THE TRUNCATED NEW-XLINDLEY DISTRIBUTION WITH APPLICATIONS

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Abstract

This work presents the truncated variants of the new-XLindley distribution and examines their properties, highlighting the monotonic behavior of the density and hazard functions. It also talks about the quantile function, order statistics, moments, and other statistical features. Additionally, the paper develops maximum likelihood estimators for the unknown parameters of the upper, lower, and double truncated new-XLindley distributions. To demonstrate the practicality of the proposed distribution, we apply it to analyze three different data sets related to medical data. We intend to capture researchers' attention and showcase this new distribution's versatility and potential applications.

Keywords: Truncated Lindley distribution, New-XLindley distribution, Lindley distribution, Moments, Maximum likelihood estimation.

1. Introduction

Truncated distributions are highly effective in situations where a random variable is restricted to a specific range, as is often the case in multiple disciplines. For example, in survival analysis, failures that occur within the warranty period may be excluded from consideration. Certain failures may also be omitted when items are replaced after a designated period according to replacement policies.

Consequently, many researchers were attracted to the problem of analysing truncated data encountered in various disciplines and proposed truncated versions of the usual statistical distributions. Among other things, (Ahmed et al., 2010) discussed how to improve a forecasting actuarial model by incorporating the truncated Birnbaum-Saunders (BS) distribution. In (Aban et al., 2006; Zaninetti & Ferraro, 2008), the truncated Pareto distribution was applied to the statistical analysis of star masses and asteroids' diameters. The truncated Weibull distribution is used in various fields, such as analysing the diameter data of trees, truncating data-specific threshold levels, predicting the height distribution of small trees based on incomplete laser scanning data, modelling the diameter distribution of forests, characterizing the observed Portuguese fire size distribution, and seismological data on the development of pit depths on a water pipe, etc. The book (Murthy et al., 2004) that discussed Weibull distributions and a recent article (Zhang & Xie, 2011), based on the truncated Weibull distribution, provide more detail on the truncated Weibull distribution and related references.

From the above commentary and monitoring the wide applicability of the truncated distributions, we proposed truncation in the new-XLindley distribution. The new-XLindley distribution introduced by (Khodja et al., 2023) is a mixture of exponential (θ) and gamma (2, θ) distributions with their mixing proportions $p_1 = p_2 = \frac{1}{2}$, respectively. Since the last decade, the new-XLindley distribution has been attracting the attention of researchers, scientists, and reliability probationers, and many authors extended it to the various parsimonious distributions. To name a few extensions, Two Parameter Beta-Exponential Distribution (KOUADRIA & ZEGHDOUDI, 2024), The power new XLindley distribution (Gemeay et al., 2024), The Discrete New XLindley Distribution (Maya et al., 2024), Modified XLindley distribution (Gemeay et al., 2023), The Exponentiated New XLindley Distribution (MirMostafaee, 2024), this distribution has captured the attention of researchers, scientists, and reliability experts, prompting numerous authors to explore its extensions into various parsimonious forms.

The next parts include the remainder of the paper. The upper truncated new-XLindley (UTNXL), lower truncated new-XLindley (LTNXL), and double truncated new-XLindley (DTNXL) distributions are the truncated variants of the new-XLindley distribution that are shown in section 2. In particular, it has been demonstrated that the UTNXL distribution's flexibility exhibits the properties of the hazard and probability density (pdf) functions with varying combinations of its parameter values. Section 3 deduces the (UTNXL) distribution's moments, quantile function, and order statistics. The estimates of the parameters of the (UTNXL), (LTNXL), and (DTNXL) distributions are obtained in section 4 using the maximum likelihood technique. In Section 5, three real datasets are modelled using various distributions, and their applicability is evaluated. Section 6 concludes the paper.

2. The Truncated New-XLindley Distributions

A distribution $G(x,\Theta)$ is said to be a double truncated distribution over the interval $[a,b]$ if it has the cumulative distribution function (cdf) defined as:

$$G(x,\Theta) = \frac{F(x,\Theta) - F(a,\Theta)}{F(b,\Theta) - F(a,\Theta)} \text{ with } a \le x \le b,\ -\infty < a < b < +\infty \qquad (1$$

And the corresponding probability density function (pdf) is

$$g(x;\Theta) = \frac{f(x,\Theta)}{F(b,\Theta) - F(a,\Theta)} \text{ with } a \le x \le b,\ -\infty < a < b < +\infty \qquad (2$$

where, $g(x;\Theta)$ and $G(x,\Theta)$ are the (pdf) and (cdf) of the baseline model and $\Theta \in R^n$ denotes the vector parameter of base line model. Here, three cases can be recognized as

- When $a = 0$ and $b \to +\infty$, it reduces to baseline model.

- When $a=0$, it is called the upper truncated distribution of the baseline model.
- When $b\to+\infty$, it is called the lower truncated distribution of the baseline model.

In this article, we consider the New-XLindley distribution (NXLD)(Khodja et al., 2023) as baseline model with the following (cdf) distribution function :

$$\mathrm{F(x)}=1-(\frac{\theta x}{2}+1)\mathrm{e}^{-\theta x}\quad \text{with}\ \theta,x>0 \qquad (3$$

And the corresponding (pdf) is given by:

$$f(x)=\frac{\theta}{2}\left(1+\theta x\right)e^{-\theta x}\ \text{with}\ \theta,x>0 \qquad (4$$

Using (1) and (3), the double truncated new-XLindley distribution is defined as

$$\mathrm{g}_D(\mathrm{x};\theta)=\frac{\theta\left(1+\theta x\right)e^{-\theta x}}{2\left(\mathrm{F(b},\theta)\text{-}\mathrm{F(a},\theta)\right)}\ \text{with}\ 0\le a\le x\le b<+\infty \qquad (5$$

In the following sections, we will only discuss the properties of the upper truncated new-XLindley distribution and the same procedure can be applied to study the properties of the lower truncated new-XLindley distribution as well as double truncated new-XLindley distribution. The upper truncated new-XLindley distribution has the following (pdf) is given by

$$\mathrm{g}_u(\mathrm{x};\theta)=\frac{\theta\left(1+\theta x\right)e^{-\theta x}}{2\mathrm{F(b},\theta)}=\frac{\theta\left(1+\theta x\right)e^{-\theta(x-b)}}{2\left(e^{\theta b}-1\right)-\theta b}\ ,\ \ 0\le x\le b \qquad (6$$

And the corresponding (cdf) is given by

$$G\left(x,\theta\right)=\frac{F\left(x,\theta\right)}{F\left(b,\theta\right)}=\frac{\left(2e^{\theta x}-\theta x-2\right)}{\left(2e^{\theta b}-\theta b-2\right)}e^{-\theta(x-b)}\ ,\ \ 0\le x\le b \qquad (7$$

It is denoted by UTNXLD. Note that the above (pdf) will behave like as

$$\frac{d\mathrm{g}_u(\mathrm{x};\theta)}{dx}=\frac{\theta^3 x}{\theta b-2(e^{\theta b}-1)}e^{-\theta(x-b)}$$

The expression $\theta b-2\left(e^{\theta b}-1\right)$ is negative for all $\theta b>0$, meaning that the derivative is negative for all $x>0$, then $\mathrm{g}(\mathrm{x};\theta)$ is decreasing over $\left[0,b\right]$.

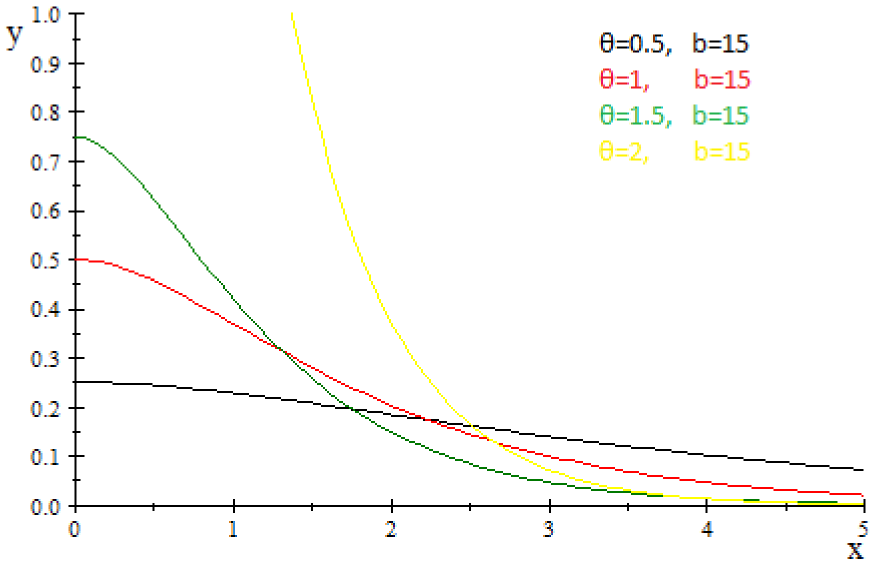

Figure1 : The density function of UTNXL distribution for $\theta = 0.5, 1, 1.5 \& 2$ and $b = 15$

The corresponding hazard function at epoch t is given by

$$\mathrm{H}(\mathrm{t},\theta)=\frac{\mathrm{f}(\mathrm{t},\theta)}{F(\mathrm{b},\theta)-F(\mathrm{t},\theta)}=\frac{\theta(1+\theta t)e^{-\theta(t-b)}}{2\left(\mathrm{F}(\mathrm{b},\theta)\text{-}\mathrm{F}(\mathrm{t},\theta)\right)} \text{ with } 0\le t\le b$$

$[9]$ used the term $\eta(x) = -\frac{f'(x)}{f(x)}$ to determine the monotonicity of the hazard function. For UTNXL distribution, we get

$$\eta_{UTNXL}(x) = -\frac{g_u'(x)}{g_u(x)} = -\frac{g_u'(x)/F(b)}{g_u(x)/F(b)} = -\frac{f'(x)}{f_u(x)} = \eta_{NXL}(x)$$

It followed that

- $H(0) = \dfrac{\theta e^{2\theta b}}{2\left(e^{\theta b}-1\right)-\theta b}$.
- $H(b) \to \infty$, when $t \to b$.
- $\eta_{NXL}(x) = -\dfrac{f'(x)}{f(x)} = \dfrac{\theta^2 x}{1+\theta x} > 0, \forall x$, it implies that the hazard rate function of UTNXL distribution is increasing in x and θ , see Figure 2.

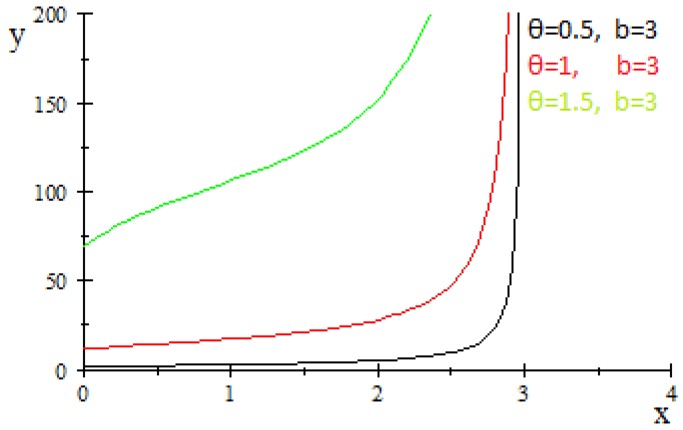

Figure2: The hazard function of UTNXL distribution for $\theta = 0.5, 1 \& 1.5$ and $b = 3$

3. Statistical properties

a) Moments and related measures

The rth moment under the upper truncated New-XLindley distribution is defined as

$$E\left[X^r\right] = \frac{\theta}{2\mathrm{F}(\mathrm{b},\theta)} \int_0^b x^r (1+\theta x) e^{-\theta x} dx$$

$$= \frac{\theta}{2\mathrm{F}(\mathrm{b},\theta)} \left(\int_0^b x^r e^{-\theta x} dx + \theta \int_0^b x^{r+1} e^{-\theta x} dx \right)$$

$$= \frac{1}{2\mathrm{F}(\mathrm{b},\theta)\theta^r} \left(\int_0^{\theta b} y^r e^{-y} dy + \int_0^{\theta b} y^{r+1} e^{-y} dy \right)$$

And using the lower incomplete gamma function defied by $\gamma(s,t) = \int_0^t x^{s-1} e^{-x} dx = (s-1)! \left(1 - e^{-t} \sum_{k=0}^{s-1} \frac{x^k}{k!} \right)$,

then we have

$$E\left[X^r\right] = \frac{\gamma(r+1, \theta b) + \gamma(r+2, \theta b)}{2\mathrm{F}(\mathrm{b},\theta)\theta^r}$$

In particular, the first two moments can be worked out as

$$E[X] = \frac{\gamma(2,\theta b) + \gamma(3,\theta b)}{2\mathrm{F}(\mathrm{b},\theta)\theta} = \frac{3 - \left(3(1+\theta b) + (\theta b)^2\right) e^{-\theta b}}{2\mathrm{F}(\mathrm{b},\theta)\theta}$$

$$E\left[X^2\right] = \frac{\gamma(3,\theta b) + \gamma(4,\theta b)}{2\mathrm{F}(\mathrm{b},\theta)\theta^2} = \frac{5\left(1 - \left(1 + \theta b + \frac{(\theta b)^2}{2} + \frac{(\theta b)^3}{10}\right) e^{-\theta b}\right)}{2\mathrm{F}(\mathrm{b},\theta)\theta^2}$$

The variance is $\sigma^2 = E\left[X^2\right] - \left(E\left[X\right]\right)^2$

Skewness, Kurtosis and Coefficient of variation of the upper truncated New-XLindley distribution

$$\text{Skewness}=\sqrt{\beta_1} = \frac{E\left[X^3\right]}{\left(\sigma^2\right)^{\frac{3}{2}}}$$

$$\text{Kurtosis}=\beta_2 = \frac{E\left[X^4\right]}{\left(\sigma^2\right)^2}$$

$$CV = \frac{\sqrt{\sigma^2}}{E\left[X\right]}$$

b) Order statistics

In this subsection, we derive the (pdf) $g_{X_{(k)}}(t)$ of the s th, ($s = 1,...,n$) order statistics $X_{(k)}$:

$$g_{X_{(k)}}(t) = \frac{1}{B(k, n-k+1)} g(t)\left(G(t)\right)^{k-1}\left(1-G(t)\right)^{n-k}$$

Where $B(k, n-k+1)$ is the beta function. Expanding the binomial expansion, we get

$$g_{X_{(k)}}(t) = \frac{1}{B(k, n-k+1)} \sum_{i=0}^{n-k} (-1)^i \binom{n-k}{i} \left(\frac{F(t,\theta)}{F(b,\theta)}\right)^{k+i} \frac{f(t,\theta)}{F(t,\theta)}$$

Where $f(t,\theta)$ and $F(t,\theta)$ are the (pdf) and (cdf) of the New-XLindley distribution. For $k = n$, the (pdf) of $X_{(n)}$ is given by

$$g_{X_{(n)}}(t) = \frac{n\theta\left(1-\left(\frac{\theta t}{2}+1\right)e^{-\theta t}\right)^{n-1}\left(1+\theta t\right)e^{-\theta t}}{2\left(1-\left(\frac{\theta n}{2}+1\right)e^{-\theta b}\right)^{n}}$$

Similarly, the pdf of $X_{(1)}$ is given by

$$g_{X_{(1)}}(t) = \frac{n\theta}{2} \sum_{i=0}^{n-1} (-1)^i \binom{n-1}{i} \frac{\left(1-\left(\frac{\theta t}{2}+1\right)e^{-\theta t}\right)^{i}\left(1+\theta t\right)e^{-\theta t}}{\left(1-\left(\frac{\theta b}{2}+1\right)e^{-\theta b}\right)^{i+1}}$$

c) Quantile function

Let X be a (UTNXLD) with (cdf) as given in (7). The quantile function $Q(p) = G^{-1}(p,\theta) = x_p$.

For that we need to solve this equation:

$$G\left(x_p,\theta\right)=p \text{ with } p\in\left(0,1\right) \quad (8$$

From (7) and (8), we have

$$-\left(\theta x_p+2\right)e^{-\left(\theta x_p+2\right)}=2e^{-2}\left(pF\left(b,\theta\right)-1\right) \quad (9$$

To solve the above equation for x_p. The Lambert W function was first used in [10] to generate random variables with the Lindley or Poisson-Lindley distribution. The Lambert W function is a multivalued complex function defined as the solution of the equation:

$$W\left(z\right)e^{W(z)}=z \text{ where } z\in £ \quad (10$$

Form (9) and (10), we obtained

$$-\left(\theta x_p+2\right)=W_{-1}\left(2e^{-2}\left(pF\left(b,\theta\right)-1\right)\right)$$

where, W_{-1} is negative branch of the Lambert W function. Then we have the quantile function of (UTNXLD)

$$x_p=Q\left(p\right)=-\frac{2}{\theta}-\frac{1}{\theta}W_{-1}\left(2e^{-2}\left(pF\left(b,\theta\right)-1\right)\right)$$

As $b\to+\infty$, we get the quantile function of new-XLindley distribution derived by [10]:

$$x_p=Q\left(p\right)=-\frac{2}{\theta}-\frac{1}{\theta}W_{-1}\left(2e^{-2}\left(p-1\right)\right)$$

The median of the UTNXL distribution can obtained as

$$x_{Med}=Q\left(\frac{1}{2}\right)=-\frac{2}{\theta}-\frac{1}{\theta}W_{-1}\left(e^{-2}\left(F\left(b,\theta\right)-2\right)\right)$$

4. Maximum Likelihood Estimation

This section explains how to get the maximum likelihood estimates (MLE) of the UTNXL parameters as well as the lower truncated New-XLindley (LTNXLD) and double truncated New-XLindley (DTNXLD) distributions based on a random sample $x=\left\{x_1,x_2,...,x_n\right\}$ of size n . Depending on the type of data, these distributions can be used to model the actual problems. In the following part, we fitted these distributions to two real datasets.

a) MLEs for UTNXLD

In the UTNXL distribution, let $x=\left\{x_1,x_2,...,x_n\right\}$ be a sample of size n that is (iid) independent and identically distributed. Given sample x , the likelihood function is provided by

$$L\left(\theta,b\middle|x\right)=\left(\frac{\theta}{2\left(e^{\theta b}-1\right)-\theta b}\right)^n\prod_{i=1}^{n}\left(1+\theta x_i\right)e^{-\theta\left(x_i-b\right)}$$

We noted here that $\sum_{i=1}^{n} x_i$ is the joint sufficient statistics for θ and b. The corresponding log-likelihood equation is given by

$$\log L\left(\theta,b\left|x\right.\right)=n\log\left(\theta\right)-n\log\left(2\left(e^{\theta b}-1\right)-\theta b\right)+\sum_{i=1}^{n}\log\left(1+\theta x_i\right)-\theta\sum_{i=1}^{n}\left(x_i-b\right) \qquad (11$$

Note that in the above log-likelihood equation, it is impossible to get an estimate of b in terms of the observed sample since b is free from x. Let $x_{(1)} < x_{(2)} < \text{L} < x_{(n)}$ be the order sample corresponding to $x_1, x_2, ..., x_n$. Then, the MLE $\hat{b}$ of b is $\hat{b} = \max\left(x_1, x_2, ..., x_n\right) = x_{(n)}$, the MLE $\hat{\theta}$ of θ can be obtained as the solution of the following non-linear equation:

$$\frac{n}{\theta}-\frac{n\hat{b}\left(2e^{\theta\hat{b}}-1\right)}{2\left(e^{\theta\hat{b}}-1\right)-\theta\hat{b}}+\sum_{i=1}^{n}\frac{x_i}{1+\theta x_i}-\sum_{i=1}^{n}\left(x_i-\hat{b}\right)=0$$

To answer the aforementioned problem, we must employ an iterative process similar to Newton's method.

b) MLEs for LTNXLD

The (pdf) of LTNXLD is given by

$$g_L\left(x,\theta\right)=\frac{\theta\left(1+\theta x\right)}{2+\theta a}e^{-\theta\left(x-a\right)}$$

So, the likelihood function based on x from LTNXL distribution is given by

$$L(\theta,a\left|x\right.)=\frac{\theta^n}{\left(2+\theta a\right)^n}\prod_{i=1}^{n}\left(1+\theta x_i\right)e^{-\theta\left(x_i-a\right)}$$

The corresponding log- likelihood function is given by

$$\log L(\theta,a\left|x\right.)=n\log\left(\theta\right)-n\log\left(2+\theta a\right)+\sum_{i=1}^{n}\log\left(1+\theta x_i\right)-\theta\sum_{i=1}^{n}\left(x_i-a\right)$$

Similarly,from the above subsection, the maximum likelihood estimate of a will be $\hat{a}=\min\left(x_1,x_2,...,x_n\right)=x_{(1)}$, smallest observation. The maximum likelihood estimate $\hat{\theta}$ of θ can be uniquely determined by solving the following non-linear equation:

$$\frac{n}{\theta}-\frac{n\hat{a}}{2+\theta\hat{a}}+\sum_{i=1}^{n}\frac{x_i}{1+\theta x_i}-\sum_{i=1}^{n}\left(x_i-\hat{a}\right)=0$$

To answer the aforementioned problem, we must employ an iterative process similar to Newton's method.

c) MLEs for DTNXLD

The likelihood function under the assumption of the double truncated New-XLindley distribution for the random variable X, is given by

$$L(\theta,a,b|x)=\left(\frac{2}{\Phi(\theta)}\right)^n\prod_{i=1}^{n}(1+\theta x_i)e^{-\theta x_i}$$

The corresponding log-likelihood function is given by

$$\log L(\theta,a,b|x)=n\log(2)-n\log(\Phi(\theta))+\sum_{i=1}^{n}\log(1+\theta x_i)-\theta\sum_{i=1}^{n}x_i$$

Where $\Phi(\theta)=\left(be^{-\theta b}-ae^{-\theta a}\right)+\frac{2}{\theta}\left(e^{-\theta b}-e^{-\theta a}\right)$, and the MLEs of a and b are $\hat{a}=x_{(1)}$ and $\hat{b}=x_{(n)}$ respectively, the MLE of θ can be obtained by solving the following non-linear equation :

$$-\frac{n\Phi'(\theta)}{\Phi(\theta)}+\sum_{i=1}^{n}\frac{x_i}{1+\theta x_i}-\sum_{i=1}^{n}x_i=0$$

5. Real Data Applications

In this section, we illustrate that the truncated new-XLindley distribution can be a better model than the exponential, Lindley, and new-XLindley distributions by using two real data sets. And to compare several distribution models, we consider criteria like AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion), -2L (-2Log-Likelihood), AICC (Consistent Akaike Information Criterion), and HQIC (Hannan-Quinn Information Criterion) for the data set. The better distribution corresponds to smaller AIC, BIC, -2L, AICC, and HQIC values.

Dataset 1: this data on recovered times (in week) of 74 Angola individuals infected with the Marburg virus (see https://www.who.int/): 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9, 9.

Table 1: The ML estimates, -2 log-likelihood, AIC, BIC,AICC, and HQIC for dataset 1.

distribution	Estimates	AIC	BIC	-2L	AICC	HQIC
exp(θ)	0.1989	388.9947	391.2987	386.9947	389.0502	389.9138
Lindley (θ)	0.3466	367.9865	370.2906	365.9865	368.0421	368.9056
XLindley (θ)	0.3128	374.8971	377.2012	372.8971	374.9527	375.8162
New-XL (θ)	0.3111	377.825	380.129	375.825	377.8805	378.7441
Zeghdoudi(θ)	0.5536	351.3284	353.6325	349.3284	351.384	352.2476
UTNXLD(θ,b)	(0.00057,9)	329.9532	334.5613	325.9532	330.1222	331.7915
LTNXLD(θ,a)	(0.4410,2)	317.2225	321.8306	313.2225	317.3915	319.0608
DTNXLD(θ,a,b)	(0.5035,2,9)	328.2239	332.832	324.2239	328.3929	330.0621

Dataset 2:A numeric vector of 15 measurements by different laboratories of the pesticide DDT in kale, in ppm (parts per million) using the multiple pesticide residue measurement.(Finsterwalder, 1976).

The data are: 2.79, 2.93, 3.22, 3.78, 3.22, 3.38, 3.18, 3.33, 3.34, 3.06, 3.07, 3.56, 3.08, 4.64, 3.34

Table 2: The ML estimates,-2 log-likelihood, AIC, BIC,AICC, and HQIC for dataset 2.

distribution	Estimates	AIC	BIC	-2L	AICC	HQIC
exp(θ)	0.3004	68.07115	68.7792	66.07115	68.37884	68.0636
Lindley (θ)	0.5007	61.84888	62.55693	59.84888	62.15658	61.84134
XLindley (θ)	0.44029	64.57162	65.27967	62.57162	64.87932	64.56408
New-XL (θ)	0.4854	64.18241	64.89046	62.18241	64.4901	64.17486
Zeghdoudi(θ)	0.81449	53.14825	53.8563	51.14825	53.45595	53.14071
UTNXLD(θ,b)	0.0006	50.12314	51.53924	46.12314	51.12314	50.10805
LTNXLD(θ,a)	2.0968	15.30998	16.72608	11.30998	16.30998	15.2949
DTNXLD(θ,a,b)	2.2577	18.09554	20.21969	12.09554	20.27736	18.07291

Dataset 3: A numeric vector with 18 determinations by different laboratories of the amount (percentage of the declared total weight) of shrimp in shrimp cocktail.(King & Ryan, 1976)

The data are: 32.2, 33.0, 30.8, 33.8, 32.2, 33.3, 31.7, 35.7, 32.4, 31.2, 26.6, 30.7, 32.5, 30.7, 31.2, 30.3, 32.3, 31.7

Table 3: The ML estimates,-2 log-likelihood, AIC, BIC,AICC, and HQIC for dataset 3.

distribution	Estimates	AIC	BIC	-2L	AICC	HQIC
exp(θ)	0.03144	162.5345	163.4249	160.5345	162.7845	162.6573
Lindley (θ)	0.06109	149.7339	150.6242	147.7339	149.9839	149.8566
XLindley (θ)	0.05943	150.6912	151.5816	148.6912	150.9412	150.814
New-XL (θ)	0.05087	157.7884	158.6788	155.7884	158.0384	157.9112
Zeghdoudi(θ)	0.09296	141.4851	142.3755	139.4851	141.7351	141.6079
UTNXLD(θ,b)	0.00046	133.303	135.0838	129.303	134.103	133.5486
LTNXLD(θ,a)	0.21758	99.04227	100.823	95.04227	99.84227	99.28781
DTNXLD(θ,a,b)	0.26181	105.8118	108.4829	99.81177	107.5261	106.1801

It is revealed that the proposed upper truncated, lower truncated, and double truncated distributions have lower values of AIC, BIC, log-likelihood, HQIC and ACIC, the goodness of-fit measure.

6. Conclusions

This article provides a summary of the truncated new XLindley distributions, particularly focusing on the upper truncated, lower truncated, and double truncated forms. We examine the characteristics of the upper truncated new XLindley distribution, emphasizing its moments, quantile function, and order statistics. Additionally, we derive maximum likelihood estimators for the unknown parameters associated with the upper truncated, lower truncated, and double truncated new XLindley distributions. A comparative assessment of the goodness-of-fit for the exponential, Lindley, XLindley, Zeghdoudi, new XLindley, and various truncated new XLindley distributions is conducted using log-likelihood, AIC, AICC, HQIC, and BIC, showing that the lower truncated new XLindley distribution provides an excellent fit for the window strength data. In conclusion, we argue that truncated distributions are highly beneficial for modeling real-world situations, and we recommend the use of truncated new XLindley distributions in various fields such as engineering, medicine, finance, and demography, where such truncated data frequently arise.

AUTHORS CONTRIBUTIONS

Mohamed Kouadria*:* Investigation; formal analysis; methodology, writing—original draft; simulation, interpretation of results; writing—review and editing.

Halim Zeghdoudi*:* methodology, writing—original draft ; Software; formal analysis; supervision; validation.

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