3-VERTEX SELF SWITCHING OF GRAPHS

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Abstract

For a finite undirected graph $G(V,E)$ and a non-empty subset $\sigma \subseteq V$, the switching of G by σ is defined as the graph $G^{\sigma}(V,E')$ which is obtained from G by removing all edges between σ and its complement $V-\sigma$ and adding as edges all non-edges between σ and its complement $V-\sigma$. A subset σ of V is said to be self switching if $G \cong G^{\sigma}$. We call it as $|\sigma|$-vertex self switching. When $|\sigma| = 3$, it is termed as 3 -vertex self switching. The set of all 3-vertex self switchings of G with cardinality 3 is denoted by $SS_3(G)$ and its cardinality by $ss_3(G)$. Switching was defined by Seidel. Seidel and Taylor provide a study on switching classes of graphs. For $\sigma = \{v\} \subseteq V$, the corresponding switching $G^{\{v\}}$, represented by G^{v}, is called as vertex switching. In this article we find the necessary condition for a graph to have 3-vertex self switching and few of its properties are studied. Also we find $ss_3(G)$ for path P_n, cycle C_n, complete K_n and complete bipartite $K_{m,n}$ graphs.

Keywords: $SS_3(G), ss_3(G)$, Switching, 3-vertex self switching

AMS Classification Number: *05C60, 05C38*

1.Introduction

For a finite undirected graph $G(V,E)$ and a non-empty subset $\sigma \subseteq V$, the switching of G by σ is defined as the graph $G^{\sigma}(V,E')$ which is obtained from G by removing all edges between σ and its complement $V-\sigma$ and adding as edges all non-edges between σ and its complement $V-\sigma$. Switching was defined by Seidel [4] and it is also called as seidel switching. For $\sigma=\{v\}\subseteq V$, the corresponding switching $G^{\{v\}}$, represented by G^{v}, is called as vertex switching. A subset σ of V is said to be self switching if $G\cong G^{\sigma}$. We also call it as $|\sigma|$-vertex self switching. When $|\sigma|=1$, it is termed as self vertex switching [2], $|\sigma|=2$, it is termed as 2 -vertex self switching [1,3] and $|\sigma|=3$, it is termed as 3 -vertex self switching where $\sigma=\{\mathrm{u},\mathrm{v},\mathrm{w}\}$. The set of all 3-vertex self switchings of G with cardinality 3 is denoted by $SS_3(G)$ and its cardinality by $ss_3(G)$. A graph H is a subgraph of graph G if $V(H)\subseteq V(G)$ and $E(H)\subseteq E(G)$. For any set σ of vertices of G, the induced subgraph $G[\sigma]$ is the maximal subgraph of G with vertex set σ. A subgraph H of G is called the spanning subgraph H of graph G if $V(H)=V(G)$. In this article, we provide the necessary condition needed for a graph to be 3-vertex self switching. We also find $ss_3(G)$ for path, cycle, complete bipartite graph and complete graph.

2.Preliminaries

Theorem 2.1. [5] *Let $G(V,E)$ be a graph and $\sigma\subseteq V$ be a self switching of G. Then the number of edges between the vertices of σ and $V-\sigma$ in G is $\frac{k(p-k)}{2}$ where $k=|\sigma|$.*

Theorem 2.2. [5] *If v is a self vertex switching of a graph G of order p, then $deg\,(v)=\frac{p-1}{2}$*

Theorem 2.3. [1] *If σ is a 2-vertex self switching of G, then*

$$deg\,(u)+deg\,(v)=\begin{cases} p & if \quad uv\in E(G)\\ p-2 & if \quad uv\notin E(G)\end{cases}.$$

Theorem 2.4. [3] *For $p\geq 3, ss_2(C_p)=\begin{cases}4 & if\, p=4\\ 3 & if\, p=6\\ 0 & otherwise\end{cases}$*

3.Main Results

Definition 3.1. A subset σ of V is said to be self switching if $G\cong G^{\sigma}$. We call it as $|\sigma|$-vertex self switching. When $|\sigma|=3$, it is termed as 3 -vertex self switching. The set of all 3-vertex self switchings of G with cardinality 3 is denoted by $SS_3(G)$ and its cardinality by $ss_3(G)$.

Example 3.2. Consider the graph P_5 shown in fig 3.1. Different 3-vertex switchings are given in fig 3.2 to 3.3 show. From these figures, we find that there are two, 3-vertex self switchings namely $\{c_1, c_3, c_4\}$ and $\{c_2, c_3, c_5\}$ and hence $ss_3(P_5) = 2$.

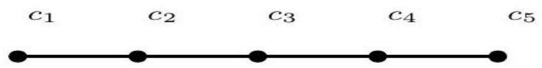

Figure 3.1. **G =** P_5

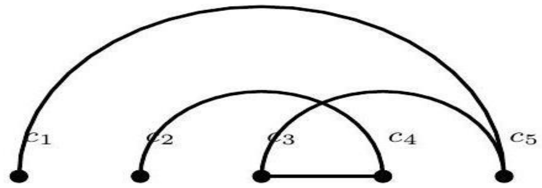

Figure 3.2. $G^{\{c_1,c_3,c_4\}}$

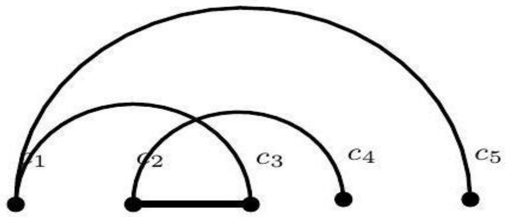

Figure 3.3. $G^{\{c_2,c_3,c_5\}}$

Theorem 3.3. *If σ is a 3-vertex self switching of G, then*

$$\deg_G(u) + \deg_G(v) + \deg_G(w) = \begin{cases} \frac{3p-5}{2} & if \quad G[\sigma] = K_2 \cup K_1 \\ \frac{3p-1}{2} & if \quad G[\sigma] = P_3 \\ \frac{3p+3}{2} & if \quad G[\sigma] = K_3 \\ \frac{3p-9}{2} & if \quad G[\sigma] = \overline{K_3} \end{cases}$$

Proof. Let $\sigma = \{u, v, w\}$ be a 3 -vertex self switching of G. This implies that $G \cong G^\sigma$ and therefore $|E(G)| = |E(G^\sigma)|$. Now, $|E(G^\sigma)| =$ number of edges in $G - \Big((\deg_G(u) + \deg_G(v) + \deg_G(w)) + \deg_{G[\sigma]}(u) + \deg_{G[\sigma]}(v) + \deg_{G[\sigma]}(w)$ +number of non-adjacent vertices of u in G - number of non-adjacent vertices of u in $G[\sigma]$ + number of non-adjacent vertices of v in G - number of non-adjacent vertices of v in

$G[\sigma]$ + number of non-adjacent vertices of w in G - number of non-adjacent vertices of w in $G[\sigma]$. We have the following four cases.

Case 1. $G[\sigma] = K_3$

Let K_2 be uv and w be the vertex of K_1 which is not adjacent to u and v. In this case $|E(G^\sigma)| = q - (\deg_G(u) + \deg_G(v) + \deg_G(w) + 2 + 2 + 2 + (p - 1 - \deg_G(u)) - 0 + (p - 1 - \deg_G(v)) - 0 + (p - 1 - \deg_G(w)) - 0$. This implies that $|E(G)| = q = q - 2\big((\deg_G(u) + \deg_G(v) + \deg_G(w)) + 6 + 3p - 3$. Therefore $\deg_G(u) + \deg_G(v) + \deg_G(w) = \frac{3p+3}{2}$.

Case 2. $G[\sigma] = P_3$

Let P_3 be uvw. In this case $|E(G^\sigma)| = q - (\deg_G(u) + \deg_G(v) + \deg_G(w)) + 1 + 2 + 1 + (p - 1 - \deg_G(u)) - 1 + (p - 1 - \deg_G(v)) + (p - 1 - \deg_G(w)) - 1$. This implies that $|E(G)| = q = q - 2\big((\deg_G(u) + \deg_G(v) + \deg_G(w)) - 1 + 3p$. Therefore $\deg_G(u) + \deg_G(v) + \deg_G(w) = \frac{3p-1}{2}$.

Case 3. $G[\sigma] = K_2 \cup K_1$

In this case $|E(G^\sigma)| = q - (\deg_G(u) + \deg_G(v) + \deg_G(w)) + 1 + 1 + 0 + (p - 1 - \deg_G(u)) - 1 + (p - 1 - \deg_G(v)) - 1 + (p - 1 - \deg_G(w)) - 2$. This implies that $|E(G)| = q = q - 2\big((\deg_G(u) + \deg_G(v) + \deg_G(w)) + 3p - 5$. Therefore $\deg_G(u) + \deg_G(v) + \deg_G(w) = \frac{3p-5}{2}$.

Case 4. $G[\sigma] = \overline{K_3}$

In this case $|E(G^\sigma)| = q - (\deg_G(u) + \deg_G(v) + \deg_G(w)) + (p - 1 - \deg_G(u)) - 2 + (p - 1 - \deg_G(v)) - 2 + (p - 1 - \deg_G(w)) - 2$. This implies that $|E(G)| = q = q - 2\big((\deg_G(u) + \deg_G(v) + \deg_G(w)) + 3p - 9$. Therefore $\deg_G(u) + \deg_G(v) + \deg_G(w) = \frac{3p-9}{2}$.

Hence theorem follows from cases 1,2,3 and 4.

Remark 3.4. *The converse of the above theorem need not be true.*

Consider the graph G given in fig 3.4. Here $\deg_G(d_1) + \deg_G(d_2) + \deg_G(d_5) = 3 + 2 + 3 = 8 = \frac{3(7)-5}{2}$. The graph $G^{\{d_1,d_2,d_5\}}$ is given in fig 3.5. Clearly, G is not isomorphic to $G^{\{d_1,d_2,d_5\}}$ and hence $\{d_1, d_2, d_5\}$ is not a 3-vertex self switching of G.

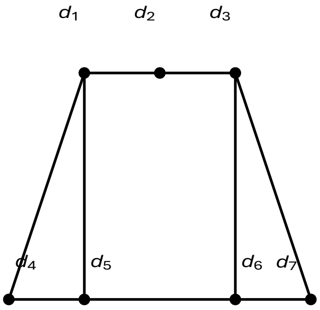

Figure 3.4. G

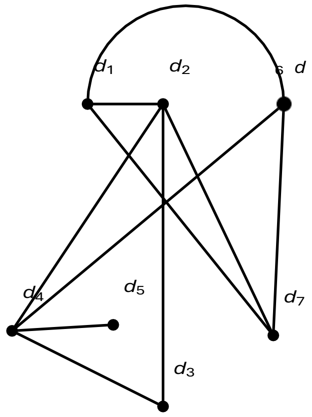

Figure 3.5. $G^{\{d_1,d_2,d_5\}}$

Theorem 3.5. *Let $G(V, E)$ be a graph and let $\sigma = \{u, v, w\} \subset V$ be a 3-vertex self switching of G. Then the number of edges between the vertices of σ and $V - \sigma$ in R is $\frac{3(p-3)}{2}$.*

Proof. Let $\sigma = \{u, v, w\} \subset V$ be a 3 -vertex self switching of G and let $G^\sigma(V, E')$ be the switching of G by σ. Then $G \cong G^\sigma$ and therefore of $|E|$=$|E'|$. This implies that the number of edges between the vertices of σ and $V - \sigma$ in G is same as G^σ. Since the number of edges between the sets σ and $V - \sigma$ both in G and G^σ is the number of edges of $K_{3,p-3}$, which is $\frac{3(p-3)}{2}$, the theorem follows.

Corollary 3.6. *If a graph has a 3-vertex self switching, then the order of the graph is odd.*

Proof. Let $G(V, E)$ be a graph and $\sigma \subset V$ be a 3-vertex self switching of G. Let $k = |\sigma|$. Then, by Theorem 3.5, $\frac{3(p-3)}{2}$ is an integer. This implies that $p - 3$ is even and therefore p is odd.

Theorem 3.7. *If G is a graph with even size, then the line graph $L(G)$ has no 3-vertex self switching.*

Proof. Let G be a graph of even size. By the definition of $L(G)$, edges of the graph G are the vertices of $L(G)$. Since G has even number of edges, the line graph $L(G)$ has even number of vertices. By Corollary 3.6, $L(G)$ has no 3 -vertex self switching.

Theorem 3.8. $\mathrm{ss}_3(\mathrm{P}_\mathrm{p}) = \begin{cases} 1 & \text{for p} = 3,7 \\ 2 & \text{for p} = 5 \\ 0 & \text{otherwise} \end{cases}$

Proof. Let P_p be the path graph. It has p vertices and $p-1$ edges. Let $\sigma = \{u, v, w\} \subseteq V(P_p)$. Then $G[\sigma]$ is either P_3 or $K_2 \cup K_1$ or $\overline{K_3}$. By Theorem 3.1, $\deg_G(u) + \deg_G(v) + \deg_G(w) \in \left\{\frac{3p-9}{2}, \frac{3p-5}{2}, \frac{3p-1}{2}\right\}$.

If $p \geq 8$, then $\deg_G(u) + \deg_G(v) + \deg_G(w) \geq \frac{3p-9}{2} \geq \frac{3(8)-9}{2} = 8$. But for any three vertices u, v and w in $P_p, \deg_G(u) + \deg_G(v) + \deg_G(w) \leq 6$ which is a contradiction. Hence, $ss_3(P_p) = 0$.

So, we calculate $ss_3(P_p)$ for $3 \leq p \leq 7$. If $p \in \{4,6\}$, then by Corollary 3.6, P_p has no 3 -vertex self switching. Let us calculate $ss_3(P_p)$ for $p \in \{3,5,7\}$.

Case 1. $G[\sigma] = K_2 \cup K_1$

Let K_2 be uv and let w be the vertex of K_1 which is non-adjacent to u and v and so p is either 5 or 7.

Subcase 1.1. $p = 5$

In this case either one or both vertices of P_2 are internal vertices of P_5. If u is an internal vertex and v is an end vertex, then $\deg_G(u)$ and $\deg_G(v) = 1$ and thereby $\deg_G(u) + \deg_G(v) + \deg_G(w) \in \{4,5\}$. If $\deg_G(u) + \deg_G(v) + \deg_G(w) = 4$, then $\deg_G(u) + \deg_G(v) + \deg_G(w) \neq \frac{3p-5}{2}$ and hence by Theorem 3.3, $\sigma = \{u, v, w\}$ is not a 3-vertex self switching of P_5. If $\deg_G(u) + \deg_G(v) + \deg_G(w) = 5$, then $\deg_G(u) + \deg_G(v) + \deg_G(w) = \frac{3p-5}{2}$ and $\deg_G(w) = 2$. This implies that $\sigma = \{u, v, w\}$ may or may not be a 3 -vertex self switching of P_5 and w is an internal vertex. Let $a \neq v$ be the vertex of degree 1 in P_5. Then a is adjacent to w in P_5 and thereby a is adjacent to both u and v in ${P_5}^{\sigma}$ and so ${P_5}^{\sigma}$ has a cycle C_3 which implies that σ is not a 3 -vertex self switching of P_5. If u and v are internal vertices of P_5, then w is an end vertex and so $\deg_G(u) = \deg_G(v) = 2$ and $\deg_G(w) = 1$. Clearly, $P_5 \cong {P_5}^{\sigma}$ and hence $\sigma = \{u, v, w\}$ is a 3 -vertex self switching of P_5.

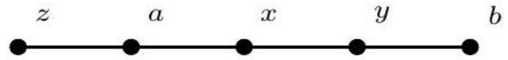

Figure 3.6. P_5

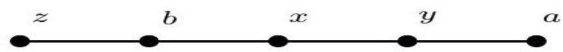

Figure 3.7. P_5^{σ}

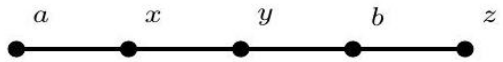

Figure 3.8. P_5

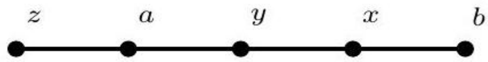

Figure 3.9. P_5^{σ}

Subcase 1.2. p = 7

If u is an internal vertex and v is an end vertex, then $\deg_G(u) = 2$ and $\deg_G(v) = 1$ and thereby $\deg_G(u) + \deg_G(v) + \deg_G(w) \in \{4,5\}$ and $\frac{3p-5}{2} = \frac{16}{2} = 8$ and so $\deg_G(u) + \deg_G(v) + \deg_G(w) \neq \frac{3p-5}{2}$. By Theorem 3.3, $\sigma = \{u, v, w\}$ is not a 3 -vertex self switching of P_7. If u and v are internal vertices, then $\deg_G(u) = \deg_G(v) = 2$ and thereby $\deg_G(u) + \deg_G(v) + \deg_G(w) \in \{5,6\}$ and $\frac{3p-5}{2} = \frac{16}{2} = 8$ and so $\deg_G(u) + \deg_G(v) + \deg_G(w) \neq \frac{3p-5}{2}$. By Theorem 3.3, $\sigma = \{u, v, w\}$ is not a 3 -vertex self switching of P_7.

Case 2. $G[\sigma] = P_3$

Subcase 2.1. $p = 3$

Since P_3 has only three vertices, $P_3 \cong {P_3}^{\sigma}$. This implies that $\sigma = \{u, v, w\}$ is the only 3 -vertex self switching of P_3.

Subcase 2.2. $p = 5$ or 7

Since $G[\sigma] = P_3$, either no vertex or one vertex in σ is an end vertex of P_p. If no vertex is an end vertex, then u, v and w are internal vertices and so $\deg_G(u) + \deg_G(v) + \deg_G(w) = 6 \neq \frac{3p-1}{2}$. If one vertex is an end vertex, then $\deg_G(u) + \deg_G(v) + \deg_G(w) = 5 \neq \frac{3p-1}{2}$ and thereby Theorem 3.3, σ is not a 3 -vertex self switching of P_p.

Case 3. $G[\sigma] = \overline{K_3}$

Subcase 3.1. $p = 5$

In this case two vertices from σ are end vertices of P_5. This implies that $\deg_G(u) + \deg_G(v) + \deg_G(w) = 4 \neq \frac{3p-9}{2}$ and hence by Theorem 3.3, $\sigma = \{u, v, w\}$ is not a 3 -vertex self switching of P_5.

Subcase 3.2. $p = 7$

Clearly, either no vertex or one vertex or two vertices of σ are end vertices of P_7 and so $\deg_G(u) + \deg_G(v) + \deg_G(w) \in \{4,5,6\}$. Now, $\frac{3p-9}{2} = \frac{21-9}{2} = \frac{12}{2} = 6$ implies that u, v and w are internal vertices. Since $G[\sigma] = \overline{K_3}$, the end vertices of P_7 are adjacent to two vertices of σ. Let w be the vertex which is non-adjacent to the end vertices, say a and b, in P_7. The graphs given in fig 3.10 and fig 3.11 are P_7 and ${P_7}^{\sigma}$. Clearly, $P_7 \cong {P_7}^{\sigma}$ and so $\sigma = \{u, v, w\}$ is the only 3 -vertex self switching of P_7.

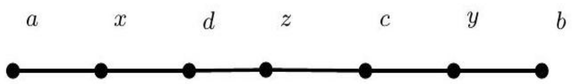

Figure 3.10. $\boldsymbol{P_7}$

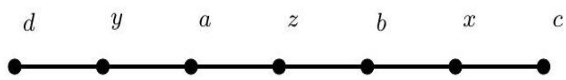

Figure 3.11. $\boldsymbol{P_7^{\sigma}}$

Hence theorem follows from above three cases.

Theorem 3.9. $ss_3(K_p) = 0$ *for* $p \geq 4$.

Proof. Let $\sigma = \{u, v, w\} \subset V(K_p)$ be such that $|\sigma| = 3$. Then $K_p{}^{\sigma} = K_3 \cup K_{p-3}$ which is a disconnected graph and so σ cannot be a 3 -vertex self switching of K_p. Hence, $ss_3(K_p) = 0$.

Theorem 3.10. $ss_3(C_p) = 0$ *for* $p \geq 4$.

Proof. Let C_p be the cycle graph. It has p vertices and p edges. Let us calculate $ss_3(C_p)$ for different values of p. If p is even, then by Corollary 3.6, C_p has no 3 -vertex self switching. Let us calculate $ss_3(C_p)$ for odd number of vertices. Let $\sigma = \{u, v, w\} \subset V(C_p)$ is a 3 -vertex self switching of C_p. Clearly, $G[\sigma]$ is either P_3 or $K_2 \cup K_1$ or $\overline{K_3}$. Also in $C_p, \deg_G(u) + \deg_G(v) + \deg_G(w) = 6$.

Case 1. $G[\sigma] = K_2 \cup K_1$

Let K_2 be uv and let w be the vertex of K_1 which is non-adjacent to u and v. Now, $\deg_G(u) + \deg_G(v) + \deg_G(w) = 6 \neq \frac{3p-5}{2}$ and hence by Theorem 3.3, $\sigma = \{u, v, w\}$ is not a 3-vertex self switching of C_p.

Case 2. $G[\sigma] = P_3$

Let P_3 be uvw. Now, $\deg_G(u) + \deg_G(v) + \deg_G(w) = 6 \neq \frac{3p-1}{2}$ and hence by Theorem 3.3, $\sigma = \{u, v, w\}$ is not a 3-vertex self switching of C_p.

Case 3. $G[\sigma] = \overline{K_3}$

Here $\deg_G(u) + \deg_G(v) + \deg_G(w) = 6 = \frac{3p-9}{2}$ for $p = 7$ only. Let a be the vertex which is adjacent to both u and v in C_7. Then a is adjacent to only w in $C_7{}^{\sigma}$ and so a has degree 1 in $C_7{}^{\sigma}$ where as C_7 has no vertex of degree 1. This implies that C_7 is not isomorphic to $C_7{}^{\sigma}$ and so $\sigma = \{u, v, w\}$ is not a 3 -vertex self switching of C_7. Hence, $ss_3(C_p) = 0$.

Theorem 3.11. $ss_3(K_{m,n}) = \begin{cases} n\binom{m}{2} & \text{for } m = n+1 \\ \binom{m}{3} & \text{for } m = n+3 \\ 0 & \text{otherwise} \end{cases}$

Proof. Let $V = V_1 \cup V_2$ where $V_1 = \{v_1, \ldots, v_m\}$ and $V_2 = \{u_1, \ldots, u_n\}$ be the bipartition of vertex set of $G = K_{m,n}$. Let $\sigma = \{u, v, w\} \subseteq V(K_{m,n})$. Then either all the 3 vertices in σ are in V_1or V_2 or one vertex in $V_1(V_2)$ or two vertices in $V_2(V_1)$. This implies that $G[\sigma]$ is either P_3 or $\overline{K_3}$. By Theorem 3.3, $\deg_G(u) + \deg_G(v) + \deg_G(w) \in \left\{\frac{3p-9}{2}, \frac{3p-1}{2}\right\}$. Without loss of generality, assume that $n \leq m$. If $m = n$, then by Corollary 3.6, $K_{m,m}$ has no 3 -vertex self switching. So, let $m > n$. Consider $m = n + t, t \geq 1$. If t is even, then by Corollary 3.6, $K_{n+t,n}$ has no 3 -vertex self switching. Let us calculate $ss_3(K_{n+t,n})$ for $p = 2n + t, t$ is odd.

Case 1. $G[\sigma] = \overline{K_3}$

If $\sigma \subseteq V_1$, then $K_{m,n}^{\sigma} = K_{m-3,n+3}$ and if $\sigma \subseteq V_2$, then $K_{m,n}^{\sigma} = K_{m+3,n-3}$. Now $K_{m-3,n+3} = K_{m,n}$ if and only if $m - 3 = n$ and $n + 3 = m$ if and only if $m - n = 3$. Also $K_{m+3,n-3} = K_{m,n}$ if and only if $m + 3 = n$ and $n - 3 = m$ if and only if $n - m = 3$. Hence $|m - n| = 3$. Thus $K_{m,n}$ has a 3 -vertex self switching if and only if $|m - n| = 3$. Let $m > n$. Then $m = n + 3$ and $K_{m,n}$ has a 3-vertex self switching if $\sigma \subseteq V_1$. Since we can choose 3 vertices from the m vertices in $\binom{m}{3}$ ways, the number of 3 -vertex self switching of $K_{m,n}$ is $\binom{m}{3}$ when $m = n + 3$.

Case 2. $G[\sigma] = P_3$

Here, $V_1 \cap \sigma \neq \varphi$ and σ contains either one vertex or two vertices of V_1.

Subcase 2.1. σ contains one vertex of V_1

Then σ contains two vertices of V_2. Now, $K_{m,n}^{\sigma} = K_{m-1+2,n-2+1} = K_{m+1,n-1}$. Hence, $K_{m,n}^{\sigma} \cong K_{m,n}$ if and only if $m = n - 1$ and $n = m + 1$ if and only if $n = m + 1$. In this case $n > m$ and $K_{m,n}$ has a 3 -vertex self switching. One vertex can be chosen from V_1 in m ways and the 2 vertices from V_2 can be chosen in $\binom{n}{2}$ ways and thereby $m\binom{n}{2}$ number of 3 -vertex self switching of $K_{m,n}$.

Subcase 2.2. σ contains two vertices of V_1

Then σ contains one vertex from V_2. Now, $K^{\sigma}_{m,n} = K_{m-2+1,n-1+2} = K_{m-1,n+1}$. Hence, $K^{\sigma}_{m,n} \cong K_{m,n}$ if and only if $m-1=n$ and $n+1=m$ if and only if $m=n+1$ and so $m>n$. One vertex from V_2 can be chosen in n ways and the 2 vertices from V_1 can be chosen in $\binom{m}{2}$ ways and thereby $n\binom{m}{2}$ number of 3-vertex self switching of $K_{m,n}$.

Hence theorem follows from above two cases.

Conclusion

In this article, we discussed the necessary conditions for a graph to be 3-vertex self switching including the few properties.

Application

The application of 3-vertex self-switching in road traffic can reduce traffic congestion by optimizing traffic flow, decrease travel times and improve traffic efficiency and enhance safety by reducing the risk of accidents caused by congestion. Vertices represent the intersections or road junctions and edges represent the roads that connect the intersections. In the context of road traffic, 3-vertex self switching means reconfiguring the roads and intersections to optimize traffic flow.

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