USING EXACT ALGORITHM FOR THE UNDIRECTED URBAN POSTMAN PROBLEM

Anshul Dubey[1], Rajan Singh[2], B. K. Singh[3], Nidhi Tiwari[4],

Tarun Gupta[5], Riddhi Garg[6], Vipin Kumar[7], Jyoti Agarwal[8]

Email: anshuldubey8063@gmail.com, rajan0779@redifmail.com, drbksingh@iftmuniversity.ac.in, nidhitiwari26@gmail.com, mathtarun26@gmail.com, riddhigarg5@gmail.com, vipinsinghhrit@gmail.com, jyotiag79@gmail.com

[1,2,3,4,5,6,7,8]Department of Mathematics, IFTM University, Moradabad, 244001, India.

ABSTRACT: We are aware of four exact algorithms for the undirected UPP. The first, due to Christofides, Campos, Corberan, and Mota (1981) uses branch-and-bound combined with Lagrangean relaxation. Corberan and Sanchis (1994) described an integer linear programming formulation solved using a branch-and-cut algorithm in which the separation problems are solved visually. Letchford (1996) added so-called path-bridge and Laporte (2000) introduced a new and more compact formulation which, when solved by branch-and-cut, yields excellent results on test problems. In what follows, we summarize some of the Ghiani and Laporte results.

KEYWORDS: Arc Routing, Exact Algorithms, Urban Postman Problem, Variable Neighbourhood Search.

INTRODUCTION: Let $C_k(k = 1, \ldots, p)$ be the k^{th} connected component of the subgraph of $G = (V, E)$ induced by the set R of required edges. Let V_R be the set of vertices v_i such that an edges (v_i, v_j) exists in R, and $V_k \subseteq V_R$ ($i = 1, \ldots, p$) the vertex set of C_k. Denote by c_e the cost of edge e $\in R$. A vertex $v_i \in R$ is R-odd (R-even) if and only if an odd (even) number of edges of R are incident to v_i. Christofides et al. (1981) proposed the following graph reduction.

Step 1. Add to $G_R = (V_R, R)$ an edge between every pair of vertices of V_R having a cost equal to that of the corresponding shortest chain on G.

Step 2. Delete one of two parallel edges if they have the same cost, and all edges $(v_i, v_j) \notin R$ such that $C_{ij} = C_{ik} + C_{kj}$ for the some v_k.

We now recall the Corberan and Sanchis (1994) formulation. Given $S \subset V$, $\delta(S)$ be the set of edges of E with one extremity in S and one in V/S, If $S = \{v\}$, then we write $\delta(v)$ instead of δ (S). Let $x_e = x_{ij}$ represent the number of additional (dead headed) copies of edge $e = (v_i, v_j)$ that must be added to G to make it Eulerian. The formulation is given:

(UUPP1)

$$Minimize \sum_{e \in E} C_e x_e \qquad (1)$$

Subject to

$$\sum_{e \in \delta(v)} x_e = 0\, mod(2) \qquad (if\ v \in V_R\ is\ R - even) \qquad (2)$$

$$\sum_{e \in \delta(v)} x_e = 1\, mod(2) \qquad (if\ v \in V_R\ is\ R - \text{odd}) \qquad (3)$$

$$\sum_{e \in \delta(v)} x_e \geq 2 \qquad (S = \bigcup_{k \in P} V_k,\ P \subset \{1, \dots, p\}, p \neq \emptyset) \qquad (4)$$

$$x_e \geq 0 \text{ odd integer} \qquad (e \in E) \qquad (5)$$

In this formulation, constraints (2) and (3) force each vertex to have an even degree, while constraints (4) ensure connectivity. In what follows, we recall some dominance relations that will enable a reformulation of the problem without the non-linear constraints (2) and (3).

Dominance relation 1. (Christofides et al.., 1981)

Every optimal UPP solution satisfies the relations

$x_e \leq 1$ (if e$\in R$) (6)

$x_e \leq 2$ (if e $\in E \backslash R$) (7)

Dominance relation 2. (Corberan and Sanchis, 1994)

Every optimal solution satisfies

$x_e \leq 1$ (if $e = (v_i, v_j)$, v_i, v_j belong to the same component C_k). (8)

Dominance relation 3. (Ghiani and Laporte, 2000).

Let $x(e^{(1)}), x(e^{(2)}),\ldots, x(e^{(\ell)})$ be the variables associated with the edges $e^{(1)}$, $e^{(2)},\ldots, e^{(\ell)}$ having exactly one end vertex in a given component C_k and exactly one end vertex in another given component C_h. Then, in an optimal solution, only the variable $x(e^{(r)})$ having a cost $c(e^{(r)})$ = min {c($e^{(1)}$), c($e^{(2)}$), …, c($e^{(\ell)}$)} can be equal to 2.

Now define a 0/1/2 edge as an edge e for which x_e can be at most equal to 2 in (UUPP1), and a 0/1 edge as an edge e for which x_e can be at most equal to 1. Denote by E_{012} and E_{01} the corresponding edge sets.

Dominance relation 4. (Ghiani and Laporte, 2000)

Let $G_C^* = (V_C, E_C)$ be an auxiliary graph having a vertex v_i' for each component C_i and, for each pair of components C_i and C_j. Then, in any optimal (UUPP1) solution, the only 0/1/2 edges belong to a Minimum Spanning Tree on G_C^* (denoted by $M\,ST_C$).

A New Binary Formulation Using Only Edge Variables: Using Dominance relation 4, formulation (UUPP1) can now be rewritten by replacing each 0/1/2 edge e belonging to a given $M\,ST_C$ by two parallel 0/1 edge e' and e''. Denote by $E'(E'')$ the sets of edges $e'(e'')$, and let $\bar{E} = E_{01} \cup E' \cup E''$. In formulation (UUPP1), constraints (13) are simply replaced by

$$x_e = 0 \text{ or } 1 \qquad (e \in \bar{E}) \qquad (9)$$

Ghiani and Laporte (2000) also replace the modulo relations (2) and (3) by the following constraints called cocircuit inequalities by Barahona and Grötschel (1986).

$$\sum_{e\in\delta(v)\setminus F} x_e \geq \sum_{e\in F} x_e - |F| + 1(v \in V, F \subseteq \delta(v),$$

$$|F| \text{ is odd if } v \text{ is R-even, } |F| \text{ is even if } v \text{ is R-odd.} \qquad (10)$$

Thus, the new undirected UPP formulation, called (UUPP2) and defined by (1), (10), (4) and (9),

is linear in the 0/1 x_e variables. Constraints (10) can be generalized to any non-empty subset S of V:

$$\sum_{e\in\delta(v)\setminus F} x_e \geq \sum_{e\in F} x_e - |F| + 1(F \subseteq \delta(S),$$

$$|F| \text{ is odd if } S \text{ is R-even, } |F| \text{ is even if } S \text{ is R-odd.} \tag{11}$$

Which are valid inequalities for (UUPP2). If S is R-odd and F= ∅, constraints (19) reduce to the known R-odd inequalities (Corberan and Sanchis, 1994)

$$\sum_{e \in \delta(S)} x_e \geq 1$$

If S is R-even and F={e_b}, they reduce to the R-even inequalities introduced by Ghiani and Laporte (2000):

$$\sum_{e \in \delta(S)/\{e_b\}} x_e \geq x_{e_b} \tag{13}$$

Ghiani and Laporte (2000) have shown that constraints (11) are facet inducing for (UUPP2). They have also developed a branch-and-cut algorithm in which connectivity constraints (4) and generalized cocircuit inequalities (11) are dynamically generated. In practice, it is sufficient to generate constraints of type (10), (12) and (13) to identify a feasible UPP solution.

Computational Results: The branch-and-cut algorithm developed by Ghaiani and Laporte (2000) was tested on several sets on randomly generated instances generated in the same manner as those of Hertz, Laporte and Nanchen-hugo (1999). Instances defined on random planar graphs with $|V|$ = 50, 100, …, 350 were solved to optimality with very few nodes in the branch-and -cut tree and within reasonable computing times. For example, the 350 vertex instances required an average of 22.4 nodes and 332.5 seconds on a PC with a Pentium processor at 90 MHz with 16 Mbytes Ram. At the root of the search tree, the average ratio of the lower bound over the optimum almost always exceeded 0.997. R-even inequalities played a key role in the problem resolution. These results outperform those reported by Christofides et al. (1981), Corberan and Sanchis (1994) and Letchforg (1996) who solved much smaller randomly generated instances ($|V| \leq 84$) with far more branching.

Conclusions: Arc routing problems lie at the heart of several real-life applications and their resolution by good heuristics or exact algorithms can translate into substantial savings. Over the past few years, there has been a revived interest in the study of these problems. In the area of heuristics, specialized procedures such as SHORTEN, DROP-ADD, and 2-OPT (Hertz, Laporte and Nanchen-Hugo, 1999) have been proposed for the undirected UPP. These can also be used for the solution of constrained versions of the UPP such as the CARP (Hertz, Laporte and Mittaz, 2000) or adopted to the directed case (Mittaz, 1999). In addition, powerful local search out: tabu search and variable neighborhood search. In the field of exact algorithms, branch-and-cut appears to be the most promising approach. In the past two decades this method has known a formidable growth and considerable success on related problems such as the TSP (see Padberg and Rinaldi, 1991; Grotschel and Holland, 1991; Junger, Reinelt and Rinaldi, 1995). Recent advances made by Corberan and Sanchis (1994), Letchford (1996) and Ghiani and Laporte (2000) indicate that this method also holds much potential for arc routing problems.

In coming years, we expect to see the development of similar heuristics and branch-and -cut algorithms to more intricate and realistic arc routing problems incorporating a wider variety of practical constraints. Extensions to the area of locating-routing problems in arc routing context (see Ghiani and Laporte, 1999) are also to be expected.

ACKNOWLEDGEMENTS: Authors acknowledge the immense help received from the scholars whose papers are cited and included in the manuscript. The authors are also grateful to authors / editors / publishers of all those articles, journals, and books from where the literature for this article has been reviewed and discussed. Authors are grateful to editorial board members and reviewers for their useful comments that lead the improvement of the manuscript.

REFERENCES

1. **Anily, S., Gendreau, M.** and **Laporte, G.** Optimal sequencing of tasks on a tree shaped structure. Ricerca Operativa 2000; 29:3-14.
2. **Assad, A.A.** and **Golden, B.L.** “Arc routing methods and applications”. In M.O. Ball, T.L. Magnanti, C.L. Monma and G.L. Nemhauser, Network Routing, Handbooks in Operation Research and Management Sciences, Amsterdam: North-Holland, 1995.
3. **Amine, K. & Djellab, R.** 2013. Industrial and Urban Applications of Eulerian and Chinese Walks, In Graph Theory for Operations Research and Management: Applications in Industrial Engineering, IGI-Global, Hershey, 271-279
4. **Barahona, F.** and **Grotschel, M.** On the cycle polytope of a binary matroid. Journal of Combinatorial Theory, Series B 1986; 40:40-62.

5. **Brucker, P.** 1981. "The Chinese Postman Problem for Mixed Graphs. Graph Theoretic Concepts in Computer Science," Springer Berlin Heidelberg.
6. **Beltrami, E.**L. and **Bodin,** L.D. Networks and vehicle routing for municipal waste collection. Network 1974; 4:65-94.
7. **Brucker, P.** 1981. "The Chinese Postman Problem for Mixed Graphs. Graph Theoretic Concepts in Computer Science," Springer Berlin Heidelberg.
8. **Corberan, A.** and **Sanchis, J.M.** A polyhedral approach to the rural postman problem. European Journal of Operational Research 1994; 79:95-114.
9. **Croes, G.A.** A method for solving traveling-salesman problems. Operations Research 1958; 6:791-812.
10. **Christofides, N., Benavent, E., Campos, V., Corberán, A., & Mota, E.** 1984. "An Optimal Method for the Mixed Postman Problem." System Modelling and Optimization. Springer Berlin Heidelberg
11. **Corberán, A., Oswald, M., Plana, I., Reinelt, G., & Sanchis, J. M.** 2012. "New Results on the Windy Postman Problem." Mathematical Programming, vol. 132, p. 309-332
12. **Christofides, N.** The optimum traversal of a graph. Omega 1973; 1:719-732.
13. **Christofides, N.** Worst-case analysis of a new heuristic for the travelling salesman problem. Report No. 388, Graduate School of Industrial Administation, Pittsburgh, PA, Carnegie Mellon University, 1976.
14. **Christofides, N., Campos, V., Corberan, A.** and **Mota, E.** An Algorithm for The Rural Postman Problem. Imperial College Report I.C.O.R. 81.5, London, 1981.
15. **DeArmom, J.S.** A comparison of heuristics for the capacitated Chinese postman problem. Master`s Thesis, University of Maryland, College Park, MD, 1981.
16. **Edmonds, J.,** & **Johnson, E. L.** 1973. "Matching, Euler Tour and the Chinese Postman Problem." Mathematical Programming, vol. 5, p. 88-124.
17. **E. Gregory** and **M. Gutin,** “Chinese Postman Problem on edge-colored multigraphs”, Discrete Applied Mathematics, vol. 217, pp. 196–202, 2017
18. **Eiselt. H.A., Gendreau, m.** and **Laporte, G.** Arc routing problems, part I: The Chinese Postman Problem. Operations Research 1995a; 43:231-242.
19. **Eiselt. H.A., Gendreau, m.** and **Laporte, G.** Arc routing problems, part II: The Rural postman problem. Operations Research 1995b; 43:399-414.

20. **Euler, L. Solutio problematis** and **Geometriam situs perinentis**. Commentarii academiae scientarum Petropolitanae 1736; 8:128-140.
21. **Frederickson**, **G.N.** Approximation algorithms for some postman problems. SIAM Journal on Computing 1979; 7:179-193.
22. **Grötschel, M.,** & **Win, Z.** 1992. "A Cutting Plane Algorithm for the Windy Postman Problem." Mathematical Programming, vol. 55, p. 339-358.
23. **Guan, M.** 1962. Graphic programming using odd or even points. Chinese Math, vol. 110, p. 273-277.
24. **Grotschel, M.** and **Holland, O.** Solution of Large-scale Symmetric Traveling Salesman Problems. Mathematical Programming 1991; 51:141:202.
25. **Greistorfer, P.** Computational experiments with heuristics for a capacitated arc routing problem. Working paper 32, Department of Business, University of Graz, Austria, 1994.
26. **Gendreau, M., Hertz, A.** and **Laporte, G.** A tabu search heuristic for the vehicle routing problem. Management Science 1994; 40:1276-1290.
27. **Ghiani, G.** and **Improta, G.** Optimizing laser-plotter beam movement. Journal of the Operational Research Society 2001, forthcoming.
28. **Ghiani, G.** and **Laprote, G.** Eulerian location problems. Network 1999; 34:291-302.
29. **Ghiani, G.** and **Laprote, G.** A Branch-and-cut Algorithm for The Undirected Rural Postman Problem. Mathematical Programming 2000; 84:467-481.
30. **Golden, B.**L., **De Armon, J.S.** and **Baker, E.K.** Computational experiments with algorithms for a class of routing problems. Computers & Operations Research 1983; 10:47-59.
31. **Haimes, Y. Y., Lasdon, L. S.,** & **Wismer, D. A.** 1971. "On a Bicriterion Formulation of the Problems of Integrated System Identification and System Optimization." IEEE Transactions on Systems Man and Cybernetics, vol. 1, p. 296-297.
32. **Hierholzer Uber die Moglichkeit einen Linienzug ohne Widerholung** and **ohne Unterbrechung zu umfahren.** Mathematische Annalen 1973; VI:30-32.
33. **Hertz, A., Laporte, G. and Nanchen-Hugo, P.** Improvement Procedures for The Undirected Rural Postman Problem. INFORMS Journal on Computer 1999; 11:53-62.
34. **Hertz, A.**, **Laporte, G.** and **Mittaz, M.** A tabu search heuristic for the capacitated arc routing problem. Operations Research 2000; 48:129-135.