CONTRA HARMONIC MEAN LABELING FOR SOME TRIANGULAR MAN RELATED GRAPHS

EZHILARASAN . M[1], KARUNAKARAN .V [2] ,K . BALASANGU[3], MALATHI . T[4]

[1] Research scholar, Thiru kolanjiappar Govt. Arts College,Vriddhachalam 606001. Tamil Nadu, India. ezhilathi@gmail.com.

[2,3,4]Assistant Professor Department of Mathematics, Thiru kolanjiappar Govt. Arts College Vriddhachalam 606001. Tamil Nadu, India.
vkmaths2010@gmail.com,balasangu76@gmail.com,mithranmalathi317@gmail.com.

ABSTRACT

In this paper new graph called Triangular man graph is presented and some activity talked about. Triangular man graph T(Mn) and its activities specifically fusion, duplication, switching, path union way association are demonstrated to contra harmonic mean labeling graph.

***Key words*:** Contra harmonic Mean graph, Triangular man graph, Fusion, Duplication, Switching, Path union.

1.INTRODUCTION

The graph S has vertex or point set P = P(S) and the edge or line set L= L(S). The set of vertices adjacent to a vertex u of S is denoted by N(u). In this paper, we consider only undirected and finite graph. For notation and terminology, we refer to J.A Bondy and U.S.R Murthy[1]. Assignments of integers to points and or lines of a graph subject to certain conditions is known as graph labeling. [3,4] Gallian refered for the latest survey of graph labelling. Contra harmonic Mean labeling was first introduced somasundaram ponraj [7].This paper attempt to prove that contra harmonic mean labeling in the context of Triangular man graph and its some operation of fusion, duplication, switching are contra harmonic mean labeling graph.

Definition:1.1:

A graph S with p vertices q edges is a contra harmonic mean graph if there is an injective function f from the vertices of G to {0,1,2,...q} such that when each edge uv is labelled with f(e = uv) = $ceiling\ \ function\ of\ \frac{f(u)2+f(v)2}{f(u)+f(v)}\rceil$ or floor function of $\frac{f(u)2+f(v)2}{f(u)+f(v)}\rfloor$ with distinct edge labels. The mapping f is called contra harmonic mean labeling of S.

Definition 1.2: Duplication of a vertex v_k of a graph S produces new graph G_1 by adding a vertex v'_k with $N(v'_k) = N(v_k)$. In other words a vertex v'_k is said to be a duplication o*f* v_k . If all the vertices which are adjacent to v_k are now adjacent to v'_k also.

Definition 1.3: Let u and v be any vertices of a graph S. A new graph S_1 is constructed by fusing two vertices are vertices u and v by a single vertex x is such that every edge which was incident with either u or v in S is now incident with x in S_1.

Definition 1.4: Let S_1, S_2, S_3,...S_n be n duplicates of a fixed graph S. The graph acquired by adding an edge among S_i and S_{i+1} for i= 1,2...n-1 is known as the path union of S

Definition 1.5: A vertex switching S_V of a graph S is obtained by taking a vertex V of S, removing all the entire incident with V and adding edges joining V to every vertex which are not adjacent to V in S..

Definition 1.6 *:* Traingular man graph $T(M_n)$ $n \geq 3$, can be constructed by joining a fan graph F_n, ($n \geq 2$) of apex vertex with K_2 and its adjacent vertices with K_1 and a cycle C_n ($n \geq 3$) with sharing a common vertex , where n is any natural numbers. The vertices of fan with attaching k_1 (pendent vertex), the vertices of fan with attaching k_2 (apex vertex) are u, u_1, u_2 , ... ,u_n , u_{n+1} , v_1, v_2 , ... v_n , v_{n+1} and cycle C_n vertex are $w_1, w_2, \ldots, w_{n-1}$.

2. MAIN RESULT:

Theorem2.1:

The $T(M_n)$ $3 \leq$ n, is contra harmonic mean labelling, where n is any natural numbers.

Proof:

Consider the smallest to highest point set and line set *be* $L(T(M_n)) = \{(u_0u_i) / 1 \leq i \leq n+1\} \cup \{u_i u_{i+1} / 1 \leq i \leq n\} \cup \{u_i v_i / 1 \leq i \leq n\} \cup (u_0v_{n+1}) \cup (u_0u_{n+1}) \cup (w_i / 1 \leq i \leq n)$.

Here *u* is the centre vertex.

Let $P(T(M_n))=\{u,u_1,u_2,\ldots u_{n+1},v_1,v_2,v_3\ldots v_{n+1},w_1,w_2,w_3,\ldots w_{n-1}\}$.$P((T(Mn)) = 3n + 2.$

$L((T(Mn)) = 4n + 1$, here n is a characteristic numbers.

Define R: $P((T(M_n)) \rightarrow \{0,1,2,3,..q\}$ as follows

$R(u) = 0$, $R(u_{n+1}) = 4n + 2$, $R(v_{n+1}) = 4n+3$.

$R(w_i) = i$, $i = 1,2,3\ldots,n-2$, $R(w_{n-1}) = n$.

$R(u_i) = n+2i-1$,

$R(v_i) = u_n+i+2$, $i = 1,2,3\ldots,n$,

Then the different edge labels are

$R(u_0w_1) = 1$,

$R(w_iw_{i+1}) = i+1, i = 1,2,3\ldots,n-2$, $R(u_0w_{n-1}) = n$,

$R(u_iu_{i+1}) = n+2i$, $i = 1,2,3\ldots,n-1$, $R(u_iv_i) = u_n+i-1$, $i = 1,2,3\ldots,n$,

$R(u_0u_i) = n+2i-1$, $i = 1,2,3\ldots,n$, $R(u_{n+1}u_0) = 4n+2$, $R(u_0v_{n+1}) = 4n+3$, $R(v_{n+1}u) = n+2$,

Hence $T(M_n)$ *is a contra harmonic mean labeling.*

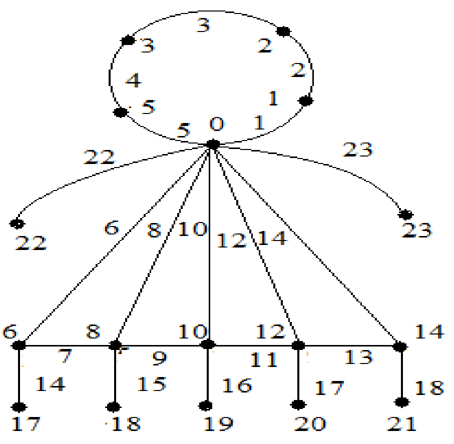


Figure. 2.2 Traingular man graph T(M_5).

Theorem2.2:

The graph contra harmonic mean labeling is created by fusing the vertex vn to un of T(Mn).

Proof:.

Sum of point set is $P((T(M_n)) = 3n+1$.

When n is a non negative integer, the sum of the edge set is L((T(Mn)) = 4n.

Define injective function R : P((T(Mn)) → {0,1,2,3,..q} as follows, u_n to v_n after combined u_n and v_n equivalent to 2n-1.

R (u) = 0,

$R(u_i) = 2i-1$, i = 1,2,3...n,

$R(w_i) = 2n+i$, i from 1 to n-2, $R(w_{n-1}) = 3n-1$

$R(v_i) = 3n+2i-1$, i = 1,2,3...n-1,

$R(u_n) = 2n-1$

$R(u_n) = R(v_n)$

$R(u_{n+1}) = 5n-2$,

$R(v_{n+1}) = 5n-1$.

Then the different edge labels are

$R(uu_i) = 2i - 1$, I varies 1 to n

$R(w_iw_{i+1}) = 2n + i$,i = 1,2,3...,n-2, $R(u_0w_{n-1}) = 3n-1$,

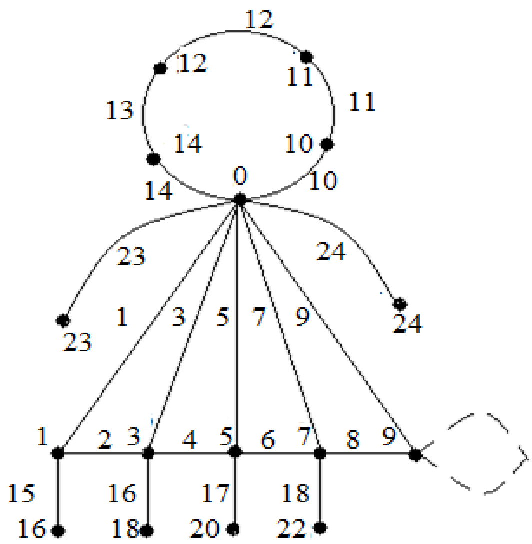
$R(u_iu_{i+1}) = 2i$, i = 1,2,3...,n-1,

$R(u_iv_i) = 3n+i-1$, i = 1,2,3...,n-1,

R $(uu_{n+1}) = 5n-2$,R $(uv_{n+1}) = 5n-1$.

$R(w_{n-1}u) = 3n-1$,

Hence the graph contra harmonic mean labeling is created by fusing the vertex vn to un of T(Mn).



Theorem 2.3 :

The graph apply by fusing the vertex u to w_1 of cycle, *$T(M_n)$* pairs are harmonic mean labeling graph..

Proof:

Now consider the point set

Le $P(T(Mn)) = \{u, u1, u2, \dots un+1, v1, v2, v3 \dots vn+1, w1, w2, w3, \dots, wn-2\}$.

Let S* be the graph obtained by fusing u and w_1 .

Sum of point set is P((T(M_n)) = 3n+1.

Sum of line set is L((T(M_n)) = 4n, here n is a natural numbers. Describe injective function R: P((T(Mn)) ⟶ {0,1,2,3,..q} as follows,

R (u) = 0, R (u) = R (w_1)

R $(ui) = 2i - 1, i = 1,2,3..., n,$

R$(vi) = 3n + 2i - 2, i = 1,2,3..., n$

R $(un + 1) = 5n - 1$, R$(vn + 1) = 5n$

R$(wi) = 2n - 1 + i, i = 1,2,3..., n - 2$,R$(wn) = 3n - 2$

License S to be the actually labelled graph. In T(Mn) we truly need to look at the general contra harmoni mean labeling of R ∗(u_0), R ∗ (u_i) , R ∗(v_i) and R ∗ (w_i).

According to this figure 2.4

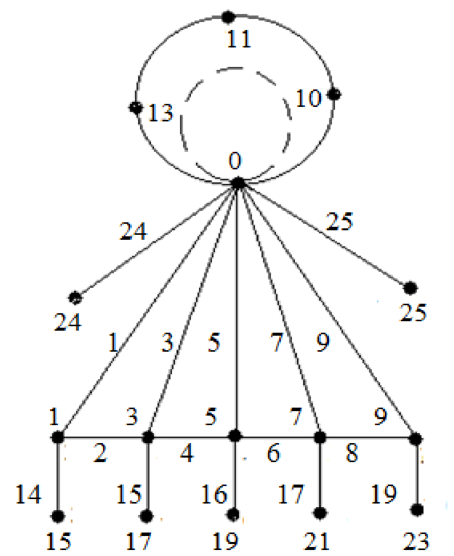


Figure. 2.4 Fusion of two subsequent vertices u to w_1 of T(M_5).

Therefore then the resulting edge labels are distinct

Then the different edge labels are

$R(uu_i)$ =2i - 1, i varies 1 to n

$R(w_iw_{i+1})$ =2n +i, i = 1,2,3...,n-2, $R(u_0w_{n-1})$ = 3n-2,

$R(u_iu_{i+1})$ = 2i, i = 1,2,3...,n-1,

$R(u_iv_i)$ = (3n-1)+i-1, i = 1,2,3...,n-1, $R(u_nu_{n-1})$ = 4n-1,

R (uu_{n+1}) = 5n-1,

R (uv_{n+1}) = 5n.

$R(w_{n-1}u)$ =3n-2, Hence *$T(M_n)$ is a contra harmonic mean labeling.*

Hence the graph contra harmonic mean labeling is created by fusing the vertex u to w1 of T(Mn).

Theorem 2.4:

Fusing the vertex v_1 to u_1 (or any two consequent points) $T(M_n)$ is contra harmonic mean labelling.

Proof:

consider S′ be the graph obtain by fusing v_1 and $u_1$in P((S)).

The Sum of vertex set is $P((T(M_n))$ = 3n+1.

Characterize R : $P((T(M_n)) \rightarrow \{1,2,3,..3n+1\}$ in this way,

R (u_1) = 1, R (u_1) = R (v_1), R (u_1) = 0

R $(ui) = 2i - 1, i = 1,2,3\ldots, n,$

R$(vi) = 3n + 2i - 2, i = 2,3\ldots, n$

R $(un + 1) = 5n,$ R$(vn + 1) = 5n - 1$

R$(wi) = 2n - 1 + i, i = 1,2,3\ldots, n - 2,$ R$(wn) = 3n - 1$

License S to be the actually labelled graph. In T(Mn) we truly need to look at the general contra harmonic mean labeling of R(u_0), R(u_i) , R(v_i) and R(w_i).

According to this figure 2.4

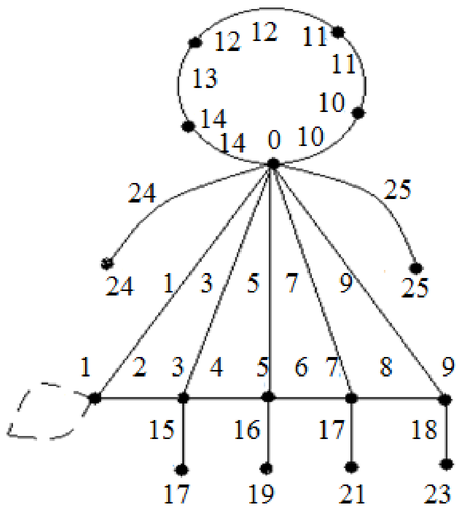


Figure.2.5 Fusion of two starting vertices v_1 to u_1 *T(M_6).*

Then the different edge labels are

R(uu_i) =2i - 1,i varies 1 to n; R(uw_1) =2n

R(w_iw_{i+1}) =2n +i, i = 1,2,3...,n-2, R(u_0w_{n-1}) = 3n-1,

R(u_iu_{i+1}) = 2i, i = 1,2,3...,n-1,

$R(u_iv_i)$ = 3n+i-2, i = 2,3...,n-1, $R(u_nu_{n-1})$ = 4n-1,

R (uu_{n+1}) = 5n-1, R (uv_{n+1}) = 5n.

$R(w_{n-1}u)$ =3n-1.Hence the graph contra harmonic mean labeling is created by fusing the vertex u_1 to v_1 of T(Mn).

Theorem 2.5:

Fusing the vertex v_1 to u_1 and v_n to u_n (or any two corresponding vertices) of $T(M_n)$ is a contra harmonic mean labeling graph.

Proof:

Total number of vertex set is P(S′) = 3n, here n is a natural numbers.

Define injective function R : P(S′) → {0,1,2,3,..q} as follows

R (u_1) = 2, R(u_1) = R(v_1), R(u) = 0;

R(u_n) = 2n, R(u_n) = R(v_n),

$R(ui) = 2i, i = 1,2,3\ldots, n,$

$R(vi) = 2i-3, i = 2,3\ldots, n-1$

$R\ (un+1) = 5n+1,\qquad R(vn+1) = 5n+2$

$R(wi) = 2n+i, i = 1,2,3\ldots, n-2$, $R(wn-1) = 3n.$

Then the different edge labels are

$R(uu_1)$ =1,$R(uu_i)$ =2i ,i varies2 to n; $R(uw_1)$ =2n

$R(w_iw_{i+1})$ =2n +i, i = 1,2,3...,n-2, $R(u_0w_{n-1})$ = 3n,

$R(u_iu_{i+1})$ = 2i+1, i = 1,2,3...,n-1,

$R(u_iv_i)$ = 2i-1, i = 2,3...,n-1,

R (uu_{n+1}) = 3n+1, R (uv_{n+1}) = 3n+2.

Hence the graph contra harmonic mean labeling is created by fusing the vertex u_1 to v_1 and v_n to u_n of T(Mn).

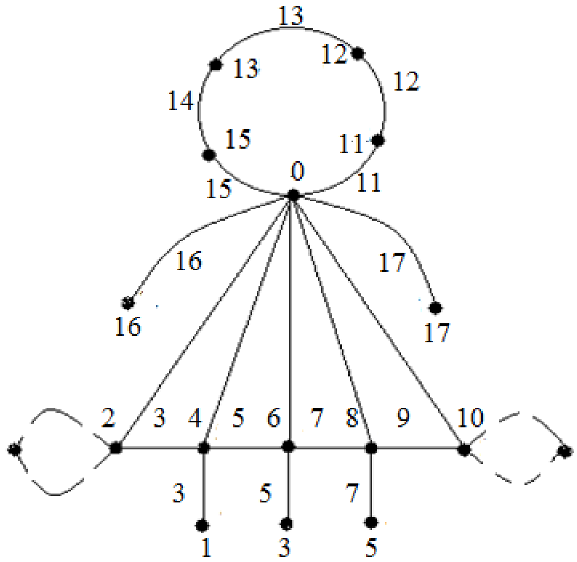


Figure. 2.6 Fusion starting vertices(fan graph) v_1 to u_1 and v_n to u_n of $T(M_5)$.

Theorem 2.6:

The graph get by duplicating to intial u_1 pendent vertex u_k of $T(M_n)$ is a contra harmonic mean graph .

***Proof* :**

Now S′ be the graph obtained by duplicating u_1 of $P(T(M_n))$. . Total number of vertex set is P(S′) = 3n+3, here n equal to natural numbers. Let the new vertex be W'_k.

Define *injective function* R: PT(S′) → {0,1,2,3,..3n+3} as follows, R(u) = 0;

R(wk) = 5n+2;

R $(ui) = 2i - 1, i = 1,2,3\ldots, n,$

R$(vi) = (4n - 3) + 2i - 2, i = 1,2,3\ldots, n,$

R $(un + 1) = 3n$,

R$(vn + 1) = 3n + 1$

R$(wi) = 2n + i, i = 1,2,3 \ldots, n - 2,$

R$(wn - 1) = 3n - 1.$

Then the different edge labels are

R(uu_i) = 2i-1 ,i varies1 to n;

R(w_iw_{i+1}) = 2n +i, i = 1,2,3...,n-2,

R(uw_1) = 2n,

R(uw_{n-1}) = 3n-1,

R(u_iu_{i+1}) = 2i-1, i = 1,2,3...,n,

R(u_iv_i) = 3n+2i+1, i = 1,2,3...,n,

R (uu_{n+1}) = 3n,

R (uv_{n+1}) = 3n+1.

R(uwk) = 5n+2;

Hence the graph contra harmonic mean labeling is created by duplicated the vertex u_1 to u_k of T(Mn).

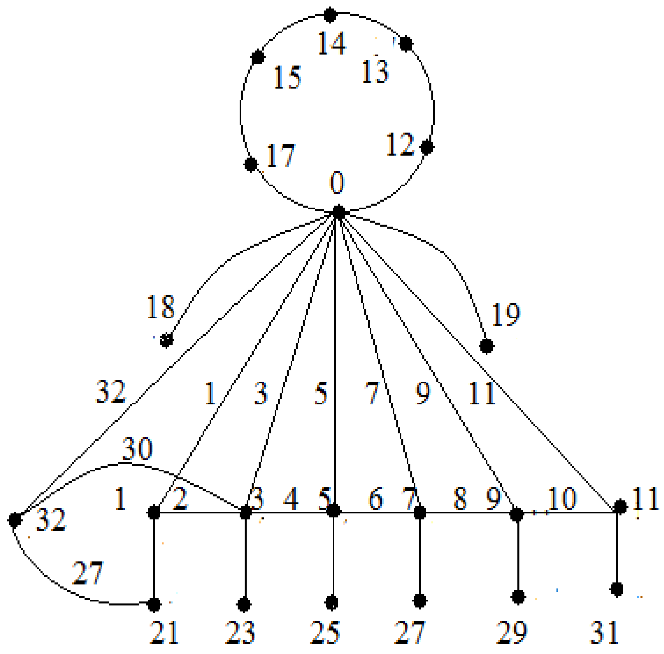


Figure.2. 7 Duplication of a vertex to pendent vertex u_1 of $T(M_6)$.

Theorem 2.7:

The graph acquired by duplicating a vertex u of w_k in the cycle of $T(M_n)$ is a contra harmonic mean labeling graph.

Proof:

Take S′ be the graph obtained by duplicating u. Total number of point set is

$P((T(M_n))$ = 3n+3,

Define $injective\ function\ R$: PT(S′) → {1,2,3,..3n+3} as follows,

R(u) = 0; R(wk) = 5n+2;

R $(ui) = 2i - 1, i = 1,2,3\ldots, n,$

R$(vi) = (4n - 3) + 2i - 2, i = 1,2,3\ldots, n,$

R $(un + 1) = 3n,$ R$(vn + 1) = 3n + 1$

R$(wi) = 2n + i, i = 1,2,3\ldots, n - 2,$ R$(wn - 1) = 3n - 1.$

Then the different edge labels are

$R(uu_i)$ =2i-1 ,i varies1 to n;

$R(w_iw_{i+1})$ =2n +i, i = 1,2,3...,n-2,

$R(uw_1)$ = 2n,

$R(uw_{n-1})$ = 3n-1,

$R(uw_{n-1})$ = 3n,

$R(u_iu_{i+1})$ = 2i-1, i = 1,2,3...,n,

$R(u_iv_i)$ = 3n+2i+1, i = 2,3...,n-1,

R (uu_{n+1}) = 3n, R (uv_{n+1}) = 3n+1.

R(uwk) = 5n+2. Hence the graph contra harmonic mean labeling is created by duplicate the vertex w_{n-1} to v_k of T(Mn).

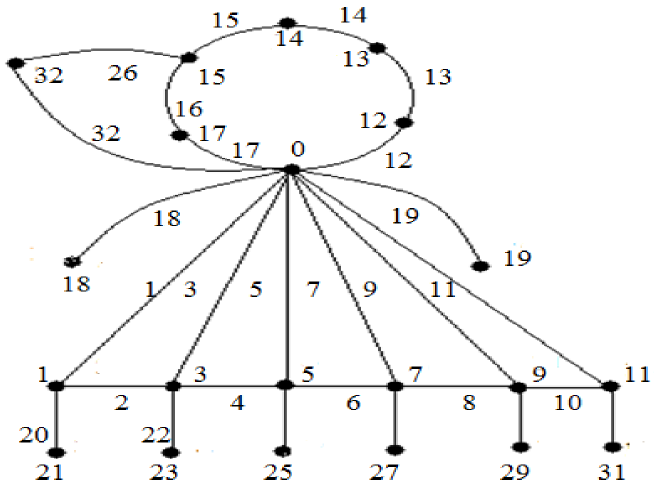


Figure. 2.8 : Duplication of a apex pendent vertex u of w_5 in *$T(M_6)$*.

Theorem 2.8:

The graph acquired by duplicating of two vertex w'_k and w''_k in the cycle of the $T(M_n)$ contra harmonic mean graph.

Proof:

Allow $P(T(M_n)) = \{u, u_1, u_2, \dots u_{n+1}; v_1, v_2, v_3 \dots v_{n+1}; w_1, w_2, w_3, \dots w_{n+1}\}$.

Let S* be the graph obtained by duplicating w_k. Sum of vertex set is $P(T(M_n)) = 3n+3$, here n is a positive integers. Let the new two point be $w'_{k,}$, w''_k

Define R : $S(T(M_n)) \rightarrow \{0,1,2,3,..q\}$ as follows,

$R(u) = 0;$

$R(w'k) = 5n+2;$

$R(w''k) = 5n+3;$

$R\ (ui) = 2i - 1, i = 1,2,3 \dots, n,$

$\mathrm{R}(vi) = (4n + 3) + 2i - 2, i = 1,2,3\ldots, n,$

$\mathrm{R}\ (un + 1) = 3n\,, \qquad \mathrm{R}(vn + 1) = 3n + 1$

$\mathrm{R}(wi) = 2n + i, i = 1,2,3 \ldots, n - 2,$

$\mathrm{R}(wn - 1) = 3n - 1.$

Then the different edge labels are

$R(uu_i) = 2i-1$,i varies1 to n;

$R(w_iw_{i+1}) = 2n + i$, i = 1,2,3...,n-2,

$R(uw_1) = 2n$,

$R(uw_{n-1}) = 3n-1$,

$R(u_iu_{i+1}) = 2i-1$, i = 1,2,3...,n,

$R(u_iv_i) = 3n+2i+1$, i = 2,3...,n,

R $(uu_{n+1}) = 3n$, R $(uv_{n+1}) = 3n+1$.

$R(uwk) = 5n+2$.

Hence the graph contra harmonic mean labeling is created by duplicate the vertex $w'_{k,}$, w''_k of T(Mn).Hence the graph S′ is a contra harmonic mean labeling graph.

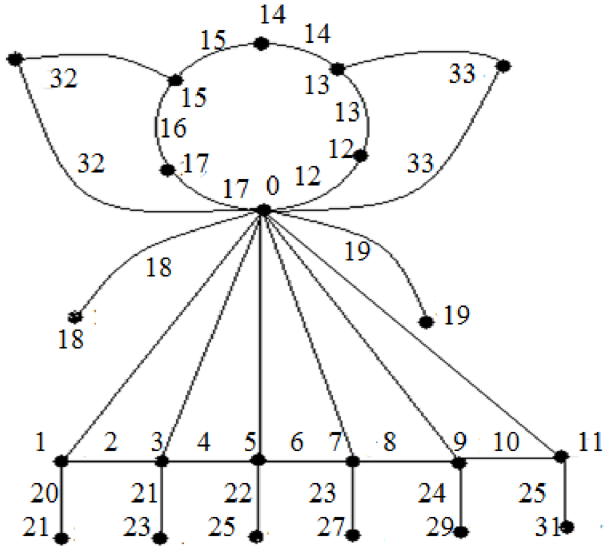


Figure. 2.8: Duplication of a pendent K_2 vertex w'_5 and w'_1in $T(M_6)$.

Theorem 2.9:

The graph S* acquired by switching of the centre vertex u in the $T(M_n)$ is a contra harmonic mean labeling .

Proof:

Let S′ be the graph obtained by switching u in $T(M_n)$.

. Total number of vertex set is $P(T(M_n)) = 3n+2$,

Define R: $P((T(M_n)) \rightarrow \{0,1,2,3,...,q\}$ as follows

$R(u) = 0$;

$R(ui) = 2i - 1, i = 1,2,3..., n,$

$R(vi) = (4n + 1) + 2i - 2, i = 1,2,3..., n,$

$R(un + 1) = 4n - 2$, $R(vn + 1) = 4n - 1$

$R(wi) = 2n + i, i = 1,2,3 \dots, n-2,$

Then the different edge labels are

$R(uw_{i+1})$ = 2n+3+2i-1, i varies1 to n-3;

$R(w_iw_{i+1})$ =2n +2i-2 i = 1,2,3...,n-2,

$R(u_iu_{i+1})$ = 2i-1, i = 1,2,3...,n,

$R(uv_i)$ = 4n+2i-1, i = 2,3...,n,

Hence the graph contra harmonic mean labeling is created byswitch the vertex u of T(Mn). Hence the graph S′ is a contra harmonic mean labeling graph.

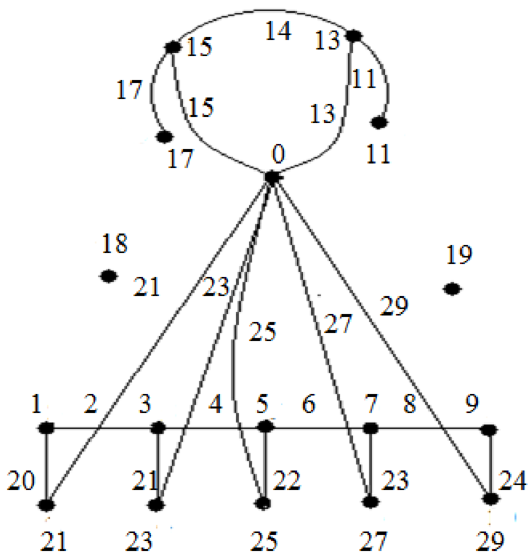


Figure. 2.12: Switching of a vertex u in $T(M_6)$.

Theorem:2.11

The graph S* apply by union path of two copies triangular man graph $T(M_n)$ is a contra harmonic mean labeling graph. Here n is a natural numbers.

Proof:

Consider the vertex set and edge be

$P(T(M_n))=\{u,u_1,u_2,...u_{n+1},v_1,v_2,v_3...v_{n+1},w_1,w_2,w_3,...w_{n-1}\}$. Let S* be the graph obtained by path union of $T(M_n)$.

Let $S(T^*(M_n))=\{u',u'_1,u'_2,...u'_{n+1},v'_1,v'_2,v'_3...v'_{n+1},w'_1,w'_2,w'_3,...w'_{n-1}\}$.

Let S* the graph acquired by union path of $T(M_n)$.

. Sum *of* vertex set is $P((T(M_n)) = 6n+4$,

Define R: $P((T(M_n)) \rightarrow \{0,1,2,3,...,q\}$ as follows

$R(u) = 0$, $R(u_{n+1}) = 3n$, $R(v_{n+1}) = 5n+1$.

$R(u_i) = 2i-1$, $i = 1,2,3...,n-1$, $R(w_i) = 2n -1+i$.

$R(v_i) = 3n+2i$, $i = 1,2,3...,n$.

$R(u') = 0$, $R(u'_{n+1}) = 5n+3$, $R(v'_{n+1}) = 6n+4$.

$R(u'_i) = (6n+4)+2i-1$, $i = 1,2,3...,n-1$, $R(v'_i) = (9n +5)+2i-2$

$R(w'_i) = 5n+3+i$, $i = 1,2,3...,n-1$,

Then the different edge labels are

$R(uu_{i+1}) = 2i-1$, $i = 1,2,3...,n$,

$R(u_iu_{i+1}) = 2i$, $i = 1,2,3...,n$,

$R(u_iv_i) = 3n+i$, $i = 1,2,3...,n$,

$R(uw_1) =2n$,

$R(w_iw_{i+1}) =2n + i$,i= 2,3...,n-1,

$R(uu_{n+1}) = 3n$,

$R(uv_{n+1}) = 5n+1$,

$R(uw_{n-1}) = 3n-1$,

$R(v'_{n+1}\, u'_{n+1}) = 5n+2$,

$R(u'_i\, u'_{i+1}) = 6n+4+ 2i$, i = 1,2,3...,n-1,

$R(u'u'_{i+1}) = 6n+4+ 2i-2$, i = 1,2,3...,n-1,

$R(u'_i v'_i) = 3n+i$, i = 1,2,3...,n,

$R(u'w'_1) = 5n+ 4$,

$R(w'_i w'_{i+1}) = 5n +4+ i$,i= 2,3...,n-1,

$R(u'u'_{n+1}) = 5n+ 3$, $R(u'v'_{n+1}) = 6n+4$, $R(u'w'_{n-1}) = 6n+3$, Hence the graph S* apply by union path of two copies triangular man graph $T(M_n)$ is a contra harmonic mean labeling graph.

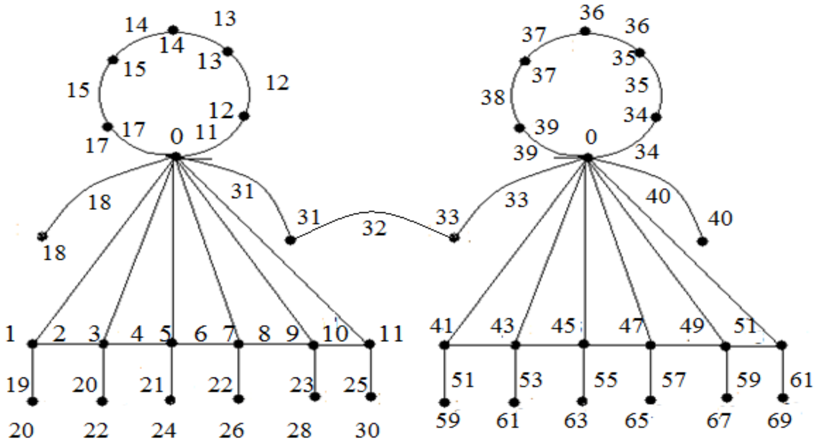


Figure. 2.12 : Path union of two copies $T(M_6)$ is a contra harmonic mean graph.

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