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(1,2)*- Delta Generalized β-Closed Maps in Bitopological Spaces

C. Nanammal[1] and K. Alli[2]

[1]Research Scholar (Reg.No.: 22221072092008), PG & Research Department of Mathematics, The M.D.T. Hindu College, Pettai, Tirunelveli, Tamil Nadu, India. Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, Tamil Nadu, India. E-mail: cnanammal@gmail.com.

[2]Associate Professor, PG & Research Department of Mathematics, The M.D.T. Hindu College, Pettai, Tirunelveli, Tamil Nadu, India. Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, Tamil Nadu, India. E-mail: allimdt@gmail.com

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Abstract

This paper introduce the concepts of (1,2)*-δgβ-closed and (1,2)*-δgβ-open maps between bitopological spaces. These maps are defined by the property that the image of any $\zeta_{1,2}$-closed or $\zeta_{1,2}$-open set under these maps remains (1,2)*-δgβ-closed or (1,2)*-δgβ-open respectively. The paper explores various characterizations of these sets and maps, examines their properties in bitopology, and investigates their connections to other bitopological concepts. The goal is to extend classical notions of closedness and openness into the context of bitopology, providing new insights into the structure and continuity of such spaces.

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Key words: (1,2)*-δgβ-closed set, (1,2)*-δgβ-open set, (1,2)*-δgβ-continuous map, (1,2)*-δgβ-closed map and (1,2)*-δgβ-open map.

1. Introduction

In the year 1980, T. Noiri [10] presented the notion of δ-closed set using δ-closure and defined a δ-open set as the complement of a δ-closed set. In this work, the concept of a δ-continuous function was introduced, and several of its properties were established.

Malghan [9] introduced and examined generalized closed mappings. Long [8] and Gnanambal [5] respectively introduced and studied regular closed maps and gpr-closed maps. Additionally, Lellis Thivagar and Ravi.O [7] initiated the exploration of (1,2)*-g-closed maps, (1,2)*-sg-closed maps, and (1,2)*-gs-closed maps in bitopological spaces.

A new type of maps, known as (1,2)*-δgβ-closed maps, is introduced and their various properties are studied in this paper. It is demonstrated that the composition of two (1,2)*-δgβ-closed maps may not necessarily be a (1,2)*-δgβ closed map. As well, significant results are obtained within bitopological settings. Throughout this paper $(\mathcal{P}, \zeta_1, \zeta_2)$, $(\mathcal{Q}, \xi_1, \xi_2)$, and $(\mathcal{R}, \varrho_1, \varrho_2)$ (briefly $\mathcal{P}$, $\mathcal{Q}$, and $\mathcal{R}$) represented the bitopological spaces.

2. Preliminaries

In this study, $\zeta_{1,2}$- $\mathcal{O}(\mathcal{P})$ and $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$ represent the set of all $\zeta_{1,2}$-open ($\zeta_{1,2}$-closed) sets in $\mathcal{P}$.

Definition 2.1: [6]

A subset $\mathcal{T}$ of a bitopological space $\mathcal{P}$ is said to be $\zeta_{1,2}$-open if $\mathcal{T} = \mathcal{E} \cup \mathcal{F}$, where $\mathcal{E} \in \zeta_1$ and $\mathcal{F} \in \zeta_2$. A subset $\mathcal{T}$ of $\mathcal{P}$ is called $\zeta_{1,2}$-closed if the complement of $\mathcal{T}$ is $\zeta_{1,2}$-open.

Definition 2.2:[6]

Let $\mathcal{T}$ be a subset of the bitopological space $\mathcal{P}$. Then

(i). The $\zeta_{1,2}$-interior of $\mathcal{T}$, denoted by $\zeta_{1,2}$- $int(\mathcal{T})$ is defined by $\cup \mathcal{C}$: $\mathcal{C} \subseteq \mathcal{T}$ and $\mathcal{C}$ is $\zeta_{1,2}$-open.

(ii). The $\zeta_{1,2}$-closure of $\mathcal{T}$, denoted by $\zeta_{1,2}$-$c\ell(\mathcal{T})$ is defined by $\cap$ $\mathcal{D}$: $\mathcal{T} \subseteq \mathcal{D}$ and $\mathcal{D}$ is $\zeta_{1,2}$-closed.

Remark 2.3:[6]

It is not necessary for $\zeta_{1,2}$-open sets to create a topology.

Definition 2.4:

When $\mathcal{E}$ is a subset of a bitopological space $\mathcal{P}$, it is referred to as

(i). (1,2)*-β-closed [3] if $\zeta_{1,2}$- $int(\zeta_{1,2}$-$c\ell(\zeta_{1,2}$- $int(\mathcal{E}) \subseteq \mathcal{E}$.

(ii). (1,2)*-δ-closed [10] if $\mathcal{E}$ =(1,2)*-δ$c\ell(\mathcal{E})$,

where (1,2)*- δ$c\ell(\mathcal{E})$=$\{\wp \in \mathcal{P}$: $\zeta_{1,2}$- $int(\zeta_{1,2}$-$c\ell(\mathcal{E})) \cap \mathcal{U} \neq \emptyset$, $\mathcal{U} \in \zeta_{1,2}$, $\wp \in \mathcal{U}\}$.

(1,2)*-open set is the complement of the (1,2)*-closed set mentioned above.

Definition 2.5:

Let $\mathcal{E}$ be a subset of a bitopological space $\mathcal{P}$. Then $\mathcal{E}$ is called

(i). a (1,2)*-semi-generalized closed ((1,2)*-sg-closed) [12] if (1,2)*-$sc\ell(\mathcal{E}) \subseteq \mathcal{U}$ whenever $\mathcal{E} \subseteq \mathcal{U}$ and $\mathcal{U}$ is (1,2)*-$\mathcal{SO}(\mathcal{P})$.

(ii). a (1,2)*-generalized semi-closed ((1,2)*-gs-closed) [12] if (1,2)*- $sc\ell(\mathcal{E}) \subseteq \mathcal{U}$ whenever $\mathcal{E} \subseteq \mathcal{U}$ and $\mathcal{U}$ is $\zeta_{1,2}$-$\mathcal{O}(\mathcal{P})$.

(iii). (1,2)*-delta generalized β-closed (briefly, (1,2)*-δgβ-closed) [1] if (1,2)*-βcl$(\mathcal{E}) \subseteq \mathcal{U}$ whenever $\mathcal{E} \subseteq \mathcal{U}$ and $\mathcal{U}$ is (1,2)*-δ$\mathcal{O}(\mathcal{P})$.

This (1,2)*-generalized closed set's complement is known as the (1,2)*-generalized-open set.

Definition 2.6:

A map $\imath: \mathcal{P} \to \mathcal{Q}$ is called

(i). (1, 2)*-closed [11] if from a (1,2)*-closed subset of the domain set to a (1,2)*-closed subset of the codomain set.

(ii). (1, 2)*-g-closed [4] if from a $\zeta_{1,2}$-closed subset of the domain set to a (1,2)*-g-closed subset of the codomain set.

Definition 2.7: [2]

We refer to a map $\imath: \mathcal{P} \to \mathcal{Q}$ as (1,2)*- δgβ-continuous if, for any $\zeta_{1,2}$-closed set $\mathcal{V}$ of $\mathcal{Q}$, $\imath^{-1}(\mathcal{V})$ is (1,2)*-δgβ-closed.

3. (1, 2)*- Delta Generalized β-Closed Maps

Definition 3.1:

An $\imath: \mathcal{P} \to \mathcal{Q}$ map is described as

(i). (1,2)*-delta generalized beta closed ((1,2)*-$\delta g\beta$-closed) if from every $\zeta_{1,2}$-closed set in the domain space it is taken to a (1,2)*-$\delta g\beta$-closed set in codomain space.

(ii). (1,2)*-generalized beta closed ((1,2)*-gβ-closed) if from every (1,2)*-gβ-closed subset of the codomain and it is taken to a $\zeta_{1,2}$-closed in the domain space.

(iii). (1,2)*-beta generalized beta closed ((1,2)*-β-gβ-closed) if $\imath(\mathcal{U})$ is (1,2)*-gβ-closed in $\mathcal{Q}$ for each (1,2)*-β-closed set $\mathcal{U}$ of $\mathcal{P}$.

(iv). (1,2)*-regular generalized b-closed ((1,2)*-rgb-closed) if $\imath(\mathcal{U})$ is (1,2)*-rgb-closed in $\mathcal{Q}$ for each $\zeta_{1,2}$-closed set $\mathcal{U}$ of $\mathcal{P}$.

Definition 3.2:

A map $\imath: \mathcal{P} \to \mathcal{Q}$ is said to be

(i). (1,2)*-delta closed ((1,2)*-δ-closed) if the image of each $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$ is (1,2)*-$\delta\mathcal{C}(\mathcal{Q})$.

(ii). (1,2)*-beta closed ((1,2)*-β-closed) if $\imath(\mathcal{U})$ is (1,2)*-β-closed in $\mathcal{Q}$ for each $\zeta_{1,2}$-closed set $\mathcal{U}$ of $\mathcal{P}$.

Theorem 3.3:

Any function which is (1,2)*-δ-closed is (1,2)*-$\delta g\beta$-closed.

Proof:

Let $\imath: \mathcal{P} \to \mathcal{Q}$ be (1,2)*-δ-closed. Let $\mathcal{V}$ be any $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$. Since $\imath$ is (1,2)*-δ-closed, $\imath(\mathcal{V})$ is (1,2)*-δ$\mathcal{C}(\mathcal{Q})$. Since every (1,2)*-δ-closed set is (1,2)*-$\delta g\beta$-closed. Therefore $\imath(\mathcal{V})$ is (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$. Hence $\imath$ is (1,2)*-$\delta g\beta$-closed.

But as the following shows that the opposite need not be true:

Example 3.4:

Let $\mathcal{P} = \mathcal{Q} = \{u_1, u_2, u_3\}$ with $\zeta_1 = \{\emptyset, \mathcal{P}, \{u_2\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{u_1, u_3\}\}$ and $\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{u_2\}, \{u_1, u_3\}\}$ be a bitopology on $\mathcal{P}$, and $\xi_1 = \{\emptyset, \mathcal{Q}, \{u_1, u_2\}\}$, $\xi_2 = \{\emptyset, \mathcal{Q}, \{u_2, u_3\}\}$ and $\xi_{1,2} = \{\emptyset, \mathcal{Q}, \{u_1, u_2\}, \{u_2, u_3\}\}$ be a bitopology on $\mathcal{Q}$. Let $\imath: \mathcal{P} \to \mathcal{Q}$ be a function defined by $\imath(u_1) = u_3$, $\imath(u_2) = u_1$, $\imath(u_3) = u_2$. Where $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P}) = \{\emptyset, \mathcal{P}, \{u_2\}, \{u_1, u_3\}\}$, (1,2)*-$\delta g\beta$- $\mathcal{C}(\mathcal{Q})$=P($\mathcal{Q}$) and (1,2)*-δ$\mathcal{C}(\mathcal{Q}) = \{\emptyset, \mathcal{Q}, \{u_2\}, \{u_1, u_3\}\}$. Then $\imath$ is (1,2)*-δgβ-closed, but not (1,2)*-δ -closed, since the image of the $\zeta_{1,2}$-closed set $\{u_1, u_3\}$ in $\mathcal{P}$ is $\{u_2, u_3\}$ is not (1,2)*- $\delta\mathcal{C}(\mathcal{Q})$.

Theorem 3.5:

It is (1,2)*-$\delta g\beta$-closed for all (1,2)*-sg-closed ((1,2)*-gs-closed) maps.

Proof:

Since every (1,2)*- sg-closed ((1,2)*-gs-closed) set is (1,2)*-δgβ-closed set. The proof is straight forward.

As seen in the following example, the converse need not be true.

Example 3.6:

Let $\mathcal{P} = \mathcal{Q} = \{v_1, v_2, v_3\}$ with $\zeta_1 = \{\emptyset, \mathcal{P}, \{v_1\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{v_3\}, \{v_2, v_3\}\}$ and $\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{v_1\}, \{v_3\}, \{v_1, v_3\}, \{v_2, v_3\}\}$ be a bitopology on $\mathcal{P}$, and $\xi_1 = \{\emptyset, \mathcal{Q}, \{v_2\}, \{v_1, v_2\}\}$, $\xi_2 = \{\emptyset, \mathcal{Q}, \{v_3\}\}$ and $\xi_{1,2} = \{\emptyset, \mathcal{Q}, \{v_2\}, \{v_3\}, \{v_1, v_2\}, \{v_2, v_3\}\}$ be a bitopology on $\mathcal{Q}$. Let $\imath: \mathcal{P} \to \mathcal{Q}$ be an identity function. Where $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P}) = \{\emptyset, \mathcal{P}, \{v_1\}, \{v_2\}, \{v_1, v_2\}, \{v_2, v_3\}\}$, (1,2)*-$\delta g\beta$- $\mathcal{C}(\mathcal{Q})$=P($\mathcal{Q}$) and (1,2)*-sg-closed((1,2)*-sg-closed) in $\mathcal{Q} = \{\emptyset, \mathcal{Q}, \{v_1\}, \{v_3\}, \{v_1, v_2\}, \{v_1, v_3\}\}$. Then $\imath$ is (1,2)*-δgβ-closed, but not (1,2)*-sg -closed ((1,2)*-sg-closed) function because the image of the $\zeta_{1,2}$-closed set $\{v_2\}$ in $\mathcal{P}$ is $\{v_2\}$ is not in (1,2)*-sg -closed((1,2)*-gs-closed) set in $\mathcal{Q}$.

Theorem 3.7:

All functions that are (1,2)*-gβ-closed are also (1,2)*-δgβ-closed.

Proof:

Since every (1,2)*-gβ-closed set is (1,2)*-δgβ-closed set. The proof is straight forward.

The illustration that follows shows that the opposite need not be true.

Example 3.8:

If $\mathcal{P} = \mathcal{Q} = \{x_1, x_2, x_3\}$ with $\zeta_1 = \{\emptyset, \mathcal{P}, \{x_1\}, \{x_1, x_3\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{x_2\}\}$ and $\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{x_1\}, \{x_2\}, \{x_1, x_2\}, \{x_1, x_3\}\}$ is a bitopology on $\mathcal{P}$, and $\xi_1 = \{\emptyset, \mathcal{Q}, \{x_1, x_2\}\}$, $\xi_2 = \{\emptyset, \mathcal{Q}, \{x_2, x_3\}\}$ and $\xi_{1,2} = \{\emptyset, \mathcal{Q}, \{x_1, x_2\}, \{x_1, x_3\}$ is a bitopology on $\mathcal{Q}$. Let $\imath: \mathcal{P} \to \mathcal{Q}$ be an identity function. Where $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P}) = \{\emptyset, \mathcal{P}, \{x_2\}, \{x_3\}, \{x_1, x_3\}, \{x_2, x_3\}\}$, (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$ =P($\mathcal{Q}$) and (1,2)*-$g\beta$-$\mathcal{C}(\mathcal{Q}) = \{\emptyset, \mathcal{Q}, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_3\}$. Then $\imath$ is (1,2)*-δgβ-closed, but not (1,2)*-$g\beta$-closed map because the image of the $\zeta_{1,2}$-closed set $\{x_2, x_3\}$ in $\mathcal{P}$ is $\{x_2, x_3\}$ is not (1,2)*-$g\beta$-$\mathcal{C}(\mathcal{Q})$.

Proposition 3.9:

For each subset $\mathcal{W}$ of $\mathcal{P}$, (1,2)*-$\delta g\beta$-$c\ell(\imath(\mathcal{W}))\subset \imath(\xi_{1,2}$- $c\ell(\mathcal{W}))$ if a mapping $\imath: \mathcal{P} \to \mathcal{Q}$ is (1,2)*-$\delta g\beta$-closed.

Proof:

Assumed $\mathcal{W}\subset \mathcal{P}$. Since $\imath$ is (1,2)*-$\delta g\beta$-closed, then $\imath(\xi_{1,2}$- $c\ell(\mathcal{W}))$ is (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$. Now $\imath(\mathcal{W}) \subset \imath(\xi_{1,2}$-$c\ell(\mathcal{W}))$. Also $\imath(\mathcal{W}) \subset$ (1,2)*-$\delta g\beta$-cl($\imath(\mathcal{W})$). By definition, we have

(1,2)*-$\delta g\beta$-$c\ell(\imath(\mathcal{W})) \subset \imath(\xi_{1,2}$-$c\ell(\mathcal{W})$.

The example that follows shows that converse statements aren't always accurate.

Example 3.10:

Let $\mathcal{P} = \mathcal{Q} = \{w_1, w_2, w_3\}$, $\zeta_1 = \{\emptyset, \mathcal{P}, \{w_3\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{w_2, w_3\}\}$, $\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{w_3\}, \{w_2, w_3\}\}$ and $\xi_1 = \{\emptyset, \mathcal{Q}, \{w_1\}\}$, $\xi_2 = \{\emptyset, \mathcal{Q}, \{w_3\}\}$, $\xi_{1,2} = \{\emptyset, \mathcal{Q}, \{w_1\}, \{w_3\}, \{w_1, w_3\}\}$. The function $\imath: \mathcal{P} \to \mathcal{Q}$ is defined by $\imath(w_1) = w_1$, $\imath(w_2) = w_3$ and $\imath(w_3) = w_2$. For every subset $\mathcal{W}$ of $\mathcal{P}$, (1,2)*-$\delta g\beta$-$c\ell(\imath(\mathcal{W}))\subset\imath(\xi_{1,2}$- $c\ell$ $(\mathcal{W})$, but $\imath$ is not (1,2)*-$\delta g\beta$-closed, since $\imath(\{w_1, w_2\}) = \{w_1, w_3\}$ is not (1,2)*-$\delta g\beta$-closed.

Proposition 3.11:

If for any subset $\mathcal{B}$ of $\mathcal{P}$, $\zeta_{1,2}$-$\imath nt(\zeta_{1,2}$-$c\ell(\zeta_{1,2}$- $\imath nt(\mathcal{B}))) \subset \imath(\xi_{1,2}$- $\imath nt(\mathcal{B}))$, then a function $\imath: \mathcal{P} \to \mathcal{Q}$ is (1,2)*-$\delta g\beta$ closed.

Proof:

$\mathcal{B}$ is closed in $\mathcal{P}$. In the meantime $\zeta_{1,2}$- $\imath nt(\zeta_{1,2}$-$c\ell(\zeta_{1,2}$- $\imath nt$ $(\mathcal{B}))) \subset \imath(\sigma_{1,2}$-$\imath nt$ $(\mathcal{B}))\subset\imath(\mathcal{B})$. $\imath(\mathcal{B})$ is (1,2)*-β closed. Thus $\imath(\mathcal{B})$ is (1,2)*-$\delta g\beta$ closed. As the following example shows, the opposite of the previously stated assertion might not be true.

Example 3.12:

Let $\mathcal{P} = \mathcal{Q} = \{c_1, c_2, c_3\}$, $\zeta_1 = \{\emptyset, \mathcal{P}, \{c_2\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{c_1, c_3\}\}$, $\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{c_2\}, \{c_1, c_3\}\}$ and $\xi_1 = \{\emptyset, \mathcal{Q}, \{c_1, c_2\}\}$, $\xi_2 = \{\emptyset, \mathcal{Q}, \{c_2, c_3\}\}$, $\xi_{1,2} = \{\emptyset, \mathcal{Q}, \{c_1, c_2\}, \{c_2, c_3\}\}$. The function $\imath: \mathcal{P} \to \mathcal{Q}$ is an identity function. Then $\imath$ is (1,2)*-$\delta g\beta$ closed, but $\zeta_{1,2}$- $\imath nt(\tau_{1,2}$-$c\ell(\zeta_{1,2}$- $\imath nt$ $(\mathcal{B}))) \subset\imath(\xi_{1,2}$- $\imath nt(\mathcal{B}))$ is not true. Since $\zeta_{1,2}$-$\imath nt(\zeta_{1,2}$-$c\ell(\zeta_{1,2}$-$\imath nt(\{c_1\})))\nsubseteq \imath(\xi_{1,2}$- $\imath nt(\{c_1\}))= \emptyset$.

Preposition 3.13:

A map $\imath: \mathcal{P} \to \mathcal{Q}$ is (1,2)*-$\delta g\beta$-closed if and only if for any subset $\mathcal{S}$ of $\mathcal{Q}$ and any open set $\mathcal{L}$ containing $\imath^{-1}(\mathcal{S}) \subset \mathcal{L}$, then there is a (1,2)*-$\delta g\beta$-open set of $\mathcal{Q}$ such that $\mathcal{S} \subset \mathcal{M}$.

Proof:

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Presume that $\imath$ is (1,2)*-$\delta g\beta$-closed. Let $\mathcal{S} \subset \mathcal{Q}$ and $\mathcal{L}$ be an $\zeta_{1,2}$-$\mathcal{O}(\mathcal{P})$ such that $\imath^{-1}(\mathcal{S}) \subset \mathcal{L}$. Now $\mathcal{P} - \mathcal{L}$ is $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$. Subsequently $\imath$ is (1,2)*-$\delta g\beta$-closed, $\imath(\mathcal{P} - \mathcal{L})$ is (1,2)*-$\delta g\beta$-closed in $\mathcal{Q}$. Then $\mathcal{M} = \mathcal{Q} - \imath(\mathcal{P} - \mathcal{L})$ is a (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$.

Since $\imath^{-1}(\mathcal{S}) \subset \mathcal{L}$ implies $\mathcal{S} \subset \mathcal{M}$ and $\imath(\mathcal{M}) = \mathcal{P} - \imath^{-1}\big(\imath(\mathcal{P} - \mathcal{L})\big) \subset \mathcal{P} - (\mathcal{P} - \mathcal{L}) = \mathcal{L}$.

That is $\imath^{-1}(\mathcal{M}) \subset \mathcal{L}$.

Contrarywise, If $\mathcal{H}$ is $\zeta_{1,2}$-$\mathcal{O}(\mathcal{P})$. Then $\imath^{-1}(\overline{\imath(\mathcal{H})}) \subset \bar{\mathcal{H}}$ is an $\zeta_{1,2}$-$\mathcal{O}(\mathcal{P})$. By supposition, there exists $\mathcal{M}$ is a (1,2)*-$\delta g\beta$- $\mathcal{O}(\mathcal{Q})$ such that $\overline{\imath(\mathcal{H})} \subset \mathcal{M}$ and hence $\mathcal{H} \subset \overline{\imath^{-1}(\mathcal{M})}$. Hence $\bar{\mathcal{M}} \subset \imath(\mathcal{H}) \subset \imath(\overline{\imath^{-1}(\mathcal{M})} \subset \bar{\mathcal{M}}$. We get $\imath(\mathcal{H}) \subset \bar{\mathcal{M}}$. Since $\bar{\mathcal{M}}$ is (1,2)*-$\delta g\beta$-closed, $\imath(\mathcal{H})$ is (1,2)*-$\delta g\beta$-closed. That is $\imath(\mathcal{H})$ is (1,2)*-$\delta g\beta$- $\mathcal{C}(\mathcal{Q})$, therefore $\imath$ is (1,2)*-$\delta g\beta$-closed.

Preposition 3.14:

A bijection mapping $\imath: \mathcal{P} \to \mathcal{Q}$ is (1,2)*-$\delta g\beta$-closed if and only if $\imath(\mathcal{T})$ is a (1,2)*-$\delta g\beta$ -$\mathcal{O}(\mathcal{Q})$ for every $\mathcal{T}$ is $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$.

Proof:

Assume $\imath: \mathcal{P} \to \mathcal{Q}$ is (1,2)*-$\delta g\beta$-closed and $\mathcal{T}$ be any $\zeta_{1,2}$-$\mathcal{O}(\mathcal{P})$. Then $\bar{\mathcal{T}}$ is a $\zeta_{1,2}$-closed set in $\mathcal{P}$. Therefore by supposition, $\imath(\bar{\mathcal{T}})$ is (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$. Since $\imath$ is bijective, $\imath(\bar{\mathcal{T}}) = \overline{\imath(\mathcal{T})}$ is (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$. Conversely, let $\mathcal{T}$ be any $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$. Then $\bar{\mathcal{T}}$ is a $\zeta_{1,2}$-$\mathcal{O}(\mathcal{P})$. Also,$\imath(\bar{\mathcal{T}})$ is (1,2)*-$\delta g\beta$-$\mathcal{O}(\mathcal{Q})$. Since $\imath$ is a bijection map, $\imath(\bar{\mathcal{T}}) = \overline{\imath(\mathcal{T})}$. Thus $\imath(\mathcal{T})$ is (1,2)*-$\delta g\beta$- $\mathcal{C}(\mathcal{Q})$. Hence $\imath$ is (1,2)*-$\delta g\beta$-closed.

Remark 3.15:

The (1,2)*-$\delta g\beta$-closed map and (1,2)*-$\delta g\beta$-continuity are independent as shown by the following examples.

Example 3.16:

Let $\mathcal{P} = \mathcal{Q} = \{y_1, y_2, y_3\}$ with $\zeta_1 = \{\emptyset, \mathcal{P}, \{y_2\}, \{y_2, y_3\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{y_3\}\}$ and $\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{y_2\}, \{y_3\}, \{y_2, y_3\}\}$ be a bitopology on $\mathcal{P}$, and $\xi_1 = \{\emptyset, \mathcal{Q}, \{y_1\}\}$, $\xi_2 = \{\emptyset, \mathcal{Q}, \{y_3\}, \{y_2, y_3\}\}$ and $\xi_{1,2} = \{\emptyset, \mathcal{Q}, \{y_1\}, \{y_3\}, \{y_1, y_3\}, \{y_2, y_3\}\}$ be a bitopology on $\mathcal{Q}$. Let $\imath: \mathcal{P} \to \mathcal{Q}$ be an identity function. Where

$\xi_{1,2}$-$\mathcal{C}(\mathcal{Q}) = \{\emptyset, \mathcal{P}, \{y_1\}, \{y_2\}, \{y_1, y_3\}, \{y_2, y_3\}\}$ and

(1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{P})$ =$\{\emptyset, X, \{y_1\}, \{y_2\}, \{y_3\}, \{y_1, y_2\}, \{y_1, y_3\}\}$. Then $\imath$ is (1,2)*-$\delta g\beta$-closed but not (1,2)*-$\delta g\beta$-continuous since the inverse image of $\{y_2, y_3\}$ in $\mathcal{Q}$ is $\{y_2, y_3\}$is not (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{P})$.

Example 3.17:

Let $\mathcal{P} = \mathcal{Q} = \{z_1, z_2, z_3, z_4\}$ with $\zeta_1 = \{\emptyset, \mathcal{P}, \{z_1\}, \{z_2\}, \{z_1, z_2\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{z_1, z_2, z_3\}\}$ and $\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{z_1\}, \{z_2\}, \{z_1, z_2\}, \{z_1, z_2, z_3\}\}$ be a bitopology on $\mathcal{P}$, and

$\xi_1 = \{\emptyset, \mathcal{Q}, \{z_1\}, \{z_2, z_3\}, \{z_1, z_4\}, \{z_1, z_2, z_3\}\}$, $\xi_2 = \{\emptyset, \mathcal{Q}, \{z_1, z_2\}, \{z_3\}, \{z_1, z_2, z_3\}\}$ and $\xi_{1,2} = \{\emptyset, \mathcal{Q}, \{z_1\}, \{z_3\}, \{z_1, z_2\}, \{z_1, z_3\}, \{z_1, z_4\}, \{z_2, z_3\}, \{z_1, z_2, z_3\}, \{z_1, z_2, z_4\}, \{z_1, z_3, z_4\}\}$ be a bitopology on $\mathcal{Q}$. Let $\imath: \mathcal{P} \to \mathcal{Q}$ be an identity function. Where

$\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$ =$\{\emptyset, \mathcal{P}, \{z_4\}, \{z_3, z_4\}, \{z_1, z_3, z_4\}, \{\{z_2\}, z_3, z_4\}\}$,

$\xi_{1,2}$- $\mathcal{C}(\mathcal{Q})$ $= \{\emptyset, \mathcal{Q}, \{z_2\}, \{z_3\}, \{z_4\}, \{z_1, z_4\}, \{z_2, z_3\}, \{z_2, z_4\}, \{z_3, z_4\}, \{z_1, z_2, z_4\}, \{z_2, z_3, z_4\}\}$, (1,2)*-$\delta g\beta$- $\mathcal{C}(\mathcal{P}) = \{\emptyset, \mathcal{P}, \{z_1\}, \{z_2\}, \{z_3\}, \{z_4\}, \{z_1, z_4\}, \{z_1, z_4\}, \{z_2, z_3\}, \{z_2, z_4\}$, $\{z_3, z_4\}, \{z_1, z_2, z_3\}, \{z_1, z_2, z_4\}, \{z_1, z_3, z_4\}, \{z_2, z_3, z_4\}\}$,

(1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q}\ )$ =$\{\emptyset, \mathcal{Q}, \{z_1\}, \{z_2\}, \{z_3\}, \{z_4\}, \{z_1, z_4\}, \{z_2, z_3\}, \{z_2, z_4\}, \{z_3, z_4\}, \{z_1, z_2, z_3\}$ $\{z_1, z_2, z_4\}, \{z_2, z_3, z_4\}\}$.

Then $\imath$ is (1,2)*-$\delta g\beta$-continuous but not (1,2)*-$\delta g\beta$-closed map since the image of $\zeta_{1,2}$-closed set $\{z_1, z_3, z_4\}$ in $\mathcal{P}$ is $\{z_1, z_3, z_4\}$is not a (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$.

Then $\imath$ is (1,2)*-$\delta g\beta$-continuous but not (1,2)*-$\delta g\beta$-closed since the image of $\zeta_{1,2}$-closed set $\{z_1, z_3, z_4\}$ in $\mathcal{P}$ is $\{z_1, z_3, z_4\}$is not a (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$.

Theorem 3.18:

If $\imath: \mathcal{P} \to \mathcal{Q}$ is $\zeta_{1,2}$-closed and $\jmath: \mathcal{Q} \to \mathcal{R}$ is (1,2)*-$\delta g\beta$-closed, then the composition $\jmath \circ \imath: \mathcal{P} \to \mathcal{R}$ is (1,2)*-$\delta g\beta$-closed.

Proof:

Let $\mathcal{F}$ be any $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$. Since $\imath$ is $\zeta_{1,2}$-closed map, then $\imath(\mathcal{F})$ is $\zeta_{1,2}$- $\mathcal{C}(\mathcal{Q})$. Since $\jmath$ is (1,2)*-$\delta g\beta$-closed map, then $\jmath(\imath(\mathcal{F}))$ is (1,2)*-$\delta g\beta$-closed set in $\mathcal{R}$. That is $(\jmath \circ \imath)(\mathcal{F}) = \jmath(\imath(\mathcal{F}))$ is (1,2)*-$\delta g\beta$-closed. Hence $\jmath \circ \imath$ is (1,2)*-$\delta g\beta$-closed map.

Remark 3.19:

If $\imath: \mathcal{P} \to \mathcal{Q}$ is (1,2)*-$\delta g\beta$-closed map and $\jmath: \mathcal{Q} \to \mathcal{R}$ is $\zeta_{1,2}$-closed map, then the composition need not be (1,2)*-$\delta g\beta$-closed map as seen in the following example.

Example 3.20:

Let $\mathcal{P} = \mathcal{Q} = \mathcal{R} = \{a_1, a_2, a_3, a_4\}$ with $\zeta_1 = \{\emptyset, \mathcal{P}, \{a_2\}, \{a_3\}, \{a_2, a_3\}, \{a_2, a_3, a_4\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{a_2, a_4\}\}$, and $\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{a_2\}, \{a_3\}, \{a_2, a_3\}, \{a_2, a_4\}, \{a_2, a_3, a_4\}\}$ be a bitopology on $\mathcal{P}$, $\xi_1 = \{\emptyset, \mathcal{Q}, \{a_2\}, \{a_3\}, \{a_2, a_3\}, \{a_2, a_3, a_4\}\}$, $\xi_2 = \{\emptyset, \mathcal{Q}, \{a_1, a_2\}, \{a_1, a_2, a_3\}\}$ and $\xi_{1,2} = \{\emptyset, \mathcal{Q}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_2, a_3, a_4\}\}$ be a bitopology on $\mathcal{Q}$, $\varrho_1 = \{\emptyset, \mathcal{R}, \{a_1\}, \{a_3\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}\}$, $\varrho_2 = \{\emptyset, \mathcal{R}, \{a_1, a_2\}, \{a_1, a_2, a_3\}\}$, $\varrho_{1,2} = \{\emptyset, \mathcal{R}, \{a_1\}, \{a_3\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_1, a_2, a_3\}, \{a_1, a_3, a_4\}\}$. Let $\imath: \mathcal{P} \to \mathcal{Q}$ be an identity function and $\jmath: \mathcal{Q} \to \mathcal{R}$ be a function defined by $\jmath(a_1) = a_2, \jmath(a_2) = a_1, \jmath(a_3) = a_3$ and $\jmath(a_4) = a_4$. Then $\imath$ is a (1,2)*-$\delta g\beta$-closed map and $\jmath$ is a $\zeta_{1,2}$-closed map, but their composition $\jmath \circ \imath: \mathcal{P} \to$

$\mathcal{R}$ need not be a (1,2)*-$\delta g\beta$-closed map because the image of $\{a_1, a_3\}$ in $\mathcal{P}$ is $\{a_2, a_3\}$ is not a (1,2)*-$\delta g\beta$-closed in $\mathcal{R}$.

Theorem 3.21:

Let $\mathcal{P}$, $\mathcal{R}$ be bitopological spaces, $\mathcal{Q}$ the bitopological space where every (1,2)*- $\delta g\beta$-closed subset is $\zeta_{1,2}$-closed. Then the composition $j \circ i: \mathcal{P} \to \mathcal{R}$ of the (1,2)*-δgβ-closed, $i: \mathcal{P} \to \mathcal{Q}$ and $j: \mathcal{Q} \to \mathcal{R}$ is (1,2)*-$\delta g\beta$-closed.

Proof:

Let $\mathcal{D}$ be $\zeta_{1,2}$-closed set of $\mathcal{P}$. Later i is (1,2)*-$\delta g\beta$-closed,$i(\mathcal{D})$ is (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$. Then by statement $i(\mathcal{D})$ is is $\zeta_{1,2}$-closed. Since j is (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{R})$ and $j(i(\mathcal{D})$ is $) = (j \circ i)(\mathcal{D})$ is (1,2)*-$\delta g\beta$-closed. Hence $j \circ i$ is (1,2)*-$\delta g\beta$-closed.

Theorem 3.22:

If $i: \mathcal{P} \to \mathcal{Q}$ is (1,2)*-g-closed, $j: \mathcal{Q} \to \mathcal{R}$ be (1,2)*-$\delta g\beta$-closed and $\mathcal{Q}$ is (1,2)*-$\delta g\beta$-$T_{1/2}$-space then their composite map $(j \circ i): \mathcal{P} \to \mathcal{R}$ is (1,2)*-$\delta g\beta$-closed.

Proof:

Let $\mathcal{D}$ be a $\zeta_{1,2}$-closed set of $\mathcal{P}$. Meanwhile$i$ is (1,2)*- g-closed, $i(\mathcal{D})$ is (1,2)*-g- $\mathcal{C}(\mathcal{Q})$. Since $\mathcal{Q}$ is (1,2)*-$\delta g\beta$-$T_{1/2}$-space, $i(\mathcal{D})$ is $\zeta_{1,2}$-$\mathcal{C}(\mathcal{Q})$. Since j is (1,2)*-$\delta g\beta$-closed, $j(i(\mathcal{D}))$ is (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{R})$ and $j(i(\mathcal{D})) = (j \circ i)(\mathcal{D})$. Therefore $j \circ i$ is (1,2)*-$\delta g\beta$-closed.

Theorem 3.23:

If a map $i: \mathcal{P} \to \mathcal{Q}$ is (1,2)*-$\delta g\beta$ -closed and $\mathcal{B}$ is $\zeta_{1,2}$-closed set of $\mathcal{P}$, then $i_{\mathcal{B}}: (\mathcal{B}, \zeta_1/\mathcal{B}, \zeta_2/\mathcal{B} \to (\mathcal{Q}, \xi_1, \xi_2)$ is (1,2)*-$\delta g\beta$-closed.

Proof:

Let $\mathcal{H}$ be a $\zeta_{1,2}$-closed set of $\mathcal{B}$. Then $\mathcal{H} = \mathcal{B} \cap \mathcal{R}$, where $\mathcal{R}$ is any $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$ and so $\mathcal{H}$ is $\zeta_{1,2}$- $\mathcal{C}(\mathcal{P})$. Since i is (1,2)*-$\delta g\beta$-closed, $i(\mathcal{H})$ is (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$. But $i(\mathcal{H}) = i_{\mathcal{B}}(\mathcal{H})$ and consequently $i_{\mathcal{B}}: (\mathcal{B}, \zeta_1/\mathcal{B}, \zeta_2/\mathcal{B}) \to (\mathcal{Q}, \xi_1, \xi_2)$ is (1,2)*-$\delta g\beta$-closed.

Theorem 3.24:

All of the maps that have (1,2)*-β-closed is also (1,2)*-$\delta g\beta$-closed.

Proof:

Since every (1,2)*-β-closed set is (1,2)*-δgβ-closed set. The proof is straight forward.

However, the following illustration shows that the opposite of the above need not be true:

Example 3.25:

Let $\mathcal{P} = \mathcal{Q} = \{b_1, b_2, b_3\}$ with $\zeta_1 = \{\emptyset, \mathcal{P}, \{b_2\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{b_1, b_3\}\}$ and $\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{b_2\}, \{b_1, b_3\}\}$ be a bitopology on $\mathcal{P}$, and $\xi_1 = \{\emptyset, \mathcal{Q}, \{b_1, b_2\}\}$, $\xi_2 = \{\emptyset, \mathcal{Q}, \{b_2, b_3\}\}$, $\xi_{1,2} = \{\emptyset, \mathcal{Q}, \{b_1, b_2\}, \{b_2, b_3\}\}$ be a bitopology on $\mathcal{Q}$. Let $i: \mathcal{P} \to \mathcal{Q}$ be defined by $i(b_1) = b_3$, $i(b_2) = b_1$ and $i(b_3) = b_2$. Where $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P}) = \{\emptyset, \mathcal{P}, \{b_1\}, \{b_2\}, \{b_3\}, \{b_1, b_3\}\}$, (1,2)*-$\delta g\beta$- $\mathcal{C}(Q) = P(Q)$ and (1,2)*-β-$\mathcal{C}(Q) = \{\emptyset, Q, \{b_1\}, \{b_3\}, \{b_1, b_2\}, \{b_1, b_3\}\}$. Then $\imath$ is (1,2)*-δgβ-closed, but not (1,2)*-β-closed map because the image of the $\zeta_{1,2}$-closed set $\{b_1, b_3\}$ in $\mathcal{P}$ is $\{b_2, b_3\}$ is not in (1,2)*-β-$\mathcal{C}(Q)$.

Theorem 3.26:

Each of the maps that have (1,2)*-$\delta g\beta$-closed map is also(1,2)*-rgb-closed.

Proof:

Let $\imath: \mathcal{P} \to Q$ be (1,2)*-$\delta g\beta$-closed and $\mathcal{W}$ be an $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$ then $\imath(\mathcal{W})$ is (1,2)*-$\delta g\beta$-$\mathcal{C}(Q)$. Since every (1,2)*-$\delta g\beta$-closed set is (1,2)*-rgb-closed set. Hence $\imath(\mathcal{W})$ is (1,2)*-rgb closed in Q. Then $\imath$ is (1,2)*-rgb-closed. The following experiment demonstrates that the above theorem's converse need not be true.

Example 3.27:

Consider $\mathcal{P} = Q = \{c_1, c_2, c_3\}$, $\zeta_1 = \{\emptyset, \mathcal{P}, \{c_1\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{c_2, c_3\}\}$, $\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{c_1\}, \{c_2, c_3\}\}$ and $\xi_1 = \{\emptyset, Q, \{c_2\}\}$, $\xi_2 = \{\emptyset, Q, \{c_1\}, \{c_2, c_3\}\}$, $\xi_{1,2} = \{\emptyset, Q, \{c_2\}, \{c_3\}, \{c_2, c_3\}\}$. Let $\imath: \mathcal{P} \to Q$ be an identity map. Where $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P}) = \{\emptyset, \mathcal{P}, \{c_1\}, \{c_2, c_3\}$.The function is (1,2)*-$rgb$-closed but not (1,2)*-$\delta g\beta$-closed as the image of $\{c_2, c_3\}$ in $\mathcal{P}$ is $\{c_2, c_3\}$ is not (1,2)*-$\delta g\beta$-$\mathcal{C}(Q)$.

4. (1,2)*- Delta Generalized β-Open Maps

Definition 4.1:

A map $\imath: \mathcal{P} \to Q$ is said to be (1,2)*-delta generalized beta open map ((1,2)*-$\delta g\beta$-open) if from a $\zeta_{1,2}$-open subset of the domain set to a (1,2)*-$\delta g\beta$-open subset of the codomain set.

Theorem 4.2:

For any bijection map : $\mathcal{P} \to Q$, the following statements are equivalent:

(i). $\imath^{-1}: Q \to \mathcal{P}$ is (1,2)*-$\delta g\beta$-continous.

(ii). $\imath$ is (1,2)*-$\delta g\beta$-open map and

(iii). $\imath$ is (1,2)*-$\delta g\beta$-closed map.

Proof:

(i) ⇒ (ii): If $\mathcal{U}$ is $\zeta_{1,2}$-$\mathcal{O}(\mathcal{P})$. Presumably, $(\imath^{-1})^{-1}(\mathcal{U})= \imath(\mathcal{U})$ is (1,2)*-$\delta g\beta$-$\mathcal{O}(Q)$ and so $\imath$ is (1,2)*-$\delta g\beta$-open.

(ii) ⇒ (iii): If $\mathcal{L}$ is a $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$. Then $\bar{\mathcal{L}}$ is $\zeta_{1,2}$-$\mathcal{Q}(\mathcal{P})$. Assuming, $\imath(\bar{\mathcal{L}})$ is (1,2)*-$\delta g\beta$-$\mathcal{O}(Q)$. That is $\imath(\bar{\mathcal{L}})= \overline{\imath(\mathcal{L})}$ is (1,2)*-$\delta g\beta$-$\mathcal{O}(Q)$ and therefore $\imath(\mathcal{L})$ is (1,2)*-$\delta g\beta$-$\mathcal{C}(Q)$. Hence $\imath$ is (1,2)*-$\delta g\beta$-closed.

(iii) ⇒ (i) Let $\mathcal{L}$ be a $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$. Supposition, $\imath(\mathcal{L})$ is (1,2)*-$\delta g\beta$-$\mathcal{C}(Q)$. But $\imath(\mathcal{L}) = (\imath^{-1})^{-1}(\mathcal{L})$, so $\imath^{-1}$ is (1,2)*-$\delta g\beta$-continuous.

Theorem 4.3:

For any subset $\mathcal{S}$ of $\mathcal{Q}$ and any $\zeta_{1,2}$-$\mathcal{O}(\mathcal{P})$ containing $\imath^{-1}(\mathcal{S})$, a function $\imath: \mathcal{P} \to \mathcal{Q}$ is (1,2)*-$\delta g\beta$-open if and only if there exists a (1,2)*-$\delta g\beta$-closed set $\mathcal{K}$ of $\mathcal{Q}$ containing S such that $\imath^{-1}(\mathcal{K}) \subset \mathcal{H}$.

Proof: Suppose $\imath$ is (1,2)*-$\delta g\beta$-open. Assume $\subset \mathcal{Q}$, $\mathcal{H}$ a $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$ such that $\imath^{-1}(\mathcal{K}) \subset \mathcal{H}$. Now $\mathcal{P}$-$\mathcal{H}$ is $\mathcal{O}(\mathcal{P})$. Since $\imath$ is (1,2)*-$\delta g\beta$-open, $\imath(\mathcal{P}$-$\mathcal{H})$ is (1,2)*-$\delta g\beta$-$\mathcal{O}(\mathcal{Q})$. Then $\mathcal{K}$= $\mathcal{Q}$- $\imath(\mathcal{P}$- $\mathcal{H})$ is a (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$. Since $\imath^{-1}(\mathcal{K}) \subset \mathcal{H}$ implies $\mathcal{S} \subset \mathcal{K}$ and $\imath^{-1}(\mathcal{K})$ = $\mathcal{P}$- $\imath^{-1}(\mathcal{P}$- $\mathcal{H}) \subset \mathcal{P}$-($\mathcal{P}$- $\mathcal{H}$) = $\mathcal{H}$. That is $\imath^{-1}(\mathcal{K}) \subset \mathcal{H}$.

For the contrary $\mathcal{U}$ is an $\tau_{1,2}$-$\mathcal{O}(\mathcal{P})$. Then $\imath^{-1}(\overline{\imath(\mathcal{U})}) \subset \bar{\mathcal{U}}$ and $\bar{\mathcal{U}}$ is a $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$. By hypothesis, there exists a (1,2)*-$\delta g\beta$-closed set $\mathcal{K}$ of $\mathcal{Q}$ such that $(\overline{\imath(\mathcal{U})} \subset \mathcal{K}$ and $\imath^{-1}$ $(\mathcal{K}) \subset \bar{\mathcal{U}}$ and so $\mathcal{U} \subset \overline{\imath^{-1}\ (\mathcal{K})}$. Hence $\overline{\mathcal{K}} \subset \imath(\mathcal{U}) \subset \imath(\overline{\imath^{-1}\ (\mathcal{K})})$ which implies $\imath(\mathcal{U}) = \overline{\mathcal{K}}$. Since $\overline{\mathcal{K}}$ is a (1,2)*-$\delta g\beta$-open, $\imath(\mathcal{U})$ is (1,2)*-$\delta g\beta$-open in $\mathcal{Q}$ and therefore $\imath$ is (1,2)*-$\delta g\beta$-open map.

Theorem 4.4:

If a map $\imath: \mathcal{P} \to \mathcal{Q}$ is (1,2)*-$\delta g\beta$-open, then $\imath(\zeta_{1,2}$- $int(\mathcal{D})) \subset$(1,2)*-$\delta g\beta$- $int(\imath(\mathcal{D}))$ for every subset $\mathcal{D}$ of $\mathcal{P}$.

Proof:

Let $\imath: \mathcal{P} \to \mathcal{Q}$ be a $\zeta_{1,2}$-open map and $\mathcal{D}$ be any subset of $\mathcal{P}$. Then $\zeta_{1,2}$-int($\mathcal{D}$) is $\zeta_{1,2}$-open in $\mathcal{P}$ and so $\imath(\zeta_{1,2}$- $int(\mathcal{D}))$ is (1,2)*-$\delta g\beta$-open in $\mathcal{Q}$. We have $\imath(\zeta_{1,2}$- $int(\mathcal{D})) \subset$ $\imath(\mathcal{D})$. Therefore we have $\imath(\zeta_{1,2}$- $int(\mathcal{D})) \subset$ (1,2)*-$\delta g\beta$- $int(\imath(\mathcal{D}))$.

Remark 4.5

The opposite of what was said above, The following example demonstrates that a theorem need not hold true universally.

Example 4.6:

Consider $\mathcal{P} = \mathcal{Q} = \{d_1, d_2, d_3\}$, $\zeta_1 = \{\emptyset, \mathcal{P}, \{d_3\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{d_1, d_3\}\}$,
$\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{d_3\}, \{d_1, d_3\}\}$ and $\xi_1 = \{\emptyset, \mathcal{Q}, \{d_2\}\}, \}$, $\xi_2 = \{\emptyset, \mathcal{Q}, \{d_1\}, \{d_1, d_2\}\}$,
$\xi_{1,2} = \{\emptyset, \mathcal{Q}, \{d_1\}, \{d_2\}, \{d_1, d_2\}\}$. Let $\imath: \mathcal{P} \to \mathcal{Q}$ be an identity map. In $\mathcal{Q}$, (1,2)*-$\delta g\beta$- $int(\imath(\mathcal{B}))$= $\imath(\mathcal{B})$ for each subset $\mathcal{B}$ of $\mathcal{P}$.

So $\imath(\zeta_{1,2}$- $int(\mathcal{B})) \subset \imath(\mathcal{B})$= (1,2)*-$\delta g\beta$- $int(\imath(\mathcal{B}))$ any subset $\mathcal{B}$ of $\mathcal{P}$. But $\imath$ is not (1,2)*-$\delta g\beta$-open, since aimed at $\zeta_{1,2}$-open set $\{d_3\}$ of $\mathcal{P}$, $\imath(\{d_3\}) = d_3$ which is not (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$.

Theorem 4.7.

If a function $\imath: \mathcal{P} \to \mathcal{Q}$ is (1,2)*-$\delta g\beta$-open, then $\imath^{-1}$((1,2)*-$\delta g\beta$-$c\ell(\mathcal{B})) \subset \zeta_{1,2}$- $c\ell(\imath^{-1}(\mathcal{B}))$ for each subset $\mathcal{B}$ of $\mathcal{Q}$.

Proof:

Assume $\imath$ is a (1,2)*-$\delta g\beta$-open, $\mathcal{B}$ any subset of $\mathcal{Q}$. Formerly $\imath^{-1}(\mathcal{B}) \subset \zeta_{1,2}$- $c\ell(\imath^{-1}$ $(\mathcal{B}))$ and $\zeta_{1,2}$-$c\ell(\imath^{-1}(\mathcal{B}))$ is $\zeta_{1,2}$-$\mathcal{C}(\mathcal{P})$. By above Theorem 4.3, there exists a (1,2)*-$\delta g\beta$-$\mathcal{C}(\mathcal{J})$ of $\mathcal{Q}$ such that $\mathcal{B} \subset \mathcal{J}$

and $\imath(\mathcal{J}) \subset \zeta_{1,2}$-$c\ell(\imath^{-1}(\mathcal{B}))$. Now $(1,2)$*-$\delta g\beta$-$c\ell(\mathcal{B})$⊂$(1,2)$*-$\delta g\beta$-$c\ell(\mathcal{J}) = \mathcal{J}$, by theorem 3.14 in [1], $\mathcal{J}$ is $(1,2)$*-$\delta g\beta$-$\mathcal{C}(\mathcal{Q})$. Consequently $\imath^{-1}((1,2)$*-$\delta g\beta$-$c\ell(\mathcal{B})) \subset \imath^{-1}(\mathcal{J})$ and so $\imath^{-1}((1,2)$*-$\delta g\beta$-$c\ell(\mathcal{B}))$⊂$\imath^{-1}(\mathcal{J})$⊂$\zeta_{1,2}$-$c\ell(\imath^{-1}(\mathcal{B}))$. Thus $\imath^{-1}((1,2)$*-$\delta g\beta$-$c\ell(\mathcal{B}))$⊂$\zeta_{1,2}$-$c\ell(\imath^{-1}(\mathcal{B}))$ for every subset of $\mathcal{B}$ of $\mathcal{Q}$.

Remark 4.8:

The opposite of the preceding as the following example shows, a theorem need not be true universally.

Example 4.9:

Consider $\mathcal{P} = \mathcal{Q} = \{g_1, g_2, g_3\}$, $\zeta_1 = \{\emptyset, \mathcal{P}, \{g_2\}\}$, $\zeta_2 = \{\emptyset, \mathcal{P}, \{g_2, g_3\}\}$, $\zeta_{1,2} = \{\emptyset, \mathcal{P}, \{g_2\}, \{g_2, g_3\}\}$ and $\xi_1 = \{\emptyset, \mathcal{Q}, \{g_2\}\}$, $\xi_2 = \{\emptyset, \mathcal{Q}, \{g_1\}, \{g_1, g_2\}\}$, $\xi_{1,2} = \{\emptyset, \mathcal{Q}, \{g_1\}, \{g_2\}, \{g_1, g_2\}\}$. $\imath: \mathcal{P} \to \mathcal{Q}$ is a function defined by $\imath(g_1) = g_2$, $\imath(g_2) =$ and $\imath(g_3) = g_1$. In Q, each subset $\mathcal{B}$ of $\mathcal{Q}$ has (1,2)*-δgβ-$c\ell(\mathcal{B}) = \mathcal{B}$. Thus, for each subset $\mathcal{B}$ of $\mathcal{Q}$, $\imath^{-1}((1,2)$*-$\delta g\beta$- $c\ell(\mathcal{B})) = \imath^{-1}(\mathcal{B}) \subset c\ell(\imath^{-1}(\mathcal{B}))$. However, for the $\zeta_{1,2}$-open set $\{g_2\}$ of P, $\imath(\{g_2\}) = \{g_3\}$, which is not (1,2)*-δgβ-open in Q, hence $\imath$ is not (1,2)*-δgβ-open.

Conclusion:

Finally, this paper introduces the concepts of $(1,2)$*-$\delta g\beta$-closed maps in bitopology, offering new insights into the behaviour of sets and maps under two topologies. These definitions enrich their interaction with classical notions of continuity, openness and closedness. The study highlights how properties of topological spaces and preserved under dual topologies, unveiling phenomena absent in classical topology and expanding the field's scope.

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