{2}\rfloor + |Y| \times \lceil\frac{d_2}{2}\rceil + \lceil\frac{d_2-1}{4}\rceil$ .Thus γ_{ns}'(M(d1, d2)) $= |X| \times \lfloor\frac{d_2}{2}\rfloor + |Y| \times \lceil\frac{d_2}{2}\rceil + \lceil\frac{d_2-1}{4}\rceil$.

Subcase (ii):

Suppose B = $\{a_4, a_8, a_{12}, a_{16}, \ldots.\}$ and d_2 is even then the inverse nonsplit dominating set D'. D_{ns}'(M(d1,d2)) = $\{\bigcup_{i=1}^{d_1}\bigcup_{j=1}^{d_2}(a_i, b_j)$ / $a_i \in$ X, j is odd$\}$ $\cup$ $\{\bigcup_{i=1}^{d_1}\bigcup_{j=1}^{d_2}(a_i, b_j) / a_i \in$ Y, j is even$\}$ $\cup$ $\{(a_{d_1}, b_j)/ a_{d_1} \in X \; and \; j = 2,6,10,\ldots.. d_2 - 2$. Then D' = |X| × {odd vertices in P_{d_1}} + |Y| × {even vertices in P_{d_2}} .Therefore D' $= |X| \times \lfloor\frac{d_2}{2}\rfloor + |Y| \times \lceil\frac{d_2}{2}\rceil + \lceil\frac{d_2}{4}\rceil$. Thus γ_{ns}'(M(d1, d2)) $= |X| \times \lfloor\frac{d_2}{2}\rfloor + |Y| \times \lceil\frac{d_2}{2}\rceil + \lceil\frac{d_2}{4}\rceil$.

If d_2 is odd then the inverse nonsplit dominating set D'. D_{ns}'(M(d1,d2)) = $\{\bigcup_{i=1}^{d_1}\bigcup_{j=1}^{d_2}(a_i, b_j)$ / $a_i \in$ X, j is odd$\}$ $\cup$ $\{\bigcup_{i=1}^{d_1}\bigcup_{j=1}^{d_2}(a_i, b_j) / a_i \in$ Y, j is even$\}$ $\cup$ $\{(a_{d_1}, b_j)/ a_{d_1} \in X \; and \; j = 2,6,10,\ldots.. d_2 - 1\}$. Then D' = |X| × {odd vertices in P_{d_1}} + |Y| × {even vertices in P_{d_2}} .Therefore D' $= |X| \times \lfloor\frac{d_2}{2}\rfloor + |Y| \times \lceil\frac{d_2}{2}\rceil + \lceil\frac{d_2}{4}\rceil$ .Thus γ_{ns}'(M(d1, d2)) $= |X| \times \lfloor\frac{d_2}{2}\rfloor + |Y| \times \lceil\frac{d_2}{2}\rceil + \lceil\frac{d_2}{4}\rceil$.

Definition:2.7

We denote n- dimensional generalized Torus network as T($d_1 d_2, \ldots\ldots, d_n$), d_i *is an integer* , $d_i \geq 2$, I = 1, 2, …., n. We can easily describe it as T ($d_1 d_2, \ldots\ldots, d_n$) $= C_{d_1} \times C_{d_2} \times \ldots \times C_{d_n}$. The vertex set V = $\{a_1 a_2 \ldots\ldots a_n : a_i \in \{0,1, \ldots, d_i - 1\}$. The vertices a = $a_1 a_2 \ldots\ldots a_n$ and $b_1 b_2 \ldots\ldots b_n$ are adjacent if and only if the following conditions are holds (i) $\sum_{i=1}^{n} |a_i - b_i| = 1$ and (ii) $\sum_{i=1}^{n} |a_i - b_i|$

$= d_i$-1 for i = 1,2, n.

Using the above theorem 2.6 , we obtain the value of the inverse nonsplit domination number of 2D Mesh Network for $2 \leq d_1 \leq d_2 \leq 20$ as shown in Table 1.

d_1d_2	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
2	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11
3		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
4			5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
5				7	9	10	11	12	14	15	16	17	19	20	21	22	24	25	26
6					11	12	14	15	17	18	20	21	23	24	26	27	29	30	32
7						14	16	17	19	21	23	24	26	28	30	31	33	35	37
8							18	20	22	24	26	28	30	32	34	36	38	40	42
9								22	25	27	29	31	34	36	38	40	43	45	47
10									28	30	33	35	38	40	43	45	48	50	53
11										33	36	38	41	44	47	49	52	55	58
12											39	42	45	48	51	54	57	60	63
13												45	49	52	55	58	62	65	68
14													53	56	60	63	67	70	74
15														60	64	67	71	75	79
16															68	72	76	80	84
17																76	81	85	89
18																	86	90	95
19																		95	100
20																			105

Table 1: Inverse Nonsplit Domination Number of 2D Mesh Network

Example:2.7.1

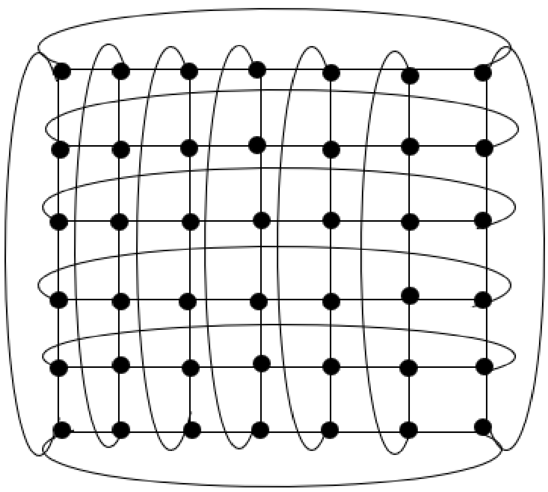

Figure 2: Two dimensional Torus Network T (6,7)

Theorem:2.8

For n=2, Independent number of T (d_1, d_2) torus network is

$$\beta_0(T(d_1, d_2)) = \begin{cases} d_2 & if \quad d_1 < d_2 \\ d_1 & if \quad d_1 \geq d_2 \end{cases}$$

Proof:

Let D be the largest independent set of T (d_1, d_2) . According tp the structural characteristics of the generalized torus network T (d_1, d_2) = $C_{d_1} \times C_{d_2}$, the vertex set is of the form

The vertex set is {0a: a $\in \{0,1, \ldots, (d_2 - 1)\}\}$ ∪ {1a: a $\in \{0,1, \ldots, (d_2 - 1)\}\}$ ∪........∪ {(d_1 −1)a: a $\in \{0,1, \ldots, (d_2 - 1)\}\}$. The vertex set can form a $d_1 \times d_2$ matrix.

It is clear that there is a second coordinate difference in each row's vertices. So each row can generate a cycle graph P_{d_2}. If $d_1 \geq d_2$, now to create an independent set D, it is sufficient to choose non adjacent vertices in each matrix row. Since the number of row's is maximum. |D| = $\bigcup_{i=0}^{d_1-1} \bigcup_{j=0}^{d_2-1}\{ij\} = d_2$.

It is clear that there is a first coordinate difference in each column's vertices. So each column can generate a cycle graph P_{d_1}. If $d_1 < d_2$, now to create an independent set D, it is sufficient to choose non adjacent vertices in each matrix column. Since the number of column's is maximum. |D| = $\bigcup_{i=0}^{d_1-1} \bigcup_{j=0}^{d_2-1}\{ij\} = d_1$.

Theorem: 2.9

Let G be any T (d_1, d_2) torus network with $d_1, d_2 \geq 2$ and $d_1, d_2 \neq 3$. $1 \leq i \leq d_1$ Then

$$\gamma_{ns}'(T(d_1, d_2)) = \begin{cases} |X| \times \lfloor \frac{d_2}{2} \rfloor + |Y| \times \lceil \frac{d_2}{2} \rceil & if \ a_{d_1} \notin A \\ |X| \times \lfloor \frac{d_2}{2} \rfloor + |Y| \times \lceil \frac{d_2}{2} \rceil + \lceil \frac{d_2}{4} \rceil & if \ a_{d_1} \in A \end{cases}$$

Proof:

Let V (M (d_1, d_2)) = {(a_i, b_j) / $1 \leq i \leq d_1, 1 \leq j \leq d_2$}. Consider X = {$a_1$, a_5, a_9, a_{13}, a_{17}, ……} and Y= {a_3, a_7, a_{11}, a_{15}, …….} are the set of odd vertices in T (d_1, d_2). Without loss of generality

assume that A = {a_6, a_{10}, a_{14}, a_{18}, ….} and B = {a_4, a_8, a_{12}, a_{16}, a_{20}, ….} are the collection of even vertices in T (d_1, d_2).

Case (i): $a_{d_1} \notin A$

Consider the inverse nonsplit dominating set D' of T (d_1, d_2).

D_{ns}'(T(d_1,d_2)) = $\{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{d_2}(a_i, b_j) \ / \ a_i \in X, \ j \text{ is odd}\} \cup \{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{d_2}(a_i, b_j) \ / \ a_i \in Y, \ j \text{ is even}\}$. Then D' = |X| × {odd vertices in P_{d_1}} + |Y| × {even vertices in P_{d_2}}

Therefore D' = |X| $\times \lfloor \frac{d_2}{2} \rfloor + |Y| \times \lceil \frac{d_2}{2} \rceil$.

Case (ii): $a_{d_1} \in A$

If d_2 is odd then the inverse nonsplit dominating set D' of T(d_1, d_2) is D_{ns}'(M(d_1,d_2)) = $\{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{d_2}(a_i, b_j) / a_i \in X, \ j \text{ is odd}\} \cup \{\bigcup_{i=1}^{d_1} \bigcup_{j=1}^{d_2}(a_i, b_j) \ / \ a_i \in Y, \ j \text{ is even}\} \cup \{(a_{d_1}, b_j)/ \ a_{d_1} \in X \ and \ j = 3,7,11 \ldots.. d_2 - 2\}$.. Then D' = |X| × {odd vertices in P_{d_1}} + |Y| × {even vertices in P_{d_2}} .Therefore D' = $|X| \times \lfloor \frac{d_2}{2} \rfloor + |Y| \times \lceil \frac{d_2}{2} \rceil + \lceil \frac{d_2}{4} \rceil$. Thus γ_{ns}'(M(d_1, d_2)) = $|X| \times \lfloor \frac{d_2}{2} \rfloor + |Y| \times \lceil \frac{d_2}{2} \rceil + \lceil \frac{d_2}{4} \rceil$.

If d_2 is even then the inverse nonsplit dominating set D' of T(d_1, d_2) is D_{ns}'(M(d_1,d_2)) =

$\{\cup_{i=1}^{d_1}\cup_{j=1}^{d_2}(a_i, b_j)$ / a_i $\in$ X, j is odd$\}$ $\cup$ $\{\cup_{i=1}^{d_1}\cup_{j=1}^{d_2}(a_i, b_j) / a_i \in$ Y, j is even$\}$ $\cup$ $\{(a_{d_1}, b_j)/ a_{d_1} \in X\ and\ j = 3,7,11,15,\ldots.. d_2 - 1\}$. Then D' = |X| × {odd vertices in P_{d_1}} + |Y| × {even vertices in P_{d_2}} .Therefore D' = $|X| \times \lfloor\frac{d_2}{2}\rfloor + |Y| \times \lceil\frac{d_2}{2}\rceil + \lceil\frac{d_2}{4}\rceil$.Thus γ_{ns}'(M(d₁, d₂)) = $|X| \times \lfloor\frac{d_2}{2}\rfloor + |Y| \times \lceil\frac{d_2}{2}\rceil + \lceil\frac{d_2}{4}\rceil$.

Using the above theorem 2.9 , we obtain the value of the inverse nonsplit domination number of 2D Torus Network for $3 \leq d_1 \leq d_2 \leq 20$ as shown in Table 2.

d₁d₂	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
3	3	3	4	5	6	6	7	8	9	9	10	11	12	12	13	14	15	15
4		4	6	7	8	8	10	11	12	12	14	15	16	16	18	19	20	20
5			7	9	10	10	12	14	15	15	17	19	20	20	22	24	25	25
6				11	12	12	15	17	18	18	21	23	24	24	27	29	30	30
7					14	14	17	19	21	21	24	26	28	28	31	33	35	35
8						16	20	22	24	24	28	30	32	32	36	38	40	40
9							22	25	27	27	31	34	36	36	40	43	45	45
10								28	30	30	35	38	40	40	45	48	50	50
11									33	33	38	41	44	44	49	52	55	55
12										36	42	45	48	48	54	57	60	62
13											45	49	52	52	58	62	65	65
14												53	56	56	63	67	70	70
15													60	60	67	71	75	75
16														64	72	76	80	80
17															76	81	85	85
18																86	90	90
19																	95	95
20																		100

Table 2: Inverse Nonsplit Domination number of 2D Torus Network

Definition: 2.10

We denote n- dimensional generalized hypercube network as Q $(d_1 d_2, \ldots\ldots, d_n)$, d_i *is an integer* , $d_i \geq 2$, i = 1, 2, …., n. We can easily describe it as Q $(d_1 d_2, \ldots\ldots, d_n)$ = $K_{d_1} \times K_{d_2} \times \ldots \times K_{d_n}$. The vertex set V = $\{a_1 a_2 \ldots\ldots a_n : a_i \in \{0,1, \ldots, d_i - 1\}$. The vertices a = $a_1 a_2 \ldots\ldots a_n$ and $b_1 b_2 \ldots\ldots b_n$ are adjacent if and only if a and b vary in exactly one coordinate.

Example: 2.10.1

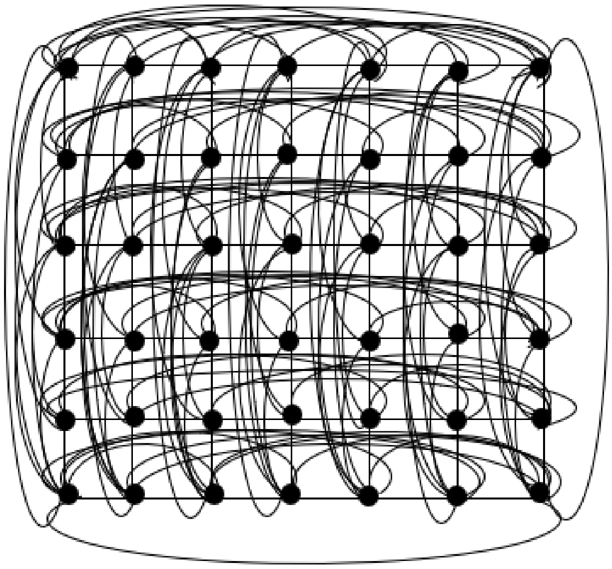

Figure 2: Two dimensional Generalized Hypercube Network Q (6,7)

Theorem:2.11

For 2- dimensional Generalized Hypercube Network γ_{ns}'(Q(d_1, d_2)) = $\begin{cases} d_1 & if \quad d_1 < d_2 \\ d_2 & if \quad d_1 \geq d_2 \end{cases}$

Proof:

For n=2, then the GHN Q (d_1, d_2) = $K_{d_1} \times K_{d_2}$. The vertex set is of the form {0u: u ∈ {0, 1, …, (d_2 – 1)}} ∪ {1u: u ∈ {0, 1, …, (d_2 – 1)}} ∪…….∪ {(d_1-1) u: u ∈ {0, 1, …, (d_2 – 1)}}. This vertex set can form a $d_1 \times d_2$ matrix.

Case (i): Suppose $d_1 < d_2$

It is clear that there is a single co-ordinate difference between each row's vertices in the second one. Consequently, every vertex in a row is adjacent to every other vertex in that same row. As a result, every row can generate a complete graph K_{d_2}. Now to create a dominating set it is sufficient to choose single vertex from each row of $d_1 \times d_2$ matrix such that D = $\bigcup_{i=1}^{d_1-1} \bigcup_{j=1}^{d_2-1} ij$ where i's are distinct and j's are distinct. Since row is minimum, |D| = d_1. Clearly <V (Q (d_1, d_2)) – D> is connected. Therefore D is a nonsplit Dominating set. We can find one more nonsplit dominating set with same cardinality. Hence γ_{ns}'(Q(d_1, d_2)) = d_1.

Case (i): Suppose $d_1 \geq d_2$

If $d_1 = d_2$, then the vertex set can form a $d_1 \times d_2$ square matrix. The minimum nonsplit dominating set D = {00, 11, …, (d_1 -1) (d_2-1)}. Clearly we can find one more dominating set D'
with |D| = |D'|. where D' is an inverse nonsplit dominating set of V (Q (d_1, d_2)). If $d_1 > d_2$, It is clear that there is a single co-ordinate difference between each column's vertices in the second one. Consequently, every vertex in a column is adjacent to every other vertex in that same column. As a result, every column can generate a complete graph K_{d_1}. Now to create a dominating set it is sufficient to choose single vertex from each column of $d_1 \times d_2$ matrix such that D = $\bigcup_{i=1}^{d_1-1} \bigcup_{j=1}^{d_2-1} ij$ where i's are distinct and j's are distinct. Since column is minimum, |D| = d_1.

Clearly <V (Q (d_1, d_2)) – D> is connected. Therefore D is a nonsplit Dominating set. We can find one more nonsplit dominating set with same cardinality. Hence γ_{ns}'(Q(d_1, d_2)) = d_2.

Observation:2.12

Evaluating four or more numbers can be challenging. When the values of d_1, d_2, ..., d_n are substantial, visualizing the graph becomes intricate. If we swap the d_i's, the structures of the graphs remain isomorphic.

Theorem: 2.13

For n ≥ 3, The Inverse Nonsplit Domination Number of n dimensional Generalized Hypercube Network is γ_{ns}'(Q (d_1, d_2, d_3, ..., d_{n-1}, d_n)) = $d_1.d_2.d_3...d_{n-2}$. γ_{ns}'(Q(d_{n-1},d_n))

Proof:

Using Mathematical Induction on n we prove this result.
Let n = 3, Q(d_1,d_2,d_3) = $K_{d_1} \times K_{d_2} \times K_{d_3}$ can be divided into d_1 groups according to the first coordinate. Then each group can form $d_2 \times d_3$ matrix. Let D be the nonsplit dominating set of Q(d_1, d_2, d_3).

Case (i): Suppose $d_2 < d_3$
By Previous Theorem, The minimum inverse nonsplit dominating set is d_2. Since $d_2 \times d_3$ matrix can be formed into d_1 times, we get γ_{ns}'(Q(d_1, d_2,d_3)) = d_2d_1.

Case (ii): Suppose $d_2 \geq d_3$
Subcase (i): $d_2 > d_3$
By Previous Theorem, The minimum inverse nonsplit dominating set is d_3. Since $d_2 \times d_3$ matrix can be formed into d_1 times, we get γ_{ns}'(Q(d ,d_2,d_3)) = d_3d_1.
Subcase (ii): $d_2 = d_3$
By Previous Theorem, The minimum inverse nonsplit dominating set is d_3. Since $d_2 \times d_3$ matrix can be formed into d_1 times, we get γ_{ns}'(Q(d ,d_2,d_3)) = d_3d_1. From the above cases, we conclude that γ_{ns}'(Q(d_1,d_2,d_3)) = d_1.min(d_2,d_3) = d_1. γ_{ns}'(Q(d_2, d_3)).
Let n = 4, then Q(d_1, d_2, d_3) = $K_{d_1} \times K_{d_2} \times K_{d_3} \times K_{d_4}$. According to the structural characteristics of the generalized hypercube network, first we consider the $K_{d_3} \times K_{d_4}$. It can form $d_3 \times d_4$ matrix. Clearly γ_{ns}' (Q(d_3, d_4)) = min(d_3,d_4). Next we consider Q(d_2, d_3,d_4). It can be divided into d_2 groups according to the first coordinate. Then each group can form a $d_3 \times d_4$ matrix. Then Clearly γ_{ns} ′ (Q(d_2, d_3, d_4)) = d_2 min(d_3,d_4). Finally, Consider Q(d_1,d_2,d_3,d_4) =$K_{d_1} \times K_{d_2} \times K_{d_3} \times K_{d_4}$. The vertex set of Q($d_1$, d_2,d_3,d_4) can be divided into d_1d_2 groups according to first coordinate. Then each group can form a $d_3 \times d_4$ matrix. Therefore γ_{ns}' (Q(d_1, d_2, d_3, d_4)) = $d_1.d_2$.min(d_3,d_4) = $d_1.d_2$. γ_{ns}' (Q(d_3, d_4)). The Statement is true for n = 3,4. Now assume that the statement is true for n−1. Thus γ_{ns}' (Q(d_1, d_2, d_3,....,d_{n-2}, d_{n-1})) = $d_1.d_2.d_3$ d_{n-3}. γ_{ns} ′ (Q(d_{n-2}, d_{n-1})). Now to prove the statement is true for n. For Generalized Hypercube Network Q(d_1, d_2,...d_{n-1},d_n) =$K_{d_1} \times K_{d_2} \times ... \times K_{d_n}$. = K_{d_1}.Q(d_2,d_3, ,d_{n-1},d_n). Since the result is true for n − 1, we get γ_{ns}'(Q(d_2, d_3, d_4,....,d_{n-1})) = $d_2.d_3$ d_{n-2}. γ_{ns}' (Q(d_{n-1}, d_n)). Now the vertex set of Q(d_1, d_2,d_3,. ,d_{n-1},d_n) can be divided into d1 groups and each group can form a $d_{n-1} \times d_n$ matrix. Therefore γ_{ns}'(Q(d_1, d_2, d_3, d_4,....,d_{n-1}, d_n)) = $d_1.d_2.d_3....d_{n-2}$. γ_{ns}'((Q(d_{n-1}, d_n)). Hence the statement is true for all n.

3 Conclusion:

Network reliability is one of the key factors in designing network topology, which can greatly increase the cost of performance of network performance. This paper discusses the inverse nonsplit domination of multi dimensional networks, determines the exact values of it. Besides, there are many other properties, which will be studied further.

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