Odd Prime labeling of Franklin graph

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Abstract – A graph G = (V, E) with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceed n such that the label of each pair of adjacent vertices are relatively prime. A Graph G which admits prime labeling is called prime graph. In this paper we investigate prime labeling for some classes of graph. In particular we discussed on prime labeling of Franklin graph.

Key words–Franklin graph, graph labeling, prime labeling, Duplication, Switching and Path union.

1. Introduction

All graphs considered here are finite, simple, undirected, connected and non – trivial graph. The graph G has vertex set V=$V(G)$ and the edge set E=$E(G)$. The number of elements of V,denoted as |V| called the order of the graph while the number of elements of E, denoted as |E| called the size of the graph. For notation and terminology we refer to J.A Bondy and U.S.R.Murthy [1]. The notion of the prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout.A(1982P365-368) [2].Lee S(1998P59-67)[6] have proved that wheel W_n is a prime graph iff n is even. In [5] S.Meena and Vaithelingam have proved that the prime labeling for some fan related graphs .In [8] Dr V.Ganesan etal "Prime labeling of split graph of Star K1,n".In [9] Dr V.Ganesan proved "prime labeling of split graph of cycle C_n".

We will give brief summary of definitions and other information which are useful for the present task.

1. Preliminary definitions

Definition 1.1

Let G=(V(G),E(G)) be graph with P vertices. A bijection $f: V(G) \to \{1,2,\dots\dots\dots|V|\}$ is called a prime labeling if for each edge$e = uv, \gcd\big(f(u), f(v)\big) = 1$. A graph which admits prime labeling is called prime graph.

Definition 1.2

The Franklin graph is a 3-regular graph with 12 vertices and 18 edges.

Definition 1.3

Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v_k' with $N(v_k') = N(v_k)$. In other words a vertex v_k' is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v_k'.

Definition 1.4

A Vertex Switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.5

Let$G_1, G_2, G_3, \ldots \ldots G_n$be n copies of a fixed graph G. The graph obtained by adding an edge between G_i and G_{i+1} for i =1,2,…..n-1 is called the path union of G.

Illustration 1.6

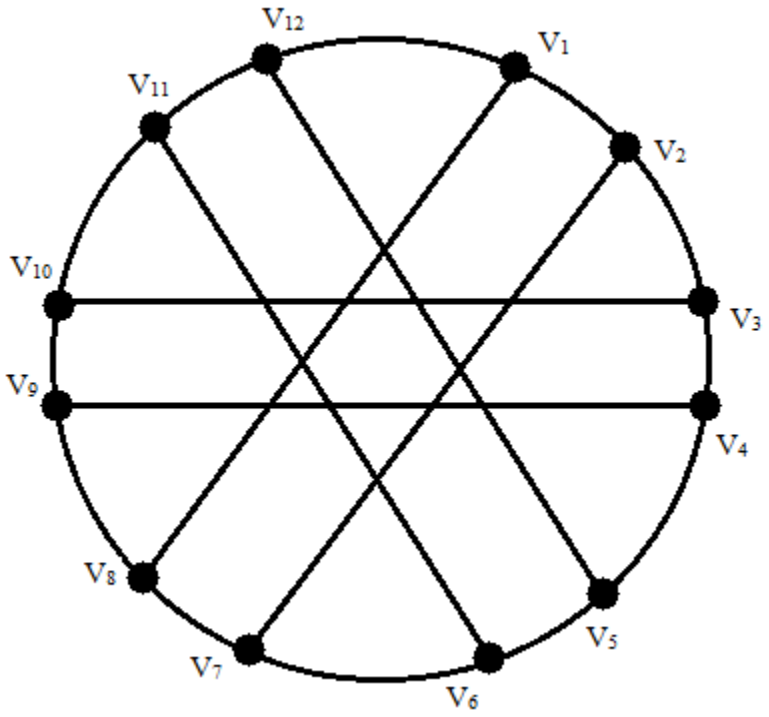


Figure 1.1 The Franklin graph

Theorem-1

Odd prime labeling is admitted by franklin graph

Proof

Let FR_G be franklin graph having 12 vertrices and 18 edges

The vertices and edges of Franklin graph are

$A(FR_G)=\{a_1, a_2 \ldots\ldots..a_{12}\}$ and $B(FR_G)=\{\{a_i, a_{i+1}/1\leq i\leq 11\}\cup a_{12}a_1\{a_ia_{9-i}/1\leq i\leq 2\}\cup\{a_ia_{13-i}/3\leq i\leq 4\}\cup\{a_ia_{17-i}/5\leq i\leq 6\}\}$

Hence $|A(FR_G)|=12$ And $|B(FR_G)|=18$

The labeling is defined by $\varphi:A(FR_G)\rightarrow \{1,3,5,\ldots.23\}$

$\varphi(a_1)=1$

$\varphi(a_{i+1})=2i+1$ for $1\leq i\leq 11$

We should check relative prime of adjacent vertices

gcd $(\varphi(a_i), \varphi(a_{i+1}))=1$for $1\leq i\leq 11$

gcd $(\varphi(a_{12}), \varphi(a_1))=1$

gcd $(\varphi(a_i), \varphi(a_{9-i}))=1$for $1\leq i\leq 2$

gcd $(\varphi(a_i), \varphi(a_{13-i}))=1$for $3\leq i\leq 4$

gcd $(\varphi(a_i), \varphi(a_{17-i}))=1$ for $5\leq i\leq 6$

Therefore FR_G admits odd prime labeling. Hence Franklin graph is an odd prime graph

Example,

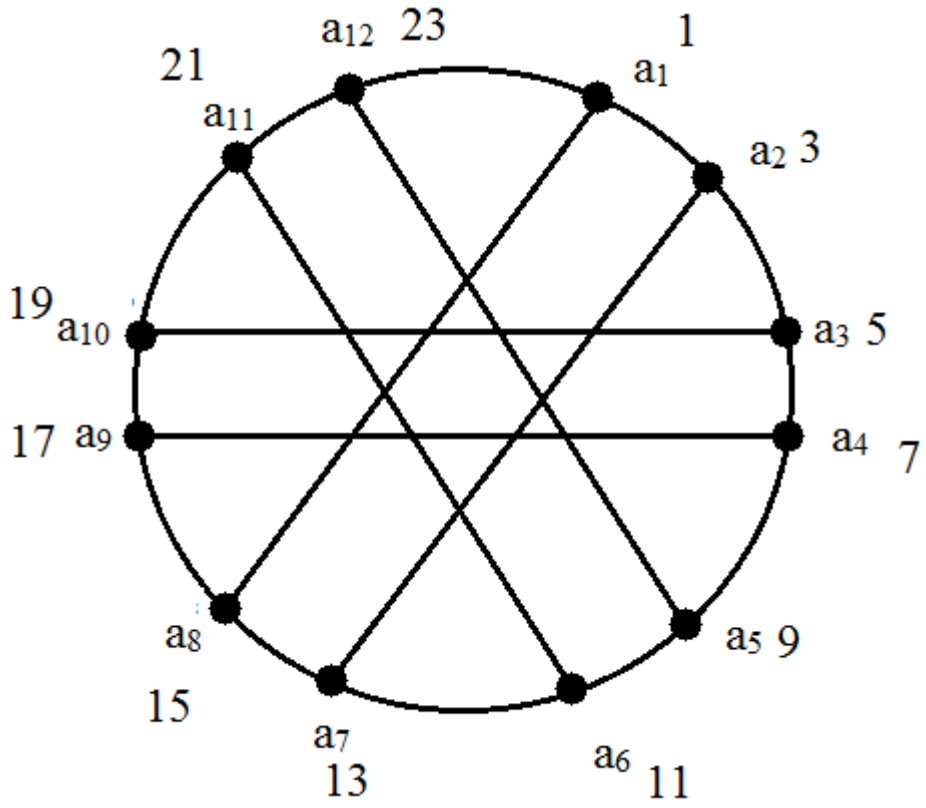


Figure 1.2 Odd Prime Labeling of Franklin Graph

Theorem – 2

A Switching of any vertex in Franklin graph is an odd prime labeling

Proof

Let FR_G be the Franklin graph

The Franklin graph is a 3- regular graph

The graph consists of 12 vertices and 18 edges

Take G_S be the graph which obtained from

F R_G by switching any vertex a_i in Franklin graph

The vertices and edges of G_S are

$A(G_S) = \{a_1, a_2 \ldots a_{12}\}$

$B(G_S) = \{\{ a_i a_{i+1} / 2 \leq i \leq 11\} \cup a_2 a_7 \cup \{a_i a_{13-i} / 3 \leq i \leq 4\}$

$\cup \{a_i a_{17-i} / 5 \leq i \leq 6\} \cup \{a_1 a_{2+i} / 1 \leq i \leq 5\}$

U{{a_1a_{8+i} / 1≤i≤4}}

Then | A (G_S) | = 12 and | B (G_S) | = 23

Define a labeling φ: A (G_S) → { 1, 3, 5 ... 23 }

φ (a_i)=1

φ (a_{i+1})=2i+1for1≤i≤11

To verify the relative prime of adjacent vertices

1) gcd (φ (a_i), φ (a_{i+1})) = 1for 2≤i≤11

gcd (φ (a_i), φ (a_{13-i})) = 1for 3≤i≤4

gcd (φ (a_i), φ (a_{17-i})) = 1

2) gcd (φ (a_2), φ (a_7)) = 1for 5≤i≤6

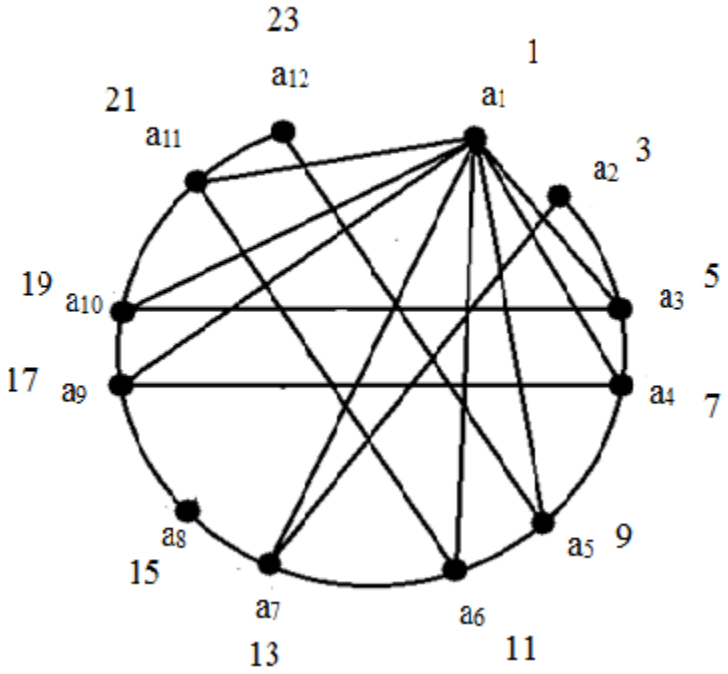
3) gcd (φ (a_1), φ (a_{2+i})) = 1for 1≤i≤5

gcd (φ (a_1), φ (a_{8+i})) = 1for 1≤i≤4

Therefore φ is an odd prime labeling consequently G_S is an odd prime graph.

Hence Switching of any vertex in a Franklingraph admits odd prime labeling

Example



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