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Permeable values with applications in BE-algebras

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Abstract. The notions of energetic subsets and (anti) permeable values are introduced, and related properties are investigated. These notions are applied to the theory of BE-algebras. Regarding (anti) fuzzy subalgebras/filters and energetic subsets are investigated.

1. Introduction

As a generalization of a BCK-algebra, the notion of BE-algebras has been introduced by H. S. Kim and Y. H. Kim in [5]. The study of BE-algebras has been continued in papers [1], [2], and [6]. Jun et al. [3] introduced the notions of S-energetic subsets and I-energetic subsets in BCK/BCI-algebras, and investigated several properties. Jun et.al [4] defined the notions of a C-energetic subset and (anti) permeable C-value in BCK-algebras and studied some related properties of them.

In this paper, we introduce the notions of energetic subsets and (anti) permeable values, and investigate some related properties. These notions are applied to the theory of BE-algebras. Regarding (anti) fuzzy subalgebras/filters and energetic subsets are investigated.

2. Preliminaries

We display basic notions on BE-algebras. We refer the reader to the papers [2, 5] for further information regarding BE-algebras.

By a *BE-algebra* [5] we mean a system $(X; *, 1)$ of type $(2, 0)$ which the following axioms hold:
(BE1) $(\forall x \in X)\,(x * x = 1)$,
(BE2) $(\forall x \in X)\,(x * 1 = 1)$,
(BE3) $(\forall x \in X)\,(1 * x = x)$,
(BE4) $(\forall x, y, z \in X)\,(x * (y * z) = y * (x * z))$ (exchange).

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We introduce a relation " $\leq$ " on X by $x \leq y$ if and only if $x * y = 1$.

A BE-algebra $(X; *, 1)$ is said to be *transitive* if it satisfies: for any $x, y, z \in X$, $y * z \leq (x * y) * (x * z)$. A BE-algebra $(X; *, 1)$ is said to be *self distributive* if it satisfies: for any $x, y, z \in X$, $x * (y * z) = (x * y) * (x * z)$. Note that every self distributive BE-algebra is transitive, but the converse is not true in general [5].

Every self distributive BE-algebra $(X; *, 1)$ satisfies the following properties:

(2.1) $(\forall x, y, z \in X)\,(x \leq y \Rightarrow z * x \leq z * y$ and $y * z \leq x * z)$,
(2.2) $(\forall x, y \in X)\,(x * (x * y) = x * y)$,
(2.3) $(\forall x, y, z \in X)\,(x * y \leq (z * x) * (z * y))$.

Definition 2.1. Let $(X; *, 1)$ be a BE-algebra and let F be a non-empty subset of X. Then F is a *filter* [5] of X if

(i) $1 \in F$;
(ii) $(\forall x, y \in X)(x * y, x \in F \Rightarrow y \in F)$.

The concept of fuzzy sets was introduced by Zadeh [7]. Let X be a set. The mapping $f : X \to [0, 1]$ is called a *fuzzy set* in X. A fuzzy set f in a BE-algebra X is called a *fuzzy subalgebra* of X if it satisfies

(F_0) $(\forall x, y \in X)(f(x * y) \geq \min\{f(x), f(y)\})$.

A fuzzy set f in a BE-algebra X is called a fuzzy filter of X if it satisfies

(F_1) $(\forall x \in X)(f(1) \geq f(x))$;
(F_2) $(\forall x, y \in X)(f(y) \geq \min\{f(x * y), f(x)\})$.

Note that every fuzzy filter f of a BE-algebra X satisfies

$$(\forall x, y \in X)(x \leq y \Rightarrow f(y) \geq f(x)).$$

For a fuzzy set f in X and $t \in [0, 1]$, the (strong) upper (resp. lower) t-level sets are defined as follows:

$$U(f; t) := \{x \in X | f(x) \geq t\}, \quad U^*(f; t) := \{x \in X | f(x) > t\},$$
$$L(f; t) := \{x \in X | f(x) \leq t\}, \quad L^*(f; t) := \{x \in X | f(x) < t\}.$$

3. Energetic subsets

In what follows, let X denote a BE-algebra unless otherwise specified.

Definition 3.1. A nonempty subset A of a BE-algebra X is said to be S-*energetic* if it satisfies

(S) $(\forall a, b \in X)(a * b \in A \Rightarrow \{a, b\} \cap A \neq \emptyset)$.

Definition 3.2. A nonempty subset A of a BE-algebra X is said to be F-*energetic* if it satisfies

(F) $(\forall x, y \in X)(y \in A \Rightarrow \{x * y, x\} \cap A \neq \emptyset)$.

Example 3.3. (1) Let $X := \{1, a, b, c\}$ be a BE-algebra with the following Cayley table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	a	a
b	1	1	1	a
c	1	1	a	1

It is easy to show that $A := \{b, c\}$ is a S-energetic subset of X. But $B := \{a\}$ is not an S-energetic subset of X since $c * b = a \in B$ and $\{c, b\} \cap B = \emptyset$. It is routine to verify that $C := \{c\}$ is an S-energetic subset of X. But it is not an F-energetic subset of X, since $c \in C$ and $\{b * c, b\} \cap C = \emptyset$.
(2) Let $X := \{1, a, b, c\}$ be a BE-algebra with the following Cayley table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	a	a
b	1	1	1	a
c	1	a	a	1

It is easy to show that $A := \{a, b\}$ is an F-energetic subset of X.

Theorem 3.4. *For any nonempty subset A of X, if A is a subalgebra of a BE-algebra X, then $X \setminus A$ is an S-energetic a subset of X.*

Proof. Let $a, b \in X$ be such that $a * b \in X \setminus A$. If $\{a, b\} \cap (X \setminus A) = \emptyset$, then $a, b \in A$ and so $a * b \in A$ since A is a subalgebra of X. This is a contradiction. Thus $\{a, b\} \cap (X \setminus A) \neq \emptyset$. Therefore $X \setminus A$ is an S-energetic subset of X. □

Theorem 3.5. *For any nonempty subset A of X, if A is a filter of a BE-algebra X, then $X \setminus A$ is an F-energetic a subset of X.*

Proof. Let $x, y \in X$ be such that $y \in X \setminus A$. If $\{x * y, x\} \cap X \setminus A = \emptyset$, then $x * y, x \in A$ and so $y \in A$, since A is a filter of X. This is a contradiction. Therefore $\{x * y, x\} \cap X \setminus A \neq \emptyset$. Thus $X \setminus A$ is an F-energetic subset of X. □

Theorem 3.6. *Let A be a nonempty subset of a BE-algebra X with $1 \notin A$. If A is F-energetic, then $X \setminus A$ is a filter of X.*

Proof. Obviously, $1 \in X \setminus A$. Let $x, y \in X$ be such that $x * y, x \in X \setminus A$. Assume that $y \in A$. Then $\{x * y, x\} \cap A \neq \emptyset$ by (F). Hence $x * y \in A$ or $x \in A$, which is a contradiction. Therefore $y \in X \setminus A$. This completes the proof. □

Theorem 3.7. *If f is a fuzzy filter of a BE-algebra X, then the nonempty lower t-level set $L(f; t)$ is an F-energetic subset of X for all $t \in [0, 1]$.*

Proof. Assume that $L(f;t) \neq \emptyset$ for $t \in [0,1]$ and let $x, y \in X$ be such that $y \in L(f;t)$. Then $t \geq f(y) \geq \min\{f(x*y), f(x)\}$. Hence $f(x*y) \leq t$ or $f(x) \leq t$, i.e., $x*y \in L(f;t)$ or $x \in L(f;t)$. Thus $\{x*y, x\} \cap L(f;t) \neq \emptyset$. Therefore $L(f;t)$ is an F-energetic subset of X. □

Corollary 3.8. *If f is a fuzzy filter of a BE-algebra X, then the nonempty stronger lower t-level set $L^*(f;t)$ is an F-energetic subset of X.*

Since $L(f;t) \cup U^*(f;t) = X$ and $L(f;t) \cap U^*(f;t) = \emptyset$ for all $t \in [0,1]$, we have the following corollary.

Corollary 3.9. *If f is a fuzzy filter of a BE-algebra X, then $U^*(f;t)$ is empty set or a filter of X for all $t \in [0,1]$.*

For any $a, b \in X$, we consider sets

$$X_a^b := \{x \in X | a*(b*x) = 1\} \text{ and } A_a^b := X \setminus X_a^b.$$

Obviously, $a, b \notin A_a^b, A_a^b = A_b^a$ and $1 \notin A_a^b$. In the following example, we know that there exist $a, b \in X$ such that A_a^b may not be F-energetic.

Example 3.10. Let $X := \{1, a, b, c, d, 0\}$ be a BE-algebra [2] with the following Cayley table

$*$	1	a	b	c	d	0
1	1	a	b	c	d	0
a	1	1	a	c	c	d
b	1	1	1	c	c	c
c	1	a	b	1	a	b
d	1	1	a	1	1	a
0	1	1	1	1	1	1

Then $A_c^d = \{0, b\}$ and it is not F-energetic since $b \in A_c^d$ but $\{a*b, a\} \cap A_c^d = \emptyset$.

We consider conditions for the set A_a^b to be F-energetic.

Theorem 3.11. *If X is a self distributive BE-algebra X, then A_a^b is F-energetic for all $a, b \in X$.*

Proof. Let $y \in A_a^b$ for any $a, b, y \in X$. Assume that $\{x*y, x\} \cap A_a^b = \emptyset$ for any $x \in X$. Then $x*y \notin A_a^b$ and $x \notin A_a^b$ and so $a*(b*(x*y)) = 1$ and $a*(b*x) = 1$. Using (BE3) and the self distributivity of X, we have

$$\begin{aligned} 1 =& a*(b*(x*y)) = a*((b*x)*(b*y)) \\ =& (a*(b*x))*(a*(b*y)) = 1*(a*(b*y)) = a*(b*y) \end{aligned}$$

and so $y \notin A_a^b$. This is a contradiction, and therefore $\{a*b, a\} \cap A_a^b \neq \emptyset$. Hence A_a^b is an F-energetic subset of X for all $a, b \in X$. □

Definition 3.12. A fuzzy set f in a BE-algebra X is called an *anti fuzzy subalgebra* of X if $f(x*y) \leq \max\{f(x), f(y)\}$ for all $x, y \in X$. A fuzzy set f in a BE-algebra X is called an *anti fuzzy filter* of X if it satisfies

(AF_1) $(\forall x \in X)(f(1) \leq f(x))$;
(AF_2) $(\forall x, y \in X)(f(y) \leq \max\{f(x*y), f(x)\})$.

Proposition 3.13. *For any anti fuzzy filter of a BE-algebra X, then following are valid.*

(i) $(\forall x, y \in X)(x \leq y \Rightarrow f(y) \leq f(x))$;
(ii) $(\forall x, y, z \in X)(f(x*z) \leq \max\{f(x*(y*z)), f(y)\})$;
(iii) $(\forall a, x \in X)(f((a*x)*x) \leq f(a))$.

Proof. (i) Let $x, y \in X$ be such that $x \leq y$. Then $x*y = 1$. It follows from Definition 3.12 that $f(y) \leq \max\{f(x*y), f(x)\} = \max\{f(1), f(x)\} = f(x)$.
(ii) Using (AF_2) and (BE4), we have $f(x*z) \leq \max\{f(y*(x*z)), f(y)\} = \max\{f(x*(y*z)), f(y)\}$ for any $x, y, z \in X$.
(iii) Taking $y := (a*x)*x$ and $x := a$ in (AF_2), we have $f((a*x)*x) \leq \max\{f(a*((a*x)*x)), f(a)\} = \max\{f((a*x)*(a*x)), f(a)\} = \max\{f(1), f(a)\} = f(a)$ for any $a, x \in X$. □

Theorem 3.14. *Any fuzzy set of a BE-algebra X satisfying (AF_1) and Proposition 3.13 (ii) is an anti fuzzy filter of X.*

Proof. Taking $x := 1$ in Proposition 3.13 (ii) and (BE3), we have $f(z) = f(1*z) \leq \max\{f(1*(y*z)), f(y)\} = \max\{f(y*z), f(y)\}$ for all $y, z \in X$. Hence f is an anti fuzzy filter of X. □

Corollary 3.15. *For any fuzzy set f of a BE-algebra X, f is an anti fuzzy filter of X if and only if it satisfies (AF_1) and Proposition 3.13 (ii).*

Theorem 3.16. *Any fuzzy set f of a BE-algebra X is an anti fuzzy filter of X if and only if it satisfies the following conditions:*

(i) $(\forall x, y \in X)(f(y*x) \leq f(x))$;
(ii) $(\forall x, a, b \in X)(f((a*(b*x))*x) \leq \max\{f(a), f(b)\})$.

Proof. Assume that f is an anti fuzzy filter of X. It follows from Definition 3.12 that $f(y*x) \leq \max\{f(x*(y*x)), f(x)\} = \max\{f(1), f(x)\} = f(x)$ for all $x, y \in X$. Using Proposition 3.13, we have $f((a*(b*x))*x) \leq \max\{f((a*(b*x))*(b*x)), f(b)\} \leq \max\{f(a), f(b)\}$ for any $a, b, x \in X$.

Conversely, let f be a fuzzy set satisfying conditions (i) and (ii). Setting $y := x$ in (i), we have $f(x*x) = f(1) \leq f(x)$ for all $x \in X$. Using (ii), we obtain $f(y) = f(1*y) = f((x*y)*(x*y))*y) \leq \max\{f(x*y), f(y)\}$ for all $x, y \in X$. Hence f is an anti fuzzy filter of X. □

Proposition 3.17. *For any fuzzy set of a BE-algebra X, then f is an anti fuzzy filter of X if and only if*

$(*)$ $(\forall x, y, z \in X)(z \leq x*y \Rightarrow f(y) \leq \max\{f(x), f(z)\})$.

Proof. Assume that f is an anti fuzzy filter of X. Let $x, y, z \in X$ be such that $z \leq x * y$. By Proposition 3.13, we have $f(y) \leq \max\{f(x * y), f(x)\} \leq \max\{f(z), f(x)\}$.

Conversely, suppose that f satisfies $(*)$. By (BE2), we have $x \leq x * 1 = 1$. Using $(*)$, we have $f(1) \leq f(x)$ for all $x \in X$. It follows from (BE1) and (BE4) that $x \leq (x * y) * y$ for all $x, y \in X$. Using $(*)$, we have $f(y) \leq \max\{f(x * y), f(x)\}$. Therefore f is an anti fuzzy filter of X. ☐

4. Permeable values in BE-algebras

Definition 4.1. Let f be a fuzzy set in a BE-algebra X. A number $t \in [0, 1]$ is called a *permeable S-value* for f if $U(f; t) \neq \emptyset$ and the following assertion is valid.

(4.1) $(\forall a, b \in X)(f(a * b) \geq t \Rightarrow \max\{f(a), f(b)\} \geq t)$.

Example 4.2. Consider a BE-algebra $X = \{1, a, b, c\}$ as in Example 3.3 (1). Let f be a fuzzy set of X defined by $f(1) = 0.2, f(a) = 0.3$, and $f(b) = f(c) = 0.6$. Take $t \in (0.3, 0.6]$. Then $U(f; t) = \{b, c\}$. It is easy to check that t is a permeable S-value for f.

Theorem 4.3. *Let f be a fuzzy subalgebra of a BE-algebra X. If $t \in [0, 1]$ is a permeable S-value for f, then the nonempty upper t-level set $U(f; t)$ is an S-energetic subset of X.*

Proof. Let $a, b \in X$ be such that $a * b \in U(f; t)$. Then $f(a * b) \geq t$ and so $\max\{f(a), f(b)\} \geq t$. Therefore $f(a) \geq t$ or $f(b) \geq t$, i.e., $a \in U(f; t)$ or $b \in U(f; t)$. Hence $\{a, b\} \cap U(f; t) \neq \emptyset$. Thus $U(f; t)$ is an S-energetic subset of X. ☐

Since $U(f; t) \cup L^*(f; t) = X$ and $U(f; t) \cap L^*(f; t) = \emptyset$ for all $t \in [0, 1]$, we have the following corollary.

Corollary 4.4. *Let f be a fuzzy subalgebra of a BE-algebra X. If $t \in [0, 1]$ is a permeable S-value for f, then $L^*(f; t)$ is empty or a subalgebra of X.*

Definition 4.5. Let f be a fuzzy set in a BE-algebra X. A number $t \in [0, 1]$ is called an *anti permeable S-value* for f if $L(f; t) \neq \emptyset$ and the following assertion is valid.

(4.2) $(\forall a, b \in X)(f(a * b) \leq t \Rightarrow \min\{f(a), f(b)\} \leq t)$.

Example 4.6. Consider a BE-algebra $X = \{1, a, b, c\}$ as in Example 3.3 (1). Let f be a fuzzy set of X defined by $f(1) = 0.4, f(a) = f(b) = 0.5$, and $f(c) = 0.3$. Take $t \in [0.3, 0.4)$. Then $L(f; t) = \{c\}$. It is easy to check that t is an anti permeable S-value for f.

Theorem 4.7. *Let f be an anti fuzzy subalgebra of a BE-algebra X. For any anti permeable S-value $t \in [0, 1]$ for f, we have $L(f; t) \neq \emptyset \Rightarrow L(f; t)$ is an S-energetic subset of X.*

Proof. Let $a, b \in X$ be such that $a * b \in L(f; t)$. Then $f(a * b) \leq t$ and so $\min\{f(a), f(b)\} \leq t$. Thus $f(a) \leq t$ or $f(b) \leq t$, i.e., $a \in L(f; t)$ or $b \in L(f; t)$. Hence $\{a, b\} \cap L(f; t) \neq \emptyset$. Therefore $L(f; t)$ is an S-energetic subset of X. ☐

Theorem 4.8. *Let f be a fuzzy subalgebra of a BE-algebra X and let $t \in [0,1]$ be such that $L(f;t) \neq \emptyset$. Then t is an anti permeable S-value for f.*

Proof. Let $a, b \in X$ be such that $f(a*b) \leq t$ for all $t \in [0,1]$. Then $\min\{f(a), f(b)\} \leq f(a*b) \leq t$. Therefore t is an anti permeable S-value for f. □

Definition 4.9. Let f be a fuzzy set in a BE-algebra X. A number $t \in [0,1]$ is called a *permeable F-value* for f if $U(f;t) \neq \emptyset$ and the following assertion is valid.

(4.3) $(\forall x, y \in X)(f(y) \geq t \Rightarrow \max\{f(x*y), f(x)\} \geq t)$.

Example 4.10. Consider the BE-algebra $X = \{1, a, b, c, d, 0\}$ as in Example 3.10. Let f be a fuzzy set in X defined by $f(1) = 0.2, f(a) = f(b) = 0.4$, and $f(c) = f(d) = f(0) = 0.7$. If $t \in (0.4, 0.7]$, then $U(f;t) = \{0, c, d\}$ and it is easy to check that t is a permeable F-value for f.

Theorem 4.11. *Let f be a fuzzy filter of a BE-algebra X. If $t \in [0,1]$ is a permeable F-value for f, then the nonempty upper t-level set $U(f;t)$ is an F-energetic subset of X.*

Proof. Assume that $U(f;t) \neq \emptyset$ for $t \in [0,1]$. Let $y \in X$ be such that $y \in U(f;t)$. Then $t \leq f(y)$. It follows from (4.3) that $t \leq \max\{f(x*y), f(x)\}$ for all $x \in X$. Hence $f(x*y) \geq t$ or $f(x) \geq t$, i.e., $x*y \in U(f;t)$ or $x \in U(f;t)$. Hence $\{x*y, x\} \cap U(f;t) \neq \emptyset$. Therefore $U(f;t)$ is an F-energetic subset of X. □

Since $U(f;t) \cup L^*(f;t) = X$ and $U(f;t) \cap L^*(f;t) = \emptyset$ for all $t \in [0,1]$, we have the following corollary.

Corollary 4.12. *Let f be a fuzzy filter of a BE-algebra X. If $t \in [0,1]$ is a permeable F-value for f, then $L^*(f;t)$ is empty or a filter of X.*

Theorem 4.13. *For a fuzzy set f in a BE-algebra X, if there exists a subset K of $[0,1]$ such that $\{U(f;t), L^*(f;t)\}$ is a partition of X and $L^*(f;t)$ is a filter of X for all $t \in K$, then t is a permeable F-value for f.*

Proof. Assume that $f(y) \geq t$ for any $y \in X$. Then $y \in U(f;t)$ and so $\{x*y, x\} \cap U(f;t) \neq \emptyset$ for any $x \in X$, since $U(f;t)$ is an F-energetic subset of X. Hence $x*y \in U(f;t)$ or $x \in U(f;t)$ and so $\max\{f(x*y), f(x)\} \geq t$. Therefore t is a permeable F-value for f. □

Theorem 4.14. *Let f be a fuzzy set in a BE-algebra X with $U(f;t) \neq \emptyset$ for $t \in [0,1]$. If f is an anti fuzzy filter of X, then t is a permeable F-value for f.*

Proof. Let $y \in X$ be such that $f(y) \geq t$. Then $t \leq f(y) \leq \max\{f(x*y), f(x)\}$ for all $x \in X$. Hence t is a permeable F-value for f. □

Theorem 4.15. *Let f be an anti fuzzy filter of a BE-algebra X. Then the following assertion is valid.*

$$(\forall t \in [0,1])(U(f;t) \neq \emptyset \Rightarrow U(f;t) \text{ is an } F\text{-energetic subset of } X).$$

Proof. Let $y \in X$ be such that $y \in U(f;t)$. Then $f(y) \geq t$. By (AF_2), we have $t \leq f(y) \leq \max\{f(x*y), f(x)\}$ for all $x \in X$. Hence $f(x*y) \geq t$ or $f(x) \geq t$, i.e., $x*y \in U(f;t)$ or $x \in U(f;t)$. Therefore $\{x*y, x\} \cap U(f;t) \neq \emptyset$. Thus $U(f;t)$ is an F-energetic subset of X. □

Definition 4.16. Let f be a fuzzy set in a BE-algebra X. A number $t \in [0,1]$ is called an *anti permeable F-value* for f if $L(f;t) \neq \emptyset$ and the following assertion is valid.

(4.4) $(\forall x, y \in X)(f(y) \leq t \Rightarrow \min\{f(x*y), f(x)\} \leq t)$.

Theorem 4.17. *Let f be a fuzzy set in a BE-algebra X with $L(f;t) \neq \emptyset$ for $t \in [0,1]$. If f is a fuzzy filter of X, then t is an anti permeable F-value for f.*

Proof. Let $y \in X$ be such that $f(y) \leq t$. Then $\min\{f(x*y), f(x)\} \leq f(y) \leq t$ for all $x \in X$. Hence t is an anti permeable F-value for f. □

Theorem 4.18. *Let f be an anti fuzzy filter of a BE-algebra X. If $t \in [0,1]$ is an anti permeable F-value for f, then the lower t-level set $L(f;t)$ is an F-energetic subset of X.*

Proof. Let $y \in X$ be such that $y \in L(f;t)$. Then $f(y) \leq t$. It follows from (4.4) that $\min\{f(x*y), f(x)\} \leq t$ for all $x \in X$. Hence $x*y \in L(f;t)$ or $x \in L(f;t)$ and so $\{x*y, x\} \cap L(f;t) \neq \emptyset$. Therefore $L(f;t)$ is an F-energetic subset of X. □

Corollary 4.19. *Let f be an anti fuzzy filter of a BE-algebra X. If $t \in [0,1]$ is an anti permeable F-value for f, then $U^*(f;t)$ is empty or a filter of X.*

Theorem 4.20. *For a fuzzy set f in a BE-algebra X, if there exists a subset K of $[0,1]$ such that $\{U^*(f;t), L(f;t)\}$ is a partition of X and $U^*(f;t)$ is a filter of X for all $t \in K$, then t is an anti permeable F-value for f.*

Proof. Assume that $f(y) \leq t$ for any $y \in X$. Then $y \in L(f;t)$ and so $\{x*y, x\} \cap L(f;t)\} \neq \emptyset$ for all $x \in X$, since $L(f;t)$ is an F-energetic subset of X. Hence $f(x*y) \leq t$ or $f(x) \leq t$ and so $\min\{f(x*y), f(x)\} \leq t$. Therefore t is anti permeable F-value for f. □

References

[1] S. S. Ahn, Y. H. Kim, J. M. Ko, *Filters in commutative BE-algerbas,* Commun. Korean Math. Soc. **27** (2012), no. 2, 233–242.

[2] S. S. Ahn, K. S. So, *On ideals and upper sets in BE-algerbas,* Sci. Math. Jpn. **68** (2008), no. 2, 279–285.

[3] Y. B. Jun, S. S. Ahn, E. H. Roh, *Energetic subsets and permeable values with applications in BCK/BCI-algebras,* Appl. Math. Sci. **7** (2013), no. 89, 4425–4438.

[4] Y. B. Jun, E. H. Roh and S. Z. Song, *Commutative energetic subsets of BCK-algebras*, Bulletin of the Section of Logic **45** (2016), 53–63.

[5] H. S. Kim, Y. H. Kim, *On BE-algerbas,* Sci. Math. Jpn. **66** (2007), no. 1, 113–116.

[6] H. S. Kim, K. J. Lee, *Extended upper sets in BE-algerbas,* Bull. Malays. Math. Sci. Soc. **34** (2011), no. 3, 511–520.

[7] L. A. Zadeh, *Fuzzy sets*, Information and Control **8** (1965), 338-353.